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ABSTRACT

This paper deals with the empirical relationships between public capital and aggregate economic growth in the United States, and in particular the question of whether or not public capital is productive. It develops a theoretical framework which allows for full consideration of feedback among variables without imposing a priori dynamic structural constraints. Parameter estimates are obtained through a VARMA model. This approach departs from the current literature, which relies on a single equation approach to estimate production functions and implicitly assumes the absence of feedback relations.

In this paper estimates for the period 1956-1989 suggest that public capital has a substantial effect on production as well as on private capital formation and on labor. Furthermore, decisions on public capital seem to follow a policy rule that relates the current stock of public capital positively with lagged output and negatively with lagged labor. Our results are shown to be compatible with different specifications of production functions, in which public capital may or may not be present. Therefore, current interpretations of the importance of the effects of public capital in terms of the size of the estimated parameters in a production function framework are not conclusive.

RESUMEN

En este artículo se estudian las relaciones empíricas entre el capital público y el crecimiento agregado de la economía de Estados Unidos. Se desarrolla un marco teórico que permite tener en cuenta el conjunto total de relaciones dinámicas entre las variables, sin imponer a priori restricciones dinámicas estructurales. Las estimaciones de los parámetros se obtienen a través de un modelo ARMA vectorial. Este enfoque difiere del enfoque uniecuacional convencional basado en la estimación de funciones de producción y que implícitamente supone la ausencia de relaciones de retroalimentación.

Nuestras estimaciones para el período 1956-1989 sugieren que el capital público tiene un efecto substancial sobre la producción, la formación bruta de capital privada y el empleo. Además, decisiones sobre el capital público parecen seguir una regla de política que relaciona positivamente, el stock corriente de capital público con el producto retardado y negativamente, dicho stock con el empleo retardado.

Por último se muestra como nuestros resultados son compatibles con diferentes especificaciones de funciones de producción, en las que el capital público puede o puede no estar presente como input. Por consiguiente, las interpretaciones de la importancia de los efectos de capital público en términos del tamaño de los parámetros estimados en una función de producción no son concluyentes.

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Public Capital and Aggregate Growth in the United States:  
Is Public Capital Productive?

1. Introduction

The objective of this paper is to provide an analysis of the empirical relationships between public capital and aggregate growth in the United States, and in particular to establish whether or not public capital is productive. The issue of the possible effects of public capital on private output is currently at the center of the political debate in the U.S..

It has been suggested that the American economy is stagnating on a crumbling infrastructure base. In fact, between 1960 and 1991, federal public investments in the U.S. decreased from 2.3% of the GDP to just 1% and are currently well below the levels of public investment in the European Community, at 3%, Germany, at 2.4%, and Japan, at 3%, of the respective GDPs. In part reflecting this view of the state of the U.S. economy, the commitment to developing and expanding the network of public capital as a central feature of an economic package to promote long term growth has been a trademark of the Clinton Administration.

The issue of the empirical evaluation of the effects of public capital on output growth, i.e., of whether or not public capital is productive, has been brought to the limelight by the seminal work of Aschauer (1989a, 1989b), Munnell (1990) and Munnell with Cook (1990).<sup>1/</sup> In their work, investment in public capital has been suggested as a powerful instrument to promote economic growth, and, in particular, the decline in infrastructure investment in the last two decades has been identified as a major culprit for the decline in productivity. More recent contributions have challenged these views (see for

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example Aaron, 1990, Eberts, 1990, Hulten and Schwab, 1991, Jorgenson, 1991, and Tatom, 1991).

The debate on whether or not public capital is productive has focused on the size of the elasticity of output with respect to public capital as implied by estimates of static production functions contemporaneously relating labor, private capital, and public capital to output.<sup>2/</sup> If the elasticity is zero it is said that public capital has no effect on production. If the elasticity is significantly different from zero then public capital is said to affect output, i.e. to be productive.

The issue has been addressed in the context of aggregate output in the U.S. (see for example Aschauer, 1989a, 1989b, Munnell, 1990a, and Holtz-Eakin, 1988), at the state level, at the industry level, or at both levels, in the U.S. (see for example Costa et al., 1987, Duffy-Deno and Eberts, 1991, Garcia-Mila and McGuire, 1992, Holtz-Eakin, 1992, Hulten and Schwab, 1992, and Munnell, 1990b), and in the context of aggregate international comparisons (see Barro, 1991, Berndt and Hansson, 1991, Ford and Poret, 1991, Shah, 1992, and Baffes and Shah, 1992).

No clear conclusions arise from the literature. No agreement has been reached as to the importance of public capital, i.e., the size and significance of the associated coefficient. The estimated elasticities range from .39 in Aschauer (1989a) to .03 in Eberts (1990) and zero in Tatom (1991) (see Munnell, 1992, for details).

In this paper we argue that the focus of the debate on the size of elasticities obtained from single equation estimates is not adequate to provide a conclusive answer to the question of whether or not public capital is productive. This is because the single equation framework excludes the likely presence of dynamic feedbacks among the relevant variables. This is of paramount importance since if feedbacks exist, the size of the elasticity of output with respect to public capital as obtained from a single equation production function estimate does not answer the question of whether or not public capital is productive.

In fact, a zero elasticity with respect to public capital is neither necessary nor sufficient for public capital to have no effect on output. Indeed, public capital can affect output directly, i.e., as an additional input in the production function, or indirectly through capital and labor. At the same time, output can also affect labor as well as private and public capital. Finally, labor and private capital formation can affect public capital formation. The final effect of public capital on output is the result of a direct effect and many indirect effects. If feedback relationships do exist among output, labor, private and public capital, then the single equation approach is misleading and a multivariate stochastic approach is required.

In this paper we address the issue of whether or not public capital is productive in a theoretical framework which allows for fully consideration of feedbacks among output, labor, private and public capital, without imposing a priori structural constraints on the dynamic relationships among these variables.

The multivariate stochastic framework is particularly appropriate for dealing with other technical problems previously recognized in the literature (see Munnell (1992) and Tatom (1991) for a comprehensive discussion of these problems). The main criticisms come in terms of the econometric specification of the production function (the same criticisms would apply to a cost function approach). First, labor and private capital inputs are endogenous variables which generate simultaneous equation biases and invalidate OLS estimates. Second, statistical properties of the different time series have been ignored, i.e., non-stationarity and the possible existence of cointegration among the variables. Third, the issue of reverse causality, i.e., the possibility that output may be causing public capital and not the other way around, has not been conclusively addressed.

