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ABSTRACT

Some markets are characterized by a systematic relation between how costly it is for consumers to observe prices, market power, and incentives to reduce costs. This paper offers a model of such markets and discusses incentives to invest in cost reduction. I develop two partial equilibrium models, one static, other dynamic, where technology is determined endogenoussly throug stochastic investment, firms set prices, entry is free, and consumers search for prices. I use the static model to discuss market power and price controls, and the dynamic model to discuss cost volatility and predation.

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**Visiting Professor.

1 Introduction

Some markets are characterized by a systematic relation between how costly it is for consumers to observe prices[1], market power, and firms' incentives to reduce costs. Costly consumer search gives firms market power, since consumers accept prices above the minimum charged in the market, but affects firms with different costs differently. Low cost firms are not constrained by consumer search because they charge the lowest price, but high cost firms are. Thus, a rise in incentives to search (i.e., a fall in the reservation price), forces high cost firms to lower their prices, and redistributes consumers to low cost firms, which increases the marginal benefit of cost reduction.

The following retail industry cases illustrate this relation, although neither is a perfect example. Chrysler's Bob Eaton (The Economist, 14 Feb. 1998, pp. 71-3) reports that 1/4 of American car buyers will use the Internet in 98 and 1/2 by 2000. Most will use the Internet only to obtain information (e.g., how much a dealer pays for a car), that allows them to bargain lower prices out of local traditional dealers. That is, Internet reduces serach costs and thereby market power. Other consumers, however, will buy directly from dealers selling cars on-line at lower prices than traditional retailers[2]. Those dealers afford cheaper rates, because developing an on-line sales system amounts to creating a new, lower cost retail technology, with a smaller sales force, savings on shop and warehouse costs, and better inventory management. It is the Internet that both

[1] This market imperfection due to consumes having imperfect information about prices generates price dispersion (McMillan & Rothschild (92), Benabou (93), Stahl (89), Rob (85), Burdett & Judd (83), Carlson & McAfee (83), Varian (80), MacMinn (80), Reinganum (79)).

[2] Independently of this a consolidation process is also taking place. The model presented ahead predicts also that lower search costs lead

reduces serach costs and allows the new technology, but these are different things as the example shows. Similar examples can be found in the retail brokerage industry, or the book and compact disc retail industries.

This paper offers a model of markets characterized by a systematic relation between consumer search, market power, and cost reducing investment, and discusses incentives to reduce costs. I develop two partial equilibrium models, one static other dynamic, where technology is determined endogenously through stochastic investment, firms set prices, entry is free, and consumers search for prices. I use the static model to discuss market power (the ability to rise price above marginal cost) and price controls, and the dynamic model to discuss cost volatility and predation.

The basic static model is a three stage game. In the first stage firms invest in cost reduction; in the second stage firms choose prices; and in the third stage consumers search and buy. The dynamic model has two periods, each composed of three stages. The first period unfolds as the static model. In the second, first firms suffer a (technological or monetary) cost shock, and then pricing and search are repeated. The model is dynamic because consumers buy repeatedly (Benabou (93), Cressy (83), Fishman & Rob (95), McMillan & Morgan (84), Tommasi (94)), and marginal costs vary over time.

The paper makes five observations. First, that stochastic investment is a way of endogeneizing cost heterogeneity in the Reinganum (79)-MacMinn (80) model. Suppose firms can produce at a basic unit cost, which through an investment, whose outcome is uncertain, they can try to reduce. Cost heterogeneity is then the result of investment's randomness. This, embodies two concepts: first the creation of a new production process involves irreducible *ex ante* uncertainty (Lippman & Rumelt (82)), and second, firms can influence, at a cost, their technological type. My model can be interpreted as a ***double search*** model, where firms search for technology (Evenson & Kislev (76), Telser (80), Reinganum (82), Nelson (82)), and consumers search for prices.

There are two opposing views on how market power affects incentives to reduce costs, on the ***research and development*** (R&D) literature, and the ***agency*** literature that investigates if the market reduces agency problems. One view holds that downward pressure on prices is needed to induce cost reduction. A firm can always profit from reducing costs[3]. But when high cost firms can charge high prices, cost reduction is less valuable. While downward pressure on prices, erodes high cost firms' profits, making cost reduction more valuable. I call this the ***Classical conjecture***. The second view consists of two parts. First, market power is necessary to induce cost reduction. Market power generates the quasi-rents required to cover investment expenditures. And since it is the prospect of high profits that induces cost reduction, and since profits are higher the bigger the market power, the higher the market power, the bigger the incentives to reduce costs. Second, the welfare gain from cost reduction compensates the loss from price differing from marginal cost (Tandon (84), Nelson & Winter (82)). I call this *Schumpeter's conjecture* (47).[4]

The second observation is that the relation between market power and cost reduction depends on the context. This clarifies under which conditions it is one of the two opposing views that best describes a market, and agrees with the ambiguity of the empirical literature on R&D (Cohen & Levin (89)). I show that search cost shifts lead market power and investment to move in ***opposite*** directions, while demand shifts can lead them to move in the ***same*** direction. Market power and cost reducing investment are both functions of the parameters of the model (Dasgupta & Stiglitz (80)), and the many sources of market power (fixed costs, costly search, transportation costs, patents, etc) affect the marginal benefit of investment differently. The reasoning underlying *Schumpeter's conjecture* is that a fall in market power reduces total profit, and thereby incentives to invest. But what matters is ***marginal,*** not ***total*** profit, and they need not move in the same direction. A fall in market power can both rise the marginal profit and reduce total profit, if it impacts firms asymmetrically, eroding high cost firms' profits relatively more.

Cost reduction under the form of R&D is an important component of growth, and object of many policy measures, particularly in the European Community. The previous discussion, calls attention to the fact that the relation between market power and cost reduction depends on the nature of an industry. Policy measures should reflect this.

The third observation is that in search markets, price controls may not affect cost reduction monotonically. As long as price controls hurt relatively more high cost firms, they stimulate cost reduction. Otherwise, they can reduce investment.

The fourth observation is that with switching costs, cost volatility can lead firms to charge below marginal cost, without predatory motives. I assume it is cheaper for consumers to observe the current price of the firms they bought from previously, than from another firm. This creates a ***switching cost*** that tends to lock consumers to their period 1 suppliers. Due to the ***lock-in effect,*** firms need to establish a customer base, which might involve pricing below marginal cost in the first period, if the reservation price is sufficiently low. Notice also that sales maximization in period 1 is part of an intertemporal profit maximizing strategy, as in Klemperer (87a, 87b).

The fifth observation is that marginal cost volatility can lower cost reduction. I analyze two types of cost volatility: ***idiosyncratic cost volatility,*** which is a change in the ratio of the firm's cost to the lowest cost in the industry, and, ***industry wide cost volatility,*** which is an equal rise in all firms' costs.

Idiosyncratic cost volatility impacts investment through three effects, the more interesting of which is the following. When a product is bought repeatedly, finding a low price firm is valuable for both current and future consumption, if current prices convey information about future prices. A rise in intertemporal price variability, induced by relative cost volatility, lowers the informativeness of current prices about future prices. This reduces incentives to search, lowering investment.

Industry wide cost volatility impacts cost reduction in two ways. First, it reduces equally the markup of both types of firms. Since low cost firms have larger sales, they are relatively more affected. Second, it reduces incentives to search in period 2, further decreasing investment.

There is empirical evidence suggesting inflation is positively correlated with relative price variability, both across markets (Domberger (87)) and within markets (van Hoomissen, (88), Tommasi (93), Lach & Tsiddon (92), Vining & Elwertowski (76), Glejser, (65), Parks (78)). Based on these studies Tommasi (94) and Fishman & Rob (95) identify idiosyncratic cost volatility with the result of the nonuniform propagation of inflation across the economy. If one accepts this, then inflation lowers cost reduction, and rises average costs.

Next I integrate the paper on the literature. Bagwell & Ramey (96) propose an alternative way of endogeneizing cost heterogeneity. They use a Rosenthal (80)-Varian (80)-Stahl (89) model, where identical firms pay a fixed cost to make a deterministic cost reduction investment.

A vast literature (Arrow (62), Dasgupta & Stiglitz (80), Futia (80), Flaherty (80), Spence (84)), developed models that reproduce *Schumpeter's conjecture*. For example, Dasgupta & Stiglitz (80) showed that for a symmetric Cournot oligopoly, with free entry, where the marginal cost is determined by a deterministic investment, the degree of concentration is positively correlated with investment. Since in their model concentration is positively correlated with the price level, market power and investment vary positively. Here market power results from the fixed cost nature of R&D expenditures, and that *ex post* firms are identical.

