EngOpt 2008 - International Conference on Engineering Optimization
Rio de Janeiro, Brazil, 01 - 05 June 2008.

Validation of a new variance-expected compliance model for structural optimization

Miguel Carrasco [1], Benjamin Ivorra[2], Angel Manuel Ramos[2], Felipe Alvarez[3]

[1] Facultad de Ingeniería,
Universidad de los Andes
Av. San Carlos de Apoquindo 2200, Santiago de Chile, Chile

[2] Departamento de Matemática Aplicada,
Universidad Complutense de Madrid
Plaza de Ciencias, 3, 28040–Madrid, Spain

[3] Centro de Modelamiento Matemático,
Universidad de Chile
Av. Blanco Encalada 2120, Santiago de Chile, Chile

Abstract: In this paper, we first remember the mathematical formulation of an original variance-expected compliance model used for structural optimization. It allows to find structures robust for the main load and its perturbations. In the second part, we valid this model on a 3-D benchmark test case and compare the obtained results with those given by a classical expected compliance model.

Keywords: Structural optimization, truss modeling, expected compliance model, variance-compliance model, global optimization.

1 Introduction

Trusses are mechanical structures consisting of an ensemble of slender bars, connecting some pairs of nodal points in $\mathbb{R}^d$ with either $d = 2$ or $d = 3$. Bars are supposed to be made of a linearly elastic, isotropic and homogeneous material; long bars overlapping small ones are neglected. They are designed to support some external nodal loads as well as self-weight loads, taking into account certain mechanical properties of the bar material.

Using the *ground structure approach* [13], we focus on the case where the goal is to find bar volumes (topology) which minimize the *compliance* of the truss under mechanical equilibrium and total volume constraints; see problem (2) in Section 2.

Typically, optimal trusses are unstable under load perturbations (see for example [4, 8]). Therefore, in order to design robust truss structures it is necessary to include this perturbations into the model. In this context, several approaches might be considered (among others). For example: the *multiload model*, which, in its standard form, consist of minimizing a weighted average of compliances associated with a finite set of loading scenarios [3]. In the *worst case design* the objective is to minimize the maximum compliance under a set of discrete loading scenarios ([1, 2]). Finally, in the same direction, the *ellipsoid method* considers a continuum of primary and secondary loads defined by a particular ellipsoid ([8]).

Assuming that loads are random variables, Alvarez and Carrasco [4] proposed the problem of finding the truss of minimum *expected compliance* (see problem $(D_{Var}; \mathbb{P}; \alpha; 0)$ in Section 3). They show that this problem can be reformulated as a multiload-like problem, but, with a different interpretation on the scenarios. Since multiload problems can be equivalently formulated as a convex finite-dimensional problem, then the expected-compliance model may be efficiently solved, see [3, 7, 23, 9]

The stochastic setting given in [4] allows to consider the *minimum variance-compliance problem*, where the variance of the compliance is included into the model. This stochastic problem is also equivalent to a nonlinear programming problem. However, the variance formulation introduces a non convex term, that makes this kind of problems harder to solve numerically than the minimum expected compliance model. This paper focuses on this variance formulation. Using a global optimization algorithm [17], we analyze numerically a truss from the point of view of its mechanical stability.

We point out that *multilevel stochastic programming* problems have also been proposed to deal with mechanical stability of trusses ([24, 14]). In this approach approximation and discretization schemes are required in order to estimate expected values. In our formulation, since we compute explicitly the expected values, this type of approximation techniques are not needed at all. Nevertheless, the non-convex term associated to the variance prevents to use it for a large amount of variables.

In Section 2 we recall the well known *minimum compliance truss* and the *multiload model* [1, 8]. In Section 3, we present the mathematical formulation of the *variance-expected compliance* model, based on [4, 11]. Finally, in Section 4, the results are analyzed and compared, in term of robustness, between each other.

2 Standard minimum compliance truss design

Set $n = d \cdot N - s$, the number of degrees of freedom of a ground structure consisting of $N \geq 2$ nodes, where $s \geq 0$ is the number of fixed nodal coordinate directions (i.e., coordinates corresponding to support conditions are removed). Let $m \geq n$ be the number of potential bars in the truss structure (of course, $m \leq N(N-1)/2$), and denote by $\lambda_i \geq 0$ the volume (normalized) of the i-th bar with $i \in \{1, \ldots, m\}$. External loads are applied only at nodal points, and are described in global reduced coordinates by a vector $f \in \mathbb{R}^n$. Under the assumption that each bar is subjected to only axial tension or compression (neglecting thus large deflections and bending effects), the mechanical response of the truss is described by the elastic equilibrium equation (see e.g. [1])

$$K(\lambda)u = f,$$

where $u \in \mathbb{R}^n$ is the nodal displacements vector in global reduced coordinates and $K(\lambda)$ is the *stiffness matrix* of the truss, which has the form

$$K(\lambda) = \sum_{i=1}^{m} \lambda_i K_i. \tag{1}$$

Here, $\lambda_i \geq 0$ is the volume of the i-th bar and $K_i \in \mathbb{R}^{n \times n}$ is a positive semi-definite matrix which corresponds to the specific stiffness matrix of the i-th bar in global coordinates.