In this paper we use aggregate U.S. data<sup>3/</sup> for private output, labor, private capital, and public capital for the period 1956-1989. The theoretical model is estimated using a VARMA model. Estimates suggest that public capital have a substantial effect on long term production as well as on private capital formation and labor, implying that public

capital is productive. Furthermore, decisions on public capital seem to follow a policy rule that relates the current stock of public capital positively with lagged output and negatively with lagged labor. Finally, our results of strong long term effects of public capital on production are shown to be compatible, under different assumptions on the contemporaneous effects among variables, with different specifications of production functions in which public capital may or may not be present. Therefore, we claim that current interpretations of the importance of the effects of public capital in terms of the size of the estimated parameter in a production function are not conclusive.

The rest of the paper is organized as follows. In Section 2. we present the theoretical model. In Section 3. we report and analyze the empirical results, including the study of the effects of public capital through orthogonalized impulse-response functions. Finally, in Section 4. we provide concluding remarks and suggestions for future work.

## 2. Theoretical Framework

We propose a conceptual model for the treatment of the effects of public capital on the performance of the economy (see Treadway, 1986, and Flores, 1990, for early applications of this type of conceptual framework). The theoretical framework is designed to minimize the number of assumptions and to achieve the greatest degree of generality. Accordingly, the theoretical model has a number of desirable characteristics. First, the model is presented as a multivariate time series model. This facilitates the integration of basic economic theory concepts into a statistical framework. Second, it does not impose a priori restrictions on the dynamic relations among the different variables. In particular, it allows for feedbacks between output, labor and private capital, and public capital (exogeneity of public capital is not imposed), and among output, labor and private capital. Third, it requires minimal structural assumptions in order to study the effects of public

capital on output, labor and private capital formation. In particular, it does not require the a priori specification of any production function, even though it may be compatible with different specifications.

Moreover, this formulation allows us to address in a systematic way some of the criticisms of the previous empirical literature, namely the endogeneity of private capital and labor, the non-stationarity of the different variables, and the possibility of reverse causation from output to public capital.

### 2.1 The Model

To address the impact of public capital formation on output, we consider the same type of economic variables as in the previous literature, i.e., output  $Y_t$ , labor  $L_t$ , private capital  $K_t$ , and public capital  $PK_t$  (hereafter, lower case letters denote the logarithmic rate of change). We assume that there are two sectors in the economy: the private sector and the public sector. The two sectors are different in that they have control over different variables. The private sector controls  $y_t$ ,  $l_t$ , and  $k_t$ , or in vector notation  $z_t = (y_t \ l_t \ k_t)'$ , while the public sector determines  $pk_t$ .

Let us consider now the behavior of both private and public sector variables.

**Private Sector** – The private sector determines  $y_t$ ,  $l_t$ , and  $k_t$ , using information on past values of all the variables as well as the current value of  $pk_t$ . Mathematically, this can be expressed in the following way:

$$(1) \quad z_t = \nu_z(B) pk_t + \epsilon_{zt}$$

$$\pi_z(B) \epsilon_{zt} = a_{zt}$$

where,

$$\nu_z(B) = (\nu_y(B) \ \nu_l(B) \ \nu_k(B))'$$

is a  $3 \times 1$  vector of stable transfer functions with

$$\nu_j(B) = \nu_{j0} + \nu_{j1}B + \nu_{j2}B^2 + \dots, \text{ for } j = y, l, k;$$

$$\epsilon_{zt} = (\epsilon_{yt} \ \epsilon_{lt} \ \epsilon_{kt})' \text{ is a } 3 \times 1 \text{ noise vector;}$$

$\pi_z(B) = I - \pi_1 B - \pi_2 B^2 - \dots$  is a  $3 \times 3$  polynomial matrix with  $\pi_i$  as lag- $i$  coefficient matrices (the roots of the determinant of  $\pi_z(B)$  must lie on or outside the unit circle); and

$a_{zt} = (a_{yt} \ a_{lt} \ a_{kt})'$  is a  $3 \times 1$  white noise vector with contemporaneous covariance matrix  $\Sigma_z$ .

**Public Sector** – The public sector determines  $pk_t$ , using information about the past values of all the variables. Mathematically, this can be expressed in the following way:

$$(2) \quad pk_t = \nu_p(B)z_t + \epsilon_{pt}$$

$$\pi_p(B) \epsilon_{pt} = a_{pt}$$

where,

$$\nu_p(B) = (\nu_{py}(B) \ \nu_{pl}(B) \ \nu_{pk}(B)), \text{ is a } 1 \times 3 \text{ vector of stable transfer functions,}$$

$$\epsilon_{pt} \text{ is a noise scalar,}$$

$$\pi_p(B) = I - \pi_{p1}B - \pi_{p2}B^2 - \dots, \text{ has roots on or outside the unit circle, and}$$

$$a_{pt} \text{ is a white noise scalar with variance } \sigma_p^2.$$

Notice that, if the public sector does not use information on the previous values of the private sector variables, no feedback rules exist and public capital is truly an exogenous variable.

## 2.2 Basic Assumptions

In the model formulation there are two basic assumptions which we will call asymmetry and independence. These assumptions jointly represent sufficient condition for the parameters of the theoretical model to be exactly identified.

### Assumption 1 – Asymmetry

Let  $\Omega_{zt}$  and  $\Omega_{pt}$  be the information sets of the private and public sectors at  $t$ .

These sets can be defined as follows:

$$\Omega_{zt} = \{ z_{t-j}, pk_{t-j}, pk_t \}, j = 1, 2, \dots$$

$$\Omega_{pt} = \{ z_{t-j}, pk_{t-j} \}, j = 1, 2, \dots$$

This assumption can be interpreted as follows. First, both the private and the public sectors are assumed to know at the beginning of the period all the past values of all the variables determined in both sectors. Empirical results will determine whether or not this information has been used by either the private or the public sectors in their decision making.