Several authors (Hart (83), Scharfstein (88), Hermalin (92), Martin (93), Schmidt (97)) modeled, with mixed results, how the market could reduce agency problems. For example, Schmidt (97) showed that a rise in the number of competitors, on the one hand, increases the probability of liquidation, which rises managerial effort, on the other hand, reduces the firm's profits, which decreases managerial effort.

Fishman & Rob (95), Tommasi (94), Benabou & Gertner (93), Benabou (93), Benabou (92), Benabou (88), showed that in search markets a raise in relative price variability or inflation lowers the incentives to search. This allows high cost firms to rise their prices leading to a higher price expectation and variance.

I develop the static model in section 2, and in section 3 I characterize its equilibrium. I discuss market power in section 4, price controls in section 5, and present two extensions to the static model in section 6. I develop the dynamic model in section 7, and in section 8 I describe its equilibrium. In section 9 I discuss cost volatility, predation and inflation. The proofs of Lemma 2 and Propositions 2 and 3 are given in appendix since they consist of straightforward applications of the implicit function theorem. The dynamic model is formally development in a technical appendix available from the author upon request.

2 The Static Model

In this section I present the static model.

Consider a market for a homogeneous (search) good that opens for one period.

[Insert figure 1 here]

The game consists of three stages (figure 1). In stage 1 firms simultaneously choose investments; each firm observes only its cost realization. In stage 2 firms simultaneously choose prices. In stage 3 consumers simultaneously make their search and purchase decisions; then production and delivery take place instantaneously, agents receive their payoffs, and the market closes.

There is a continuum of consumers of unit measure. Consumers are identical and risk neutral. A consumer who buys at price p demands $x(p;\delta)$, where $x(.): (0,+\infty)\times(0,+\infty) \longrightarrow (0,+\infty)$ is a twice differentiable, bounded function, with a bounded inverse, decreasing and strictly concave on price and increasing on δ, a parameter that measures the level of demand. Denote the surplus of a consumer who pays p by $S(p;\delta) := \int_p x(t;\delta)\,dt$.

To obtain a price quote from a firm a consumer must pay a constant amount $\sigma\in(0,+\infty)$: the *search cost*. Search is instantaneous, a consumer may solicit any number of price quotes, and may at any time accept any offer received to date. I assume:

(A.1) Each consumer picks at random which firm to sample, from the set of firms whose price he does not know.

A consumer's *information set* just after his k-th search (or return) step consists of all the prices previously observed. A consumer's stage 3 *strategy* is a stopping rule, that for every possible search cost, and sequence of observations, says whether search should stop or continue; denote it by s. A consumer's *payoff* is the expected consumer surplus, net of the search expenditure.

There is a continuum of firms of unit measure. Firms are risk neutral and may differ in their marginal production costs.

A firm's ***cost reducing*** investment $a\in[0,+\infty)$ generates marginal cost level c_l with probability $\mu(a)$, and marginal cost level c_h with probability $1 - \mu(a)$, where $0 \le c_l < c_h < +\infty$. This specification contains four assumptions: first, marginal production costs are constant; second, the cost type distribution is at most binary[5]; third, each firm's probability of having a low cost depends only on its investment[6] and fourth, the support of the cost distribution is independent of the firms' individual and aggregate investments. I assume $\mu(.): [0,+\infty) \longrightarrow [0,1]$ is a twice differentiable function, with: $\mu''(a) < 0 < \mu'(a)$, $\forall a\ge 0$; $\mu(0) = 0$; $\mu(a) < 1$, $\forall a<+\infty$; and $\lim_{a\to+\infty} \mu(a) = 1$. These assumptions imply that investment induces first order stochastic (leftward) shifts in the cost distribution; no investment induces a degenerate distribution at $c = c_h$; and firms can never get a low cost with probability 1.

Cost reducing investment could be a process innovation that reduces the cost of producing existing products, a managerial innovation that rises productivity, or an investment in capital that increases the marginal productivity of labor[7].

Denote the per consumer profit of a firm with cost c_τ who charges price p by $\pi(p;c_\tau) := (p - c_\tau)x(p;\delta)$ ($\tau = l, h$). Let $p_\tau^m := \arg\max_p \pi(p;c_\tau)$. I call p_τ^m the cost c_τ firm's ***monopoly price***. By strict concavity of demand p_τ^m is unique and strictly increasing in c_τ. Denote the price of a firm with cost c_τ by p_τ, and the expected consumer measure (or share) of a firm that cahrges price p by $\varphi(p)$. A firm's expected profit equals the firm's per consumer profit, times the firm's expected consumer measure; denote it by: $\Pi(p_\tau;c_\tau,\varphi(p_\tau)) := \pi(p_\tau;c_\tau)\varphi(p_\tau)$. I assume only low cost firms can charge p_l^m without losing money, i.e., $p_l^m < c_h$.

When a firm chooses to charge a price higher than the maximum consumers are willing to pay, and hence forgoes the chance of selling its product, I say the firm is ***inactive***; otherwise I say the firm is ***active***. I assume that consumers can only learn whether a firm is inactive through search.

A firm's ***information set*** just before stage 2 consists of the firm's investment and cost realization. A firm's stage 1 ***strategy*** is an investment level. A firm's stage 2 ***strategy*** is a pricing rule that for each possible history, says which price the firm should charge. A firm's ***payoff*** is expected profit, net of the investment expenditure.

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[5] It is straightforward to generalize the model for non-decreasing marginal costs and a continuum of cost types.

[6] That is, there are no externalities. A simple way of modeling externalities is: $\mu(a(t),A)$, $A = \int a(t)dt$, $\mu_a > 0$, $\mu_A > 0$.

[7] Since the low cost technology can be used by all firms that succeed at cost reduction, but only those, I am implicitly assuming that patent protection is unavailable but imitation is as costly as research. I make this assumption due to the encompassing notion of cost reduction, and to avoid externality issues. The problems associated with patents (i.e., appropriability) are essentially externalities problems (Spence (84)). The relation between concentration, appropriability, and incentives to cost reduction is analyzed by the patent

The solution concept is a refinement of Nash equilibrium. First I restrict attention to symmetric pure strategies. Recall that consumersare identical, and that after uncertainty is resolved there are two firm types. Next I introduce the two remaining restrictions. Consumersdo not know the prices charged by individual firms. However, they hold beliefs about the price distribution across firms. I assume[8]:

(A.2) Consumers' search strategy satisfies ***sequential rationality***, i.e., consumers choose whether to search again to maximize net expected surplus, given the previously observed prices and their conjecture of the price distribution at the unsearched firms, conditional onany observed information.

(A.3) Consumers'beliefs about the price distribution satisfy the ***independent prices conjecture***, i.e., consumers believe firms choose prices independently and maintain this assumption throughout the search process.

The first condition implies that consumers behave optimally at every information set, given their beliefs about the firms' strategies. The second implies that consumersdo not change their beliefs regardless of what prices they observe[9], and that on the equilibrium path, the consumers' beliefs agree with the price distribution induced by the firms investment and pricing strategies.

Let the cumulative distribution function, $F(.)$, give the consumers' beliefs about the (unconditional) market price distribution[10], and let $\underline{p}$ and $\overline{p}$ denote the lowest and highest prices onthe support of $F(.)$.

An ***equilibrium*** is: a stopping rule, consumerbeliefs, a pricing rule for each cost type, and an investment level, $\{s^*, F^*(.), p_l^*, p_h^*, a^*\}$, such that:

(i) Given beliefs $F^*(.)$ and the search cost σ, consumerschoose stopping rule s^* to maximize net expected surplus;

(ii) Given stopping rule s^*, and cost level c_τ, firms choose pricing rule p_τ^m, and investment level a^*, to maximize net expected profit, i.e., to respectively solve the problems:

$$\max_p \Pi(p;c_\tau,\phi(p)), \ \tau=l,h$$

$$\max_a \mu(a)\ \Pi(p;c_l,\phi(p)) + [1-\mu(a)]\ \Pi(p;c_h,\phi(p)) - a$$

(iii) Beliefs $F^*(.)$ agree with the price distribution induced by investment level a^*, and pricing rules p_τ^m.