The problem of finding the minimum *compliance* truss for a normalized volume constraint of material is given by (see e.g. [3, 10])

$$\min_{\lambda \in \Delta_m} \left\{ \frac{1}{2} f^T u \mid K(\lambda)u = f, u \in \mathbb{R}^n \right\}, \tag{2}$$

where $\Delta_m = \{\lambda \in \mathbb{R}^m \mid \lambda \geq 0, \ \sum_{i=1}^m \lambda_i = 1\}$. This problem is known as *single load model*.

Remark 2.1. For the sake of simplicity, the *self-weight* of the bars are not included in the formulation of the model. If we consider the self-weight we have to replace f by $f + g(\lambda)$ in problem (2), where $g(\lambda) = \sum_1^m \lambda_i g_i$, with g_i being a specific nodal gravitation force vector (see [10] §4).

We remark also that compliance, i.e., the value of the objective function in (2), does not depend on the choice of the equilibrium displacement vector $u \in \mathbb{R}^n$ such that $K(\lambda)u = f$.

Taking into account the particular structure of the matrices K_i in (1), it is possible to show that the *single load* model (2) is equivalent to a linear programming problem, and therefore might be efficiently solved. Nevertheless, numerical results using this model show that optimal solutions may be unstable with respect to the mechanical equilibrium, even under small perturbations in the principal load ([1, 8]). In fact, there are several examples showing some optimal structures which, under small perturbations, give infinite compliance.

In order to handle this inconvenient we may consider a *Multiload Model* (see [3, 7]) that we recall here.

$$\min_{\lambda \in \Delta_m} \left\{ \frac{1}{2} \sum_{j=1}^{k} \gamma_j (f^j)^T u^j \mid K(\lambda)u^j = f^j, \ j = 1, ..., k \right\}. \tag{3}$$

where $\gamma_j > 0$ corresponds to the influence of the scenario j into the model. In this formulation we minimize a weighted average of the compliances associated with k different loads scenarios. The multiload model can be transformed to an equivalent convex quadratic problem and might be solved efficiently. ([23, 9]).

3 Minimization stochastic problem

According to definitions of the previous section we define the function $\Psi\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ as

$$\Psi(\xi,\lambda) = \begin{cases} \frac{1}{2}(f+\xi)^T u & \text{if } \lambda \in \Delta_m \text{ and } \exists\ u \in \mathbb{R}^n \text{ such that } \ K(\lambda)u = f+\xi. \\ +\infty & \text{otherwise.} \end{cases} \tag{4}$$

The function Ψ turns out to be proper (i.e. $\Psi \not\equiv +\infty$), lower semi-continuous and convex (see [4]). Therefore, for each $\lambda \in \Delta_m$, the function

$$\Psi(\cdot,\lambda)\colon \big(\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n)\big) \to \big(\mathbb{R}\cup\{+\infty\}, \overline{\mathcal{B}}(\mathbb{R})\big)$$

is measurable. Here, $\mathcal{B}(\mathbb{R}^n)$ and $\overline{\mathcal{B}}(\mathbb{R})$ stand for the Borel σ-algebra of $\mathbb{R}^n$ and $\mathbb{R}\cup\{+\infty\}$ respectively.

Next, let us assume that ξ is a random variable corresponding to an uncertain perturbation of Ψ. More precisely, let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space and consider a measurable function

$$\begin{array}{cccc} \xi\colon & (\Omega,\mathcal{A}) & \to & (\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n)) \\ & \omega & \mapsto & \xi(\omega). \end{array}$$

According to this settings we study the following stochastic minimization problem:

$$\min_{\lambda\in\Delta_m} \{\alpha \mathbb{E}_\xi[\Psi(\xi,\lambda)] + \beta \operatorname{Var}_\xi[\Psi(\xi,\lambda)]\}, \tag{$\mathcal{D};\mathbb{P};\alpha;\beta$}$$

where Ψ is defined by (4), $\alpha, \beta \geq 0$ and $\mathbb{E}_\xi(\cdot), \operatorname{Var}_\xi(\cdot)$ stands, respectively, for the expected value and the variance of a random function.

Remark 3.1. Let $\lambda \in \Delta_m$ and we consider $V_\lambda = \operatorname{Im} K(\lambda)$ if $\mathbb{P}\{f+\xi \notin V_\lambda\} > 0$ then the value of $\alpha\mathbb{E}_\xi[\Psi(\xi,\lambda)] + \beta \operatorname{Var}_\xi[\Psi(\xi,\lambda)]\} = +\infty$ and the corresponding volume is not a feasible point in $(\mathcal{D};\mathbb{P};\alpha;\beta)$.

