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Topological solutions in ungauged supergravity

A. de la Cruz-Dombriz,[1,2,*] M. Montero,[3,†] and C. S. Shahbazi[3,‡]

[1]*Astrophysics, Cosmology and Gravity Centre (ACGC) and Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa*

[2]*Instituto de Ciencias del Espacio (ICE/CSIC) and Institut d'Estudis Espacials de Catalunya (IEEC), Campus UAB, Facultat de Ciències, Torre C5-Par-2a, 08193 Bellaterra (Barcelona), Spain*

[3]*Instituto de Física Teórica UAM/CSIC C/Nicolás Cabrera, 13-15, Cantoblanco, 28049 Madrid, Spain*

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A new general class of solutions of ungauged four-dimensional supergravity, in one-to-one correspondence with spherically symmetric, static black-hole solutions and Lifshitz solutions with hyperscaling violation is studied. The causal structure of the space-time is then elucidated.

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I. INTRODUCTION

The dimensional reduction performed in [1] allows us, when considering a spherically symmetric and static background, to write down the equations of motion of any four-dimensional ungauged supergravity as an effective, one-dimensional, system of differential equations for the scalars fields $\{\phi^i, i = 1, \ldots, n_v\}$ and the metric warp factor U, since the vector fields and one of the two arbitrary functions of the metric can be explicitly integrated.

In [2], a remarkable fact was found: given a solution (U, ϕ^i) of the one-dimensional equations of motion, a solution of the complete four-dimensional theory can be constructed not only using the spherically symmetric, static space-time metric, but also using two other different space-time metrics. In other words, given a solution (U, ϕ^i) of the one-dimensional equations of motion, we can choose three different space-time metrics such that the complete four-dimensional solution obeys the equations of motion of the original theory.

One of these three choices is, of course, the spherically symmetric and static space-time metric describing a black-hole solution, which we shall denote by C_1. The second one was previously investigated in [2] and corresponds to Lifshitz solutions with hyperscaling violation, and will be denoted by C_2. The third and final choice C_3 remains to be completely identified, and its study is the leitmotif of this article. We shall find that this class of solutions corresponds to a specific kind of naked singularity in either static or time-dependent solutions, depending on the values of the solution's parameters, which we shall illustrate by studying two simple examples. Therefore, the solutions belonging to C_3 are not topological black holes, in the sense that it is commonly understood in the literature [3]. However, they are still topological solutions, i.e., they represent static space-times with a *topological* spacelike slicing. In other words, the space-time is foliated by a family of two-dimensional surfaces, each being locally isometric to the hyperbolic plane, which can, in principle, be of an arbitrary genus, depending on the existence of global identifications as shown in [3].

In any case, a *triality* among three general classes (C_1, C_2, and C_3) of solutions in four-dimensional supergravity can be established in terms of a 1-1-1 map: i.e., for any solution $s_1 \in C_1$ there is one and only one corresponding solution $s_2 \in C_2/\mathbb{Z}_2$[1] and one and only one solution $s_3 \in C_3$ such that s_1, s_2, and s_3 are constructed in terms of the same (U, ϕ^i) appearing in the one-dimensional equations of motion.

Finally, as a consequence of the triality, all the methods developed to obtain black-hole solutions in ungauged four-dimensional supergravity [4–7], as well as the new results concerning the effective one-dimensional equations of motion [8–10], can be applied to solutions belonging to the classes C_2 and C_3.

The new class C_3 of solutions are relevant for several reasons. It is a class of solutions which can be easily embedded in string theory, for example, by means of type-II fluxless Calabi-Yau compactifications, and therefore they correspond to states in the full-fledged string theory, after being appropriately corrected. In addition, they are a nontrivial example which exhibits the attractor mechanism, different from all the previous solutions where the attractor mechanism was proven to hold [1,11–15]. The attractor mechanism was of outermost importance in supergravity and string theory in order to check the macroscopic computation, at strong coupling, of the entropy of a black hole versus the microscopic calculation, at weak coupling, where the black hole becomes a configuration of D-branes and other objects [16,17]. Since it is possible to associate to

*alvaro.delacruzdombriz@uct.ac.za
†mig.montero@estudiante.uam.es
‡Carlos.Shabazi@uam.es

