SOME RESULTS ON FUZZY SYSTEMS

F. J. Montero & J. Tejada

Dept. Estadística e I.O., Fac. C.C. Matemáticas
Universidad Complutense
28040 Madrid (Spain)

Generalization of classical structure function of a system is considered in order to represent non-probabilistic uncertainties attached to systems with degrees of performance between perfect functioning and failed. Some new concepts for static and non-static fuzzy systems are developed.

INTRODUCTION

In traditional System Theory, a system with n components is described by a binary structure function $\phi : \{0,1\}^n \to \{0,1\}$ in such a way that the state ϕ of the system is determined uniquely by the states of its components, $\phi = \phi(x_1,\ldots,x_n)$. Each component and the system itself is supposed to be in either two states: perfect functioning "1" and failed "0" (see [1]). Multistate systems of multistate components has been developed when a finite number of states is supposed (see [5]), but in this paper we consider the whole range of levels between "0" and "1", representing the degrees of performance of functioning. In this context, new concepts are suggested. Examples of fuzzy systems can be easily found in electrical engineering. The study of fuzzy systems will be relevant in the analysis of complex systems (nuclear power plants, social systems,...).

PERFORMANCE IN STATIC SYSTEMS

A fuzzy system can be denoted by (S,ϕ), where $S = \{1,\ldots,n\}$ is the set of components and $\phi : [0,1]^n \to [0,1]$ a fuzzy structure function. (S,ϕ) is said to be:

- Monotonic, if ϕ is non-decreasing

- Continuous, if ϕ is continuous.
- Standard, if $\phi(\vec{0}) = 0$ and $\phi(\vec{1}) = 1$
- Dichotomous, if $\phi(\vec{x}) \in \{0,1\}$ $\forall \vec{x} \in [0,1]^n$.

Let us denote $(y_i,\vec{x}^i) = (x_1,\ldots,x_{i-1},y_i,x_{i+1},\ldots,x_n)$. Given (X,ϕ), a component $i \in S$ is irrelevant if $\phi(y_i,\vec{x}^i) = \phi(z_i,\vec{x}^i)$ $\forall y_i,z_i \in [0,1]$, $\forall \vec{x}^i$. A monotonic standard system with no irrelevant components is said coherent. Given a partition $A_1,\ldots,A_k$ of S, a fuzzy system (S,ϕ) is constructed from the fuzzy subsystems $(A_1,\phi_1),\ldots,(A_k,\phi_k)$ if there exists an integration system (K,ψ), where $K = \{1,\ldots,k\}$, such that $\phi(\vec{x}) = \psi(\phi_1(\vec{x}_1),\ldots,\phi_k(\vec{x}_k))$, $\vec{x}_i$ being the state vector of components in A_i. In crisp theory, systems constructed from coherent systems are coherent. But in our context this property is not necessarily true, since the condition of relevance does not propagate without impossing some aditional hypothesis, like continuity of ϕ's and a more restricted concept of relevance on (K,ψ), for example. The following concepts will be useful in the next section: a fuzzy system (S,ϕ) is said to be of type 1 (type 2) if $\phi(\vec{x}^a) \geq \phi^a(x)$ $(\phi(\vec{x}^a) \leq \phi^a(\vec{x}))$, $\forall \vec{x} \in [0,1]^n$, $\forall a \in (0,1)$, being $\vec{x}^a = (x_1^a,\ldots,x_n^a)$. These mathematical properties propagates through monotonic integration systems. For example, any weigthed mean defines a fuzzy coherent system of type 2. Minimum and and maximum structure functions define two fuzzy coherent systems, simultaneously of type 1 and type 2. Any monotonic and dichotomous system will be of type 1.

PERFORMANCE OF NON-STATIC SYSTEMS

In the previous section, performance has been considered in terms of the degree of functioning at a fixed time. In this section aging will be considered. Fuzzy performance of a non-static system can be understood as a mapping $P:[0,\infty) \to [0,1]$, $P(t)$ meaning the degree of performance at time t (the system is supposed to be connected at time $t=0$). Standard components will be characterized by a non-increasing performance $x:[0,\infty) \to [0,1]$ such that $x(0) = 1$ and $x(\infty) = 0$. If $x_i(t)$ represents the performance of component i at time t, the fuzzy performance of a system (S,ϕ) with n components will be defined by
$P(t) = \phi(x_1(t),\ldots,x_n(t))$. But it is easy to prove

that coherent systems of standard components does not necessarily verify $P(\infty) = 0$. When performance is a non-increasing function being absolutely continuous, the concept of wear rate at time t can be defined as follows: $\tau(t) = -x'(t)/x(t)$, being x' the derivative of x. Wear rate must not be understood as a stochastic conditional failure rate; it represents the intensity rate of the decreasing performance, usually due to mechanical wear. Analogously to mathematical formulation of classical IFR and IFRA distributions, we can propose the concepts of increasing wear rate and increasing wear rate in average performances. A performance function x is said IWR (DWR) when $x(t+h)/x(t)$ is non-increasing (non-decreasing) in $t \geq 0$ for each $h \geq 0$ -as a consequence, we obtain that $\tau(t)$ is non-decreasing (non-increasing) when the derivative exists-; a performance function x is said IWRA (DWRA) if $-(1/t)\ln x(t)$ is non-decreasing (non-increasing) in t. An IWRA performance x is characterized by the fact that $x(a,t) \geq x^a(t)$ $\forall a \in (0,1)$, $\forall t$; a performance function x will be DWRA if and only if $x(a,t) \leq x^a(t)$ $\forall a \in (0,1)$, $\forall t$. Obviously, an IWR (DWR) performance function is IWRA (DWRA), but the converse is not necessarily true. Then the following closure theorem holds:

Theorem.4 Let $(S,\emptyset)$ be a monotonic system of type 1 (type 2) such that each relevant component has an IWRA (DWRA) performance function. Then the system itself has an IWRA (DWRA) performance function.

In particular, coherent systems where $\emptyset$ is a fuzzy switching function (see |4|) with IWRA (DWRA) components will be IWRA (DRWA).

FINAL COMMENTS

Some definitions relative to fuzzy systems have been translated from binary systems, but new concepts appear in our context. It must be remarked that performance function is not understood as a stochastic property. Moreover, many properties of finite multistate systems can be generalized. In particular, when performance is understood as a stochastic property, probabilistic sets (see |3|) can be considered in order to develope the properties of random cut vectors (see |2|).

References

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