We note that taking ξ be a random variable with finite support, $\xi_1,\ldots,\xi_k$, then, denoting $\gamma_j = \mathbb{P}\{\xi = \xi^j\}$ and $f^j = f + \xi^j$, $j = 1,\ldots,k$ we obtain the model described in (3). Therefore, the previous model extends the well known *multiload model*.

The next result gives an explicit expression for $(\mathcal{D};\mathbb{P};\alpha;\beta)$, with $\alpha = 1$ and $\beta = 0$ when the perturbation ξ is a continuous random variable (without atoms). In the sequel, given a square matrix $A = (a_{ij})$, we denote by $\operatorname{Tr}(A)$ the trace of A, i.e., $\operatorname{Tr}(A) = \sum a_{ii}$.

Theorem 3.1 (Alvarez, Carrasco [4])**.** *Let $\xi : \Omega \to \mathbb{R}^n$ be a continuous random variable with mean vector $\mathbb{E}(\xi) = 0$ and covariance matrix $\operatorname{Var}(\xi) = PP^T$ with $P \in \mathbb{R}^{n\times k}$ for some $k \geq 1$. Then the corresponding* minimum expected compliance design problem $(\mathcal{D};\mathbb{P};1;0)$ *is given by*

$$\min_{\lambda\in\Delta_m} \left\{ \frac{1}{2} f^T u + \frac{1}{2}\operatorname{Tr}(P^T U) \right\} \tag{5}$$

$$\text{s.t. } K(\lambda)u = f, \tag{6}$$

$$K(\lambda)U = P. \tag{7}$$

Here, the value of the objective function is independent of the choice of $u \in \mathbb{R}^n$ and $U \in \mathbb{R}^{n\times k}$ satisfying (6) *and* (7) *respectively.*

We note that (see [4]) (5)-(7) may be written as a multiload like problem (3) but, the loading scenarios has a new interpretation. Thus, in order to construct robust structures in this continuous model, is not necessary to consider explicitly the all loading scenarios but a good representation of them, according to the covariance matrix P^tP.

3.1 Minimization stochastic problem, including variance

In this section we treat the case of $\beta \neq 0$ in the formulation of $(\mathcal{D}; \mathbb{P}; \alpha; \beta)$, we recall the following result about the formulation of the stochastic model $(\mathcal{D}; \mathbb{P}; \alpha; \beta)$ as a mathematical programming problem (see [4]). We make the proof here only for completeness

Theorem 3.2 (Alvarez, Carrasco [4]). *Suppose $\xi \sim \mathcal{N}_n(0, PP^T)$, i.e., the distribution of ξ is a n-multivariate normal with mean vector 0 and covariance matrix PP^T. Taking for simplicity, $\alpha = 0$ and $\beta = 1$, we have that $(D_{Var}; \mathbb{P}; \alpha; \beta)$ is given by*

$$\min_{\lambda \in \Delta_m} \left\{ \frac{1}{2} \operatorname{Tr}(P^T U)^2 + f^T U U^T f \mid K(\lambda)u = f,\ K(\lambda)U = P \right\}.$$

It is worth pointing out the highly nonlinearity of these problem, it would be interesting to develop a primal-dual formulation of $(\mathcal{D}_{Var}; \mathbb{P}; \alpha; \beta)$ which permits to implement efficient numerical resolution methods for this alternative.

Proof. We will use the two technical lemmas below whose proof can be found it in [4].

Let $\lambda \in \Delta_m$ be a feasible for $(\mathcal{D}; \mathbb{P}; \alpha; \beta)$, i.e. $\mathbb{P}(f + \xi \in V_\lambda) = 1$, where $V_\lambda = \operatorname{Im} K(\lambda)$. By Lemma 3.1 we have that $f \in V_\lambda$ and $\xi \in V_\lambda$ $\mathbb{P}$- a.s., then, there exist u such that $K(\lambda)u = f$ and a measurable function $x\colon \Omega \to \mathbb{R}^n$ such that $K(\lambda)x = \xi$ $\mathbb{P}$-a.s. The existence of such a function is ensured by classical results on measurable selections (see for instance [26, Ch. 14]). Then $\Psi(\xi(\omega), \lambda) = \frac{1}{2}(f + \xi(\omega))^T (u + x(\omega))$ for $\mathbb{P}$-a.e. $\omega \in \Omega$. By Lemma 3.2, we get

$$\mathbb{E}_\xi[\Psi(\xi, \lambda)] = \frac{1}{2} f^T u + \frac{1}{2} \operatorname{Tr}(P^T U),$$
$$\operatorname{Var}_\xi[\Psi(\xi, \lambda)] = \frac{1}{2} \operatorname{Tr}(P^T U)^2 + f^T U U^T f.$$

where U is such that $K(\lambda)U = P$. This completes the proof. □

Lemma 3.1. *Let V be a nonempty vector subspace of $\mathbb{R}^n$. Under the assumptions of Theorem 3.1, we have:*
(i) $\mathbb{P}\{f + \xi \in V\} = 1$ *iff* $f \in V$ *and* $\mathbb{P}\{\xi \in V\} = 1$.
(ii) $\mathbb{P}\{\xi \in V\} = 1$ *iff the columns of P are vectors in V.*

