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This paper shows that price level trends in many of the EMU countries evolve with different patterns and that these patterns will not converge in the long-run .We propose that the hypothesis of price convergence should be evaluated and tested employing the relative prices. To this aim, we: (i) define the asymptotic price level convergence in mean and variance, (ii) provide a model for relative price levels that includes a transition path, and (iii) show how to properly test the definitions stated. Our results show that only French and German price levels converge in mean to a zero gap in the EMU while some others, not many, converge to a nonzero significant gap. This should be a matter of concern for the European monetary policy makers as it implies that the monetary policy does not affect all the EMU members equally.

Keywords Price convergence, Price levels, relative price, inflation, EMU

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Price-Level Convergence in the Eurozone

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Abstract

This paper shows that price level trends in many of the EMU countries evolve with different patterns and that these patterns will not converge in the long-run. We propose that the hypothesis of price convergence should be evaluated and tested employing the relative prices. To this aim, we: (i) define the asymptotic price level convergence in mean and variance, (ii) provide a model for relative price levels that includes a transition path, and (iii) show how to properly test the definitions stated. Our results show that only French and German price levels converge in mean to a zero gap in the EMU while some others, not many, converge to a nonzero significant gap. This should be a matter of concern for the European monetary policy makers as it implies that the monetary policy does not affect all the EMU members equally.

1 Introduction

The European Union is currently facing huge challenges related to economic integration and the combined economy. This is especially dramatic for the Eurozone, the Economic and Monetary Union (EMU), where the probability of disintegration is still not negligible (see, for instance, Alvarez and Dixit, 2014; Draghi, 2014). The common wisdom is that the problems arise from a segmented and poorly coordinated fiscal policy. However, this paper argues that the integration problems can also be related to monetary markets and monetary policy.

We present empirical evidence that most of the price level trends in the EMU are evolving differently and, what could be more dangerous, they will not converge in the long run, given the current patterns. This fact makes it very difficult to implement

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a coordinated monetary policy, especially when there is a single currency and from a monetary authority with a single inflation target. In extreme cases, wide and persistent inflation gaps could cause wide differentials in long-run real interest rates, exacerbating the problem. Meanwhile, as it is noted in Weber (2012), there is a lack of conclusive literature on the effects of a single common inflation target, when there are different price levels and persistent inflation differentials in a monetary union. Therefore, we suggest that the optimal policy framework of the European Central Bank (ECB) should not be restricted to a single common inflation target, "one-size-fits-all," and it should include a criterion for price level convergence in mean and, probably, in variance. In short, the ECB should have a target oriented to reduce the economic heterogeneities represented by different paths in the EMU price levels.

Most of the related literature refers to the existence -or absence- of inflation convergence in the EMU. In spite of this, there is a certain consensus that the inflation differentials across EMU are, in most cases, different from zero and tend to remain stable since the adoption of the single currency. Empirical evidence on the persistence of inflation differentials in the EMU countries were shown by Rogers (2007), Berk and Swank (2002), and Engel and Rogers (2004), among others. In the same way, Busetti et al. (2006, 2007), using time series techniques, find inflation convergence before the introduction of the euro, but lack of convergence (or even divergence, in some cases) after that. Only Lopez and Papell (2012), employing new panel-data techniques and a larger sample size, find some evidence of inflation convergence after and before the euro. Nevertheless, all these results should be carefully considered, given the apparently slow speed of convergence in the inflation differentials, in addition to the short sample data then available after the physical appearance of the euro. Cecchetti et al. (2002), with a much larger sample -from 1918 to 1996-, find empirical evidence on a slow inflation speed of convergence (around nine years for half of the effects) between some U.S. cities.

However, the existence of persistent inflation differentials in the EMU area can be explained by price level convergence, as an effect of the common monetary policy and the common currency. In this sense, the ECB defends the hypothesis that the price levels convergence across the EMU area would imply higher inflation differentials where prices were initially low; see Duisenberg (2000). Thus, a relevant question here is: are these different trends converging asymptotically to a common trend or will they remain divergent in the long run? The main contribution of this paper is to provide a framework to test this thesis and answer the question posted.

Many articles on inflation convergence relates convergence and cointegration in a time-series approach. Therefore, they are based on the study of the steady state of the underlying processes, under the strong assumption that the convergence was already reached.

But if the convergence process is on course, usually named as "catching-up convergence", common cointegration analysis fails to capture asymptotic price level convergence.[1] At this methodological level, our paper also looks at price level convergence from a time-series perspective. We present three contributions in this sense. First, we formally define a generalized notion of asymptotic price level convergence in mean and variance, based on the property of cointegration, which adapts to both existing types of convergence: (1) as catching-up, which includes a transition path, and (2) as steady-state. Second, we provide an encompassing model for relative price levels which allows a convergence path represented by an exogenous and simple deterministic input. Third, we show how to appropriately test convergence implied by the definitions proposed employing the model previously built.

Besides these methodological contributions, empirical analyses are presented for 40 quarterly observations between IV/2001 to III/2011 of eleven EMU national indexes of nominal prices, often employed to measure inflation. These analyses are designed to enlighten about convergence in the price levels between the EMU countries, after the Euro entered circulation, physically, on the first day of 2002.

According to our results, only 13% of the relative prices (7 out of 55) analyzed for the EMU present a stable convergence path: France/Germany, Portugal/Germany, Austria/Italy, Spain/Austria, Portugal/France, Spain/Greece, and Spain/Neetherlands. From them, only the France/Germany relative price reveals convergence in mean, in the strict sense we define in Section 2. On the other hand, the pairwise Austria/Italy is the only case that shows evidence of convergence in variance. Although, the gap between these price levels is still positive and significant. Interestingly, France and Portugal are the only countries that present a stable convergence path with respect to Germany, the largest economy in the EMU, which could be a matter of concern.

The remainder of the paper is organized as follows. Section 2 introduces our theoretical framework and presents some definitions used later. Section 3 describes the model, the econometric representation of convergence, and the hypothesis testing procedures. Section 4 shows the empirical results, and Section 5 concludes.

2 Theoretical framework

This section details and links the concepts of inflation, price level convergence and the structure of markets. Next, we establish assumptions about the relationship between the countries where the price levels are analysed.

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[1] For the formal definition of convergence as "steady state" or "catching up", see Bernard and Durlauf (1996).

Let $P_{i,t}$ stand for the aggregate price level of country i at time t. The inflation, in this case, is the logarithmic difference in the aggregate price level, $\pi_{i,t} = \Delta \ln P_{i,t} = \ln P_{i,t} - \ln P_{i,t-1}$. If $P_{j,t}$ is the price level of country j at time t, then the relative price levels of the country i with respect to the country j could be represented by the logarithmic of the relative price levels, $r_{i,t} = \ln(P_{i,t}/P_{j,t}) = \ln P_{i,t} - \ln P_{j,t}$. For simplicity, hereinafter the log price level(s) will be referred to as price(s) and the logarithm of the relative price levels as relative prices.[2]

We build on the idea that prices need at least one first difference to be stationary. This implies that inflation is either, stationary or non-stationary. Drawing on economic theory, it is expected that any market-clearing nominal price follows a non-stationary process over time. This reflects the idea that shifts in supply or demand imply price adjustments to clear the market in the long run. Given that the price level is an aggregate, the aggregation of stochastically non-stationary processes should produce a non-stationary process.

