Validation of a variance-expected compliance model for truss design

Miguel Carrasco[1], Benjamin Ivorra[2], Angel Manuel Ramos[2]

[1] Facultad de Ingeniería,
Universidad de los Andes
Av. San Carlos de Apoquindo 2200, Santiago de Chile, Chile

[2] Departamento de Matemática Aplicada,
Universidad Complutense de Madrid
Plaza de Ciencias, 3, 28040, Madrid, Spain

Abstract: In this paper, we first remember the mathematical formulation of an original variance-expected compliance model used for structural optimization. It allows us to find robust structures for the main load and its perturbations. In the second part, we valid this model on a 3-D benchmark test case and compare the results obtained to those given by a classical expected compliance model.

Keywords: Structural optimization, truss modelling, expected compliance model, variance-compliance model, global optimization.

1 Introduction

Trusses are mechanical structures consisting of an ensemble of slender bars, connecting some pairs of nodal points in $\mathbb{R}^d$ with either $d = 2$ or $d = 3$. The bars are supposed to be made of a linearly elastic, isotropic and homogeneous material, and long bars overlapping small ones are neglected. They are designed to support some external nodal loads as well as self-weight loads, taking into account certain mechanical properties of the bar material.

Using the *ground structure approach* [14], we focus on the case where the goal is to find bar volumes (topology) which minimize the *compliance* of the truss under mechanical equilibrium, and total volume constraints. See problem (2) in Section 2.

Typically, optimal trusses are unstable under load perturbations (see for example [4, 8]). Therefore, in order to design robust truss structures it is necessary to include theses perturbations into the model. In this context, several approaches might be considered. For instance, the *multiload model* which, in its standard form, consists of minimizing a weighted average of compliances associated with a finite set of loading scenarios [3]. Secondly, in the *worst case design* the objective is to minimize the maximum compliance under a set of discrete loading scenarios ([1, 2]). Finally, in the same direction, the *ellipsoid method* considers a continuum of primary and secondary loads defined by a particular ellipsoid ([8]).

Assuming that loads are random variables, Alvarez and Carrasco [4] propose the problem of finding the truss of minimum *expected compliance* (see problem $(D_{Var}; \mathbb{P}; \alpha; 0)$ in Section 3). They show that this problem can be reformulated as a multiload-type problem, but with a different interpretation of the scenarios. Since multiload problems can be equivalently formulated as a convex finite-dimensional problem, then the expected-compliance model may be efficiently solved, see [3, 7, 23, 9]

The stochastic setting given in [4] allows us to consider the *minimum variance-compliance problem*, where the variance of the compliance is included into the model. This stochastic model is also equivalent to a nonlinear programming problem. However, the variance formulation introduces a non-convex term, that makes this type of problems harder to solve numerically than the minimum expected compliance model. This paper focuses on this variance formulation. Using a global optimization algorithm [25, 20], we analyze numerically a truss from the point of view of its mechanical stability.

We point out that *multilevel stochastic programming* problems have also been proposed to deal with mechanical stability of trusses ([24, 15]). In this approach, approximation and discretization schemes are required in order to estimate expected values. In our formulation, since we compute explicitly the expected values, these types of approximation techniques are not needed at all. Nevertheless, the non-convex term associated to the variance prevents to use it for a large amount of variables.

In Section 2 we recall the well-known *minimum compliance truss* and the *multiload model* [1, 8]. In Section 3, we present the mathematical formulation of the *variance-expected compliance* model, based on [4, 12]. Finally, in Section 4, the results are analyzed and compared in terms of robustness among them.

2 Standard minimum compliance truss design

Set $n = d \cdot N - s$, the number of degrees of freedom on a ground structure consisting of $N \geq 2$ nodes, where $s \geq 0$ is the number of fixed nodal coordinate directions (i.e., coordinates corresponding to support conditions are removed). Let $m \geq n$ be the number of potential bars in the truss structure (of course, $m \leq N(N-1)/2$), and denote by $\lambda_i \geq 0$ the volume (normalized) of the i-th bar with $i \in \{1, \dots, m\}$. External loads are applied only at nodal points and are described in global reduced coordinates by a vector $f \in \mathbb{R}^n$. Under the assumption that each bar is subjected only to axial tension or compression (thus neglecting large deflections and bending effects), the mechanical response of the truss is described by the elastic equilibrium equation (see e.g. [1])

$$K(\lambda)u = f,$$

where $u \in \mathbb{R}^n$ is the nodal displacements vector in global reduced coordinates and $K(\lambda)$ is the *stiffness matrix* of the truss, which has the form

$$K(\lambda) = \sum_{i=1}^{m} \lambda_i K_i. \tag{1}$$

Here $\lambda_i \geq 0$ is the volume of the i-th bar and $K_i \in \mathbb{R}^{n\times n}$ is a positive semi-definite matrix, which corresponds to the specific stiffness matrix of the i-th bar in global coordinates.

The problem of finding the minimum *compliance* truss for a normalized volume constraint of material is given by (see e.g. [3, 10])

$$\min_{\lambda\in\Delta_m} \left\{ \frac{1}{2} f^T u \mid K(\lambda)u = f, u \in \mathbb{R}^n \right\}, \tag{2}$$

where $\Delta_m = \{\lambda \in \mathbb{R}^m \mid \lambda \geq 0, \ \sum_{i=1}^m \lambda_i = 1\}$. This problem is known as *single load model.*

Remark 2.1. For the sake of simplicity, the *self-weight* of the bars is not included in the formulation of the model. If we consider the self-weight we have to replace f by $f + g(\lambda)$ in problem (2), where $g(\lambda) = \sum_1^m \lambda_i g_i$, with g_i being a specific nodal gravitation force vector (see [10] §4).