Lemma 3.2. *Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix. Under the assumptions of Theorem 3.1, if $\mathbb{P}\{\xi \in \operatorname{Im} A\} = 1$ and $x : \Omega \to \mathbb{R}^n$ is a measurable function satisfying $Ax(\omega) = \xi(\omega)$ for $\mathbb{P}$-a.e. $\omega \in \Omega$, then $\mathbb{E}(\xi^T x) = \operatorname{Tr}(P^T U)$, where $U \in \mathbb{R}^{n \times k}$ is any matrix satisfying $AU = P$. Also if we suppose that $\xi \sim \mathcal{N}_n(0, PP^T)$ then $\operatorname{Var}(\xi^T x) = 2 \operatorname{Tr}((A\Gamma)^2) + 4\mu^T A \Gamma A^T \mu$ (see [27]).*

4 Numerical study

4.1 Numerical problem

In order to perform a numerical study of the interest of the formulation $(D_{Var}; \mathbb{P}; \alpha; \beta)$ presented in Section 3 and the choice of the parameters (α, β), we consider a 3-D benchmark structure (related to [23]) composed by a set of 3×3×3 nodes in which all nodes are connected by bars. This structure is depicted by Figure 1-Left. A main vertical load f is applied at a particular node.

This problem can be solved problem as it using the dual approach presented in [3, 1]. The obtained solution, depicted by Figure 1-Right, is resilient to f but is highly unstable in cases of small perturbations of this load [11]. Thus, as we are interested to generate structures resilient to small perturbations of f, we consider a random load ξ of law $\mathcal{N}_n(0, PP^T)$ applied at the same node as f and situated in the plane orthogonal to f.

This problem is solved considering the formulation $(D_{Var}; \mathbb{P}; \alpha; \beta)$ considering $S \in \mathbb{N}$ set of values $(\alpha, \beta) \in \Sigma = \{(\alpha_1, \beta_1), ..., (\alpha_S, \beta_S)\}$. In order to solve $(D_{Var}; \mathbb{P}; \alpha; \beta)$, which seems to be a non-convex problem (the cost function associated to the problem to have various local minima, see Figure 2), we

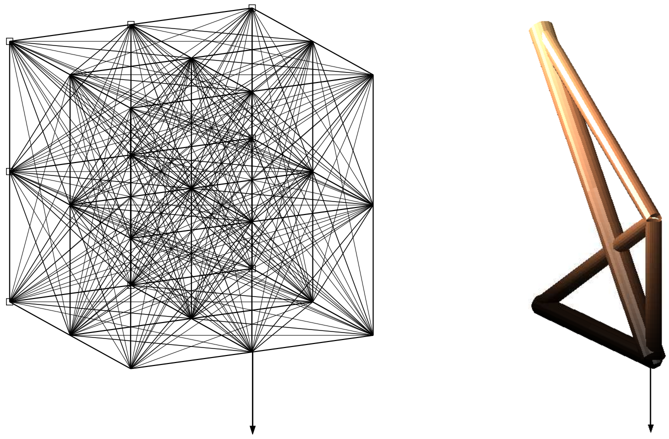

Figure 1: (**Left**) Considered 3D-benchmark structure and (**Right**) associated single-load solution.

use a particular global optimization algorithm based on the steepest descent algorithm where the initial condition is generated using the secant method [25]. A complete description and validation of this algorithm can be found in the following literature [20, 18, 12, 22, 21, 15, 16]. The obtained solutions are denoted by $\lambda_{(\alpha,\beta)}$. It is interesting to notice that for $(D_{Var}; \mathbb{P}; 1; 0)$, $\lambda_{(1,0)}$ is the solution as the one obtained using the dual approach described in [11].