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[8] I follow Bagwell & Ramey (96). The following restrictions are variations of Kreps & Wilson's (82) concepts of ***sequential rationality*** and ***consistent beliefs***. They are a generalization of subgame perfection to incomplete information games, whose purpose is to rule out unreasonable Nash equilibria. For example, suppose consumers reject all prices above p_l^-. Given the consumers' behavior, it is an equilibrium for firms to charge p_l^-. Thus, the consumers' behavior is optimal along the equilibrium path. To rule out such equilibria (see equation (3)) two conditions are required. First consumers should optimize from each point on. Sequential rationality ensures this. But, if when they observe a price higher than p_l^- consumers revise their exceptions to believe that the other firms charge much lower prices, it is still sequentially rational for them to reject any price above p_l^-. Thus, additionally consumers should not revise their expectations when faced with an unexpected price, i.e., the consumers' beliefs should satisfy the independent prices conjecture.

[9] There is a large number of small *ex ante* identical firms. Hence, a price observation does not provide information that a consumer can use to revise expectations. In addition, pricing decisions are viewed as independent. Hence, a consumer who observes an unexpected price will not change his expectations over the prices charged by other firms.

3 Characterization of Equilibrium

In this section I construct the equilibrium by working backwards. First, given the consumers' beliefs about the price distribution, I derive the consumers' equilibrium search behavior, which consists of holding a reservation price. Second, given consumers' equilibrium search behavior and firms' costs, I derive the firms' equilibrium pricing behavior. Low cost firms are always active and charge their monopoly price. High cost firms are sometimes active and others inactive, which allows for two types of equilibria. When high cost firms are active they charge the minimum of the reservation price and their monopoly price. Third, given firms' equilibrium pricing behavior, I derive the firms' equilibrium investment behavior, which consists of equating expected marginal benefit to marginal cost. Finally, I establish existence of equilibrium of the whole game.

3.1 Stage 3: The Search Game

In this sub-section I characterize the consumers'search equilibrium.

Given **(A.2)** consumersoptimize with respect to beliefs, which given **(A.3)** do not depend of the prices observed. Thus, the consumer'ssearch problem can be solved using dynamic programming. Under my assumptions sequential search is optimal (Morgan & Manning (85), proposition 3).

Denote the maximum expected surplus, net of the search expenditure, of a consumer who's best available offer is p and behaves optimally by $V(p;\sigma,\delta)$. After each draw a consumermust choose between one of two actions: accept the best available offer and terminate search, the value of which is $S(p;\delta)$, or, draw a new price at cost σ, and subsequently behave optimally, the expected value of which is $K(\sigma) := -\sigma + \int V(p^*p,\delta)\ dF(p')$, where $p^* = \min\{p,p'\}$. Search should stop when a sufficiently attractive price is observed. The Bellman equation of the consumerproblem is:

$$V(p;\sigma,\delta) = \max\{S(p;\delta),\ K(\sigma)\} \tag{1}$$

Given that demand is boundedand **(A.1)**, (1) has a well defined and finite optimal value function, and optimal search terminates in a finite numberof steps with probability 1 (De Groot: Lemma 1, p. 350, Th. 1, pp. 347).

Given that $S_p < 0$, that $S(+\infty;\delta) \le K(+\infty) < K(\sigma) < S(\underline{p};\delta)$, for $\sigma \in (0,+\infty)$, and that the value of search is decreasing in σ, it follows from the intermediate value theorem that for every $\sigma \in (0,\bar{\sigma}]$, $\bar{\sigma} \le +\infty$, equation

$$S(\rho;\delta) = -\sigma + \int_{\underline{p}}^{\overline{p}} V(p^*;\sigma,\delta)dF(p) \tag{2}$$

has a uniquesolution $\rho \in (0,+\infty)$. Given ρ, and using (2) and $S_p < 0$, it follows that:

$$V(p;\sigma,\delta) = \begin{cases} S(p;\delta) & \Leftarrow \ p < \rho \\ S(\rho;\delta) & \Leftarrow \ p > \rho \end{cases} \tag{3}$$

and that the optimal search strategy consists of holding a reservation price ρ. Using (3) on (2):

$$\int_{\underline{p}}^{\rho}[S(p;\delta) - S(\rho;\delta)]\,dF(p) = \sigma \tag{4}$$

Equation (4) states that the reservation price, ρ, is chosen to equate the marginal cost of search, σ, to the expected marginal benefit (left side). From (4) it follows that for every strictly positive search cost, the reservation price is strictly bigger that the lowest price charged in the market: $\forall\sigma>0,\ \underline{p} < \rho$. That is, costly search gives firms market power, since consumers accept prices above the minimum charged in the market.

3.2 Second Stage: The Pricing Game

In this sub-section I characterize the prices charged in equilibrium.

If a firm charges a price higher than the reservation price, $p > \rho$, it makes no sales. If a firm charges a price no bigger than the reservation price, $p \le \rho$, given **(A.1)** and that there is a continuum of consumers and firms, it gets an expected measure of consumers equal to the measure of consumers divided by the measure of active firms. Thus, the expected measure of consumers of a firm that charges p is:

$$\phi(p,\rho) = \begin{cases} 0 & \Leftarrow\ p > \rho \\ \dfrac{1}{n} & \Leftarrow\ p \le \rho \end{cases}$$

where n is the measure of active firms (I omit n in ϕ). Since $\underline{p} < \rho$, $n > 0$.

Lemma 1: In equilibrium[11]: **(i)** Price is non-decreasing in the cost level: $\underline{p} = p_l \le p_h = \bar{p}$; **(ii)** The low cost firms' price is strictly lower than the reservation price: $p_l < \rho$; **(iii)** Low cost firms charge their monopoly price: $p_l = p_l^m$; **(iv)** When the reservation price is no smaller than the high cost level, high cost firms charge the minimum of the reservation price and their monopoly price; otherwise, high cost firms become inactive:

$$p_h = \begin{cases} \min\{\rho, p_h^m\} & \Leftarrow\ c_h \le \rho \\ \hat{p} \in (\rho, +\infty) & \Leftarrow\ c_h > \rho \end{cases}$$

Proof:

(i) Suppose $p_l > p_h$. By definition of p_v, $\prod(p_l;\rho,c_l) \ge \prod(p_h;\rho,c_l)$ and $\prod(p_h;\rho,c_h) \ge \prod(p_l;\rho,c_h)$. Adding the inequalities and using the definition of $\prod(p_v;\rho,c_v)$ one gets $(c_h - c_l)[D(p_l;\delta)\phi(p_l,\rho) - D(p_h;\delta)\phi(p_h,\rho)] \ge 0$, which is false since $\phi(.,\rho)$ is non-increasing in p and $D(.;\delta)$ is strictly decreasing in p. Thus $p_l \le p_h$.

(ii) Follows from $\sigma>0$, $\underline{p} < \rho$, and **(i)**.

(iii) Given (ii) and the definition of $\phi(p,\rho)$, from the low cost firms' perspective their consumer share is given. Thus, only per consumer profit matters for the determination of their optimal price. Suppose $p_l \ne p_l^m$. Consider first $p_l = p' < p_l^m$. There is a $\varepsilon > 0$ sufficiently small such that $p' + \varepsilon < \rho$. Thus, if a low cost firm deviates and charges $p' + \varepsilon$, it loses no customers, and by strict concavity of the per consumer profit it rises its profit. Thus, $p_l \ge p_l^m$. Now suppose, $p_l = p' > p_l^m$. If a low cost firm deviates and charges $p_l = p_l^m$, given the independent prices conjecture, and by definition p_l^m it increases its profit. Thus, $\underline{p} \le p_l^m$, and therefore, $p_l = p_l^m$.

(iv) Consider first $c_h < \rho$. When $\rho > p_h^m$ it follows that $p_h = p_h^m$ by the reasoning in (iii). So consider $\rho \le p_l^m$. Firms can make a positive profit so they never charge $p_h > \rho$. And, if they charged $p_h < \rho$, then they could, as in (iii), rise profit without losing customers by rising price to ρ. It follows that $p_h = \min\{\rho,\ p_h^m\}$ for $c_h \le \rho$. When $\rho < c_h$ high cost firms can charge any price higher than the reservation price and make a zero profit; otherwise they make a negative profit. ¶

Denote the realized value of variable μ by $\tilde{\mu}$. Using **Lemma 1**, when the reservation price is no smaller than the high cost level, all firms are active; otherwise only low cost firms are active:

$$n = \begin{cases} 1 & \Leftarrow\ \rho \in [c_h, +\infty) \\ \tilde{\mu} & \Leftarrow\ \rho \in (p_l^m, c_h) \end{cases}$$

[Insert figure 2 here]

Since low cost firms always charge the lowest price in the market, they are never constrained by consumer search and always charge their monopoly price p_l^m (figure 2).[12]

High cost firms also benefit from the market power generated by costly search, by charging a higher price than low cost firms. However, they may still be disciplined by consumer search, depending on how credible the threat of a second search is. When the reservation price is high, i.e., $\rho > p_h^m$, the threat is not credible, so high cost firms charge their monopoly price, p_h^m. For intermediate values of the reservation price, i.e., $c_h \le \rho < p_h^m$, high cost firms are active, but the threat is credible, so they are forced to reduce their price below the monopoly level. When the reservation price is low, i.e., $\rho < c_h$, high cost firms become inactive.