Second, the private sector knows the current values of public capital while the public sector does not know the current values of the variables determined in the private sector, hence the term asymmetry to describe this assumption. This assumption is consistent with the observation that the public sector, through the bureaucratic budgetary process announces in advance, i.e., at the beginning of the period, what public capital expenditures will be during the period. Therefore, the information of public capital formation for the period is available when the private sector makes its decisions. Furthermore, since public capital decisions are announced at the beginning of the period, current realizations of the private sector variables cannot be in the information set of the public sector.

Third, the private sector clearly has information about the plans for public capital formation announced in the beginning of the period by the public sector. There is a question as to how accurately the public sector plans are expected to be implemented, and in general as to whether or not the private sector actually uses this information. Our strategy is to allow the data to tell us whether or not information on the current levels of public capital is actually used by the private sector. Alternatively, current information

could be excluded a priori and symmetry assumed. This would be, in our judgment unnecessarily restrictive.

#### Assumption 2 — Independence

Let  $a_{yt}$ ,  $a_{lt}$ ,  $a_{kt}$ , and  $a_{pt}$  be the white noise residuals associated with the equations for output, labor, private capital, and public capital, respectively. We assume that  $a_{pt}$  is independent from  $a_{yt}$ ,  $a_{lt}$ , and  $a_{kt}$ .

This independence assumption postulates that random shocks in the evolution of public capital are independent from shocks in the variables determined in the private sector (notice that random shocks in the private sector,  $a_{yt}$ ,  $a_{lt}$ , and  $a_{kt}$ , are assumed to be contemporaneously correlated). The assumption of independence is directly related to the separation of the private and public sectors. Indeed, it is not possible to talk about two separate sectors with different tasks unless we also assume that the sector-specific shocks are independent. In particular, it implies that the public sector has full discretionary control over  $pk_t$ .

It can be argued that omitted variables and measurement errors can result in contemporaneously correlated structural shocks. Assuming contemporaneously correlated structural shocks leads to identification problems which are often solved by imposing a priori constraints on the parameters of the structural model, usually on the dynamic structure. However, this assumption not only does not solve the problems with omitted variables and measurement errors but also comes at a cost of imposing a priori constraints in the dynamics of the model. Such an alternative would be particularly inadequate in the context of our model, since the objective of the paper is to study the dynamic relationships among variables.

### 2.3 Impulse-Response Functions

We are now interested in considering the reaction of private sector variables to a shock in  $pk_t$ . From equations (1) and (2), we can express  $z_t$  as

$$(3) \quad z_t = \Psi_p(B) a_{pt} + \Psi_z(B) a_{zt}$$

where,

$$(4) \quad \Psi_p(B) = [I - \nu_z(B)\nu_p(B)]^{-1} \nu_z(B)\pi_p(B)^{-1} \\ = \Psi_{p0} + \Psi_{p1}B + \Psi_{p2}B^2 + \dots$$

$$(5) \quad \Psi_z(B) = [I - \nu_z(B)\nu_p(B)]^{-1} \pi_z(B)^{-1} \\ = I + \Psi_{z1}B + \Psi_{z2}B^2 + \dots$$

The sequence of coefficients associated with the lag polynomial  $\Psi_p(B)$  is to be interpreted as the orthogonalized impulse-response function of  $z_t$  versus  $a_{pt}$ , i.e.,  $\partial z_t / \partial a_{pt-j}$  for  $j=0, 1, 2, \dots$ . This function gives the response of  $z_{t+j}$  to a unitary change in  $a_{pt}$ . It identifies the dynamic consequences for the private sector variables if the public sector were to change  $pk_t$  in a marginal and transitory fashion, beyond what is implied by the estimated rule of behavior for  $pk_t$  in (2). This is a key for describing the effects of public capital upon the performance of the private sector.

Notice that by Assumption 2  $a_{pt}$  is independent from the shocks in (1). Therefore, the impulse-response function of  $z_t$  versus  $a_{pt}$  does not depend on the contemporaneous correlations among the variables in  $z_t$ . Thus, in order to study the effects of changes in  $pk_t$  on  $z_t$ , it is not necessary to specify a whole structural equations model and the model given by (1) and (2) with the underlying assumptions is all that is needed.

Additionally, one may be interested in considering the effects of shocks to  $k_t$  and  $l_t$  on  $y_t$ . As in the case of shocks to  $pk_t$ , the study of these effects requires the use of

orthogonalized impulse-response functions. The residuals in  $a_{zt}$  are assumed to be contemporaneously correlated. Therefore,  $\Psi_z(B)$  does not have the same interpretation as  $\Psi_p(B)$ . An orthogonalization of the residuals  $a_{zt}$  is necessary before the individual effects from each shock can be disentangled and the orthogonalized impulse-response function can be obtained.

#### 2.4 Estimation Strategy

Notice that by Assumption 1,  $\nu_{py}(B)$ ,  $\nu_{pl}(B)$ , and  $\nu_{pk}(B)$  are such that  $\nu_{py0} = \nu_{pl0} = \nu_{pk0} = 0$ , so that current values of  $z_t$  do not affect  $pk_t$ , and that by Assumption 2,  $a_{pt}$  is independent from  $a_{yt}$ ,  $a_{lt}$ ,  $a_{kt}$ .

The model (1)–(2) together with Assumptions 1 and 2 can be written in matrix form as  $\Pi_w(B) w_t = a_{wt}$  with  $\Sigma$  as the matrix of contemporaneous correlations of  $a_{wt}$ , or:

$$(6) \quad \begin{bmatrix} \pi_z(B) & -\pi_z(B) \nu_z(B) \\ -\pi_p(B) \nu_p(B) & \pi_p(B) \end{bmatrix} \begin{bmatrix} z_t \\ pk_t \end{bmatrix} = \begin{bmatrix} a_{zt} \\ a_{pt} \end{bmatrix}$$

$$(7) \quad \Sigma = \begin{bmatrix} \Sigma_z & 0 \\ 0 & \sigma_p^2 \end{bmatrix}$$

The multivariate stochastic model (6) is not normalized to have  $\Pi_w(0) = I$ . Rather,

$$(8) \quad \Pi_w(0) = V = \begin{bmatrix} 1 & -\nu_{z0} \\ 0 & 1 \end{bmatrix},$$

where  $\nu_{z0} = (\nu_{y0} \nu_{l0} \nu_{k0})'$  is the vector of contemporaneous effects of  $pk_t$  on  $z_t$ .