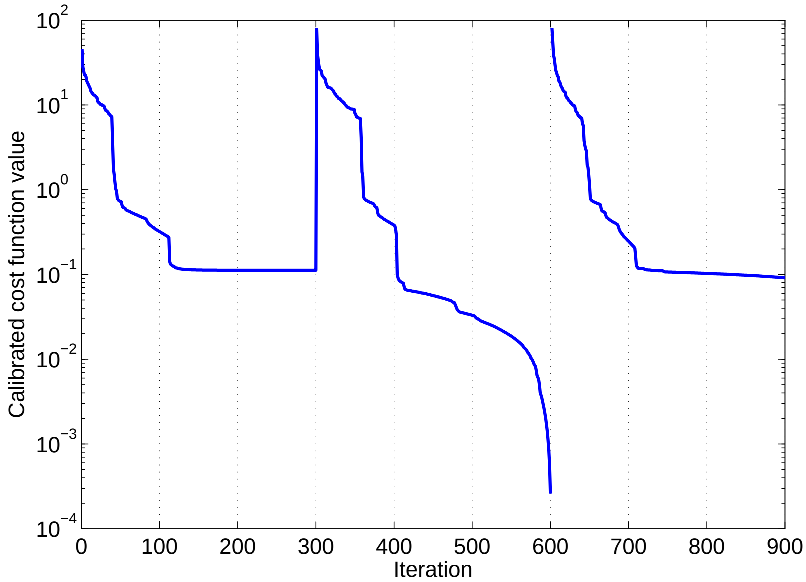

Figure 2: Part of the convergence history obtained with the considered global optimization algorithm used to solve problem $(D_{Var}; \mathbb{P}; (1, 0.25))$. The value of the cost function to minimize is calibrated in order to obtain 0 as the minimum value. As we can observe, the steepest descent algorithm visit various local attraction basin (i.e. the cost function seems to have several local minima).

In order to have a qualitative comparison of those structures $(\lambda_{(\alpha,\beta)})_{(\alpha,\beta)\in\Sigma}$, we analyze their robustness when they are submitted to random loads, their topology and their compliance iso-contours in function of loads.

α	1	1	1	1	1	0.75	0.5	0.25	0
β	0	0.25	0.5	0.75	1	1	1	1	1
Av. Comp.	32.2	34.0	36.3	38.3	40.2	42.6	47.7	56.3	8e+7
E. Comp.	28.1	31.3	33.7	35.8	37.8	40.2	45.3	54.1	8e+7
C-VaR$_{0.01}$	54.3	49.6	50.3	51.3	52.9	54.7	59	66.5	8e+7
Vari.	55	30.7	24.2	20.9	18.7	16.7	13.8	10.8	1.8

Table 1: Summary of the results obtained for each set of parameters (α, β): Average compliance (**Av. Comp.**), Expected compliance (**E. Comp.**), Coherent Value at Risk (**C-VaR$_{0.01}$**) and Variance (**Vari.**) of the structure $\lambda_{(\alpha,\beta)}$. The double bar represent changes in the topology (with apparition/vanishing of bars).

More precisely, to study the robustness of each structure $\lambda_{(\alpha,\beta)}$, we consider the random variable $\Psi_{(\alpha,\beta)} = \Psi(\xi, \lambda_{(\alpha,\beta)})$ and we approximate its density function $\rho_{\Psi_{(\alpha,\beta)}}$ using a Monte-Carlo approach [17] (i.e. generating $M \in \mathbb{N}$ values of ξ). Then we compute the minimum, maximum and average values of $\rho_{\Psi_{(\alpha,\beta)}}$ and a particular risk measure of $\Psi_{\alpha,\beta}$. *Risk measures* on $L^\infty(\Omega, \mathcal{A}, \mathbb{P})$ are mapping $\varpi : L^\infty(\Omega, \mathcal{A}, \mathbb{P}) \to \mathbb{R}$ where $(\Omega, \mathcal{A}, \mathbb{P})$ is a probability space (a complete presentation can be found in [6]). They are used in various areas, such as in financial analysis [19], in order to study the value of the worst scenarios (in our case, the random loads which generates the highest compliances of the structure). Here, we focus on a particular and popular risk measure called the Coherent-Value at Risk (*C-VaR*) [5] and defined as:

$$\text{C-VaR}_\gamma(X) = \frac{1}{\gamma}\int_0^\gamma \inf[Z| \int_0^Z \rho_X(x)dx > (1-p)]dp \tag{8}$$

where γ is a percentile, $X \in L^\infty(\Omega, \Im, \mathbb{P})$, ρ_X is the density function of X. The C-VaR$_\gamma$ corresponds to the average value of the worst γ % scenarios of X.

Finally, to study the compliance iso-contours of a structure $\lambda_{(\alpha,\beta)}$ in function of the loads, we consider the function $J_{\lambda_{(\alpha,\beta)}} : \mathbb{R}^3 \to \mathbb{R}$ defined by:

$$J_{\lambda_{(\alpha,\beta)}}(x_1, x_2, x_3) = \frac{1}{2}\bar{f}^T(x_1, x_2, x_3)u \tag{9}$$

where $\bar{f}(x_1, x_2, x_3) = x_1 f + x_2 d^1 + x_3 d^2$ where d^1 and d^2 are two orthogonal loads situated in the plane orthogonal to the main load and $u \in \mathbb{R}^n$ is a solution of $K(\lambda_{(\alpha,\beta)})u = \bar{f}$. Then, fixing $r \in \mathbb{R}$, we compute numerically one iso-contour of this function considering $J_{\lambda_{(\alpha,\beta)}}(x) = r$.