Thus, there can be two types of price equilibria. At a *One-Price* equilibrium, which occurs when the reservation price is below the high cost level, $\rho < c_h$, only low cost firms are active and charge their monopoly

[12] This is *Diamond's Paradox* (71) (Davis & Holt (96)): low cost firms charge their monopoly price, regardless of how high it is, how low the search cost is, and how many firms there are. This is not needed for my results. Stahl's model, does not necessarily have Diamond's paradox and gives similar results. My model's advantage is that it has equilibrium in symmetric pure strategies, and allows cost

price. At a *Two-Price* equilibrium, which occurs when the reservation price is no smaller than the high cost level, $c_h \leq \rho$, both types of firms are active; low cost firms charge their monopoly price, and high cost firms charge the minimum of their monopoly price and the reservation price. I will call a *Two-Price* equilibrium ***constrained*** or ***unconstrained***, depending on whether the reservation price is lower or higher than the high cost firms' monopoly price, respectively. The firms' expected consumer shares are constant within each type of equilibrium, and change as the model switches between types of equilibrium. When high cost firms are active consumers search only once. When high cost firms are inactive, on average consumers search more than once.

Using **Lemma 1**, the market cumulative distribution function of prices is:

$$\text{Prob}[P \leq p \mid \tilde{\mu}] = \begin{cases} 0 & \Leftarrow \quad p < p_l \\ \tilde{\mu} & \Leftarrow \quad p_l \leq p < p_h \\ 1 & \Leftarrow \quad p \geq p_h \end{cases} \tag{5}$$

3.3 Stage 1: The Investment Game

In this sub-section I characterize the investment equilibrium.

Assume $\mu'(0)$ is big enough[13] to guarantee it is never optimal to set investment to zero. The necessary condition for the investment problem is:

$$\mu'(a^*)\ \left[\Pi(p_l;\rho,c_l) - \Pi(p_h;\rho,c_h)\right] - 1 = 0 \tag{6}$$

Condition (6) states that the optimal investment, a^*, equates the expected marginal benefit to marginal cost, where marginal benefit is the difference between a firm's profit levels of when it has a low and a high cost.

3.4 Equilibrium of the Whole Game: Existence and Stability

In this sub-section I prove existence and discuss stability.

Using **Lemma 1** equation (6) defines the firms' investment best response function:

$$a = A(\rho;\delta) \tag{7}$$

At *One-Price* or ***unconstrained*** *Two-Price* equilibria, investment[14] does not depend on the reservation price. At ***constrained*** *Two-Price* equilibria, investment falls with the reservation price. When a rise in the reservation price causes high cost firms to become active, investment falls discontinuously[15]. Investment rises with demand. Let $\underline{a} := A(p_h^m;0)$, $\bar{a} := A(c_h - \varepsilon;+\infty)$, $\varepsilon>0$. The investment best response function is of the form: $A(.)$: $\left[p_l^m,+\infty\right) \times (0,+\infty) \longrightarrow \left[\underline{a},\bar{a}\right]$, and is differentiable, except at $\rho = c_h$, where it has a downward discontinuity: $A(c_h;\delta) < \lim_{\rho \longrightarrow c_h^-} A(\rho;\delta)$.

I assume the realized measure of low cost firms equals the expected measure of low cost firms: $\tilde{\mu} = \int_0 \mu(a(i))\ di$.[16] Given symmetry it follows that:

$$\tilde{\mu} = \int_0^1 \mu(a(i))\,di = \mu(a) \tag{8}$$

, i.e., $\bar{\mu} = \tilde{\mu}(a)$, with $\tilde{\mu}'' < 0 < \tilde{\mu}'$.

Using (5) and (8), equation (4) defines the search best response function:

$$\rho = R(a;\delta,\sigma) \tag{9}$$

The reservation price falls with investment and demand, rises with the search cost, is of the form $R(.)$: $[0,+\infty) \times (0,+\infty) \times (0,+\infty) \longrightarrow \left[p_l^m,+\infty\right)$, and is continuously differentiable.

Equilibrium is given by equations (7) and (9).

[Insert figure 3 here]

To discuss stability (figure 3) consider the following adjustment process, consisting of a succession of rounds, each composed of two stages. In the first stage of each round, firms choose an investment which is a best response to the reservation price chosen by the consumers in the previous round. In the second stage, consumers choose a reservation price which is a best responses to the investment chosen by firms in the first stage of that round[17]. A steady state $\{a^*,\rho^*\}$ of the adjustment process is an equilibrium: $\{a^*,\rho^*\} = \{A(\rho^*;\delta), R(a^*;\delta,\sigma)\}$. An equilibrium $\{a^*,\rho^*\}$ is ***locally asymptotically stable*** for the adjustment process, if there exists a neighborhood of $\{a^*,\rho^*\}$ such that for any initial point on the neighborhood, the adjustment process converges to $\{a^*,\rho^*\}$; otherwise an equilibrium is ***unstable***. Denote the slope of the investment and search best response curves on the investment and reservation price space $(da/d\rho)_i$, $i = A, R$, respectively.

Proposition 1: (i) Equilibrium exists[18,19]. **(ii)** A sufficient condition for uniqueness, is that globally the search best response curve is steeper than the investment best response curve[20], i.e.,

$$\left(\frac{da}{d\rho}\right)_A - \left(\frac{da}{d\rho}\right)_R > 0 \tag{10}$$

(iii) When (10) holds locally, equilibria are ***locally asymptotically stable***, otherwise equilibria are ***unstable***..

[13] That is, $\mu'(0) > \{1/[\pi(p_l^m;\delta,c_l) - \pi(p_h^m;\delta,c_h)]\}$.

[14] The characterization of the investment and search best response functions follows from the implicit function theorem.

[16] This assumption is justifiable because the investment trials are made independently.

[17] The process is myopic since agents ignore the way that their current action will influence their opponent's action in the next round.

[18] In stage 2, when the reservation price equals the high cost level, there are two additional types of equilibria. In both low cost firms play a symmetric pure strategy. High cost firms, in one equilibrium randomize between charging the reservation price and becoming inactive, and in the other play an asymmetric pure strategy.

[19] It is possible to show that *Two-Price* and *One-Price* equilibria overlap for a non-empty set of search costs (figure 2).

[20] The marginal benefit of search is relatively more sensitive to the reservation price than to investment, and the marginal benefit of

Proof: I omit δ in the notation. Let $a_h := R^{-1}(c_h;\sigma)$.[21] Replacing (9) on (7) one gets the mapping $\hat{A}(a;\sigma) := A(R(a;\sigma))$ which is of the form $\hat{A}(.)$: $[0,+\infty) \times (0,+\infty) \longrightarrow [\underline{a},\overline{a}]$, and is continuous with respect to a, except at a_h, where it has upward discontinuity $\hat{A}(a_h;\sigma) < \lim_{a \longrightarrow c_h^+} \hat{A}(a;\sigma)$ ($A(.)$ falls with ρ, and $R(.)$ with a). Let $z(a;\sigma) := \hat{A}(a;\sigma) - a$, z: $[0,+\infty) \times (0,+\infty) \longrightarrow \Re$.

(i) By Tarski's fixed point theorem $\hat{A}(.)$, has a fixed point for every σ on $(0,+\infty)$.

(ii) Since $z(0;\sigma) > 0$, $z(+\infty;\sigma) \leq 0$, $\forall \sigma \in (0,+\infty)$, a sufficient condition for uniqueness is $\partial z/\partial a < 0$, i.e., $\partial z/\partial a = (\partial A/\partial \rho)(\partial R/\partial a) - 1 < 0$, or, $\partial A/\partial \rho = (\partial a/\partial \rho)_A > (\partial a/\partial \rho)_R = (\partial R^{-1}/\partial \rho)$.