Premultiplying (6) by  $V^{-1}$  we can obtain the normalized multivariate stochastic model

$$(9) \quad \Pi_w^*(B) w_t = a_{wt}^*$$

where,

$$\begin{aligned} \Pi_w^*(B) &= V^{-1} \Pi_w(B), \\ a_{wt}^* &= V^{-1} a_{wt}. \end{aligned}$$

The matrix of contemporaneous covariances of  $a_{wt}^*$  is  $\Sigma^*$ , which is given by:

$$(10) \quad \Sigma^* = V^{-1} \Sigma V^{-1'} = \begin{bmatrix} \Sigma_z + \nu_{z0} \nu_{z0}' \sigma_p^2 & \nu_{z0} \sigma_p^2 \\ \nu_{z0}' \sigma_p^2 & \sigma_p^2 \end{bmatrix}$$

The model in (9) and (10) is a general multivariate stochastic model. A VARMA representation of this model can be constructed from the available data using the methodology developed by Tiao and Box (1981) and Jenkins and Alavi (1981). Consider the estimated version of (9) and (10). From (10) we can estimate  $V$ , since  $\nu_{z0}$  can be obtained by multiplying partition (1,2) by  $1/\hat{\sigma}_p^2$ . Once  $V$  has been obtained, it is possible to estimate  $\Pi_w(B)$  by premultiplying the estimated version of (9) by  $V$ .

### 3. Empirical Analysis

#### 3.1 Data: sources and description

The data set used in this paper is obtained from Ford and Poret (1991). The data covers the sample period of 1956 to 1989, and therefore contains 34 yearly observations (see Appendix). Private output and (gross) private capital stock both measured in billion 1982 dollars, as well as employment measured in ten-thousand workers, are obtained from the OECD Analytical Data Base.

The (gross) public capital stock, also measured in billion 1982 dollars, is obtained from OECD Flow and Stock of Fixed Capital. Public capital is defined as the capital stock of "producers of government services", a definition close to the concept of public capital in Aschauer (1989a, 1989b).

Public capital includes Federal, state, and local government capital goods. It includes: core infrastructure; buildings and equipment; and, conservation and development structures (in 1989 these three categories comprise approximately 63.4%, 28.4%, and 8.2% of total public capital, respectively). Core infrastructure includes highways, streets and roads, mass transit, airport facilities, electric, gas and water supply facilities and distribution systems, and sewer facilities. In turn, buildings and equipment refer to activities like education, hospital, police, fire, justice, and administration. Our definition of public capital does not include: private buildings and equipment in transport and communications; private buildings and equipment in electricity, gas, and water; and, military capital stock.

### 3.2 Estimation results and diagnostic tests

The univariate analyses of the different series (see Box and Jenkins, 1976), which are not presented here, suggest that  $\ln Y$  and  $\ln L$  are  $I(1)$  while  $\ln K$  and  $\ln PK$  are  $I(2)$ . Nevertheless, the univariate analysis applied to the variables  $\ln(L/Y)$ ,  $\ln(K/Y)$  and  $\ln(PK/Y)$  suggests that these variables are  $I(1)$ . These results cast doubts on the order of integration of variables  $\ln K$  and  $\ln PK$ . In fact, if  $\ln Y$ ,  $\ln(K/Y)$ , and  $\ln(PK/Y)$  are truly  $I(1)$  this implies that  $\ln K$  and  $\ln PK$  are also  $I(1)$ .

In order to decide on the order of integration of the variables we test the null hypothesis of an extra unit root on the rates of growth  $y$ ,  $l-y$ ,  $k-y$ , and  $pk-y$ , through the Augmented Dickey-Fuller (ADF)  $t$ -test. The results of the test are reported in Table 1. The first column shows the values of the ADF  $t$ -test from a regression of the left-hand side variable versus this variable lagged one year, while the second column shows the

values of the ADF  $t$ -test from a regression of the left-hand side variable versus this variable lagged one year and its first difference.

In all cases the value of the ADF  $t$ -test is lower than the 5% critical value, which is  $-3$ . Therefore, the null hypothesis of an extra unit root is rejected for all the variables. Accordingly, the variables  $\ln Y$ ,  $\ln(L/Y)$ ,  $\ln(K/Y)$ , and  $\ln(PK/Y)$  are  $I(1)$  and therefore  $\ln Y$ ,  $\ln L$ ,  $\ln K$ , and  $\ln PK$  are also  $I(1)$ .

In order to investigate the existence of cointegration among  $\ln Y$ ,  $\ln L$ ,  $\ln K$ , and  $\ln PK$ , (see Granger, 1981, 1986, and Engle and Granger, 1987), the ADF  $t$ -test was applied to the residuals ( $r_{yt}$ ,  $r_{lt}$ ,  $r_{kt}$ ,  $r_{pt}$ ) from the regressions in log-levels of each variable on the remaining variables. The results are shown in Table 2. The columns of this table show the ADF  $t$ -test values with one, two, three, and four lags of the first difference of the left-hand side variable.

In all cases the value of the  $t$ -statistic is larger than the 5% critical value, i.e.  $-4.71$ , which means that the null hypothesis of a unit root in the residuals cannot be rejected. Therefore, there is no evidence of cointegration among these variables.

Now that we have determined that all the variables have the same order of integration and there are no relations of cointegration, we can develop a multivariate stochastic model free of any cointegration constraints. Following the methodology in Tiao and Box (1981) and Jenkins and Alavi (1981) a VAR(1) specification was selected from the class of VARMA models as the most adequate representation of the correlation structure in the data set. The estimated system corresponding to (9) is (standard deviations are shown in parenthesis):

$$(9') \quad \begin{bmatrix} 1-1.04B & 1.36B & -.64B & 0 \\ (.22) & (.29) & (.17) & \\ \hline -.87B & 1+.82B & -.72B & .66B \\ (.18) & (.25) & (.17) & (.14) \\ \hline -.18B & .13B & 1-.90B & 0 \\ (.04) & (.05) & (.03) & \\ \hline -.05B & .05B & 0 & 1-.96B \\ (.02) & (.03) & & (.02) \end{bmatrix} \begin{bmatrix} y_t \\ l_t \\ k_t \\ pk_t \end{bmatrix} = \begin{bmatrix} a_{yt}^* \\ a_{lt}^* \\ a_{kt}^* \\ a_{pkt}^* \end{bmatrix}$$

In turn, the estimated matrix of contemporaneous correlations, R is

$$(10') \quad R = \begin{bmatrix} 1 & .84 & .80 & -.06 \\ .84 & 1 & .88 & -.23 \\ .80 & .88 & 1 & -.24 \\ .06 & -.23 & -.24 & 1 \end{bmatrix}$$

The likelihood ratio test of over-identifying restrictions in (9'), which is distributed as a  $\chi^2(3)$ , assumes the value 2.15. This shows that the zero constraints in (9') are adequate.