4.2 Numerical results

We set $f = (0, 0, 1)$, $P = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (both in space coordinate), $\Sigma = \{(1, 0), (1, .25), (1, .5), (1, .75), (1, 1), (.75, 1), (.5, 1), (.25, 1), (0, 1)\}$, $M = 10^6$ and $\gamma = 0.01$ (this level is often used as it reduce the impact of too extreme scenarios [19]).

All results are reported on Table 1. The different topology configurations and their compliance iso-contour in function of the loads, obtained considering $r = 0.1$, are presented by Figure 3. A boxplot [28] representation of the densities $\rho_{\Psi_{(\alpha,\beta)}}$ is presented in Figure 4.

As we can observe on Table 1 and Figure 4, the solution $\lambda_{(1,0)}$ is less resilient to perturbations of the main load, considering the 0.01 % worst scenarios, than the solutions $\lambda_{(1,0.25)}$ up to $\lambda_{(1,1)}$. This is confirmed considering the compliance of the worst scenario which is more important for $\lambda_{(1,0)}$ than all other structures, expect $\lambda_{(0,1)}$. Furthermore, the variance of the compliance decreases with the increase of β. This is intuitive as this value is controlled by our optimization problem. However, the average and the expected compliance values raise with the increase of the β proportion. For high values of the β proportion, the structure becomes meaningfully less resilient to the main load.

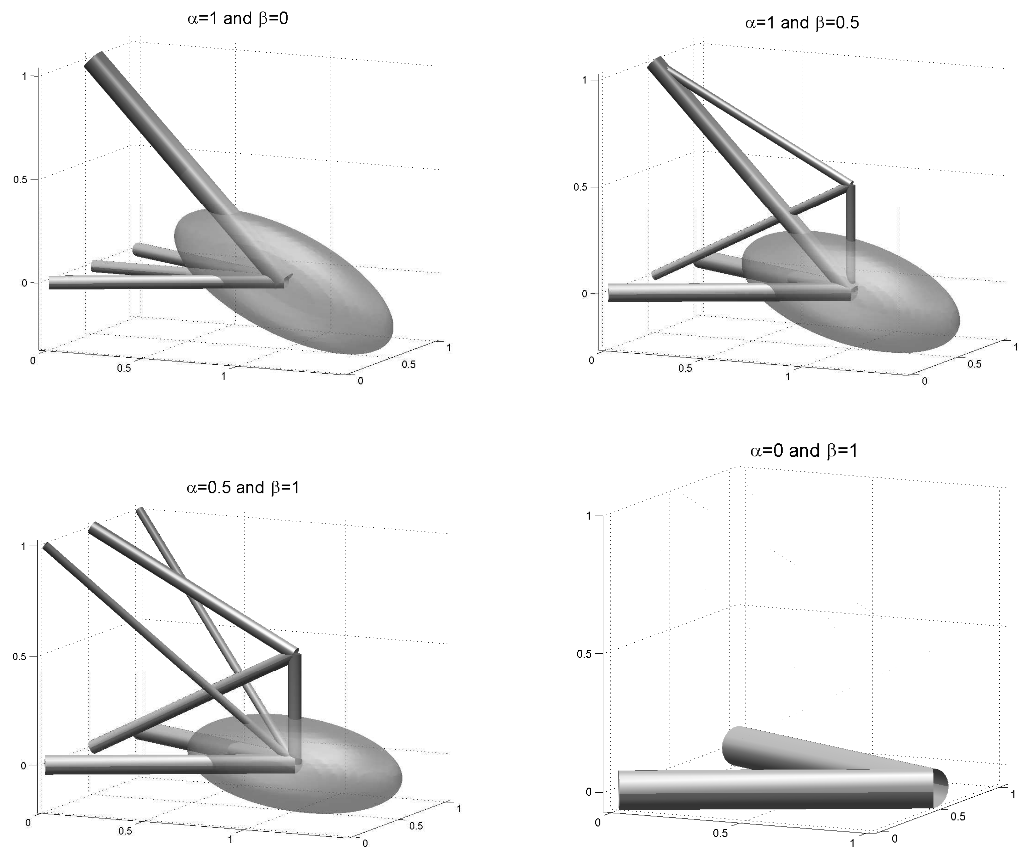

Figure 3: Topology and compliance iso-contour in function of the loads of the structures $\lambda_{\alpha,\beta}$ with $((\alpha, \beta))$ set to (**Top-Left**)(1,0), (**Top-Right**) (1,0.5), (**Bottom-Left**) (0.5,1) and (**Bottom-Right**) (0,1).

Figures 3 and Table 1 points that the topology of the structure changes with the evolution of the coefficients α and β. There is four topology configurations. In fact, it seems to have a mass transfer phenomena, which occurs when β is rising, from the upper bars to the lower bars. The final topology, with two main bars, tends to confirm this situation. This is intuitive as those two bars provide a good resistance to horizontal loads and thus to the perturbations of the main load. In counterpart, they are not resilient to the main load. As we can observe on Figure 3, considered iso-contours are ellipses which tend to straighten and to flatten in the orthogonal plane of the main load f as the β proportion increases. This geometrical evolution confirms that the structure becomes more robust for the horizontal perturbations of the main load but more fragile to the main load. Currently, considering the discrete model used during this work, our understanding of this mass transfer phenomena is limited. In Section 5, we present some issues for this limitation.