[1]The $\mathbb{Z}_2$ identification is needed to relate pairs of solutions in C_2 whose transverse part is related by a change of sign in the radial coordinate.

each black-hole solution a unique topological solution, it would be really interesting to see what is the microscopic picture of these solutions in string theory. In doing so we could compare the microscopic description of the black hole and the microscopic description of the corresponding topological solution, which will give us information about what corresponds intrinsically to the microscopic picture of a black hole, which possesses an event horizon. Furthermore, the topological solutions provide evidence about the existence of different new brane solutions in higher dimensions, which will provide us with the higher dimensional supergravity objects necessary to obtain the topological four-dimensional solutions by appropriate intersection and dimensional reduction, very much in the style of what happens for black-hole solutions in four dimensions that can be obtained from a particular intersection of brane solutions to supergravity in higher dimensions.

This article is organized as follows: in Sec. II we introduce the Ferrara-Gibbons-Kallosh formalism, developed in [1], and the effective one-dimensional equations of motion governing the theory. Section III is focused in the topological Schwarzschild-like solution,[2] where we distinguish two cases depending on the sign of the available arbitrary coefficient. Then in Sec. IV we study the topological Reissner-Norström-like solution and depict its Carter-Penrose diagram.

II. THE GENERALIZED FERRARA-GIBBONS-KALLOSH FORMALISM

Following Ref. [1], let us consider the action

$$I=\int d^4x\sqrt{|g|}(R+\mathcal{G}_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j+2\mathfrak{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}{}_{\mu\nu}F^{\Sigma\mu\nu}-2\mathfrak{Re}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}{}_{\mu\nu}\star F^{\Sigma\mu\nu}), \tag{2.1}$$

where $\mathcal{N}_{\Lambda\Sigma}$ is the complex, scalar-dependent, (*period*) matrix. The bosonic sector of any ungauged supergravity theory in four dimensions can be expressed through this action. The scalars are labeled by $i, j, \ldots = 1, \ldots, n_s$, and the vector fields by $\Lambda, \Sigma, \ldots = 0, \ldots, n_v$. The scalar metric $\mathcal{G}_{ij}$ and the period matrix $\mathcal{N}_{\Lambda\Sigma}$ depend on the particular theory under consideration.

Since we are interested in obtaining static solutions, let us consider the metric

$$ds^2=e^{2U}dt^2-e^{-2U}\gamma_{\underline{mn}}dx^{\underline{m}}dx^{\underline{n}}, \tag{2.2}$$

where $\gamma_{\underline{mn}}$ is a three-dimensional (*transverse*) Riemannian metric, to be specified later. Using Eq. (2.2) and the assumption of staticity for all the fields, we perform a dimensional reduction over time in the equations of motion that follow from the aforementioned general action. Thus, we obtain a set of reduced equations of motion that we can write in the form [1]

$$\nabla_{\underline{m}}(\mathcal{G}_{AB}\partial^{\underline{m}}\tilde{\phi}^B)-\frac{1}{2}\partial_A\mathcal{G}_{BC}\partial_{\underline{m}}\tilde{\phi}^B\partial^{\underline{m}}\tilde{\phi}^C=0, \tag{2.3}$$

$$R_{\underline{mn}}+\mathcal{G}_{AB}\partial_{\underline{m}}\tilde{\phi}^A\partial_{\underline{n}}\tilde{\phi}^B=0, \tag{2.4}$$

$$\partial_{[\underline{m}}\psi^{\Lambda}\partial_{\underline{n}]}\chi_{\Lambda}=0, \tag{2.5}$$

where all the tensor quantities refer to the three-dimensional metric $\gamma_{\underline{mn}}$ and where we have defined the metric $\mathcal{G}_{AB}$ as follows:

$$\mathcal{G}_{AB}\equiv\begin{pmatrix}2 & & \\ & \mathcal{G}_{ij} & \\ & & 4e^{-2U}\mathcal{M}_{MN}\end{pmatrix}, \tag{2.6}$$

in the *extended* manifold of coordinates $\tilde{\phi}^A=(U,\phi^i,\psi^\Lambda,\chi_\Lambda)$, where