The contemporaneous correlations between the error term in the equation for public capital and the error terms in the other equations are very small. Given their standard deviation of .175 they are not statistically different from zero. This implies  $\nu_{z0} = 0$ , which means that the private sector does not use contemporaneous information on the public capital in its decision making (see section 2.4 above). Accordingly, the data suggests a symmetric use of information. Recall that, for the sake of generality, Assumption 1 allowed for asymmetric information sets.

The diagnostic tests on the residuals of (9') are shown below in Table 3. The values for the means and standard deviations suggest that the means are not statistically different from zero. The autocorrelation function up to lag four together with the Box-Pierce

statistic indicates a white noise structure. The ARCH  $\chi^2$ -tests indicates the absence of conditional heteroskedasticity. Finally, the Jarque-Bera  $\chi^2$ -tests suggests normality of the residuals.

The cross correlation functions among residuals up to order four, which are not shown in this table, are not statistically different from zero, which indicates the absence of additional structure. Nevertheless, as an overparameterization exercise a VAR(2) specification was estimated. No estimated parameter associated to lags of order two was significantly different from zero at the 5% level.

In order to investigate the robustness of the estimated model, we re-estimate the VAR(1) model without the last five observations. The estimation results are reported below.

$$(9'') \quad \begin{bmatrix} 1-1.04B & 1.44B & -.62B & 0 \\ (.23) & (.31) & (.18) & \\ \hline -.86B & 1+.80B & -.66B & .61B \\ (.20) & (.26) & (.18) & (.14) \\ \hline -.18B & .12B & 1-.90B & 0 \\ (.04) & (.05) & (.03) & \\ \hline -.05B & .06B & 0 & 1-.96B \\ (.03) & (.03) & & (.02) \end{bmatrix} \begin{bmatrix} y_t \\ l_t \\ k_t \\ pk_t \end{bmatrix} = \begin{bmatrix} a_{yt}^* \\ a_{lt}^* \\ a_{kt}^* \\ a_{pkt}^* \end{bmatrix}$$

As can be seen from a cursory inspection of (9') and (9''), variations in the estimated parameters are negligible. The Chow test of parameter stability distributed as  $F(20,24)$  assumes the value 1.44 which implies that the null hypothesis of parameter constancy cannot be rejected at the 5% significance level. Also, we perform an out-of-the-sample forecasting exercise to investigate the forecasting ability of the model for the period 1985-1989. The cumulative  $\chi^2(20)/20$  test for an horizon of five years has a value of 1.6, which by being lower than 2. indicates a good performance of the estimated

model. The  $\chi^2(4)/4$  test for one-period ahead forecast is .58, which indicates that the model performs especially well for one-period forecasts. These statistics confirm the stability of parameter estimates.

### 3.3 Interpretation of the empirical results

Using the theoretical model in Section 2, and the empirical estimates as reported in Section 3.2 we are now in a position to analyze the nature of the relationship between public capital and the private sector variables.

Estimation results suggest that public capital cannot be considered an exogenous variable. In fact, the growth rate of public capital is positively related to the growth rate of output lagged one period and negatively related to the growth rate of labor lagged one period. This can be interpreted as a policy rule for public capital which associates decisions on the evolution of public capital to the performance of the economy in the previous period.

Public capital grows faster if output has increased faster in the previous years. Intuitively, greater output leads to greater availability of funds through the taxation mechanisms and therefore to greater public investment. This result confirms some claims in the literature (see Munnell, 1992) on the possibility of reverse causation. Furthermore, public capital grows faster if labor demand has increased more slowly. Intuitively, public investment has been used as a countercyclical tool in what refers to the evolution of the labor markets.

The effects of output and labor on public capital formation, taken together, confirm the conjecture in Ford and Poret (1991) that public capital seems to respond to labor productivity with a lag. Finally, it should be noted that we could not find any lagged feedback from private capital to public capital.

Let us now consider the effects of shocks in the policy rule for public capital formation on the evolution of the private sector variables. The response of private sector

variables to public capital is not instantaneous, but rather subject to a one-year lag. This comes from the fact that the estimated coefficients of  $\nu_{z0}$  are zero. This means that the private sector does not seem to make immediate use of the information on the public sector decisions. Intuitively, the private sector waits until the public investment plans have been carried out, which takes place during the budget year. This is consistent with some of the observations in the literature that no one would expect the evolution in public capital in one year to be correlated with the evolution of output in the same year (see Munnell, 1992).

The effects on the private sector variables of a one percentage point change in the rate of growth of public capital is given by (3). See Figure 1 for a depiction of the impulse-response functions for the rate of growth of output, labor and private capital.

In the very short-run, the impulse-response functions suggest that the rate of growth of output and private capital are relatively rigid and that an increase in the rate of growth of public capital tends to reduce, temporarily, the rate of growth of the labor input. However, after this initial reaction the effects of the original change in policy rule are consistently positive. The positive effects on the private sector variables reach a peak within the first twenty years. After this point, the rates of change of the private sector variables slowly converge to their initial position (zero).

It should be pointed out that the convergence back to the initial position is very slow. For example, one hundred years after the original shock, the rate of growth of output is still over .2 percentage points above the original state. In fact, values arbitrarily close to the original position are only reached after a period of two hundred years. This reflects a great degree of inertia in the system. This inertia is explained by the fact that changes in the evolution of public capital affect the private sector variables, and changes in the private sector variables in turn feeds back into the evolution of public capital formation.

The response of the private sector growth to a shock in the rate of growth of public capital can also be measured in terms of the levels of the private sector variables. See

Figure 2. In particular, a temporary positive shock in the rate of growth public capital has permanent positive effects on the levels of all private sector variables.