2.1 Inflation background

In the past, the definition of inflation has been a hot topic in economics. Bronfenbrenner and Holzman (1963) discussed around 13 possible definitions of inflation, which was a small sample of the definitions available at that time. Twelve years later, Laidler and Parkin (1975) presented another survey on inflation, but just one definition was used: "Inflation is a process of continuously rising prices, or equivalently, of a continuously falling value of money". Thus, inflation is defined as a pure monetary phenomenon.[3] Recently, on the contrary, there has been a lack of debate about the inflation concept, and no revised definition has been presented. There seems to be a consensus among scholars about the appropriate definition that was stated by Friedman (1963): "By inflation, I shall mean a steady and sustained rise in prices."[4] However, the terms "steady" and "sustained" are ambiguous, and are not formally defined in Friedman (1963). It is common to interpret these terms in relation to the long-run statistical equilibrium. Related to this, Beveridge and Nelson (1981) propose a decomposition of non-stationary time series between a transitory and a permanent component, where the permanent component is represented by the long-run forecast.[5] Then the inflation rate, $\pi_{i,t}$, can be decomposed as a (stochastic) trend, $\pi^*_{i,t}$, plus an inflation gap $\varepsilon_{i,t} = \pi_{i,t} - \pi^*_{i,t}$. Combining the Beveridge-

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[2] The natural-base logarithm is used for each of the variables of this paper.

[3] See Laidler and Parkin (1975) p. 741.

[4] See Friedman (1963) p. 39.

[5] Also, Friedman (1963) proposed a decomposition into transitory and permanent components, based in exponential smoothing. However the Beveridge and Nelson (1981) decomposition offers a more general framework, in which the Friedman's approach is only a special case.

Nelson decomposition with Friedman's inflation definition, we define the trend inflation as:

Definition 1 *The inflation permanent component,* π_t^*, *is the expected variation of the price level in the long-run, written formally as:*

$$\pi_t^* = \lim_{k\to\infty} \mathbb{E}\big[\pi_{t+k}|\mathcal{F}_t\big],$$

where $\mathcal{F}_t$ denotes all information available at t.

Therefore, we associate Laidler and Parkin (1975) and Friedman (1963) inflation definitions as a pure monetary phenomenon to the trend inflation, $\pi_{i,t}^*$, and we assume that the transitory component, $\varepsilon_{i,t}$, is driven by non-monetary shocks (as changes in raw materials prices, etc). Definition 1 coincides with Friedman's when prices are I(1) with drift. In such a case, the trend inflation is stationary and the long-run rise in prices would be "steady" and "sustained".

Recently, several authors, e.g. Ireland (2007), Stock and Watson (2007), Cogley and Sbordone (2008) and Cogley et al. (2010), among others, have used this inflation decomposition. Their works differ on the assumptions about the permanent and transitory components. A driftless random walk is usually used as trend inflation while a stationary serially un- or correlated noise is used for the inflation gap. We will do no assumptions on the components as we think these will depend on the data and sample analyzed. We will extract this information later from the data in our empirical exercise.

The previous definition has also implication in terms of inflation control. As it is exposed in Ireland (2007), if the inflation is "always and everywhere" a monetary phenomenon, then "permanent changes in the inflation rate cannot occur without corresponding changes in the central bank inflation target".[6] This statement is the same independently of whether or not the target is publicly announced. This means that having inflation under control implies that the inflation permanent component, from Definition 1, should be constant, $\pi_t^* = \pi^*$, when the inflation target is a constant rate. This should be the case in the EMU as the Governing Council of the ECB clarified that, "in the pursuit of price stability, it aims to maintain inflation rates below, but close to, 2% over the medium term".[7]

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[6] The dictatum of Friedman "always and everywhere" is presented in a formal way in the model by Ireland (2007), p. 1859.

[7] see, in `https://www.ecb.europa.eu/mopo/strategy/pricestab/html/index.en.html`, the definition of price stability.

2.2 Relative Price Level

In a monetary union with different fiscal policies, productivities, etc, as is the case of the EMU, measuring and controlling inflation is different from a single country situation. In such a case, price level convergence should be tested. For the bivariate case, i.e., two countries, the analogous concept of trend inflation is the long-run gap between the price levels, which is equivalent to the relative price level rate in the long-run. Inflation is a relative rate in the time domain, but here we also look at the long-run relative rate in the space domain. This notion is about how much expensive a country is with respect to another one and how this relation will evolve in the long-run. Notice that, as stated in Cecchetti et al. (2002), "if the relative price levels contains a unit root, it would mean that the nominal price levels would wonder apart indefinitely" and then any prior shocks would have a permanent impact.

According to this and building on Definition 1, we define the trend relative price as:

Definition 2 *The relative price permanent component, $\tau_{ij,t}$, for country i with respect to country j, is the expected (log) relative price level in the long-run, formally:*

$$\tau_{ij,t} = \lim_{k\to\infty} \mathbb{E}\big[r_{ij,t+k}|\mathcal{F}_t\big]$$

Lets now hypothesize what would happens if prices in two different countries are I(1). If the relative price level contains a unit root this will imply that the nominal prices are not sharing a common trend. In this case, (log) price differentials evolves across time according to the relative economic performance, different monetary and/or fiscal policies, etc. On the contrary, if the relative price level is stationary, then its permanent component (as in Definition 2) will be constant over time, i.e., $\tau_{ij,t} = \tau_{ij}\ \forall t$, and which implies that the nominal prices are sharing a common trend. As nominal prices are assumed to be I(1), then they will be cointegrated with cointegrating vector (1,-1).[8]

2.3 Asymptotic Price Level Convergence

In what follows we will assume that all nominal prices are I(1) and, therefore, inflations are stationary. This will be formally tested with the our data in the empirical exercise. Under this assumption and the Law of One Price, in a new monetary union price levels should converge to a long-run statistical equilibrium, sharing a long-run trend. Thus, the relative Purchasing Power Parity (PPP) should hold, at least, asymptotically. Unless

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[8] For our case, a different value from unity for the cointegration parameter makes no sense as the countries share the same monetary units and neutrality of the money in the long run is assumed. However, it is easy to extend this framework to a pool of countries not sharing the same monetary units, or if the analyst wants to test the neutrality of money in the long-run.

this requirement is met, the monetary authority will not be in a position to establish a common monetary policy. Large inflation differentials can cause real interest-rates differences and a persistent loss of relative competitiveness, which is not sustainable in the long term; see for example Cecchetti et al. (2002) and Engel and Rogers (2004). Consequently, having lower inflation rates across the new monetary union should not be a sufficient requirement for the monetary authority.

Moreover, when the price level in a monetary union is multiple, so that the relative PPP does not hold, then the monetary policy could also be ineffective. Lets assume there are different I(1) price levels in a monetary union but they are divided into two groups sharing one trend each. In absence of an active monetary policy, this is not necessarily a problem. However, any attempt to carry out a coordinated monetary policy could cause different problems. Notice that a contractionary monetary policy oriented to reduce the inflation on one side of the union could cause a recession on the other side. In contrast, the need of an expansionary monetary policy to avoid a recession on one side of the union could raise the inflation on the other.

Therefore, a real coordinated monetary policy requires the trend relative price to be a constant. Nominal prices could be different but their non-stationary factor should be common to all of them. Price levels for all members should be cointegrated so that the monetary authority can keep control over all of them simultaneously. Further, price level trends could have been different for a while but their long-run path should converge, at least, to a constant gap that could be explained by structural differences. To capture this idea and following the definitions of convergence in output presented by Bernard and Durlauf (1995, 1996), we define the asymptotic price level convergence in mean as:

Definition 3 *For Asymptotic Price Level Convergence in Mean (PCM), the price levels in countries i and j converge asymptotically if:*

$$\lim_{k \to \infty} \mathbb{E}\big[r_{ij,t+k} | \mathcal{F}_t\big] = \tau_{ij},$$

where τ_{ij} is the long-term forecasts of (log) price level differentials, which is a constant that captures the price level differential in the long-run. Using the same terms as Durlauf and Quah (1999), we say that there is *relative* convergence (relative PPP) if τ_{ij} is a non-zero constant, and the convergence is said to be *absolute* (absolute PPP) if the long-run gap is zero, $\tau_{ij} = 0$. This definition allows one to represent a process in which the price series have already converged, named *convergence as steady state*, or as a *catching up*, permitting a price differentials to converge to a constant.