$$(\mathcal{M}_{MN})\equiv\begin{pmatrix}\mathfrak{I}+\mathfrak{R}\mathfrak{I}^{-1}\mathfrak{R})_{\Lambda\Sigma} & -(\mathfrak{R}\mathfrak{I}^{-1})_{\Lambda}{}^{\Sigma}\\ -(\mathfrak{I}^{-1}\mathfrak{R})^{\Lambda}{}_{\Sigma} & (\mathfrak{I}^{-1})^{\Lambda\Sigma}\end{pmatrix},$$
$$\mathfrak{R}_{\Lambda\Sigma}\equiv\mathfrak{Re}\mathcal{N}_{\Lambda\Sigma}\quad\text{and}\quad\mathfrak{I}_{\Lambda\Sigma}\equiv\mathfrak{Im}\mathcal{N}_{\Lambda\Sigma}. \tag{2.7}$$

Equations (2.3) and (2.4) can be obtained from the three-dimensional effective action

$$I=\int d^3x\sqrt{|\gamma|}\{R+\mathcal{G}_{AB}\partial_{\underline{m}}\tilde{\phi}^A\partial^{\underline{m}}\tilde{\phi}^B\}, \tag{2.8}$$

once the constraint given by Eq. (2.5) has been added.

In order to further dimensionally reduce the theory to a mechanical one-dimensional problem, we introduce the following transverse metric:

$$\gamma_{\underline{mn}}dx^{\underline{m}}dx^{\underline{n}}=\frac{d\tau^2}{W_\kappa^4}+\frac{d\Omega_\kappa^2}{W_\kappa^2}, \tag{2.9}$$

where W_κ is a function of τ and $d\Omega_\kappa^2$ is the metric of the two-dimensional symmetric space of curvature $\kappa=-1$, 0, 1 and unit radius, respectively, as follows:

$$d\Omega_{(1)}^2\equiv d\theta^2+\sin^2\theta d\phi^2, \tag{2.10}$$

$$d\Omega_{(-1)}^2\equiv d\theta^2+\sinh^2\theta d\phi^2, \tag{2.11}$$

$$d\Omega_{(0)}^2\equiv d\theta^2+d\phi^2. \tag{2.12}$$

In these three cases the (θ,θ) or the (ϕ,ϕ) component of the Einstein equations can be solved for $W_\kappa(\tau)$, giving

[2]By "topological Schwarzschild-like solution" we mean the solution in C_3 obtained by using the (U,ϕ^i) effective solution corresponding to the Schwarzschild black hole in C_1. Similar considerations apply to the topological Reissner-Nordström-like solution.

$$W_1 = \frac{\sinh r_0\tau}{r_0}, \tag{2.13}$$

$$W_{-1} = \frac{\cosh r_0\tau}{r_0}, \tag{2.14}$$

$$W_0^{\pm} = a e^{\mp r_0\tau}. \tag{2.15}$$

where a is an arbitrary real constant with dimensions of inverse length and r_0 is an integration constant whose interpretation depends on κ. The case $\kappa = 1$ has been widely studied in the literature and corresponds to asymptotically flat, spherically symmetric, static black holes [1,7,18,19]. The case $\kappa = 0$ has been recently studied in [2] and provides a rich spectrum corresponding to Lifshitz-like solutions with hyperscaling violation. Thus, the goal of this article is to study the case $\kappa = -1$.

For the three cases (2.13), (2.14), and (2.15) we are left with the same equations for the one-dimensional fields, which can be written as follows:

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\mathcal{G}_{AB}\frac{\mathrm{d}\tilde{\phi}^B}{\mathrm{d}\tau}\right) - \frac{1}{2}\partial_A\mathcal{G}_{BC}\frac{\mathrm{d}\tilde{\phi}^B}{\mathrm{d}\tau}\frac{\mathrm{d}\tilde{\phi}^C}{\mathrm{d}\tau} = 0, \tag{2.16}$$

$$\mathcal{G}_{BC}\frac{\mathrm{d}\tilde{\phi}^B}{\mathrm{d}\tau}\frac{\mathrm{d}\tilde{\phi}^C}{\mathrm{d}\tau} = 2r_0^2. \tag{2.17}$$