From previous results, we can deduce that considering the formulation $(D_{Var}; \mathbb{P}; \alpha; \beta)$ with a reasonable proportion of β can help to generate structures more robust for the perturbation of the main load. For example, in our particular case, $(\alpha, \beta) = (0.25, 1)$ is a good compromise as it generates the lowest risk measure C-VaR$_{0.01}$, reduces the variance by 45 % and have a compliance value (31) close to the best one (28).

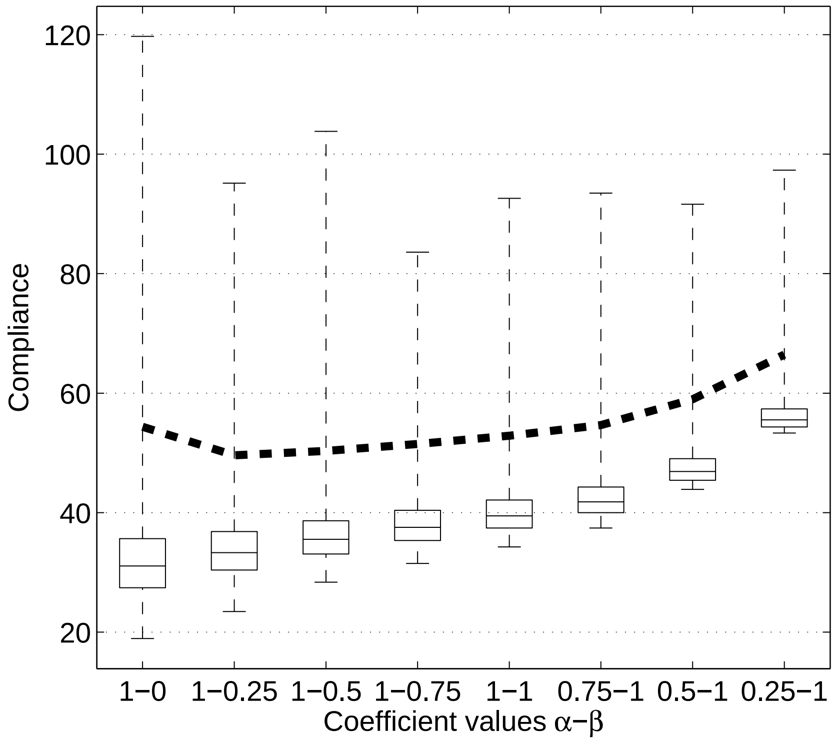

Figure 4: Boxplot representation of the density function of the compliance of the solutions $(\lambda_{(\alpha,\beta)})_{(\alpha,\beta)\in\Sigma-(0,1)}$. C-Var (- -) is also reported.

5 Conclusions

A new variance-expected compliance formulation has been validated numerically on a 3-D benchmark test. This new formulation allows to generate structures more resilient to perturbations of the main load. However, a good choice of the balance between compliance variance and expectancy weights should be chosen in order to avoid structures fragile to the main load.

After this paper, we intend to generalize the formulation and theorems to a continuous model (see [10]) and to perform new numerical tests.

Acknowledgments

This work was carried out with financial support from the Chilian "Centro de Modelamiento Matemático of the Universidad de Chile"; the Spanish "Ministry of Education and Science" under the projects No. MTM2007-64540 and "Ingenio Matematica (i-MATH)" No. CSD2006-00032 (Consolider-Ingenio 2010); and the "Dirección General de Universidades e Investigación de la Consejería de Educación de la Comunidad de Madrid" and the "Universidad Complutense de Madrid" in Spain, under the project No. CCG07-UCM/ESP-2787. The first author was partially suported by FONDECYT grant 3080037.

References

[1] W. Achtziger. Topology optimization of discrete structures: an introduction in view of computational and nonsmooth aspects. In *Topology optimization in structural mechanics*, volume 374 of

CISM Courses and Lectures, pages 57–100. Springer, Vienna, 1997.

[2] W. Achtziger. Multiple-load truss topology and sizing optimization: some properties of minimax compliance. *J. Optim. Theory Appl.*, 98(2):255–280, 1998.

[3] W. Achtziger, M. Bendsøe, A. Ben-Tal, and J. Zowe. Equivalent displacement based formulations for maximum strength truss topology design. *Impact Comput. Sci. Engrg.*, 4(4):315–345, 1992.

[4] F. Alvarez and M. Carrasco. Minimization of the expected compliance as an alternative approach to multiload truss optimization. *Struct. Multidiscip. Optim.*, 29(6):470–476, 2005.