(iii) Equations $a^t = A(\rho^{t-1})$, and $\rho^t = R(a^t;\sigma)$ can be collapsed into $a^t = A(R(a^{t-1};\sigma))$. Taking a first order approximation around an equilibrium $\{a^*,\rho^*\}$ one gets, $a^t - (A_\rho R_a)\, a^{t-1} = \kappa$, ($\kappa$ is a constant). The condition for stability is: $|A_\rho R_a| < 1$ or $A_\rho R_a < 1$ since $A_\rho R_a > 0$. ¶

At ***One-Price*** and ***unconstrained Two-Price*** equilibria condition (10) holds trivially. But there is *a priori* no (economic) reason why condition (10) should hold uniformly at ***constrained Two-Price*** (example in sub-section 4.1). Hence, one of the implications of the model, is that for a given parameter vector, different levels of investment, prices and reservation price can arise as equilibria.

4 Market Power and Cost Reduction

In this section I discuss the relation between market power and cost reduction. I show that different parameters can induce market power and investment to vary jointly in different ways. In particular, a search cost shift leads then to move in ***opposite*** directions, contradicting the first part of ***Schumpeter's conjecture.*** I will focus on ***constrained Two-Price*** equilibria since they have more intresting comparative statics. At ***constrained Two-Price*** equilibria a fall in the reservation price reduces: the high cost firms' price, the average price, and the high cost firms' mark-up[22]. That is, a fall in the reservation price reduces (the high cost firms') market power. Thus, in an abuse of language I will identify the level of market power with the level of the reservation price. I assume ***constrained Two-Price*** equilibria exist for more than one search cost[23].

Lemma 2: At ***constrained Two-Price*** equilibria: **(i)** a shift in the search cost leads investment and the reservation price to co-vary negatively; **(ii)** if a rise in demand is not bigger the higher the price (or alternatively if it does not make the demand curve steeper), i.e.,

[21] Since $R(0;\sigma) = +\infty$ and $R(+\infty;\sigma) = 0$, for every σ on $(0,+\infty)$, there is a $a_h(\sigma)$ on $[0,+\infty)$ such that $c_h = R(a_h(\sigma);\sigma)$.

[22] A fall in the reservation price lowers average price by: reducing the high cost firms' price, and, rising investment (i.e., proportion of low cost firms), and lowers the relative mark-up by reducing the high cost firms' mark-up (the low cost firms' price is constant).

$$\frac{\partial^2 x(p;\delta)}{\partial \delta \partial p} \leq 0 \tag{11}$$

then, a shift in the demand leads investment and the reservation price to co-vary negatively[24], and, if the following conditions hold:

$$\begin{cases} A_\delta < 0 \\ -(R_\delta/R_a) < A_\delta < -R_\delta A_\rho \end{cases} \tag{12}$$

then, a shift in demand leads investment and the reservation price to co-vary positively. ¶

Lemma 2 shows that the way market power and investment vary jointly as a result of parameter shifts does not depend on the stability properties of equilibrium. I will focus on locally stable equilibria since they have more intuitive comparative statics, i.e., I will use local stability as a ***selection criterion.***

Proposition 2:[25] At locally stable ***constrained Two-Price*** equilibria: **(i)** a fall in the search cost reduces the reservation price, rises investment, and reduces total expected profit; **(ii)** a rise in demand reduces the reservation price, rises (reduces) investment if (12) ((13)[26]) holds, and leads total expected profit to vary in a potentially ambiguous way. **(iii)** When a fall in the search cost or a rise in demand lead the model to switch from a ***Two-Price*** to a ***One-Price*** equilibrium, investment rises. ¶

First I will discuss how the reservation price and investment interact. A rise in investment increases the proportion of low cost firms. This rises the expected marginal benefit of search, leading consumers to hold a lower reservation price. The ***investment effect*** is the property that a rise in investment reduces the reservation price: $\partial R/\partial a < 0$. The low cost firms' per consumer profit level, $\pi(p_l^m;\delta,c_l)$, is constant. Thus, the reservation price impacts marginal benefit of investment, if it affects: the high cost firms' price, or, the firms' consumer shares. At ***constrained Two-Price*** equilibria, a fall in the reservation price forces high cost firms to lower their price and earn a smaller per consumer profit. This rises the marginal benefit of investment. The ***price-competition effect*** is the property that at ***constrained Two-Price*** equilibria a fall in the reservation price rises investment: $\partial A/\partial \rho < 0$. When the reservation price falls below the high cost level, high cost firms become inactive. Consumers move from high to low cost firms. This raises the low cost firms' expected consumer

[24] A specification of the demand function that satisfies (12) is $x(.;\delta) = \delta\, d(.)$ with d: $(0,+\infty) \longrightarrow (0,+\infty)$ and $d' < 0$.

[25] It is possible to perform an additional comparative statics exercise. Let $\mu(a;\lambda)$ be parametrized on λ which measures the marginal productivity of investment or technological opportunities. Assume λ rises the marginal productivity of investment, i.e., $\mu_{a\lambda} > 0$. Then an increase in λ rises investment and reduces the reservation price.

[26] A rise in demand reducing the marginal benefit of investment is not a pathology peculiar to my model. Quirmbach (88) showed that

measure, increasing the marginal benefit of investment. The ***selection effect*** is the property that, when due to a fall in the reservation price, the model switches between types of equilibria, investment rises.

[Insert figure 4 here]

Next I discuss how shifts in the search cost and demand affect the reservation price and investment.

When the search cost falls (figure 4), the expected marginal benefit of search must also fall for consumers to remain in equilibrium. Given investment, i.e., the proportion of low cost firms, the reservation price falls. The fall in the reservation price rises investment, either through the ***price-competition effect*** or the ***selection effect***. The rise in investment further reduces the reservation price through the ***investment effect***. A fall in the search cost reduces total expected profit, through the reservation price.

[Insert figure 5 here]

A rise in demand (figure 5) impacts consumers directly by making it relatively more valuable to find a low cost firm. Hence, the reservation price falls. A rise in demand also increases investment indirectly, either through the ***price-competition effect*** or the ***selection effect***.

A rise in demand expands sales. The direct impact on firms is composed of two effects of potentially opposite signs. First, if demand rises uniformly, low cost firms are relatively more benefited because they have a higher mark-up. This rises the marginal benefit of investment. Second, if the demand rise is not bigger the higher the price, i.e., if (11) holds, then low cost firms are further relatively more benefited, because they charge a lower price. Thus, if (11) holds both components are positive and the direct impact is positive: $A_\delta > 0$.[27] Given (11), the direct impact reduces the reservation price indirectly, through the ***investment effect***.

If $\partial^2 x/\partial\delta\partial p$ is sufficiently positive, the direct impact on firms is negative, i.e., $A_\delta < 0$. But, the direct impact on firms being negative is not enough to lead to an overall fall in investment, while still leading to an overall decrease in the reservation price. Since the indirect impact on firms rises investment, the direct impact has to be sufficiently strong to dominate it, i.e., $A_\delta < -R_\delta A_\rho$. And since the indirect impact on consumers is positive, the direct impact has to be sufficiently strong to dominate it, i.e., $A_\delta > -(R_\delta / R_a)$.

A rise in demand has a potentially ambiguous impact on total expected profit. On the one hand sales expand, on the other, the reservation price falls.

Example: Let $x(p) = p^{-1.1}$, $\mu(a) = 1 - e^{-14.303\,a}$, $c_l = .02$, $c_h = .04$. This example illustrates two points. First that associated with a given parameter vector there can be different equilibria. Second that at locally stable

equilibria (the first and the third) a fall in the search cost reduces the reservation price and rises investment; at unstable equilibria the opposite happens.

$\sigma_1 = .01$ (*)		$\sigma_2 = .009,999$	
a	ρ	a	ρ
26900.051	226,009	26,900.260	226,008
923.945	432,998	923.549	433,099
913.747	436,375	913.959	436,273

(*) With the exception of the search cost, all variables are expressed in 10^{-6} units.

¶

The reasoning underlying ***Schumpeter's conjecture*** is that a fall in market power lowers total expected profit — the total expected benefit of investment —, reducing incentives to invest. But a decrease in market power is not necessarily associated with a fall in total expected profit, as a rise in demand shows, nor with a fall in the marginal expected profit, as a fall in the search cost shows. And what matters is marginal, not total expected profit. A decrease in market power, induced by a fall in the search cost, is associated with both a rise in the marginal expected profit and a fall in the total expected profit, because of the asymmetric way in which the fall in market power impacts firms' profits. The ***price-competition effect*** reduces the high cost firms' price and per consumer profit, while leaving the low cost firms' price constant. In addition the ***selection effect*** redistributes consumers from the high to the low cost firms. Thus, a fall in market power stimulates investment, even if it reduces total expected profit, if it erodes the high cost firms' total profit relatively more[28].

When market power and investment move in opposite directions there is no ***Schumpeterian trade-off***[29].