The electrostatic and magnetostatic potentials $\psi^\Lambda, \chi_\Lambda$ only appear through their τ derivatives. The associated conserved quantities are the magnetic and electric charges p^Λ, q_Λ that can be used to eliminate completely the potentials. The remaining equations of motion can be reorganized in the convenient form

$$U'' + e^{2U}V_{\mathrm{bh}} = 0, \tag{2.18}$$

$$(U')^2 + \frac{1}{2}\mathcal{G}_{ij}\phi^{i\prime}\phi^{j\prime} + e^{2U}V_{\mathrm{bh}} = r_0^2, \tag{2.19}$$

$$(\mathcal{G}_{ij}\phi^{j\prime})' - \frac{1}{2}\partial_i\mathcal{G}_{jk}\phi^{j\prime}\phi^{k\prime} + e^{2U}\partial_i V_{\mathrm{bh}} = 0, \tag{2.20}$$

in which the prime indicates differentiation with respect to τ and the so-called *black-hole potential* V_{bh} is given by

$$V_{\mathrm{bh}}(\phi, \mathcal{Q}) \equiv \frac{1}{2}\mathcal{Q}^M\mathcal{Q}^N\mathcal{M}_{MN}, \qquad (\mathcal{Q}^M) \equiv \begin{pmatrix} p^\Lambda \\ q_\Lambda \end{pmatrix}. \tag{2.21}$$

Equations (2.18) and (2.20) can be in fact be derived from the effective action

$$I_{\mathrm{eff}}[U, \phi^i] = \int \mathrm{d}\tau\{(U')^2 + \frac{1}{2}\mathcal{G}_{ij}\phi^{i\prime}\phi^{j\prime} - e^{2U}V_{\mathrm{bh}}\}, \tag{2.22}$$

whereas Eq. (2.19) is nothing but the conservation of the Hamiltonian (due to the absence of explicit τ dependence in the Lagrangian) with a particular value of the integration constant r_0^2.

A large number of solutions of the system (2.18), (2.19), and (2.20), for different theories of $\mathcal{N} = 2$, $d = 4$ supergravity coupled to vector supermultiplets, have been found (see, *e.g.,* Ref. [4,6,7,18,20–28]), always focusing on the case $\kappa = 1$. With this choice of transverse metric, they describe single, charged, static, spherically symmetric, asymptotically flat and nonextremal black holes. These solutions can now be studied setting $\kappa = -1$ in the transverse metric.

Using Eqs. (2.11) and (2.14), the metric can be written in this case as

$$\mathrm{d}s^2 = e^{2U}\mathrm{d}t^2 - e^{-2U}\left[\frac{r_0^4\mathrm{d}\tau^2}{\cosh^4 r_0\tau} + \frac{r_0^2}{\cosh^2 r_0\tau}\mathrm{d}\Omega^2_{(-1)}\right], \tag{2.23}$$

where $\mathrm{d}\Omega^2_{(-1)} = \mathrm{d}\theta^2 + \sinh^2\theta\mathrm{d}\phi^2$ is the two-dimensional metric of a negative constant curvature. We have introduced the integration constants q_Λ and p^Λ, which come from the explicit integration of the Maxwell equations, and we have identified them with the electric and magnetic charges of the topological solution. This identification works perfectly well in the usual black-hole case; let us see how a similar procedure would apply here. Let $i: \mathcal{N}_{t,\tau} \hookrightarrow \mathcal{M}^3$ be the two-dimensional submanifold defined in local coordinates (t, τ, θ, ϕ) by $t = \tau = c^{te}$. Then p^Λ and q_Λ are given by

$$p^\Lambda = \alpha\int_{\mathcal{N}_{t,r}} i^* F^\Lambda, \qquad q_\Lambda = \alpha\int_{\mathcal{N}_{t,r}} i^* G_\Lambda, \tag{2.24}$$

where α is a normalization real constant and

$$G_\Lambda = \Re\mathrm{e}_{\Lambda\Sigma}F^\Sigma + \Im\mathrm{m}_{\Lambda\Sigma} * F^\Sigma. \tag{2.25}$$

Notice that in the standard black-hole case, we obtain the very same equation (2.24), but there $\mathcal{N}_{t,r}$ is a spatial two-sphere which englobes the black hole and is usually taken to be at spatial infinity, $\tau \to 0$. Since here, in principle, we do not have a well-defined *spatial infinity*, we cannot take $\mathcal{N}_{t,r}$ to be at such spatial infinity by an standard limit in the coordinates (t, τ). Nevertheless, the expression (2.24) that relates the integration constants to the integration of the field strengths F^Λ and their duals G_Λ over a spatial slicing $\mathcal{N}_{t,r}$ of the space-time still holds.