Notice that the condition of absolute convergence implies that the PPP is fulfilled. However, the absolute PPP does not apply in strictly sense in the case of the price levels, because they are composed by tradable and non-tradable good prices. In the case of having price aggregates that only include tradable goods, and assuming perfect competition, no trade barriers and no transport costs, the convergence should be absolute, so τ_{ij} should be zero. However, for the price aggregates that include non-tradable goods, the price level differentials also depend on the difference between the fundamentals of the economies, such as productivity and welfare levels, so just relative convergence is expected in most of the cases, i.e., relative PPP. This is also true if there are different weights for each country's market basket, as is discussed in Dornbusch (1987).

This definition has implications for the control of inflation in a monetary union. It implies that the relative prices should follow a stationary process or, at least, an asymptotically stationary process.

Price convergence in mean (PCM), and for instance the relative PPP, is a necessary but not sufficient condition to have an efficient monetary union. Price levels in different countries can react transitorily and differently to endogenous or exogenous shocks. This implies that a certain monetary policy can have a non-permanent effect on the price level in one country that is different for the other members of the union. These asymmetries in response to shocks could make it difficult to have a coordinated monetary policy and, in a statistical sense, the convergence process incomplete. Market integration processes enhances the ability of the market locations to cope with shocks in nominal prices, and in the price levels. Therefore, one would expect that, as integration in the markets increases, the dispersion of those transitory shocks would decrease.

In contrast, market integration studies based only on cointegration analysis concentrate on the first moment condition, that is, a time ago two price levels began to comove so the log differential converged in mean and it follows this steady state since then. For example, a change in trade barriers, such as tariffs, could imply a level change in relative prices (convergence in mean). However, this could happen together with an increase in the residuals' relative price dispersion (divergence in variance).

In the other hand, the relatedness of two price levels, or monetary markets, is a matter of degree and, hence, of contemporary and lagged cross covariances. This means that it is necessary to check at least the second moment of the price distribution to conduct an appropriate monetary market integration analysis. In order to conclude positive monetary market integration, it is necessary to have price level convergence in variance.

Hence, it would be worthwhile to look at a definition of convergence that includes the notion of convergence in variance for price levels that precisely reflects this situation. Thus, we complement Definition 3 with the following:

Definition 4 *For Price Level Convergence in Variance (PCV), the price levels in coun-*

tries i and j converge asymptotically if:

$$\lim_{k\to\infty} \mathbb{E}\big[(r_{ij,t+k})^2|\mathcal{F}_t\big] = \upsilon_{ij}.$$

where υ_{ij} is a constant which represents the asymptotic expected variance of the relative prices. Given this definition, absolute convergence in distribution, $\upsilon_{ij} = 0$, implies absolute PPP in distribution, and relative convergence in distribution, $\upsilon_{ij} \neq 0$, implies relative PPP in distribution (assuming normality of the process).

If the PCM and PCV requirements hold for a pair of countries, then we have a special case of price level convergence in distribution, assuming the normality and the asymptotic expected variance of the relative prices. Obviously, this absolute convergence in distribution is hard to find in practice. However, this concept is useful for understanding the relationship between price level convergence and monetary markets integration, although they might appear to be utopian. Notice that if both requirements hold for a pair of countries, then their inflation rates are exactly the same.

3 Model and hypothesis testing

We use an encompassing model for the price differentials to represent price level convergence. Formally, the model for the relative price level for two countries, including a convergence path, may be written as:

$$\begin{aligned} r_{i,t} &= D_{ij,t} + S_{ij,t}, \\ D_{ij,t} &= \mu_{ij} + C_{ij,t}, \\ \phi_{ij,p}(B)S_{ij,t} &= \theta_{ij,q}(B)a_{ij,t} \end{aligned} \qquad (1)$$

where B is the backshift (or lag) operator, such that $B^d P_t = P_{t-d}$, and the price level differential has an additive decomposition between a deterministic component, $D_{ij,t}$, and stochastic component, $S_{ij,t}$. In the deterministic component, μ_{ij} is the constant mean, C_{ij} is the converge component.[9] The stochastic component follows a stationary process and has an ARMA(p,q) representation, strictly stationary and invertible (that is, the autoregressive and moving average polynomials have all zeros lying outside the unit circle), and $a_{ij,t}$ is a weak white noise process.

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[9] The seasonal terms have been omitted in the interest of simplicity. However, in the deterministic component $Q_{ij,t}$ is the deterministic seasonal component with annual sum zero for all t. That is, each quarter $Q_{ij,t}$ has a value, but the sum of four consecutive values is zero.The Trigonometrical Representation of Deterministic Seasonality component employed here, take the form $\alpha_1 \cos(\pi/2)t + \beta_1 \sin(\pi/2)t + \alpha_2(-1)^t$ in the case of quarterly data, which is equivalent to the traditional approach of literature using the weighted sum of five seasonal dummy variables.

3.1 Convergence Representation

In Model (1) a linear transfer function is used to represent the relation between the convergence component, $C_{ij,t}$, and an input variable $\xi_t^{t^*}$ and this relationship is written:

$$C_{ij,t} = \nu(B)\xi_t^{t^*} = \nu_0\xi_t + \nu_1\xi_{t-1} + \cdots + \nu_k\xi_{t-k} + \cdots \tag{2}$$

where $\nu(B)$ represents the convergence operator, ν_k is a real constant $\forall k$, the variable, $\xi_t^{t^*}$ describes the effects of an event that will last permanently after time t^*, as unity whenever $t > t^*$, and zero otherwise. Therefore t^* represents the starting moment of the convergence process. Notice that the convergence component describes the response of the output to a unit step input, and assigns, to each non-negative integer lag $k = 0, 1, 2, \cdots$, the value of the partial sum of the coefficients $\sum_{j=0}^{k}\nu_j$. For instance, the *long run gain* of the convergence operator is defined as:

$$g \equiv \sum_{j=0}^{\infty}\nu_j = \nu(1) \tag{3}$$

which is the limiting value, as $k \to \infty$, of the convergence operator, that is, the long run effect on the output of a unit step input. Of course, g exists (is finite) only when $\nu(B)$ converges for $B = 1$. The *long run effect* of the input variable $\xi_t^{t^*}$ on the output variable $C_{ij,t}$ is defined as the long run gain of the convergence operator relating these variables. This long run effect is obviously unique.

The *short run effect* of the input $\xi_t^{t^*}$ on the output $C_{ij,t}$ is not unique until one defines the length of the short run in question. The shortest short run is of length $k = 0$ and the corresponding effect is ν_0, the contemporary effect. But one can define a short run of length k^* periods and the corresponding effect is the value of the partial convergence at k^*, $\sum_{j=0}^{k^*}\nu_j$. It is obvious that the value of the short run effect at the short run of length k^* depends, in general, on k^*.

An important restriction that is often imposed on a convergence operator is called *stability.* It is said that a convergence operator is stable if and only if $\nu(B)$ converges for $|B| \leq 1$, where here B is taken as a complex number. In this case, g exists and the long run response of the output to a finite change in the input is itself finite.

In the Econometrics literature there is a notion related to that of the convergence operator, namely, the *lag distribution function.* A lag distribution function is a special linear transfer function satisfying certain restrictions defining a (discrete) *probability density function* (*pdf*) on the set of non-negative integer lags. Consider a stable convergence operator $\nu(B)$ with non-zero long run gain $g \equiv \nu(1) \neq 0$ and consider the (also stable) derived $\nu(B)/g$, which has a long run gain of value one. If $\nu_k/g \geq 0\ \forall k = 0, 1, \cdots$, then the convergence operator $\nu(B)$ is called a lag distribution function. Clearly it is a special case of the general linear transfer function, because it has a non-zero long run gain, whereas a general linear transfer function may have a zero long run gain, and because it has a response function that is monotonic non-decreasing (if $g > 0$) or non-increasing (if $g < 0$), whereas a general linear transfer function may have a non-monotonic response function.