A one percentage point temporary shock in the growth rate of public capital, i.e., a permanent shock of one percentage point in the level of public capital, induces after a ten-year period a change of 7.4 percentage points in the level of output, 3.8 percentage points in the level of private capital, and 1.3 percentage points in the level of labor. In the long-run (after a two-hundred year period), the effects on the levels of output, private capital and labor are 55.7, 60.8 and 30.2 percentage points, respectively. It can be said that while the effects of  $pk$  are slow in coming they are also longlasting.

The effects of a positive shock in  $pk$  are systematically greater on output than on labor, i.e., they increase the average labor productivity. Accordingly, our results are consistent with the view that a slowdown in infrastructure formation (a negative shock) is at least partly responsible for the decline in the growth of labor productivity in the last few decades (see Aschauer, 1989a).

### 3.4 On the Terms of the Current Debate: Competing Production Function Specifications

The debate on the effects of public capital on output has centered on the size of the elasticity of output with respect to public capital as implied by the estimates from a production function. If the elasticity is zero it is said that public capital has no effect on production. If the elasticity is significantly different from zero then public capital is said to affect output. We have argued that, in the presence of feedbacks, a zero elasticity of output with respect to public capital is neither necessary nor sufficient for public capital to have no effect on output. This is the basic point of this section.

The statistical results detailed above are consistent with different production function specifications. The different specifications arise from the specifics of the diagonalization of the matrix of contemporaneous correlations among the residuals of (9').

Following Flores (1990) and Flores and Pereira (1992) this matrix can be interpreted as reflecting intra-period effects. In this section we use some examples of diagonalization using triangular transformation matrices to illustrate the basic point of this section.

Let us consider a standard view of the production process. Assume that private capital is rigid in the short-run. Then, the level of private capital and desired output determine labor demand. In this case the diagonalization matrix can be obtained from:

$$(11) \quad \begin{aligned} a_{lt} &= .30 a_{yt} + 2.56 a_{kt} + u_{lt} \\ a_{yt} &= 4.60 a_{kt} + u_{yt} \\ a_{kt} &= u_{kt} \end{aligned}$$

or in matrix form

$$(12) \quad \begin{bmatrix} 1 & 0 & -4.60 & 0 \\ -.30 & 1 & -2.56 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{yt} \\ a_{lt} \\ a_{kt} \\ a_{pt} \end{bmatrix} = \begin{bmatrix} u_{yt} \\ u_{lt} \\ u_{kt} \\ u_{pt} \end{bmatrix}$$

where the numbers are OLS estimates using the residuals from (9') as variables. By construction the new vector of residuals  $(u_{yt} \ u_{lt} \ u_{kt} \ u_{pt})'$  has a diagonal contemporaneous variance-covariance matrix. The coefficient matrix in (12) is the diagonalization matrix.

Premultiplying (9') by the diagonalization matrix above, we obtain:

$$(13) \quad (1-.21B) y_t = -.76B l_t + (4.60-3.50B) k_t + u_{yt}$$

$$(14) \quad (1+.08B) l_t = (.30 +.10B) y_t + (2.56 -1.78B) k_t - .66B pk_t + u_{lt}$$

$$(15) \quad (1-.90B) k_t = .18B y_t - .13B l_t + u_{kt}$$

$$(16) \quad (1-.96B) pk_t = .05B y_t - .05B l_t + u_{pt}$$

From (15) one can obtain as a long term relationship among rates of growth (which are I(0) variables)

$$(17) \quad y = .72 l + .56 k.$$

This suggests that in the long run  $y$  is determined by  $l$  and  $k$ . Furthermore, given the standard deviation associated to the elements of (9') the hypothesis of the sum of the two coefficients being one cannot be rejected at the 5% level of significance. In this case our results are compatible with a Cobb-Douglas production function in growth rates which displays constant returns to scale on the two private inputs and which does not include public capital.

The existence of constant returns to scale on private inputs leads to the indeterminacy of output size, which is determined in the demand side of the economy. Therefore equations (13) and (14) can be interpreted as yielding implicitly the cost-minimizing input demand functions in growth rates. Notice finally that equation (16) reflects the policy rule for public capital which is invariant to the choice of diagonalization matrix.

If one were to estimate a production function and were to obtain (17), the conclusion would be that changes in public capital would lead to no long term changes in output. In fact, (17) is remarkably similar to the estimates in Tatom (1991).

Let us consider now a similar set of beliefs on the production process. Assume that private capital is rigid in the very short-run. However, now the levels of private capital and labor determine output. In this case the diagonalization matrix can be obtained from:

$$(11') \quad \begin{aligned} a_{yt} &= .75 a_{lt} + 1.66 a_{kt} + u_{yt} \\ a_{lt} &= 3.93 a_{kt} + u_{lt} \\ a_{kt} &= u_{kt} \end{aligned}$$

or in matrix form

$$(12') \quad \begin{bmatrix} 1 & -.75 & -1.66 & 0 \\ 0 & 1 & -3.93 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_{yt} \\ a_{lt} \\ a_{kt} \\ a_{pt} \end{bmatrix} = \begin{bmatrix} u_{yt} \\ u_{lt} \\ u_{kt} \\ u_{pt} \end{bmatrix}$$

where the corresponding contemporaneous variance-covariance matrix of the residuals is diagonal.

Premultiplying (9') by the diagonalization matrix in (12'), we obtain:

$$(13') \quad (1-.09B) y_t = (.75 - .53B) l_t + (1.66 - 1.39B) k_t + .49B pk_t + u_{yt}$$

$$(14') \quad (1 + 31B) l_t = (3.93 + 16B) y_t - 2.82B k_t - .66B pk_t + u_{lt}$$

$$(15) \quad (1 - .90B) k_t = .18B y_t - .13B l_t + u_{kt}$$

$$(16) \quad (1 - .96B) pk_t = .05B y_t - .05B l_t + u_{pt}$$

From (13') one can obtain as a long term relationship among growth rates

$$(17') \quad y = .24 l + .30 k + .54 pk.$$

This suggests that, in the long run,  $y$  is determined by  $l$ ,  $k$ , and  $pk$ . Furthermore, the hypothesis that the sum of the coefficients of  $l$ ,  $k$ , and  $pk$  is one cannot be rejected at the 5% level of significance. Accordingly, our results are compatible with a Cobb-Douglas production function which displays decreasing returns to scale in labor and private capital, but which displays constant returns to scale when public capital is also included.