[3]Here $\mathcal{M}$ denotes the space-time manifold.

III. THE TOPOLOGICAL SCHWARZSCHILD BLACK HOLE

The formalism developed in Sec. II applies to any Lagrangian of the form (2.1). In particular, it can be applied to the case where there are no matter fields and only the Hilbert-Einstein term remains. In this case, we are dealing with the Einstein equations in vacuum, and we obtain [18]

$$U = \sigma\tau, \tag{3.1}$$

where σ is an arbitrary integration constant, which is equal to the mass of the black hole in the asymptotically flat, spherically symmetric, static case. Thus the metric is given by

$$\mathrm{d}s^2 = e^{2\sigma\tau}\mathrm{d}t^2 - e^{-2\sigma\tau}\left[\frac{\sigma^4\mathrm{d}\tau^2}{\cosh^4\sigma\tau} + \frac{\sigma^2}{\cosh^2\sigma\tau}(\mathrm{d}\theta^2 + \sinh^2\theta\mathrm{d}\phi^2)\right]. \tag{3.2}$$

In order to write Eq. (3.2) in a more convenient way we perform the following change of variables:

$$e^{2\sigma\tau} = \frac{2\sigma}{r} - 1. \tag{3.3}$$

In these new coordinates, the metric reads

$$\mathrm{d}s^2 = \left(\frac{2\sigma}{r} - 1\right)\mathrm{d}t^2 - \left(\frac{2\sigma}{r} - 1\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2_{(-1)}. \tag{3.4}$$

Thanks to Eq. (3.4) it is easy to recognize the last metric as the so-called *AII* metric with $\kappa = -1$, found in [29] and whose interpretation was first given in [30,31]. We summarize now the principal properties of such space-time, closely following [32], where a detailed description is given.

A. Carter-Penrose diagram

Since $r = 0$ is a true singularity, it is convenient to take $r \in (0, \infty)$, allowing σ to be either positive or negative. We have therefore two different possibilities, that shall be considered separately.

(1) $\sigma > 0$

The metric (3.4) can be written as follows:

$$\mathrm{d}s^2 = \left(\frac{2|\sigma|}{r} - 1\right)\mathrm{d}t^2 - \left(\frac{2|\sigma|}{r} - 1\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2_{(-1)}. \tag{3.5}$$

For $r > 2\sigma$ the metric is time dependent, since the r coordinate becomes timelike. In $r = 2\sigma$ we have the Killing horizon related to ∂_t. The metric is static for $0 < r < 2\sigma$. The corresponding Penrose diagram is shown in Fig. 1, taking $\sigma_1 = |\sigma|$ and $\sigma_2 = 0$ in (4.5) in order to recover (3.5). It is similar to the Penrose diagram of the Schwarzschild solution [29] except for a quarter-turn tilting. This is explained by the fact that (3.5) at constant ϕ, θ is related to the Schwarzschild metric with the same restriction by an overall sign. This reverses the notions of space and timelike vectors from one metric to the other, leaving everything else unchanged. The tilt is explained then by the fact that in Penrose diagrams, timelike directions are represented upwards.