These restrictions, for an convergence operator to be a lag distribution function, are arbitrary in practically all cases in which they can be used in Econometrics. However, the case of a monotonic non-decreasing (or non-increasing) response function, is easily distinguished from other cases in practice and is, in fact, often observed in the data, as in our empirical exercise. In this case, the mean (*average*) *lag* is well-defined and is calculated by:

$$\bar{l} \equiv \frac{\sum\limits_{k=0}^{\infty} \nu_k k}{\sum\limits_{k=0}^{\infty} \nu_k} = \frac{\nu'(B)}{\nu(B)}|_{B=1} \tag{4}$$

where $\nu'(B) = d\nu(B)/d(B)$. Other measures of the speed of response, such as the median lag, are available, of course. Note, however, that the speed of a response is not, in general, a well-defined concept unless the response is monotone.

The general linear transfer function treated previously has, as it stands, in all its generality, an apparently serious limitation for the practice of empirical analysis: potentially, at least, it has an infinite number of free parameters $\{\nu_k\}$. In practice, no matter how many observations are available, their number is finite. It is, in principle, impossible to estimate an infinite number of parameters from a finite number of observations.

This limitation is not, however, really very important, because any relevant convergence operator $\nu(B)$ can be adequately approximated by a ratio of finite-order polynomials with real coefficients and argument B, which has a finite number of parameters. The approximation employed here is:

$$\nu_{ij}(B) := \frac{\omega(B)}{\delta(B)} = \frac{\omega_0}{(1 - \delta_1 B)}, \tag{5}$$

where ω_0 and δ_1 are real parameters subject to $|\omega_0| > 0$ and $0 < \delta_1 < 1$. By definition, $\omega_0 \neq 0$, except when no relationship exists, i.e. $\nu(B) \equiv 0$. Figure 1 gives a graphical description of a convergence path that can be represented with this simple function.

The concept of convergence is closely linked to stability and, consequently it is assumed that $\delta_r(B) = (1 - \delta_1 B)$ is stable. Therefore, the condition for a stable convergence process is that the root of the characteristic equation $\delta_r(B) = 0$ should lie outside the unit circle. In this case, if $\delta_1 = 1$ the the transition path represent a ramp-like behaviour,

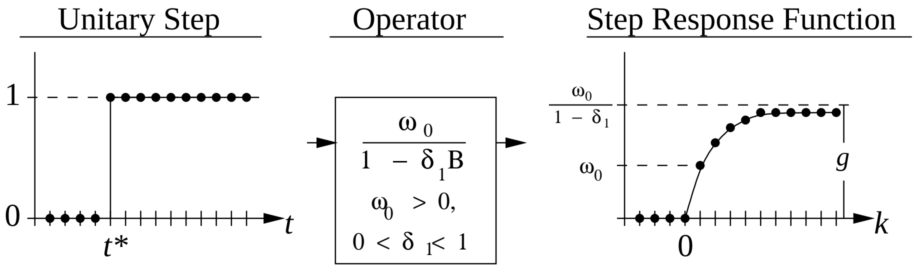

Figure 1: Example of convergence path subject to $\omega_0 > 0$ and $0 < \delta_1 < 1$.

that is a trend (positive or negative) but with a starting point in t^*. If the operator $\delta_r(B)$ is stable, the long run gain of the transition path, g, has a finite value and can be estimated as a function of the estimated parameters in (5).

In the case of (5), the steady state gain, g_{ij}, from a stable convergence process between a pairwise price level in countries i and j, is:

$$g_{ij} := \sum_{k=0}^{\infty} \nu_{ij,k} = \frac{\omega_0}{(1-\delta_1)} < \infty. \tag{6}$$

This definition implies that the backshift operator in (5) is replaced to obtain the steady state gains. The estimated steady state gain could be used for testing asymptotic convergence in mean, when a convergence path is present.

In the same way, the speed of convergence can be approximated replacing (5) in (4). Formally, the mean lag of response in this case is:

$$\bar{l}_{ij} \equiv 1 + \frac{\delta_1}{1-\delta_1} \tag{7}$$

3.2 Testing asymptotic price level convergence in mean

For the requirement of convergence in mean, both price levels will have converged asymptotically if: (i) the log-ratio $r_{ij,t}$ follows a stationary process, and (ii) $D_{ij,t}$ is equal to zero, or has converged asymptotically if $D_{ij,t}$ tends to zero as t approaches infinity.

As the price levels analysed are from the same currency area, only a cointegration vector (1, -1) is feasible between a pair of price levels. Different values for the cointegration vector imply long run monetary illusion in a common currency area, which is unlikely. Model (1) can easily be estimated for a univariate model of the relative price, $r_{ij,t}$, as well as to perform unit root tests. This not only makes the analysis simpler, but also has gains for the power of the unit root tests as the critical values are closer to zero. In this case, Saikkonen and Lutkepohl (2002), hereafter SL-GLS, present a test for a unit root with different level shifts that includes the transition path (5). They show that the convergence parameters in $\nu_{ij}(B)$, or the time at which the convergence begins, t^*, do not affect the limiting distribution of the non-stationarity test. Furthermore, the Shin and Fuller (1998) test, SF, which is more powerful than ADF-type tests in the case of ARMA structures, can also be used.

When the non-stationarity hypothesis is rejected in the univariate version of Model (1), standard inference applies. Steady state convergence, in the sense of Bernard and Durlauf (1995, 1996), arises when $D_{ij,t}$ is equal to zero for all time t. In this case, $C_{ij,t} = 0$ for all t in Model (1), and the null hypothesis is $\mu_{ij} = 0$. In contrast, there is catching-up convergence if $D_{ij,t}$ tends to zero as t approaches infinity.

For testing catching-up convergence, it is necessary to estimate the long run gains of the convergence path. With the initial conditions, represented by the constant mean, μ_{ij}, and the long run gains, g_{ij}, having the same value, but with opposite signs, there is evidence of convergence in mean (see Figure 1).

In order to test the null hypothesis $\tau_{ij} = \mu_{ij} + g_{ij} = 0$, that the long run gap is equal to zero, we use two procedures:

1. *T-student and Delta Method*:

   Assume that $\hat{g}_{ij}$ and $\hat{\mu}_{ij}$ are consistent and asymptotically normal estimators of g_{ij} and μ_{ij}, respectively, so that $\sqrt{T}(\hat{\tau}_{ij} - \tau_{ij})/\hat{\sigma}_\tau \xrightarrow{d} N(0,1)$, where $\hat{\tau}_{ij}$ and $\hat{\sigma}_\tau$ are calculated using the Delta Method.

2. *Likelihood-ratio test*:

   For the same purpose, the statistic $-2\log l(\Theta_2|P_{1,t}, P_{2,t}, \xi_t^{t^*})/l(\Theta_1|P_{1,t}, P_{2,t}, \xi_t^{t^*})$, where $\Theta_2 = \{\alpha, \omega_0, ..., \omega_s, \delta_1, ..., \delta_r, \phi_{1,ii}, ..., \phi_{p,ii},\ \theta_{1,ii}, ..., \theta_{q,ii}, \theta_{ij}\}$, and which follows a χ^2 distribution asymptotically with 1 degree of freedom, can be applied.

Whatever the test that is used, when $P_{i,t}$ and $P_{j,t}$ are cointegrated and $\tau_{ij} = 0$ cannot be rejected, then $P_{i,t}$ and $P_{j,t}$ are said to converge asymptotically in mean.

3.3 Testing asymptotic price level convergence in Variance

Testing convergence in variance (PCV) requires testing whether the residual variance in Model (1) tends to zero. We propose using the test of Breusch and Pagan (1979), which tests whether the estimated variance of the residuals is unconditionally homoscedastic. We regress the squared residuals on an exogenous variable. The test statistic, LM, is the product of the coefficient of determination (R^2) from this regression and the sample size n, namely $LM = nR^2$, where LM is the Lagrange multiplier statistic. The test statistic is asymptotically distributed as $\chi^2(1)$ under the null hypothesis of homoscedasticity.

If the null hypothesis is not rejected, there is no evidence in favor of PCV, as the variance of a_{ijt} is constant over time. In that case, PCV and, therefore, increasing prices integration through increasing monetary market efficiency, can be rejected. Unfortunately, when the null hypothesis is rejected, we cannot conclude that there is PCV, as PCV implies heteroscedasticity, but the reverse is not always true. In that case, we could have growing integration or disintegration.