[5] P. Artzner, D. Delbaen, J.M. Eber, and D. Heath. Thinking coherently. *Risk*, 10:68–71, 1997.

[6] P. Artzner, D. Delbaen, J.M. Eber, and D. Heath. Coherent measures of risk. *Risk*, 1998.

[7] A. Ben-Tal and M.P. Bendsøe. A new method for optimal truss topology design. *SIAM J. Optim.*, 3(2):322–358, 1993.

[8] A. Ben-Tal and A. Nemirovski. Robust truss topology design via semidefinite programming. *SIAM J. Optim.*, 7(4):991–1016, 1997.

[9] A. Ben-Tal and M. Zibulevsky. Penalty/barrier multiplier methods for convex programming problems. *SIAM J. Optim.*, 7(2):347–366, 1997.

[10] M. P. Bendsøe and O. Sigmund. *Topology optimization*. Springer-Verlag, Berlin, 2003. Theory, methods and applications.

[11] M. Carrasco. *Diseño óptimo de estructuras reticulares en elasticidad lineal vía teoría de la dualidad. Estudio teórico y numérico.* PhD thesis, Universidad de Chile, 2003. Engineering Degree Thesis.

[12] L. Debiane, B. Ivorra, B. Mohammadi, F. Nicoud, A. Ern, T. Poinsot, and H. Pitsch. A low-complexity global optimization algorithm for temperature and pollution control in flames with complex chemistry. *International Journal of Computational Fluid Dynamics*, 20(2):93–98, 2006, DOI: 10.1080/10618560600771758.

[13] W. Dorn, R. Gomory, and M. Greenberg. Automatic design of optimal structures. *J mechanique*, 3:25–52, 1964.

[14] A. Evgrafov, M. Patriksson, and J. Petersson. Stochastic structural topology optimization: existence of solutions and sensitivity analyses. *ZAMM Z. Angew. Math. Mech.*, 83(7):479–492, 2003.

[15] D.E. Hertzog, B. Ivorra, B. Mohammadi, O. Bakajin, and J.G. Santiago. Optimization of a microfluidic mixer for studying protein folding kinetics. *Analytical chemistry*, 78(13):4299–4306, 2006, DOI: 10.1021/ac051903j.

[16] D. Isebe, F. Bouchette, P. Azerad, B. Ivorra, and B. Mohammadi. Optimal shape design of coastal structures. *International Journal of Numerical Method in Engineering*, 74(8):1262–1277, 2008, DOI: 10.1002/nme.2209.

[17] B. Ivorra. Semi-deterministic global optimization. *PhD. University of Montpellier 2*, 2006.

[18] B. Ivorra, B. Mohammadi, L. Dumas, O. Durand, and P. Redont. Semi-Deterministic vs. Genetic Algorithms for Global Optimization of Multichannel Optical Filters. *International Journal of Computational Science for Engineering*, 2(3):170–178, 2006, DOI: 10.1504/IJCSE.2006.012769.

[19] B. Ivorra, B. Mohammadi, and A.M. Ramos. Optimization strategies in credit portfolio management. *Journal Of Global Optimization*, Accepted, in Early view, to be published, DOI: 10.1007/s10898-007-9221-6.

[20] B. Ivorra, B. Mohammadi, and A.M. Ramos. Optimizing Initial Guesses to Improve Global Minimization. *Journal Of Global Optimization*, submitted.

[21] B. Ivorra, B. Mohammadi, D.E. Santiago, and J.G. Hertzog. Semi-deterministic and genetic algorithms for global optimization of microfluidic protein folding devices. *International Journal of Numerical Method in Engineering*, 66(2):319–333, 2006, DOI: 10.1002/nme.1562.

[22] B. Ivorra, A.M. Ramos, and B. Mohammadi. Semideterministic global optimization method: Application to a control problem of the burgers equation. *Journal of Optimization Theory and Applications*, 135(3):549–561, 2007, DOI: 10.1007/s10957-007-9251-8.

[23] F. Jarre, M. Kočvara, and J. Zowe. Optimal truss design by interior-point methods. *SIAM J. Optim.*, 8(4):1084–1107 (electronic), 1998.

[24] K. Marti and G. Stöckl. Optimal (topology) design under stochastic uncertainty. *Safety and Reliability,(eds. Schueller, G.I., Kafka, P.)*, 2:1597–1602, 1999. Rotterdam-Brookfield: A.A. Balkema.

[25] B. Mohammadi and J-H. Saiac. *Pratique de la simulation numérique*. Dunod, 2002.

[26] R. Tyrrell Rockafellar and Roger J.-B. Wets. *Variational analysis*, volume 317 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 1998.

[27] G. A. F. Seber. *Linear regression analysis.* John Wiley & Sons, New York-London-Sydney, 1977. Wiley Series in Probability and Mathematical Statistics.

[28] J.W. Tukey. *Exploratory Data Analysis*. Addison-Wesley, Reading, MA., 1977.