Since the production function displays decreasing returns to scale in the private inputs, output level is well determined in a standard profit maximization problem and the long-term versions of (13') and (14') can be interpreted as the first order conditions for a

profit maximization problem, implicitly giving the optimal demands for labor and private capital in growth rates. Again notice that (16) reflects the invariant policy rule for public capital.

If one were to estimate a production function and were to obtain (17'), the conclusion would be that a permanent one-percentage point increase in rate of growth of public capital would lead to a long term increase of .54 percentage points in the rate of growth of output. In fact, the elasticity estimates in (17') are remarkably similar to the elasticity estimates in Aschauer (1989a).

The basic point illustrated by the examples above is that the substantial effects of public capital on output are consistent with production function specifications in which public capital may or may not enter. This suggests that the debate is not well framed in a production function framework with a single equation approach.

In the literature, public capital has been seen as affecting private sector performance in two different ways. First, public capital may be seen as an input to the production process, the so called direct effect. Second, public capital may affect output by affecting the marginal productivity of the private inputs capital and labor, the so called indirect effects. See Hulten and Schwab (1991) for a detailed discussion of these effects.

Our methodology allows us to identify the total effect of public capital on output, i.e., the sum of the direct and indirect effects as defined above. For the sake of comparability with the literature we calculate the effect on the long run growth rate of output of a permanent one-percentage point change in the rate of growth of public capital formation. This long run elasticity is .99.<sup>4/</sup> This result is consistent with the values obtained by Baxter and King (1993). In the context of a representative agent growth model they show that a unit increase in public investment spending may result in long-run output multipliers in excess of unity.

To decompose the effects of changes in public capital on output in direct and indirect effects, we need a structural specification. The examples above, however, illustrate

that this decomposition depends on one's beliefs on the instantaneous correlations among the variables. In equation (17) the direct effect is zero, while in (17') the direct effect is positive, and seems to be much larger than the effects of private capital or labor (see Aschauer 1989a, and Munnell, 1992 ). In both cases, however, the total effect of public capital on output is not captured within a single equation estimation procedure.

#### 4. Summary and Concluding Remarks

This paper deals with the empirical relationship between public capital and aggregate economic growth in the United States, and in particular on the question of whether or not public capital is productive. It develops a theoretical framework which allows for full consideration of feedbacks among the relevant variables without imposing a priori dynamic structural constraints. This approach departs from the previous empirical literature which relied both on the use of a single equation approach to estimate production functions and implicitly on the absence of feedback relations.

Estimates for the period 1956–1989 suggest that public capital have a substantial effect on long term production as well as on private capital formation and labor, i.e., public capital is productive. The effects are particularly strong within the first twenty years and are longlasting. Also, the effects seem to be particularly strong on private capital formation and less so on labor. Finally, decisions on public capital seem to follow a policy rule that relates the rate of growth of current public capital stock with the growth rates of lagged output (positively) and lagged labor (negatively).

We have also shown that our results of strong long term effects of public capital on production are compatible, under different assumptions on the contemporaneous effects among variables, with different specifications of production functions, in which public capital may or may not be present. Therefore, current interpretations in the literature of

the importance of the effects of public capital in terms of the size of the estimated parameter in a production function are not conclusive.

The analysis in this paper could be extended in different directions. First, and consistent with the existing literature, before one could recommend that public capital formation should be soundly established in the center stage of any long term growth package, the approach developed in this paper should be applied to disaggregated state/industry data, to other historical periods in the U.S., as well as to other countries at a different stage of development.

Second, the discussion of the effects of shocks to public capital on the private sector variables should be seen as the prelude to the study of the comparative effects of shocks to public capital, private capital, and to labor on output. After all, the crucial policy question is what the use of the marginal dollar should be. Just showing that public capital is important in the growth process, i.e., it is productive, does not imply that the best growth strategy should involve massive public capital formation and not incentives to private investment and/or labor formation.

Third, there is the outstanding question of how public capital formation would be financed. One can think of financing public capital formation through taxation, money creation or public debt. Each of these different financing mechanisms is likely to generate negative effects which may, at least partially, neutralize positive effects of infrastructures. In this sense our estimates of the effects of positive shocks are to be interpreted as upper bounds of the possible actual effects.

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## Footnotes

1/ The issue of the effects of public capital on economic performance has long received the attention of the economics profession. See earlier contributions in Arrow and Kurz (1970) and Diewert (1986). See also the recent theoretical contributions of Barro (1990) and Barro and Sala-i-Martin (1992) in the context of the literature on endogenous growth.

2/ In most cases a Cobb-Douglas production function specification is followed. See Costa et al., 1987, and Eberts, 1990, for the use of translog production functions and Berndt and Hansson, 1991, for the use of dual cost functions.

3/ While there are certain disadvantages from an aggregated approach, in particular in which respects policy recommendations, there is a fundamental question of whether or not the state/industry levels are wide enough to allow the internalization of the spillover benefits generated by public capital formation. Maybe for this very reason, the overwhelming majority of studies that suggest the irrelevance of public capital formation are disaggregated in nature.

4/ It should be noted that the computation of the effects of permanent changes in the growth rate of public capital, i.e., the computation of conventional long-run multipliers is not, strictly speaking, consistent with our estimates and is only offered to allow tentative comparisons. In fact, the estimated parameters, and in particular, the feedback from the private sector to the public sector, preclude any consideration of public capital as an exogenous variable. Rather, public capital follows a well-defined policy rule. A permanent change in the evolution of public capital formation is only meaningful if public capital were to be found exogenous.

Table 1 – Testing for Unit Roots

variable	p=0	p=1
$\Delta \ln Y_t$	-4.8	-4.9
$\Delta \ln (L_t/Y_t)$	-3.6	-3.1
$\Delta \ln (K_t/Y_t)$	-4.3	-4.3
$\Delta \ln (PK_t/Y_t)$	-4.4	-4.7

N.B. – Null hypothesis:  $\rho = 1$  in  $x_t = \rho x_{t-1} + \sum_{i=1}^p \beta \Delta x_{t-i} + e_{xt}$ , where x refers to the variables in the first column

Table 2 – Testing for Cointegration

variable	p=1	p=2	p=3	p=4
$r_{yt}$	-2.8	-2.1	-2.2	-1.9
$r_{lt}$	-3.3	-3.3	-3.3	-3.5
$r_{kt}$	-1.9	-1.6	-1.6	-1.7
$r_{pt}$	-1.3	-0.9	-1.0	-0.3

N.B. – Null hypothesis:  $\rho = 1$  in  $r_{jt} = \rho r_{jt-1} + \sum_{i=1}^p \beta \Delta r_{jt-i} + e_{jt}$ , with  $j=y, l, k, pk$ .