In the inconclusive case, we propose to observe the evolution of the standard deviation of the innovation (SDI). In this way, we could use this visual aid to decide on the evolution of the variance, the PCV and, finally, the prices integration process. If the SDI series show a decreasing pattern we can argue that there is a growing integration. This series can be estimated with a rolling window by using the residual standard deviation from Model (1).

4 Empirical Results on Price Convergence

In this section we apply the model presented in (1) to the Harmonized Consumer Price Index for the EMU countries. Our aim is to investigate whether there is price level convergence among the members of the monetary union and if that convergence was triggered by the establishment of the common currency.

4.1 Data and estimation results

We analyze Harmonized Consumer Price Index for the EMU countries, 40 quarterly observations between IV/2001-II/2011. The series were originally not seasonally adjusted monthly series and were aggregated to quarterly by taking the simple average. In order to make the analysis comparable, we change the base to December 2001, and then weight the index for the comparative price levels to Germany at the end of 2001. The series are originally produced by Eurostat.[10]

Price levels are depicted in Figure 2. As expected, different clubs appear in the EMU. The first club, with the highest levels, is made up of Ireland and Finland. The second club comprises Austria, Belgium, Germany, France, Italy and the Netherlands. Finally, Portugal, Greece and Spain show the lowest price levels in the EMU.

The relative prices in the EMU give a clearer picture about convergence -or lack thereof. Figure 3 shows the relative prices with respect to Germany. The central club seems to converge, in mean, to German prices but a more detailed statistical analysis is required to check this hypothesis. In any case, we observe that the price levels are slightly

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[10] Further details and the transformed series are in a detailed Statistical Data Appendix which is available on request from the authors

higher at the end of the sample for all the countries. Price levels relative to Germany increased most probably because of the establishment of the common currency, as there is no evidence of productivity convergence across the EMU after the euro. Looking at the Italian/Germany relative price, it had a low level (the lowest in its club) at the beginning of the sample and has practically converged to parity by the end. Price levels have become very similar in Germany and Italy 10 years after the Euro was established. For the lowest club, as expected, the positive slope of the relative price is higher. Both, Spain and Greece, exhibit a positive and (what looks like a) linear trend in their relative prices. Portugal's relative series seems to show a convergence pattern with Germany, although this occurs far from parity.

Our analysis shows that all nominal price levels have similar statistical properties. Model specification for these price level series is relatively simple. Nominal prices: (i) are integrated of order one; (ii) have an order-one, in most cases, or an order-two autoregressive structure for the stochastic part; and (iii) have a constant mean, μ_i, and a seasonal components, for the deterministic part.[11] These results are summarized in Table 1. The estimated parameters and some diagnostic tools are also reported but not those related to the seasonal modeling. All the parameters are statistically different from zero and in all the models the Q-statistic by Ljung and Box (1978) shows no sign of poor fit.

In all cases the SF unit root test rejects the null hypothesis of nonstationarity of the AR(1). Moreover, there is no evidence of an invertible representation if a second difference and a MA(1) operator to control over-differentiation are added; the GLR test cannot reject the null hypothesis of noninvertibility in these alternative models. Consequently, I(1) is confirmed in these nominal price levels. This fact implies that the inflation in these countries can be represented as a constant value; see Definition 1. For instance, the quarterly inflation rate is estimated by the mean, $\hat{\mu}$.

According to the results and the sample, EMU's countries with the lowest inflation are Germany and Finland, with quarterly inflation rates of $0.43\% \pm 0.12$ and $0.43\% \pm 0.20$ at 95% of confidence. In the opposite side, the countries with the highest inflation rates are Greece and Spain with $0.76\% \pm 0.20$ and $0.71\% \pm 0.20$, respectively.[12] Moreover, the differences in the inflation volatility are also non-negligible. Greece, Portugal and Spain show the highest quarterly volatility levels, above 0.45%. In contrast, France and Germany are slightly below 31%. Therefore, the strategy followed by the ECB of "price stability is defined as a year-on-year increase in the Harmonised Index of Consumer Prices (HICP) for the euro area of below 2%" holds for all the countries in the EMU but Spain and Greece.

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[11] The initial specification for the stochastic part is according to *pacf* values, AIC and H-Q criteria. The three criteria are in accord with the same initial specification.

[12] In order to obtain the inflation annual rates, this quarterly rates should be multiplied by four.

Unlike the inflation rates, relative prices do not seem to be stationary, see Figure 3 for the EMU relative prices with respect to Germany. In all of them, but Finland and Ireland, a deterministic convergence component seems to be necessary to get a stationary representation.

We deliberately fix t^* to the quarter at which the convergence could have started, at the beginning of the sample, the first quarter of 2002. This is because the Euro entered circulation, physically, on the first day of 2002, and we can empirically test if this fact had an effect in terms of convergence of the price levels in the EMU. In any case, despite the historical or empirical reasons that justify the use of this quarter as initial points for the convergence process, we carried out a thorough search looking for alternative starting dates within the sample. For each case presented in this section, we estimate several models with different convergence operators for later quarters, without finding any significant result.

With t^* fixed to I/2002, we fit the model in (1) to every relative price series. Note that the stochastic structure is restricted to be univariate. This is more a virtue than a drawback as making the models simpler avoids computational issues, particularly with these relatively short samples.

The estimation results, with German prices as *numéraire*, are reported in Table 2. The model identified and estimated in each case is also relatively simple: (i) an AR(1) process for the stochastic part in most of the cases, although AR(2) or AR(3) structures are estimated in a few cases; (ii) a mean, μ; and (iii) a gradual and monotone convergence path, $\omega_0/(1-\delta_1 B)$. The estimated parameters, with their standard deviations and some diagnostic tools are also presented.

In most of the EMU series a unit root was detected or it was not possible to estimate a stable convergence path. The likely reason is that these convergence processes were so slow and gradual that the most appropriate representation is very close to a positively sloped straight line.

However, adding a convergence component seems to be sufficient to represent the transition path and have a stationary representation in some particular cases. There, the estimated parameters are statistically different from zero, the convergence operator is stable in most cases, and the diagnostic statistics show no sign of poor fit. More details related to this cases will be discussed in the next section.

4.2 Testing price level convergence in mean (PCM)

Here we test the hypothesis of PCM –as stated in Definition 3– in the EMU, and we describe the convergence process to an hypothetical unique price level that could have emerged at some point after the year 2001.

Testing the PCM requires a stationary process or a transition-stationary process in which the convergence path is stable. These conditions have been previously verified, employing the estimates of model in (1) for all possible relative prices. After that, the hypothesis of PCM is tested in every relative price series in the EMU.

Table 3, Panel A, shows the results of the SF unit root tests. With Germany as numéraire, only with France and Portugal non-stationarity is rejected when a transition term is introduced from t^*. When France is the *numéraire*, non-stationarity is also rejected for common land borders Spain and Italy, besides Germany and Portugal. In short, only 30% of the total pairwise relative prices can be represented as transition-stationary processes. The same tests generally do not reject non-stationarity at any standard level for all possible relative prices when there is no convergence component in the model. This reveals evidence of asymptotic convergence in mean as "catching-up" and implicitly rejects convergence as "steady state".

Only for the relative prices that are transition-stationary, we perform the formal test for stability of the convergence operator (H_0: $\delta_1 = 1$ vs. H_A: $\delta_1 < 1$). The results of this test are shown in Table 3, Panel B. Only a few cases present a stable convergence path. The cases in which the null hypothesis is rejected in favor of the alternative are: 1) France/Germany and Portugal/Germany; 2) Italy/Austria and Spain/Austria; 3) Portugal/France; 4) Greece/Spain; and, 5) Netherlands/Spain. We can then conduct the test for PCM only for these five cases.