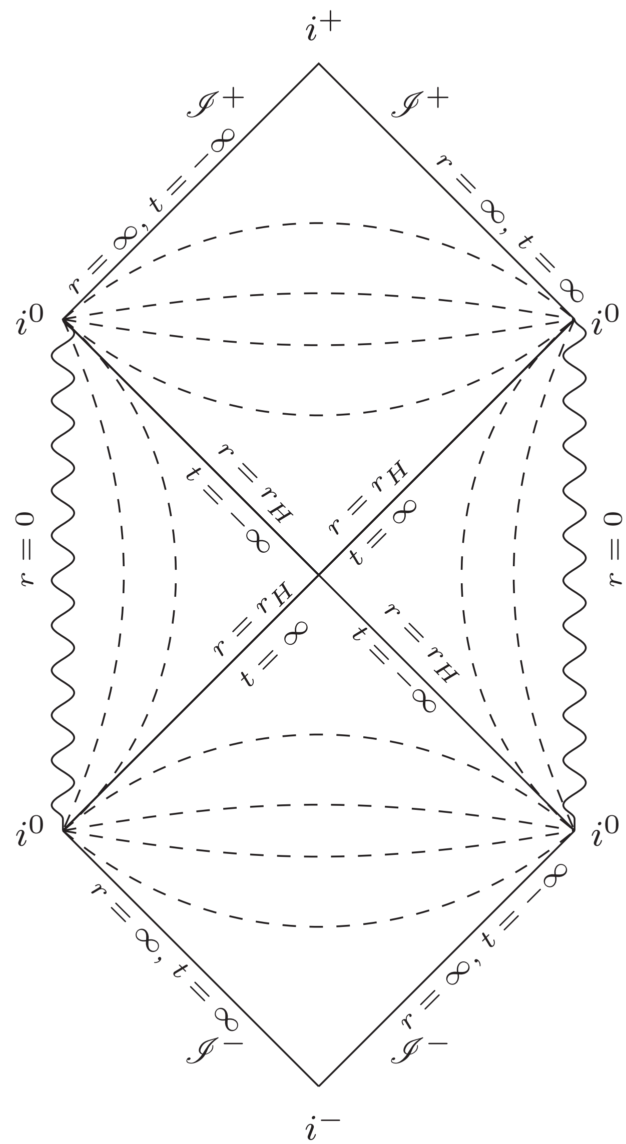

FIG. 1. Conformal diagram for metric (4.5) with $\sigma_1 > 0$. This represents sections on which angular coordinates are constant, so that each point on the diagram represents a point on an topological surface of constant negative curvature. The symbols in the figure possess their standard interpretation in this kind of diagram.

$r = 0$ represents two different timelike, naked singularities, as is apparent from Fig. 1: The coordinate singularity at $r = 2\sigma$ is not an usual event horizon (although as stated above, it is a Killing horizon), since events inside it can be seen from observers near the asymptotic future. In contrast, events taking place in this region cannot be seen from the inside, although events taking place near the asymptotic past can be seen from $r < 2\sigma$. It is possible for a particle to travel from past null infinity to future null infinity without ever encountering a singularity. Notice that $r = 2\sigma$ is still a Killing horizon.

(2) $\sigma < 0$

We can write the metric (3.4) as follows:

$$\mathrm{d}s^2 = \left(-\frac{2|\sigma|}{r} - 1\right)\mathrm{d}t^2 - \left(-\frac{2|\sigma|}{r} - 1\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2_{(-1)}, \tag{3.6}$$

and we immediately see that there is no coordinate horizon at $r = 2\sigma$; the coordinates behave properly all the way to the singularity. Also, ∂_r is now everywhere timelike. Relabeling the coordinates accordingly we obtain

$$\mathrm{d}s^2 = \left(1 + \frac{2|\sigma|}{t}\right)^{-1}\mathrm{d}t^2 - \left(1 + \frac{2|\sigma|}{t}\right)\mathrm{d}r^2 - t^2\mathrm{d}\Omega^2_{(-1)}. \tag{3.7}$$

The physical singularity is, therefore, at $t = 0$. In this case, the corresponding Penrose diagram can be seen in [32]. The solution may be regarded as a vacuum spatially homogeneous but anisotropic cosmological model that is of Bianchi-type III, in which r is a global time coordinate.

