Optimisation de Portefeuille de Crédits

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Résumé - Le travail présenté dans ce poster a été réalisé en collaboration avec l'équipe de Portfolio-Management (***'PM'***) de BNP-Paribas. L'objectif est d'étudier l'efficacité d'un nouvel algorithme d'optimisation, développé au sein de l'équipe d'optimisation numérique du laboratoire I3M, sur des problèmes de réduction de mesure de risque de Portefeuilles et sous contraintes de revenu. Les résultats obtenus sont directement applicables au milieu financier et en accord avec la théorie sur la gestion de risque.

I- Introdcution

Due to the continuous development of derivative credit products, risk management has became an important activity in asset allocation of financial structures. Basically, credit risk is the risk of a trading partners, called counterparty, not fulfilling their obligations on the due date or at any time thereafter resulting into losses for investor. This credit risk can be generated by three main factors:

- Variation of counterparty's rating: In 2000, general downgrade of telecommunication industry due to an indebtedness of the sector.
- Credit spread variation.
- A credit event: Financial fraud and Bankruptcy of Enron (2001) and WorldCom (2002).

The objective of credit risk management is multiple:

- Provide risk models and evaluation tools. One of the most important mathematical contribution in this field was the development of particular risk measures, such as Value at Risk,
- Apply those models to evaluate risk on financial products and intend to control it.

The mains difficulty is that credit losses are characterizes by a large likelihood of small earnings, coupled with a small chance of loosing a consequent amount of the investment. Thus the loss distribution are heavily skewed and functions corresponding to risk measure are highly non-linear. In literature, many works deal with the convexification of those functionals and apply those methods to simple portfolio examples .

In this poster we focus on the application of a new optimization method to the improvement under constraints of portfolio performances, in particular non-convex risk measures and profitability. The portfolio considered here corresponds to a complex category of credit portfolio, called Collateralized Debt Obligations (CDO), owned by the BNP-Paribas Portfolio Management team.

II- A short introduction to numerical optimization

Since 50's, numerical optimization methods are of great practical importance in various industrial domains such as chemistry [1], telecommunication [2] or technical engineering [3].
Optimization problem consist in improving a desired performance (called '***cost***') of an element by modifying some of its characteristics (called '***parameters***'). Any optimization problem could be decomposed in three main steps:

1-<u>Mathematical modeling of the problem:</u>
The problem is described using equations. Parameters are represented by a real vector (denoted by **'*x*'**) included in a subset Ω (called ***'admissible space'***) of $\mathbb{R}^n$, taking into account some technical restrictions (called ***'constraints'***). The cost is evaluated by a function $J:\Omega\rightarrow\mathbb{R}$ (called ***'cost function'***). Thus the considered optimization problem can be re-written as:

$$\min_{x\in\Omega} J(x) \qquad (1)$$

Or formally: Find the best element included in the subset Ω that gives the lowest value of the function J.

2-<u>Numerical resolution:</u>
In order to solve problem (1), many numerical techniques already exist and could be applied (Gradient Methods, Genetic Algorithm [4], …). Each one has its own advantages and inconveniences: For example Genetic Algorithms are efficient for a large amount of optimization problems but require a lot of computational time.

3-<u>Numerical result analysis:</u>
One important remark is that in optimization there is no theorem that can confirm the fact that we have found at the end of the numerical resolution the best element solving (1). That's why a specialist should always analyze the performance and confirm the interest of the result.

In this work, the algorithm used during step 2 is a mix of various classical methods in order to extract the advantages of each ones. This algorithm has been validated on industrial problems such as: Optical fiber design [2], DNA separator shape optimization [1] and engine pollution control [5]. This method is called Semi-Deterministic Algorithm ('***SDA***').

III- Portfolio risk measure

In order to evaluate a portfolio risk, we use in this paper a particular risk measure, called α-Value-at-Risk ('***VaR***') [6], which corresponds to the α% worst Losses that can occurs in the portfolio. Thus, we need to evaluate the portfolio loss amount and its associated density function. To perform this step we use a Monte-Carlo model that generates loss scenarios. The mains difficulty is that those credit losses are characterizes by a large likelihood of small earnings, coupled with a small chance of loosing a consequent amount of the investment. Thus, the loss distributions are heavily skewed and functions corresponding to risk measure are highly non-linear. So, to solve optimization problems involving VaR, we must use global optimization methods.

IV- Example of application in Portfolio Optimization

We are interested by minimizing the VaR (with α =0.1%) of the PM portfolio keeping the initial portfolio income.

PM portfolio is composed by 500 facilities (credit products) left again 40 Sub-Credit-Portfolios ('***SCP***') and 54 Single-Names ('***SN***' an independent credit). The nominal (money amount) of the portfolio is close to 2.000.000.000 € and the income near to 21.000.000 €.