The results of the tests for PCM (H_0: $\tau_{ij} = g_{ij} + \mu_{ij} = 0$ vs. H_0: $\tau_{ij} \neq 0$) employing the student-t statistics are presented in Table 3, Panel C.[13] Both, the student-t and LR tests strongly confirm that the price levels in Germany and France hold the PCM, which means that the remaining gap is not statistically different from zero. Since the beginning of 2002 the catching-up of the French price level was around 2% and fast (in 5.9 quarters a half of the total effect was reached).[14]

In the rest of the cases the remaining gap is statistically different from zero. For instance, the results suggest that the long-run price level in Portugal is lower than Germany's in around 17.9%. The catching-up of the Portuguese prices with the Germans' was around 5.6% , although slower than the French case (7.1 quarters to reach a half of the gain). In the case of Austria and Italy, the remaining long-run gap is small, 1.9% lower in Italy, although still significant. In this case, the reduction of the initial gap, $\hat{g}$, was 4.5%, but it took longer (13.7 quarters) to reach a half of the total catching-up.

Particularly interesting is the case of Spain, as the catching-up observed were more severe than in any other case. The long-run gain of the Spanish prices relative to Austria's

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[13] The LR test are totally consistent with the the student-t. For this reason only the student-t statistics are presented although the former are available from the authors upon request.

[14] See the estimated convergence parameters in Table 2, $\hat{g}$ for the long-run gain and $\hat{l}$ for the velocity.

and Netherlands' are roughly 12.4% and 10.3%, respectively. The speed is relative slow, around 22 quarters in the first case and 12 quarters in the second.

These results are not counterintuitive but deserve some explanation. Results obtained in the France/Germany relative price perfectly fit to the prediction of the economic theory. The two countries have relatively similar economies and their common border facilitates comercial activities. The Law of One Price applies in its strict form and we can say, ten years after the establishment of the common currency, that their prices have strictly converged in mean. But, why this does not occur between all the price pairs? Lets distinguish different cases. Those economies that show weak convergence in their prices (with lower values for the t-test in Table 5, Panel C) as Austria/Italy and Greece/Spain have also many similarities. In the case of Austria and Italy, it is probable that the North part of Italy presents more similar prices (and then mean convergence in strict form) than the South. An analysis with regional prices could confirm this thesis. With respect to Greece/Spain, the last crisis has revealed many similarities in the economies of these peripheral countries. The case of Portugal/Germany, Portugal/France, Spain/Austria and Spain/Netherlands is different. Our guess is that two different effects are acting here and, most probably, in all the relative prices. There is probably a convergence in tradable goods and services but not (or, at least, higher difficulties to reach it) in non-tradable goods and services. Our analysis is reflecting a combination of both. In any case, one of the implicit results of the analysis is that the monetary policy carried out by the ECB does not affect the economies of the EMU equally. Accordingly, one could wonder if the transmission of the monetary policy is equally efficient in all the countries of the EMU.

There are, of course, some methodological aspects that should be taken into account when analyzing the results. The fact that so many zeros appear in Table 5, Panel B, is clearly related with the sample size. In those cases the long-run gain is not finite and the test for convergence in mean cannot be performed. A longer sample will definitely improve this situation as what is now estimated as a linear transition path (that has little economic sense) will very likely be estimated as a curved with finite-gain transition. Finally, it could also happen, in some cases, that prices are indeed converging (in mean, variance or both) but our methodology is not able to capture the real transition path. Again, a longer sample will give an insight in this respect.

4.3 Testing price level convergence in variance (PCV)

For the relative prices that are transition-stationary, we performed the formal test for PCV. In order to examine PCV, we use the Breusch-Pagan test, as explained in Section 3.3. The residuals series are obtained from the models presented in Tables 3.

The results of the PCV test for the EMU prices are reported in Table 4, Panel A. The statistics for the null hypothesis that the residuals are homoscedastic are not rejected in most cases. Homoscedasticity is rejected only for the pair Italy/Austria at the 5% level and for the Netherlands/Spain at the 10% level. Thus, price level convergence in variance is only possible for the pairwise relative prices of Italy/Austria and Netherlands/Spain.

In order to see whether the heteroscedasticity detected is generated by a decreasing variance, we draw the evolution of the residual standard deviations of the natural log of relative prices for all the cases. The evolution over time of the residual standard deviations is calculated using rolling windows with a span of $t = 10$, and shown in Figure 4. This figure suggests that the standard deviation decreased for the Italy/Austria pair, countries with a common border. However, it increases for the case Netherlands/Spain. As a consequence, PCV can be rejected in the rest of the cases. Here, some considerations are needed. We could be puzzled with this results for the relative prices France/Germany. However, the volatility of the residuals of the relative price was already low around 2004, as shown in Figure 4, and kept the lowest volatility value almost for all the sample. This proves that the efficiency in the integration exhibit by the German and French markets is only comparable to that reached at the end of the sample by the pair Italy/Austria.

5 Concluding remarks

In this paper we show that the price level trends within the EMU are evolving with different patterns for some countries and that, given these patterns, they will not converge in the long-run. We propose that the hypothesis of price convergence should be evaluated and tested employing the relative prices, instead of using the inflation differentials. To this aim, the paper defines a generalized notion of asymptotic price level convergence in mean and variance, provides a model for relative price levels that includes a transition path, and shows how to properly test the definitions proposed.

According to our empirical analysis, only French and German price levels in the EMU converge in mean to a zero gap. Some, like Austria/Italy or Greece/Spain among others, converge in mean to a nonzero significant gap. But they are not many. On the other side, only Italy and Austria price levels seem to converge in variance although France and Germany keep the lowest variance during most of the sample.

The results should be a matter of concern for the European monetary policy makers as it implies that the monetary policy does not affect all the EMU members equally. The consequences on the EMU countries could be important in terms of both, economic growth and inflation.

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Table 1: Estimated univariate price models
(Quartely Prices in Log Differences)

Variable (Mnemonics)	AR(1) $\hat{\phi}_{11}$ (s.e.)	AR(2) $\hat{\phi}_{12}$ (s.e.)	$\hat{\phi}_{12}$ (s.e)	Mean (s.e.) (%)	Resid. Std.Dev. (%)	ACF$^{(1)}$ $Q_{(9)}$	SF$^{(2)}$ $H_0 : \phi_1 1 = 1$	GLR$^{(3)}$ $H_0 : \theta = 1$
Austria	0.26	–	–	0.50	0.35	14.8	14.8**	0.0
(A)	(0.14)			(0.07)				
Belgium	0.43	–	–	0.53	0.42	14.5	11.5**	0.0
(B)	(0.14)			(0.11)				
Findland	0.38	–	–	0.43	0.38	16.7	11.7**	0.2
(FI)	(0.14)			(0.10)				
France	0.31	–	–	0.48	0.31	16.4	14.6**	0.0
(FR)	(0.14)			(0.07)				
Germany	0.24	–	–	0.43	0.31	8.3	17.2**	0.0
(G)	(0.15)			(0.06)				
Greece	0.34	–	–	0.76	0.45	15.3	13.2**	0.0
(GR)	(0.14)			(0.10)				
Italy	0.44	-0.47	0.41	0.59	0.32	11.7	7.7**	0.0
(I)	(0.20)	(0.20)	(0.18)	(0.08)				
Ireland	0.73	–	–	0.52	0.39	18.6	2.2**	0.2
(I)	(0.10)			(0.21)				
Netherlands	–	0.54	-0.67	0.48	0.34	15.1	5.8**	0.0
(N)		(0.16)	(0.13)	(0.07)				
Portugal	0.38	–	–	0.58	0.58	5.3	12.8**	0.0
(P)	(0.18)			(0.11)				
Spain	0.35	–	–	0.71	0.45	6.8	13.4**	0.1
(S)	(0.15)			(0.10)				

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is that there is no autocorrelation in the first nine lags. (2) SF: Shin and Fuller (1998) statistic tests whether an AR(1) operator is nonstationary. We estimate an alternative ARIMA(3,0,1) model and test the null hypothesis. (3) GLR: Generalized Likelihood Ratio (GLR) test of Davis, Chen and Duismuir (1995) for the null hypothesis of noninvertibility of an MA(1) operator, if a second difference and a MA(1) operator to control over-differentiation are added

*Rejects the null hypothesis at the 10% level, **Rejects the null hypothesis at the 5% level.