IV. THE TOPOLOGICAL REISSNER-NORDSTRÖM BLACK HOLE

The Reissner-Nordström black hole can be embedded in pure $\mathcal{N} = 2$, $d = 4$ supergravity. The metric function of this solution in the τ coordinates is [18]

$$e^{-2U} = \frac{(M\cosh r_0\tau - r_0\sinh r_0\tau)^2}{r_0^2}, \qquad r_0^2 = M^2 - V_{\mathrm{bh}}. \tag{4.1}$$

As in the previous case, we perform a change of coordinates,

$$r = -r_0\tanh r_0\tau + M, \tag{4.2}$$

in order to rewrite the metric in a more convenient form. Thus, the metric is given by

$$\mathrm{d}s^2 = \left(-1 + \frac{2M}{r} - \frac{V_{\mathrm{bh}}}{r^2}\right)\mathrm{d}t^2 - \left(-1 + \frac{2M}{r} - \frac{V_{\mathrm{bh}}}{r^2}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2_{(-1)}, \tag{4.3}$$

where

$$V_{\mathrm{bh}} = -q^2 - \frac{p^2}{4} \tag{4.4}$$

is the black-hole potential of pure $\mathcal{N} = 2$, $d = 4$ supergravity in the chosen conventions [18]. The parameters M and V_{bh} have a clear physical interpretation in the spherically symmetric case, which, however, may not carry over to the $\kappa = -1$ case. Therefore, we rewrite Eq. (4.3) as

$$\mathrm{d}s^2 = \left(-1 + \frac{2\sigma_1}{r} + \frac{\sigma_2^2}{r^2}\right)\mathrm{d}t^2 - \left(-1 + \frac{2\sigma_1}{r} + \frac{\sigma_2^2}{r^2}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2_{(-1)}, \tag{4.5}$$

where σ_1 and σ_2 are arbitrary real parameters. Remarkably, the causal structure of the space-time is independent of the particular values of σ_1, σ_2.

A. Carter-Penrose diagram

The causal structure of more general cases, in the presence of nontrivial scalars, is analogous to the Topological Reissner-Nordström-like solution, which is therefore the relevant example which allows us to identify the space-time features of the whole class of solutions, exactly as in the spherically symmetric case.

This space-time exhibits a physical singularity at $r = 0$. Therefore, it is enough to restrict ourselves to $r > 0$, while allowing σ_1 to take any value. For the study of the Carter-Penrose diagram, let us remember that the metric (4.5) possesses two Killing horizons,

$$r_\pm \equiv \sigma_1 \pm \sqrt{\sigma_1^2 + \sigma_2^2}, \tag{4.6}$$

and only one of these,

$$r_+ = r_H \equiv \begin{cases} \sigma_1 + \sqrt{\sigma_1^2 + \sigma_2^2} & \text{if } \sigma_1 \geq 0 \\ -\sigma_1 + \sqrt{\sigma_1^2 + \sigma_2^2} & \text{if } \sigma_1 < 0, \end{cases} \tag{4.7}$$

is greater than zero. That means that there is only one Killing horizon associated to ∂_t. For $r > r_H$ the metric is time dependent, whereas for $0 < r < r_H$ it is static. This is the same behavior of the type AII metric (3.5) for $\sigma > 0$. Indeed, on their respective θ, ϕ constant slices, these two space-times are related by a conformal transformation and hence have the same causal structure and Carter-Penrose diagram. To see this, notice that the metric (4.5) is related by a global sign to the Reissner-Nordström metric with an imaginary value of the charge. Following [32], we may introduce Kruskal-Skezeres-like coordinates as follows:

$$U_+ = -\frac{2r_H^2}{r_H - r_-}\left|\frac{r}{r_H} - 1\right|^{1/2}\left|\frac{r}{r_-} - 1\right|^{\frac{r_-^2}{2r_H^2}} \exp\left[-\frac{(r_H - r_-)}{2r_H^2}(t - r)\right],$$
$$V_+ = \frac{2r_H^2}{r_H - r_-}\left|\frac{r}{r_H} - 1\right|^{1/2}\left|\frac{r}{r_-} - 1\right|^{\frac{r_-^2}{2r_H^2}} \exp\left[\frac{(r_H - r_-)}{2r_H^2}(t + r)\right], \quad (4.8)$$

in terms of which (4.5) take the form

$$\begin{aligned} ds^2 &= 4\frac{r_- r_H}{r^2}\left|\frac{r - r_-}{r_-}\right|^{1+\frac{r_-^2}{r_H^2}} \exp\left(-\frac{r_H - r_-}{r_H^2} r\right) dU_+ dV_+ \\ &\quad - r^2 d\Omega^2_{(-1)} \\ &= \Omega(r; r_H, r_-)\left[-4\frac{r_H}{r}\ \exp\left(-\frac{r}{r_H}\right) dU_+ dV_+\right] \\ &\quad - r^2 d\Omega^2_{(-1)}, \end{aligned} \quad (4.9)$$

with

$$\Omega(r; r_H, r_-) \equiv \frac{-r_-}{r}\left|\frac{r - r_-}{r_-}\right|^{1+\frac{r_-^2}{r_H^2}} \exp\left(\frac{r_-}{r_H^2} r\right). \quad (4.10)$$