5 Price Controls

In this section I show how price controls may not affect cost reduction monotonically in search markets. In markets governed by Jevon's law of one price, price controls affect prices uniformly. However, in markets with price dispersion, only prices initially above the ceiling are affected directly. Thus, low and high cost firms may not always be affected in the same way.

[28] In this model the asymmetry assumes a very sharp forms due to Diamond's paradox. See footnote 13.
[29] It is also possible to show the following. First that welfare is non-decreasing in the search cost. Second that for a fixed parameter vector, a rise in the equilibrium investment reduces expected profit, and rises welfare. The implication is that when multiplicity occurs, equilibria can be ranked by welfare. Thus, my model provides an instance of potential coordination failure in technology *creation*, parallel to the coordination failure for technology adoption (Bagwell & Ramey (94), Farrell & Saloner (85), Katz & Shapiro (85)). Third that the

Let p^c denote price control, and let the price control be between the high cost level and the minimum of the reservation and the high cost firms' monopoly price, $c_h \le p^c \le \min\{\rho, p_h^m\}$. Low cost firms are not constrained in their pricing behavior. High cost firms are active, but constrained in their pricing behavior. Thus, a fall in the price control forces them to reduce their price, rising the marginal benefit of investment.

When the price control falls below the minimum of the reservation price and the high cost level, $\min\{\rho, c_h\}$, high cost firms become inactive. Investment rises through the ***selection effect***.

When the price control is between the low cost firms' monopoly price and the minimum of the reservation price and the high cost level, $p_l^m \le p^c < \min\{\rho, c_h\}$, low cost firms continue to be unconstrained, thus the marginal benefit of investment is constant.

When the price control is between the low cost and the low cost firms' monopoly price, $c_l < p^c < p_l^m$, low cost firms become constrained. Thus, a fall in the price ceiling forces them to reduce their price. Since now the marginal benefit of investment equals the low cost firms' profit, the marginal benefit of investment falls.

When the price ceiling falls bellow the low cost level, $p^c < c_l$, the market shuts down.

6 Extensions

6.1 Free Entry and Fixed Entry Costs

In this sub-section I make two points. First, that section 4's results are independent of the measure of firms being fixed. Second that market power and the measure of firms in the industry can move in the *same* direction, contradicting usual interpretations of ***competition***. I focus on constrained ***Two-price*** equilibria.

Let the game have an additional stage 0, preceding the other three, where a continuum of firms on the positive real line decides whether to enter the industry. To enter the industry a firm must pay a fixed cost.

Proposition 3: At locally stable *Two-price* equilibria, a fall in the search cost, or a rise in the fixed cost reduces the reservation price and the measure of firms in the industry, and rises investment. ¶

A fall in the search cost reduces the reservation price and rises investment by the process described in section 4 The fall in the reservation price also reduces net expected profit, through the ***price-competition effect***. For firms to break-even the expected measure of consumers per firm must rise, which happens if the measure of firms that enter the industry falls.

A rise in the fixed cost reduces the measure of firms in the industry. The increase in the expected consumers share per firm rises the marginal benefit of investment, which through the ***investment effect*** reduces the reservation price. The fall in the reservation price, reduces net expected profit, through the ***price-competition effect***, which reinforces both the fall in the measure of firms that enter the industry, and the rise in investment. However, by raising the probability of a firm having a low cost, the increase in investment, also rises net expected profit, rising the measure of firms that enter, but this effect is dominated by the others[30].

The expected marginal benefit of investment is composed of two parts: the expected per consumer marginal benefit, and the expected consumer measure. The assumption that the measure of firms in the industry is fixed, implies that a shift in a parameter, can only change the expected per consumer marginal benefit (within each type of equilibrium). But in general, a parameter shift will induce changes in both components, and those changes could be of opposite signs. These results show that the relation between market power and incentives to invest holds even when the expected consumer measure can change due to free entry[31].

Let n denote the measure of firms that enter the industry. If one takes $1/n$ as a measure of concentration, then a rise in the entry cost rises concentration. This also occurs in Dasgupta & Stiglitz's (80) model. However, now, unlike in their model, market power and concentration co-vary negatively. This implies that concentration can be a misleading measure of market power and allocative efficiency[32].

6.2 Consumer Heterogeneity

In this sub-section I show that if consumers are heterogeneous with respect to the search cost, active firms may have different consumer measures, and, consumer search may impact investment through an additional effect: the ***consumer redistribution effect***.

Let a proportion $\beta \in [0,1]$ of consumers have a search cost s_l, and a proportion $1 - \beta$ have a search cost s_h, where $0 < s_l \le s_h < +\infty$. Each type of consumer has a reservation price. The reservation price of low search cost consumers, the low reservation price, is no bigger than that of high search cost consumers, the high reservation price, and strictly lower when the search costs are different.

Low cost firms sell to both types of consumers. However, high cost firms face a trade-off between profit margin and volume of sales. When the low reservation price is sufficiently high, or alternatively, when the proportion of low search cost consumers is sufficiently big, high cost firms prefer to sell to both types of consumers; otherwise, high cost firms prefer to sell only to high search cost consumers.

When, due to a fall in the low reservation price, high cost firms prefer to rise their price and sell only to high search cost consumers, expected consumer share is redistributed from high to low cost firms, rising the

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30 Similar results, although for different reasons, were found by: Rosenthal (80), Varian (80), Stahl (89), Stiglitz (87) Seade (80).

31 If there are externalities in investment, i.e., each firm's probability of being low cost depends also on the industry's aggregate investment expenditure, and if they are sufficiently strong, then a rise in the fixed cost still reduces the measure of firms that enter the industry, but can also decrease investment (Spence (80)), and rise the reservation price.

marginal benefit of investment. This is the ***consumer redistribution effect***. The rise in investment due to the ***consumer redistribution effect*** is discontinuous (see footnote 14).

7 The Dynamic Model

In this section I present the changes necessary to obtain the dynamic model.

Let the market open for two periods, each of which is composed of three stages. In period 1 events unfold as in the static model. In period 2, first firms suffer a shock to their marginal production costs; each firm observes only the outcome of its own shock. Second, firms simultaneously choose prices; customers learn, free of charge, the current price of their previous period supplier. And third, consumers simultaneously make their search and purchase decisions; then production and delivery (comment switching cost)take place instantaneously, agents receive their period 2 payoffs, and the market closes.

To analyze the impact of ***idiosyncratic cost volatility*** denote the probability that a firm who had a marginal production cost level c_τ in period 1 has cost level c_s in period 2 by $v_{\tau s}(\gamma)$ $(\tau, s = l, h)$. I assume the shock is independent across all firms, and identically distributed across firms with the same cost type. I assume that the probability a period 1 low cost firm remains low cost in period 2 is higher than the probability a period 1 high cost firm becomes low cost in period 2: $v_{ll} > v_{hl}$. Let the transition probability $v_{\tau s}$ be a differentiable function of $\gamma \in \mathbb{R}$, a parameter which measures the intensity of the shock. I assume that a rise in γ increases the probability of a firm changing its cost type: $v'_{ll} < 0 < v'_{hl}$. The probability of a firm having a low cost in period 2 is $m(a,\delta) := \mu(a)v_{ll}(\gamma) + [1 - \mu(a)]v_{hl}(\gamma)$.

Decompose the high cost as $c_h = c_l + \Delta$, $\Delta > 0$; c_l can be interpreted as a ***common cost*** component. I will use deterministic shifts[33] in c_l in period 2 to analyze the effects of ***industry wide cost volatility***.

8 Description of Dynamic Model's Equilibrium

In this section I make a brief description of the dynamic model's equilibrium.

The consumers' period 2 problem and hence optimal strategy is identical to that of the static model.

The firms' period 2 problem is slightly different. If a firm is active, it keeps its period 1 customers, and in addition, it gets a share of the consumers searching in period 2 (consumers that in period 1 bought from a firm that is inactive in period 2)[34]. Nevertheless the optimal strategy is identical to that of the static model.

The consumers' period 1 problem is more complicated. Because consumers learn for free the period 2 price of the firm they bought from in period 1, they tend to buy at the same firm in both periods. Due to this ***lock in effect***[35], consumers conduct a more thorough search than they would for a single purchase to benefit from future low prices and save on future search costs. Implicit in this behavior is some inference of future prices on the basis of past ones: if today's price is acceptable, one can expected that tomorrow's will also be acceptable. However, the optimal strategy still consists of holding a reservation price, that equates the sum of the expected marginal benefit of search for period 1 and 2 with the marginal search cost.