Table 2: Relative prices models with convergence path

Sample	Variable	AR(1)	AR(2)		Convergence Paramenters				Mean	Resid.	ACF$^{(1)}$	SF$^{(2)}$
	(Mnemonics)	$\hat{\phi}_1$	$\hat{\phi}_{21}$	$\hat{\phi}_{22}$	$\hat{\omega}_0$	$\hat{\delta}_1$	$\hat{l}$	$\hat{g}$	$\hat{\mu}$	Std.Dev.	$Q_{(9)}$	$H_0 : \phi = 1$
		(s.e.)	(s.e.)	(s.e.)	(s.e.)	(s.e)	(s.e)	(s.e)	(s.e.)	(%)		
Panel A: Relative price levels with Germany as Numerarie												
2001/IV	Austria	0.98	–	–	-0.0051	-0.33	–	–	-0. 18	0.26	16.6**	0.0
		(0.03)			(0.0026)	(0.34)			(1.08)			
	Belgium	0.95	0.57	–	-0.0040	0.17	–	–	-0.73	0.27	13.4**	0.0
		(0.07)	(0.17)		(0.0029)	(0.42)			(0.24)			
	Findland	0.98	–	–	-0.0046	-0.91	–	–	16.7	0.31	26.9	0.0
		(0.02)			(0.0022)	(0.12)			(0.3)			
	France	0.77	–	–	0.0030	0.86	5.9	2.0	-2.1	0.19	17.4**	1.8**
		(0.10)			(0.0010)	(0.04)	(2.1)	(0.4)	(0.3)			
	Greece	0.88	0.49	–	0.0052	0.99	–	–	-26.6	0.44	16.6**	0.0
		(0.13)	(0.22)		(0.0028)	(0.04)			(1.7)			
	Ireland	0.89	0.28	0.54	0.0071	0.86	–	–	10.2	0.31	11.8**	0.4
		(0.09)	(0.15)	(0.14)	(0.0086)	(0.08)			(2.5)			
	Italy	0.86	–	–	0.0043	0.92	–	–	-6.9	0.26	14.7**	0.2
		(0.08)			(0.0013)	(0.03)			(0.5)			
	Netherlands	0.90	–	–	0.0063	0.67	–	–	-3.2	0.29	11.2**	0.0
		(0.07)			(0.0023)	(0.12)			(0.6)			
	Portugal	0.76	–	–	0.0069	0.88	7.1	5.6	-23.5	0.33	11.3**	2.0**
		(0.10)			(0.0016)	(0.03)	(1.8)	(0.6)	(0.5)			
	Spain	0.81	–	–	0.0053	0.97	–	–	-22.5	0.25	10.8**	0.9
		(0.09)			(0.0008)	(0.01)			(0.4)			

Notes: (1) Q is the Ljung and Box (1978) statistic for the autocorrelation function (ACF). H_0 is that there is no autocorrelation in the first nine lags. (2) SF: Shin and Fuller (1998) statistic tests whether an AR(1) operator is nonstationary. *Rejects the null hypothesis at the 10% level, **Rejects the null hypothesis at the 5% level.

Table 3: Testing Asymptotic Price Convergence in Mean by Pairs

Panel A: SF Unit Root test for convergence in mean[1]

Country	Germany	Austria	Belgium	Findland	France	Greece	Ireland	Italy	Netherlands	Portugal	Spain
Germany	X	0.0	0.0	0.0	1.8**	0.0	0.4	0.2	0.0	2.0**	0.9
Austria	0.0	X	1.4*	3.8**	0.3	13.2**	0.4	3.8**	0.0	0.0	3.4**
Belgium	0.0	1.4*	X	0.1	0.0	0.0	0.1	0.1	0.0	0.0	0.4
Findland	0.0	3.8**	0.1	X	0.0	0.8	0.0	0.0	0.0	0.3	0.0
France	1.8**	0.3	0.0	0.0	X	0.0	0.1	1.1*	0.0	2.8**	1.8**
Greece	0.0	13.2**	0.0	0.8	0.0	X	0.0	0.0	0.0	15.4**	–
Ireland	0.4	0.4	0.1	0.0	0.1	0.0	X	0.2	0.1	0.7	1.0
Italy	0.2	3.8**	0.1	0.0	1.1*	0.0	0.2	X	3.8**	0.9	2.5**
Neetherlands	0.0	0.0	0.0	0.0	0.0	0.0	0.1	3.8**	X	1.9**	3.4**
Portugal	2.0**	0.0	0.0	0.3	2.8**	15.4**	0.7	0.9	1.9**	X	7.1**
Spain	0.9	3.4**	0.4	0.0	1.8**	–	1.0	2.5**	3.4**	7.1**	X

Panel B: Stability t-student Test for the Convergence Operator[2]

	Germany	Austria	Belgium	Findland	France	Greece	Ireland	Italy	Netherlands	Portugal	Spain
Germany	X	–	–	–	3.2**	–	–	–	–	4.3**	–
Austria	–	X	0.0	0.0	–	0.0	–	4.5**	–	–	6.9**
Belgium	–	0.0	X	–	–	–	–	–	–	–	–
Findland	–	0.0	–	X	–	–	–	–	–	–	–
France	3.2**	–	–	–	X	–	–	0.0	–	2.5**	0.0
Greece	–	0.0	–	–	–	X	–	–	–	0.0	2.6**
Ireland	–	–	–	–	–	–	X	–	–	–	–
Italy	–	4.5**	–	–	0.0	–	–	X	0.0	–	0.0
Neetherlands	–	–	–	–	–	–	–	0.0	X	0.6	4.5**
Portugal	4.3**	–	–	–	2.5**	0.0	–	–	0.6	X	0.0
Spain	–	6.9**	–	–	0.0	–	–	0.0	4.5**	0.0	X

Panel C: Long Run Gap Estimation Results and t-student test for convergence in mean[3]

Country	Germany	Austria	Belgium	Findland	France	Greece	Ireland	Italy	Netherlands	Portugal	Spain
Germany	X	–	–	–	-0.001	–	–	–	–	-17.4**	–
Austria	–	X	–	–	–	–	–	-1.9**	–	–	-8.3**
Belgium	–	–	X	–	–	–	–	–	–	–	–
Findland	–	–	–	X	–	–	–	–	–	–	–
France	0.001	–	–	–	X	–	–	–	–	-17.9**	–
Greece	–	–	–	–	–	X	–	–	–	–	3.5**
Ireland	–	–	–	–	–	–	X	–	–	–	–
Italy	–	1.9**	–	–	–	–	–	X	–	–	–
Neetherlands	–	–	–	–	–	–	–	–	X	–	-8.9**
Portugal	17.4**	–	–	–	17.9**	–	–	–	–	X	–
Spain	–	8.3**	–	–	–	-3.5**	–	–	8.9**	–	X

Notes: (1) SF: Shin and Fuller (1998) statistic tests whether an AR(1) operator is nonstationary, where $H_0 : \phi = 1$, i.e. the AR(1) has a unit root. (2) Stability t-student Test for the convergence operator,$\nu_{ij}(B)$, where $\mathrm{H}_0 : \delta = 1$. If the null hypothesis cannot be reject, there is evidence of a positive or negative ramp depending of the sign of $\nu_{ij}(B)$. For instance g is not finite. (3) The Tau test is a student-t test of Asymptotic Price Convergence in Mean, where $\mathrm{H}_0 : \tau_{ij} = g_{ij} + \mu_{ij} = 0$ is that the long run gap between nominal prices is zero. Only the long-run gap estimation is presented when convergence is accepted, otherwise (–) no evidence of convergence was found.

*(**)Rejects the null hypothesis at the 10% (5%) level.