Table 3 — Summary of Statistics for the Residuals of the Estimated Model

	$a_{yt}^*$	$a_{lt}^*$	$a_{kt}^*$	$a_{pt}^*$
mean	.0012	.0006	.0001	.0000
standard-deviation (%)	2.11	1.65	.37	.18
autocorrelation function				
1st lag	.06	-.09	-.18	-.17
2nd lag	.01	-.10	-.13	.18
3rd lag	-.10	.02	.05	.21
4th lag	-.18	-.08	.09	-.07
Box-Pierce Q(4) test	1.45	.82	1.9	3.7
ARCH $\chi^2(3)$ test	1.2	.49	1.86	2.27
Jarque-Bera $\chi^2(2)$ test	4.8	3.4	1.6	.83

N.B. —  $\pm 2/\sqrt{32} = \pm .35$ 

## Appendix

Table A.1 — Data Set for the United States for the Period 1956–1989

Year	Output	Labor	Capital	Public Capital
1956	1246.8	5781.6	2381.9	1130.6
1957	1263.4	5779.3	2454.9	1175.6
1958	1248.2	5576.1	2505.2	1224.7
1959	1325.2	5755.1	2565.1	1275.5
1960	1352.8	5818.5	2627.9	1327.6
1961	1387.4	5756.2	2688.6	1381.4
1962	1464.2	5840.0	2757.6	1436.8
1963	1524.9	5880.4	2831.8	1497.4
1964	1608.6	5995.6	2922.0	1564.1
1965	1703.3	6173.9	3042.2	1633.8
1966	1799.6	6365.5	3177.0	1706.8
1967	1844.7	6393.0	3306.9	1783.0
1968	1921.7	6546.0	3444.4	1859.3
1969	1967.2	6755.8	3595.4	1929.8
1970	1953.6	6751.5	3737.8	1994.0
1971	2009.6	6757.0	3871.0	2057.6
1972	2113.4	6970.2	4015.8	2121.0
1973	2229.1	7253.0	4197.6	2180.4
1974	2207.0	7382.3	4377.9	2240.1
1975	2165.5	7185.3	4513.3	2295.1
1976	2281.8	7408.6	4649.3	2344.1
1977	2397.0	7723.1	4813.8	2388.0
1978	2528.8	8131.0	5009.4	2440.4
1979	2593.6	8447.2	5222.7	2482.5
1980	2583.2	8495.3	5420.0	2530.0
1981	2643.5	8598.5	5624.5	2571.0
1982	2572.0	8479.8	5785.7	2606.2
1983	2674.4	8564.8	5932.7	2644.0
1984	2873.9	8990.5	6127.5	2682.0
1985	2973.5	9240.6	6348.3	2727.9
1986	3058.0	9413.3	6545.8	2772.9
1987	3167.8	9685.5	6731.5	2824.2
1988	3316.7	10009.7	6947.9	2875.2
1989	3400.8	10262.6	7158.2	2929.2

Units: private output, capital, and public capital — billion of 1982 dollars  
labor — ten-thousand workers;  
Sources: private output, capital, and labor — OECD's *Analytical Data Base*  
public capital — OECD's *Flows and Stocks of Fixed Capital*

Figure 1  
Impulse Response Functions

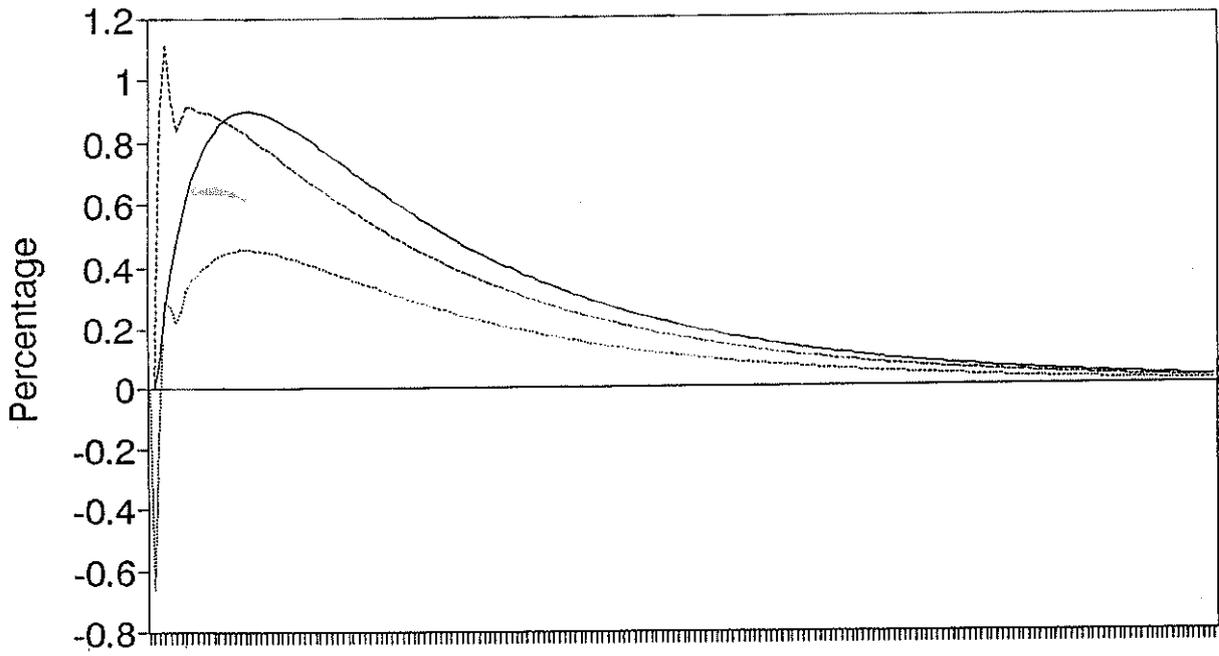


Figure 2  
Step Response Functions