In order to perform this optimization, we follow the three steps described in section I:

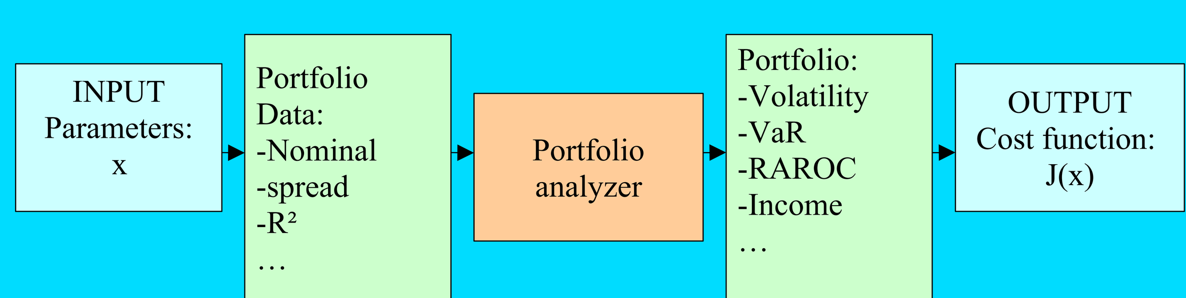
Step 1: Mathematical modeling

1- The parameters considered here are the nominal of each facility in the Portfolio. Thus vector x is the form:
x=(Nominal of Facility 1,…, Nominal of Facility 1500)

2- In order to obtain an interesting and versatile portfolio, we must impose the following constraints:
a) The portfolio income must be higher than 21.000.000€.
b) Each Facility Nominal must be inferior to a certain value, typically: 1.000.000.000€.
c) Each Facilities with a nominal lower than 5.000.000€ is abandoned (nominal is set to 0€).
d) Facilities which have to be modified must satisfy BNP investment guideline. In other cases, the nominal is kept to the initial PM portfolio value.

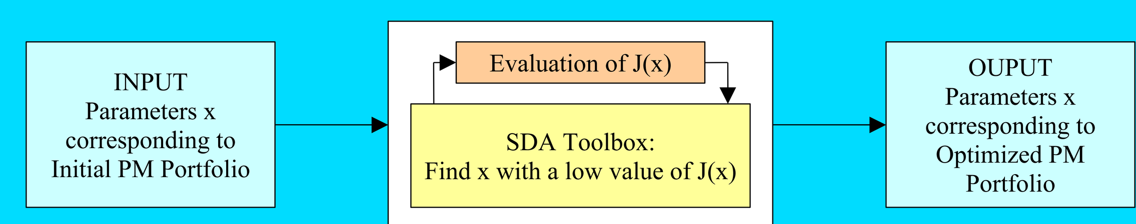
Due to those constraints, the total number of facility which can be modified or added inside the initial portfolio is about 70.

3- The cost function J(x) to be minimized is the VaR measure of the PM Portfolio associated to parameters x. J(x) is computed using a Portfolio copula based model [6] in order to evaluate the portfolio characteristics, trough the following scheme:



Step 2: Numerical resolution

As J(x) is non linear (see Section II), we apply the SDA algorithm, cited in section I, to solve our optimization problem. In a simple way, the SDA software works using the following diagram:



Step 3: Numerical results analysis

Optimized and Initial portfolios are depicted in Figure 1 and main characteristics are given in Table 1. VaR has been reduced by 30 % of its initial value. Portfolio income is kept to its initial value. This is foreseeable as the result must be situated on the constraint border: A portfolio having an income superior to 2.1e7 can be improved by projecting it on the constraint border, the risk is then reduced as each facility nominal is decreased.

Result obtained by the SDA suggests choosing a structure with a high number of different facilities. Each facility has an average nominal of 2e7 E (less for higher risk ones, more for others).

Due to the high correlation and overlap (the same facility is present in various structures) between each SCP, the total SCP nominal invested in this kind of facility is reduced (-26 %). In fact, although a simple Sub-Portfolio is robust to low loss scenarios, combining those products with a high nominal increase the high losses scenario probability: if a default occur in one SCP other SCP have higher chances to be also impacted.

In comparison, investing on diversified SN with reasonable nominal amount (around 2e7 E) decrease the chance to encounter a high loss scenario: Facility defaults in various sector and country should occur in a same scenario to raise a critical loss amount. Thus the total SN nominal is increased and divided in all eligible SN.