The factor multiplied by $\Omega(r; r_H, r_-)$ in the expression (4.9) corresponds to the $t - r$ part of the metric (3.5) in Kruskal-Skezeres-like coordinates. Since $\Omega(r; r_H, r_-) > 0$ is well defined throughout the space-time, this shows the conformal equivalence between the two metrics in the θ, ϕ constant slices.

This equivalence of conformal structures can be understood on physical grounds by considering (4.5) with a global sign change. As stated above, this corresponds to a Reissner-Nordström metric with an imaginary charge. This results in an attractive instead of a repulsive singularity at short distances, which will behave qualitatively in the same way as in Schwarzschild space-time. Therefore, (3.5) and (4.5) share the same Carter-Penrose diagram, given by Fig. 1.

The solution (4.5) can be given a physical interpretation in the limit σ_1, $\sigma_2 \to 0$, which is basically the same as that of (3.4) when $|\sigma| \to 0$. This can be found in [32]. There it is shown that, after a change of coordinates

$$T = \pm r \cosh\theta, \qquad R = r \sinh\theta, \qquad Z = t, \quad (4.11)$$

the metric becomes Minkowski in cylindrical coordinates along the Z axis, namely,

$$ds^2 = -dT^2 + dR^2 + R^2 d\phi^2 + dZ^2. \quad (4.12)$$

Since $r^2 = T^2 - R^2$, the hypersurface $r = 0$ (which naively represents the region of strong coupling since we have taken σ_1, $\sigma_2 \to 0$) corresponds to $T = 0$, $R = 0$ (the worldline of a spacelike particle moving along the Z axis) plus the cylindrical surface $T = \pm R$, which can be understood as a cylindrical wave shrinking to zero size and then expanding again at the speed of light. The resulting configuration may be interpreted as the asymptotic metric, as $r \to \infty$, of the gravitational field of a tachyon, with the $T = \pm R$ null hypersurfaces corresponding to the horizon $r \approx r_H$. The difference between the solutions of (4.5) and (3.4) would be, as far as this physical interpretation is concerned, that the tachyon of (4.5) carries some charges as dictated by (4.4).

V. ATTRACTOR MECHANISM FOR TOPOLOGICAL SOLUTIONS

The results of Sec. IV illustrate the casual structure of the supergravity class of solutions C_3, which is in 1-1-1 correspondence with the supergravity static, spherically symmetric, asymptotically flat black holes of C_1 and the hyperscaling violation solutions of $C_2/\mathbb{Z}_2$. From the very same solution (U, ϕ^i), $i = 1, \ldots, n_s$ of the system of differential equations (2.18), (2.19), and (2.20) we can build three different four-dimensional solutions $s_1 \in C_1$, $s_2 \in C_2/\mathbb{Z}_2$ and $s_3 \in C_3$ of the original theory. Since the class C_1 corresponds to spherically symmetric, static, asymptotically flat black holes, the flow of the corresponding scalars may exhibit attractors, or fixed points, at $\tau \to -\infty$ [1,11,13–15,19,33–39]. This is, in particular, ensured for supersymmetric black holes. Amazingly enough, the scalars of the related solution s_3 are the very same scalars as those of s_1, so the scalars of s_3 will have fixed points if and only if the scalars of s_1 also have them.

The previous considerations prove the attractor mechanism for a subset $C_3^{\mathrm{Att}} \subset C_3$ such that the related solutions in C_1 also exhibit an attractor mechanism. However, in the case of solutions in C_3, the scalars are not fixed at an event horizon, since the solution does not have any, but instead they are fixed at the Killing horizon.

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