The firms' period 1 problem is also more complicated. Due to the ***lock-in effect***, in period 2 a firm can only sell, either to its period 1 customers, or to consumers that are searching. Low cost firms will still choose to always be active and charge their monopoly price. However, when the period 1 reservation price falls below the high cost level: if a high cost firm chooses to be active, on the one hand, it sells its product below marginal (and average) cost, on the other hand, it secures a larger customer measure for period 2. [36] That is, high cost firms face a trade-off between current losses and future profits. For some parameter values they choose to be active, and for others inactive[37], which allows for four types of equilibria for the whole game. When high cost firms are active they charge the minimum of the reservation price and their monopoly price. But now high cost firms may choose to active even when the reservation price is below the high cost level.

The model has two interesting implications. First, that sales maximization in period 1 is part of an intertemporal profit maximization strategy. Second, that with switching costs, cost volatility can lead firms to charge below marginal cost, without predatory[38] motives. Due to the ***lock-in effect*** in period 1 firms need to establish a customer base for period 2, which might involve pricing below marginal cost, if the reservation price is sufficiently low. Also, here prices below marginal cost are charged by the smaller, higher cost firms.

9 Cost Volatility and Cost Reduction

In this section I argue that a rise in idiosyncratic or industry wide cost volatility can lead to a fall in investment, and to rise in reservation prices. I conclude by discussing an application to inflation.

[33] A stochastic shock, perfectly correlated across firms, generates the same qualitative results and is harder to handle.

[34] A firm's consumer measure depends on history. However, because consumers are identical and marginal costs constant, prices are independent of the firms' previous period consumer measures, and hence of history.

[35] For the lock-in effect to be present, all that is required is that it costs (significantly) less to learn the current price of the firm previously patronized, than of another firm.

[36] When the period 2 reservation price is above the high cost all firms are active in period 2, and no consumers search. When the period 2 reservation price is lower than the high cost, high cost firms are inactive in period 2, and the customers of firms that had a low cost in period 1 but become high cost in period 2, search. Thus, if a high cost firm chooses to be active in period 1, it will have a higher consumer share in period 2 than it would if it chose to be inactive, since it gets a share of the customers searching and keeps its period 1 customers.

[37] It is possible to ensure that the set of parameters for which high cost firms are inactive in period 1 is non-empty.

[38] Joskow & Klevorick (79): "Predatory pricing behavior involves a reduction of price in the short run so as to drive competing firms out of the market or to discourage entry of new firms in an effort to gain lager profits via higher prices in the long run that would have been

The impact of relative cost volatility on investment is composed of three effects; two of them, the ***relative price volatility effect*** and the ***proportion of cost types effect***, depend on how search behavior in period 1 and 2 are affected, respectively; however, the third, the ***direct effect***, is independent on of search behavior.

A rise in idiosyncratic cost volatility rises the proportion of firms that change their prices from period 1 to period 2 to reflect the variation in costs. This lowers the informativeness of current prices about future prices. As a result the expected marginal benefit of search in period 1 falls. Given the investment level, period 1 reservation price rises, which when firms are constrained by period 1 search, reduces investment, either through the ***price-competition*** or the ***selection effects***. I call ***relative price volatility effect*** to the property that a rise in relative cost volatility reduces investment, through its impact on period 1 reservation price.

A rise in idiosyncratic cost volatility affects the period 2 proportion of low cost firms, and thereby the period 2 reservation price. If high cost firms are active, the change in period 2 reservation price affects investment through, either the ***price-competition effect*** or the ***selection effect***. I call ***proportion of cost types effect*** to the property that relative cost volatility affects investment, through its effect on the period 2 reservation price.

Given prices, a rise in idiosyncratic cost volatility reduces a firm's period 2 expected profit conditional on the firm having a low cost in period 1, and rises a firm's period 2 expected profit conditional on the firm having a high cost in period 1. Hence, the expected marginal benefit of investment falls. I call ***direct effect*** to the property that for constant prices, a rise in relative cost volatility reduces investment.

The ***direct effect*** reinforces the ***relative price volatility effect***. When in period 2 high cost firms are active, and the period 2 proportion of low cost firms is non-increasing in ***the relative cost volatility***, $m_\delta \leq 0$, the ***proportion of cost types effect*** also reinforces the ***direct effect***. The period 2 proportion of low cost firms being non-increasing in idiosyncratic cost volatility depends on: period 1 investment generating a high proportion of low cost firms, and, a change in relative cost volatility having larger impact on the probability of a low cost firm remaining low, than the probability of a high cost firm becoming high. Thus, period 2 proportion of low cost firms is non-increasing in ***the relative cost volatility***, for example, when a change in relative cost volatility does not affect the probability of a high cost firm becoming low, $v'_{hl} = 0$.

The impact on investment of a rise in ***industry wide cost volatility*** is composed of two effects. First, given prices, a rise in ***industry wide cost volatility*** reduces the markup of both types of firms by the same amount. Since low cost firms have a larger volume of sales they are relatively more affected. Second, a rise in

industry wide cost volatility also rises both types of firms' monopoly prices, reducing the period 2 marginal benefit of search. Hence investment falls, either through the ***price-competition effect*** or the ***selection effect***..

As I mentioned in the introduction there is empirical evidence suggesting that inflation is positively correlated with relative price variability, both across markets and within markets. This leads some authors to identify idiosyncratic cost volatility with the result of the non uniform propagation of inflation across the economy. Consider an inflationary economy, where inflation is transmitted non-uniformly through the economy, so that different firms' costs are affected differently, i.e., firms are subjected to idiosyncratic and industry wide shocks coming from physical productivity or monetary cost changes. If one is prepared to accept that idiosyncratic cost volatility can be identified with inflation, then the present analysis suggests that inflation can decrease cost reducing investment, leading to a higher average cost level.

Appendix

Lemma 2

Differentiating the system $\begin{cases} a - A(\rho;\delta) = 0 \\ \rho - R(a;\rho,\delta) = 0 \end{cases}$

one gets $$\begin{bmatrix} \frac{\partial a^*}{\partial \sigma} & \frac{\partial a^*}{\partial \delta} \\ \frac{\partial \rho^*}{\partial \sigma} & \frac{\partial \rho^*}{\partial \delta} \end{bmatrix} = H^{-1} \begin{bmatrix} R_\sigma A_\rho & A_\delta + R_\delta A_\rho \\ R_\sigma & R_\delta + R_a A_\delta \end{bmatrix}$$ with $H = 1 - A_\rho R_a$

(i) From the system above it follows that $\mathrm{sgn}\{\partial\rho^*/\partial\sigma\} = -\,\mathrm{sgn}\{\partial a^*/\partial\sigma\}$.

(ii) If $A_\delta > 0$ then $R_\delta + R_a A_\delta > 0$ and $A_\delta + R_\delta A_\rho < 0$, thus $\mathrm{sgn}\{\partial\rho^*/\partial\delta\} = -\,\mathrm{sgn}\{\partial a^*/\partial\delta\}$. If $A_\delta < 0$, and $-(R_\delta/R_a) < A_\delta < -R_\delta A_\rho$, then $R_\delta + R_a A_\delta < 0$ and $A_\delta + R_\delta A_\rho < 0$, and therefore $\mathrm{sgn}\{\partial\rho^*/\partial\delta\} = \mathrm{sgn}\{\partial a^*/\partial\delta\}$. ¶

Proposition 2

(i) Follows from **Lemma 2: (i)** and the fact that at locally stable equilibria $H > 0$. Using the envelope theorem: $\partial[\mu\pi(p^m(c_l;\delta,c_l) + (1-\mu)\pi(\rho^*;\delta,c_h)]/\partial\sigma = (1-\mu)\,[\partial\pi(\rho^*;\delta,c_h)/\partial p]\,(\partial\rho^*/\partial\sigma) < 0$.