Table 4: Testing Asymptotic Price Convergence in Variance by Pairs[1]

Country	Germany	Austria	Belgium	Findland	France	Greece	Ireland	Italy	Netherlands	Portugal	Spain
Germany	X	–	–	–	1.2	–	–	–	–	1.3	–
Austria	–	X	–	–	–	–	–	5.3**	–	–	1.3
Belgium	–	–	X	–	–	–	–	–	–	–	–
Findland	–	–	–	X	–	–	–	–	–	–	–
France	1.2	–	–	–	X	–	–	–	–	1.7	–
Greece	–	–	–	–	–	X	–	–	–	–	1.9
Ireland	–	–	–	–	–	–	X	–	–	–	–
Italy	–	5.3**	–	–	–	–	–	X	–	–	–
Neetherlands	–	–	–	–	–	–	–	–	X	–	3.6*
Portugal	1.3	–	–	–	1.7	–	–	–	–	X	–
Spain	–	1.3	–	–	–	1.9	–	–	3.6*	–	X

Notes: (1) Breusch-Pagan test is a Likelihood Ratio test of Asymptotic Price Convergence in Variance, where H_0 is homoscedasticity. If the null hypothesis is rejected, there is conditional heteroscedasticity, with variance decreasing (increasing) with time starting at t^*.

*(**)Rejects the null hypothesis at the 10% (5%) level.

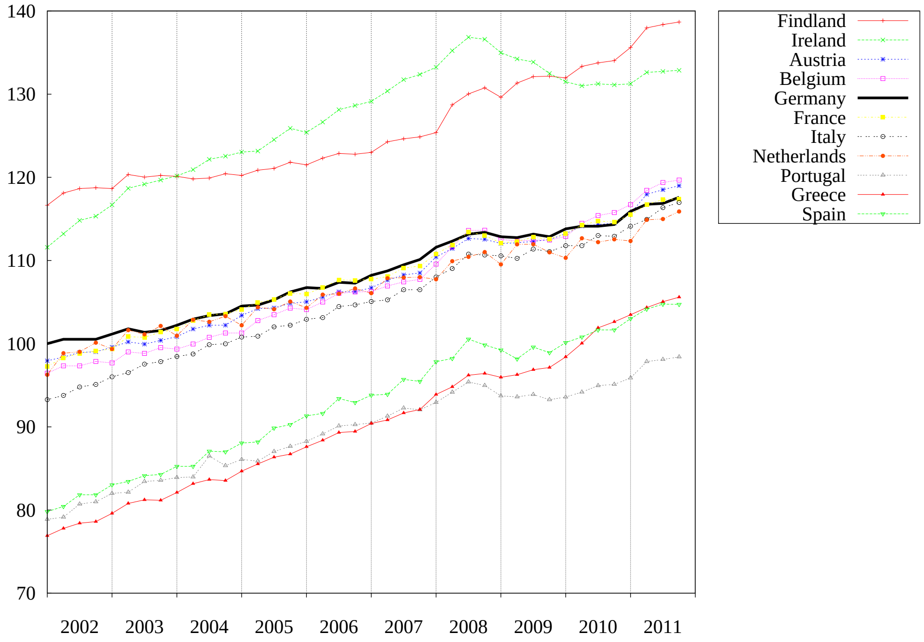

Figure 2: Price levels in the EMU

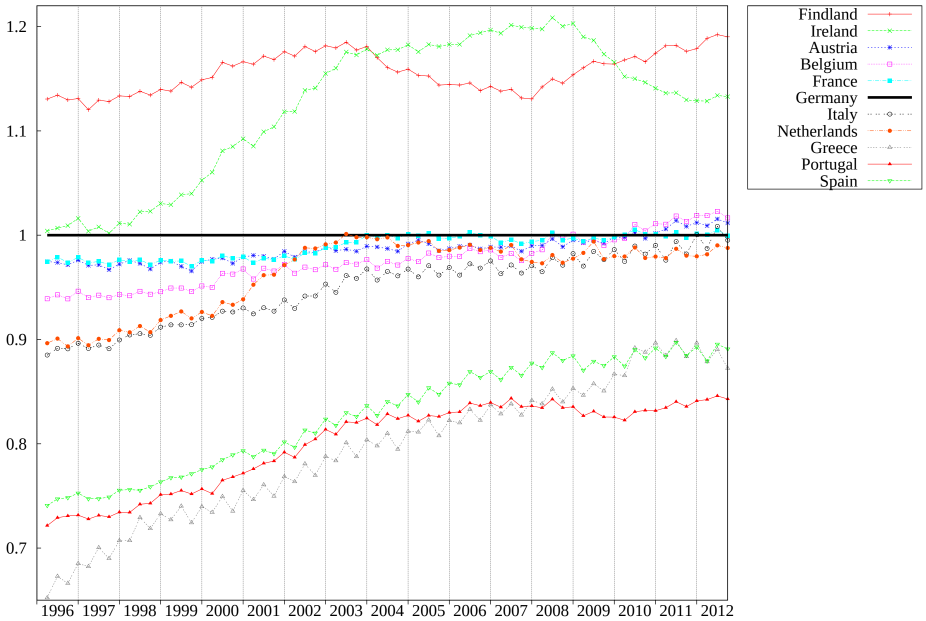

Figure 3: Relative prices in the EMU with Germany as *numéraire*

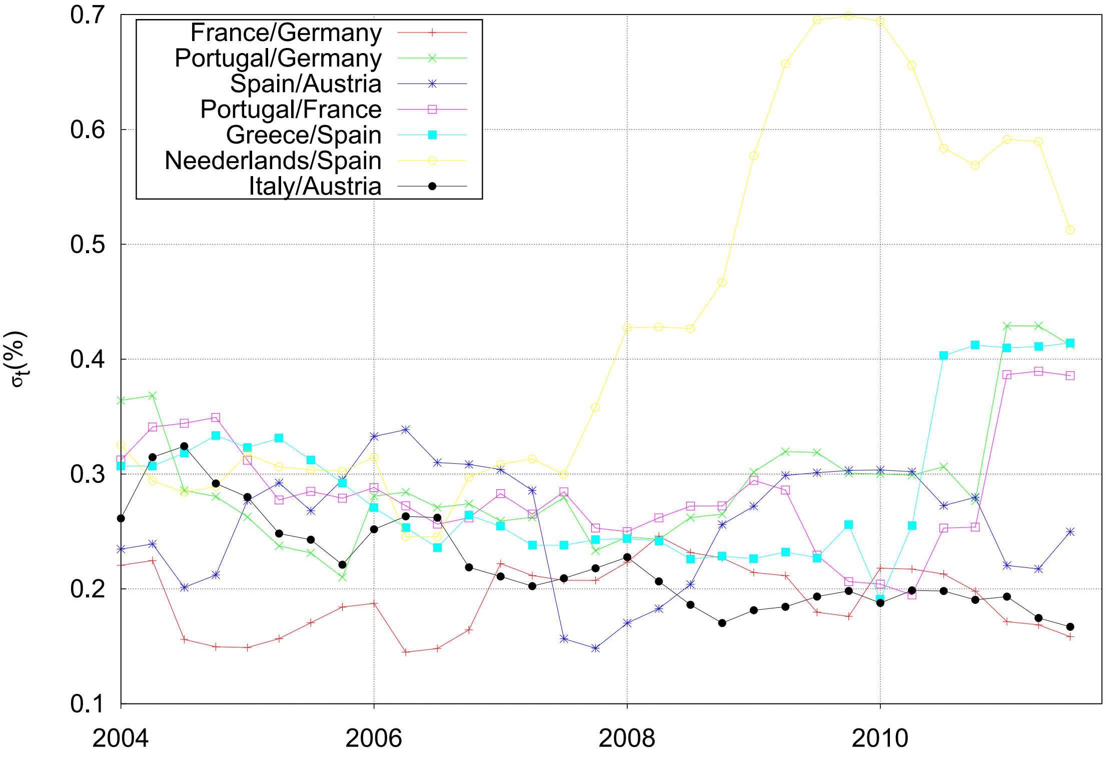

Figure 4: Price Convergence in Variance in the EMU
The series are the residual standard deviations of the natural log of relative prices calculated using rolling windows with a span of $t = 10$.