V- Conclusion

Global optimization method using a hybrid genetic optimization algorithm has been applied with success to Portfolio risk reduction under constraints. Results obtained are theorically acceptable and directly applicable to credit management. Other kinds of optimization problem have also been performed during this work, involving optimization of various Portfolio performance key indicators such as profitability.

Characteristics	Nominal	Income	Value at Risk
PM Portfolio	2.300.000.000€	21.000.000€	190.000.000€
Opt. Portfolio	2.600.000.000€	21.000.000€	130.000.000€

Tab.1 Comparison between Initial PM and Optimized Portfolio

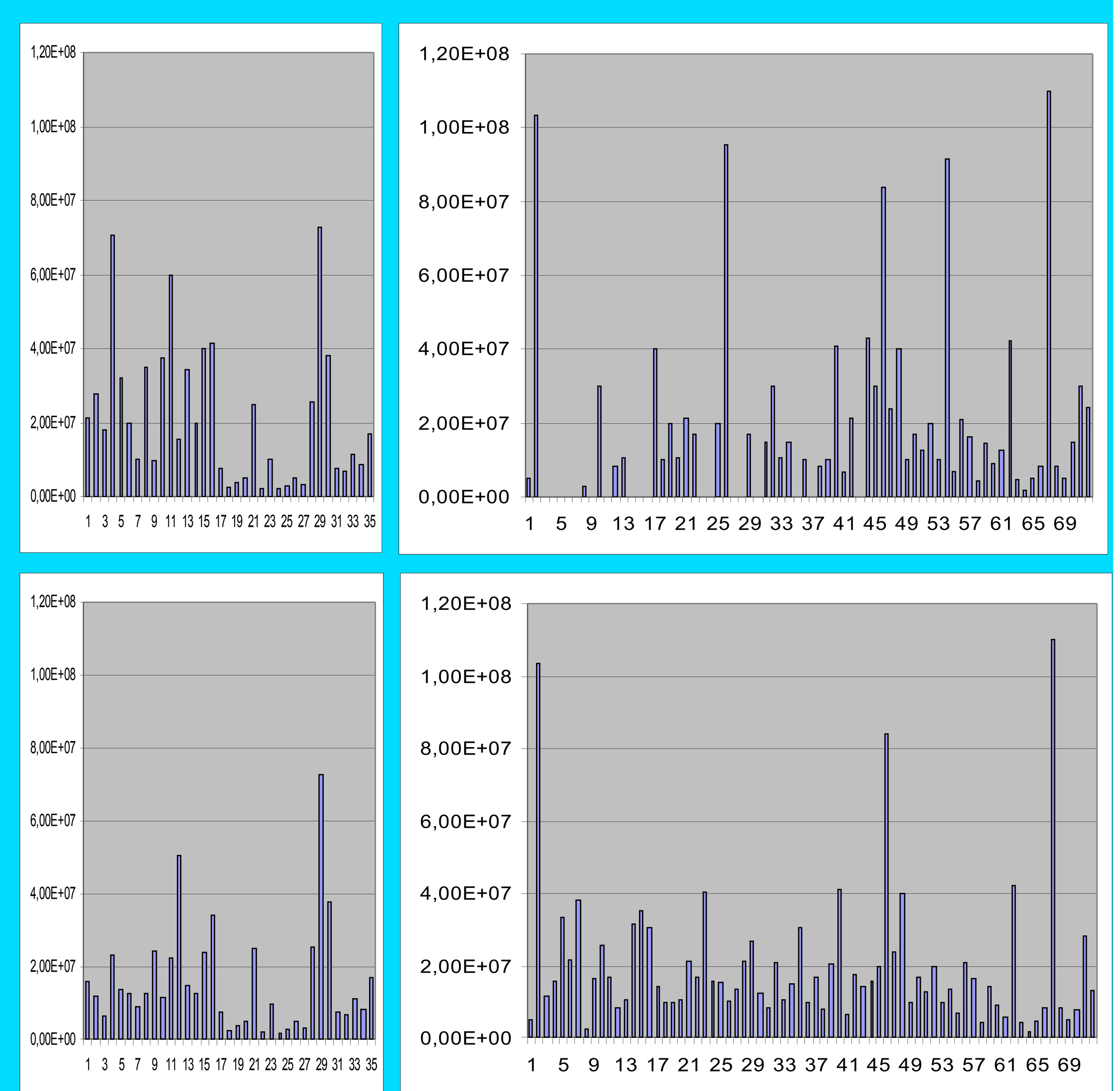


Fig.1 Nominal of SCP (Left) and SN (Right) in Initial (Top) and Optimized (Bottom) PM Portfolio (Nominal in € vs. Facility)

Bibliography
[1] http://www.ims.nus.edu.sg/preprints/2005-36.pdf
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