(ii) Follows from **Lemma 2:** (ii) and the fact that at locally stable equilibria $H > 0$. Using the envelope theorem: $\partial[\mu\pi(p^m(c_l;\delta,c_l) + (1-\mu)\pi(\rho^*;\delta,c_h)]/\partial\delta = \mu\,[\partial\pi(p^m(c_l);\delta,c_l)/\partial\delta] + (1-\mu)\,[\partial\pi(\rho^*;\delta,c_h)/\partial\delta] + (1-\mu)\,[\partial\pi(\rho^*;\delta,c_h)/\partial p]\,(\partial\rho^*/\partial\delta)$, which has a potentially ambiguous sign. ¶

Proposition 3

Denote the entry cost by F and the measure of firms in the industry by e. Equilibrium is given by:

$$\begin{cases} \frac{\tilde{\mu}'(a)}{e}\left[\pi(p_l^m, c_l) - \pi(\rho, c_h)\right] - 1 = 0 \\ \mu(a)\left[S(p_l^m) - S(\rho)\right] - \sigma = 0 \\ \frac{\tilde{\mu}(a)}{e}\left[\pi(p_l^m, c_l) - \pi(\rho, c_h)\right] - a - F = 0 \end{cases}$$ or alternatively by $$\begin{cases} a - A(\rho, e) \\ \rho - R(a;\sigma) \\ e - E(a,\rho;F) \end{cases}$$ which gives

$$\begin{bmatrix} \frac{\partial a}{\partial \sigma} & \frac{\partial a}{\partial F} \\ \frac{\partial \rho}{\partial \sigma} & \frac{\partial \rho}{\partial F} \\ \frac{\partial e}{\partial \sigma} & \frac{\partial e}{\partial F} \end{bmatrix} = M^{1} \begin{bmatrix} R_\sigma(A_\rho + A_e E_\rho) & E_F A_e \\ R_\sigma & E_F A_e R_a \\ R_\sigma E_\rho & E_F(1 - A_\rho R_a) \end{bmatrix} \qquad \text{with } M = 1 - R_a(A_\rho + A_e E_\rho) > 0$$

thus

$$\frac{\partial a}{\partial \sigma} < 0 \qquad \frac{\partial \rho}{\partial \sigma} > 0 \qquad \frac{\partial e}{\partial \sigma} > 0 \qquad \text{and} \qquad \frac{\partial a}{\partial F} > 0 \qquad \frac{\partial \rho}{\partial F} < 0 \qquad \frac{\partial e}{\partial F} < 0$$

M>0 followa from local stability where the dynamic system is given by $\begin{cases} \rho^t = R(a^t) \\ a^t = A(\rho^{t-1}, e^t) \\ e^t = E(a^{t-1}, \rho^{t-1}) \end{cases}$

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Figure 1: The Timing of the Game

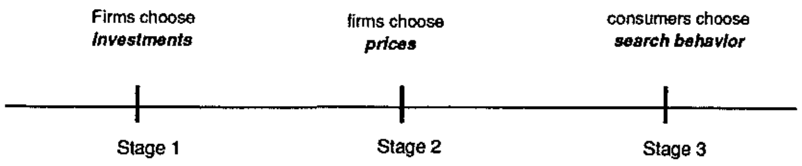
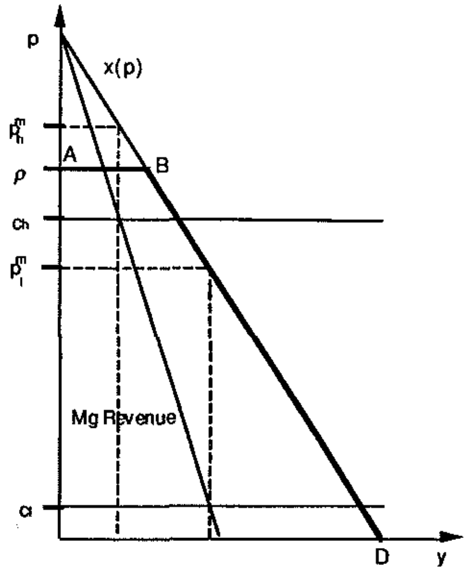
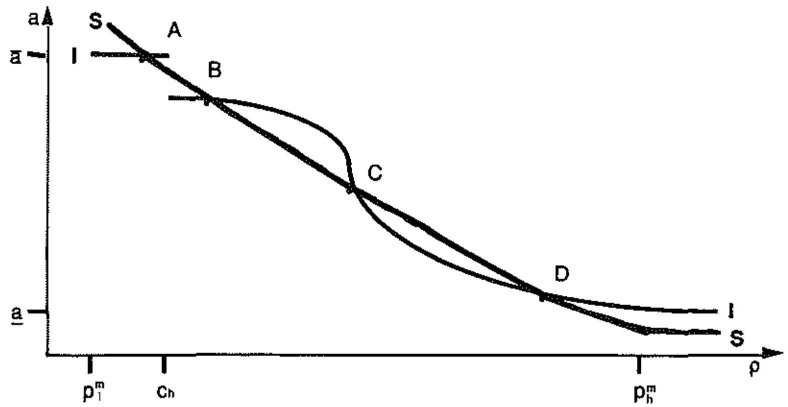


Figure 2: Pricing Equilibria



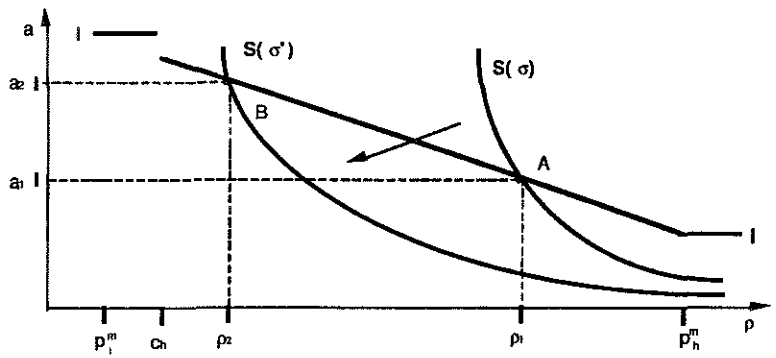
Low cost firms charge their monopoly price. When the reservation falls below the high cost firms' monopoly price, high cost firms face the truncated effective demand ***ABD***, i.e., are constrained in their pricing behavior, and charge the reservation

Figure 3: Multiple Equilibria



I is the investment best response function, and *S* is the search best response function. For the same parameter vector there are four equilibria. ***A*** is a ***One-price*** equilibrium. ***B***, ***C***, and ***D*** are constrained ***Two-price*** equilibria; ***B*** and ***D*** are localy stable, and ***C*** is unstable.

Figure 4: A decrease in the Search Cost



I is the investment best response function, and ***S(σ)*** is the search best response function when the search cost is *σ*. A rise in the search cost from *σ* to *σ'* shifts the search best response function inward from ***S(σ)*** to ***S(σ')***, which reduces the reservation price and rises investment.

Figures 5: An increase in Demand

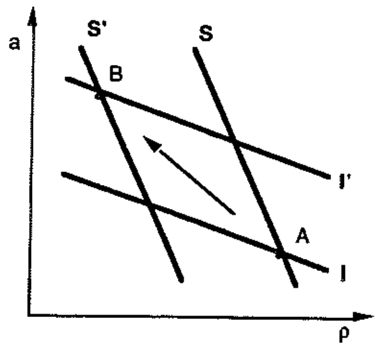


Figure 5.1: (11) holds

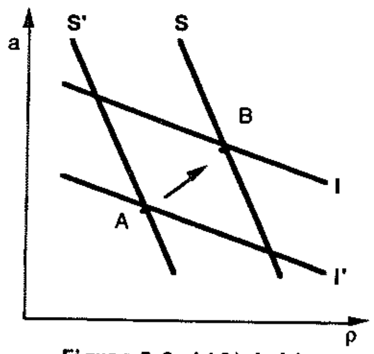


Figure 5.2: (12) holds

An rise in demand shifts the search best response function inward from S to S'. If condition (12) holds, the investment best response function shifts outward from I to I' and invesment rises and the reservation price falls. If conditions (13) hold, the investment best response function shifts inward from I to I' and both investment and the reservation price fall.

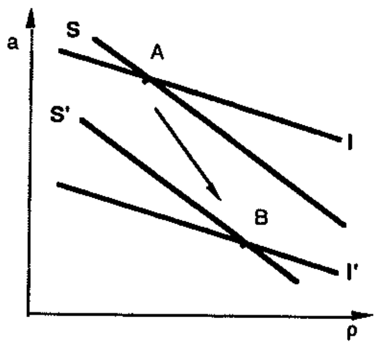


Figure 5.3: $A_s < -(R_s/R_a)$

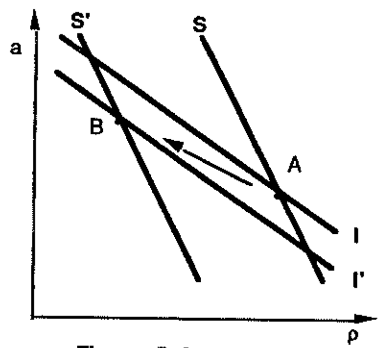


Figure 5.4: $A_s > -R_s A_p$.

In both figures 4.3 and 4.4 $A_s < 0$ holds but one of the other conditions of (13) fails to hold.