We also remark that compliance, i.e., the value of the objective function in (2), does not depend on the choice of the equilibrium displacement vector $u \in \mathbb{R}^n$ such that $K(\lambda)u = f$.

Taking into account the particular structure of the matrices K_i in (1), it is possible to show that the *single load* model (2) is equivalent to a linear programming problem, and therefore might be efficiently solved. Nevertheless, numerical results using this model show that optimal solutions may be unstable with respect to the mechanical equilibrium, even under small perturbations in the principal load ([1, 8]). In fact, there are several examples showing some optimal structures which, under small perturbations, give infinite compliance.

In order to handle this inconvenient we may consider a *Multiload Model* (see [3, 7]) that we recall here.

$$\min_{\lambda\in\Delta_m} \left\{ \frac{1}{2} \sum_{j=1}^{k} \gamma_j (f^j)^T u^j \mid K(\lambda)u^j = f^j, \ j = 1, ..., k \right\}. \tag{3}$$

where $\gamma_j > 0$ corresponds to the influence of the scenario j into the model. In this formulation we minimize a weighted average of the compliances associated with k different loads scenarios. The multiload model can be transformed to an equivalent convex quadratic problem and might be solved efficiently. ([23, 9]).

3 Minimization stochastic problem

According to definitions of the previous section we define the function $\Psi\colon \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R} \cup \{+\infty\}$ as

$$\Psi(\xi, \lambda) = \begin{cases} \frac{1}{2}(f+\xi)^T u & \text{if } \lambda \in \Delta_m \text{ and } \exists\ u \in \mathbb{R}^n \text{ such that } \ K(\lambda)u = f + \xi. \\ \quad +\infty & \text{otherwise.} \end{cases} \tag{4}$$

The function Ψ turns out to be proper (i.e. $\Psi \not\equiv +\infty$), lower semi-continuous and convex (see [4]). Therefore, for each $\lambda \in \Delta_m$, the function

$$\Psi(\cdot, \lambda)\colon \big(\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n)\big) \to \big(\mathbb{R} \cup \{+\infty\}, \overline{\mathcal{B}}(\mathbb{R})\big)$$

is measurable. Here $\mathcal{B}(\mathbb{R}^n)$ and $\overline{\mathcal{B}}(\mathbb{R})$ stand for the Borel σ-algebra of $\mathbb{R}^n$ and $\mathbb{R} \cup \{+\infty\}$ respectively.

Next, let us assume that ξ is a random variable corresponding to an uncertain perturbation of Ψ. More precisely, let $(\Omega, \mathcal{A}, \mathbb{P})$ be a probability space and consider a measurable function

$$\begin{array}{cccc} \xi\colon & (\Omega,\mathcal{A}) & \to & (\mathbb{R}^n, \mathcal{B}(\mathbb{R}^n)) \\ & \omega & \mapsto & \xi(\omega). \end{array}$$

According to this setting we study the following stochastic minimization problem:

$$\min_{\lambda\in\Delta_m} \{\alpha \mathbb{E}_\xi[\Psi(\xi,\lambda)] + \beta \operatorname{Var}_\xi[\Psi(\xi,\lambda)]\}, \qquad (\mathcal{D};\mathbb{P};\alpha;\beta)$$

where Ψ is defined by (4), $\alpha, \beta \geq 0$ and $\mathbb{E}_\xi(\cdot), \operatorname{Var}_\xi(\cdot)$ stand, respectively, for the expected value and the variance of a random function.

Remark 3.1. Let $\lambda \in \Delta_m$ and consider $V_\lambda = \operatorname{Im} K(\lambda)$ if $\mathbb{P}\{f + \xi \notin V_\lambda\} > 0$, then the value of $\alpha \mathbb{E}_\xi[\Psi(\xi,\lambda)] + \beta \operatorname{Var}_\xi[\Psi(\xi,\lambda)]\} = +\infty$ and thus the corresponding volume is not a feasible point in $(\mathcal{D};\mathbb{P};\alpha;\beta)$.

We note that taking ξ be a random variable with finite support, $\xi_1, \ldots, \xi_k$. Then, denoting $\gamma_j = \mathbb{P}\{\xi = \xi^j\}$ and $f^j = f + \xi^j$, $j = 1, \ldots, k$ we obtain the model described in (3). Therefore, the previous model extends the well-known *multiload model*.

The next result provides an explicit expression for $(\mathcal{D};\mathbb{P};\alpha;\beta)$, with $\alpha = 1$ and $\beta = 0$ when the perturbation ξ is a continuous random variable (without atoms). In the sequel, given a square matrix $A = (a_{ij})$, we denote by $\operatorname{Tr}(A)$ the trace of A, i.e., $\operatorname{Tr}(A) = \sum a_{ii}$.

Theorem 3.1 (Alvarez, Carrasco [4]). *Let $\xi\colon \Omega \to \mathbb{R}^n$ be a continuous random variable with mean vector $\mathbb{E}(\xi) = 0$, and covariance matrix $\operatorname{Var}(\xi) = PP^T$ with $P \in \mathbb{R}^{n\times k}$ for some $k \geq 1$. Then, the corresponding* minimum expected compliance design problem $(\mathcal{D};\mathbb{P};1;0)$ *is given by*

$$\min_{\lambda\in\Delta_m} \left\{ \frac{1}{2} f^T u + \frac{1}{2}\operatorname{Tr}(P^T U) \right\} \tag{5}$$

$$\text{s.t. } K(\lambda)u = f, \tag{6}$$

$$K(\lambda)U = P. \tag{7}$$

Here the value of the objective function is independent of the choice of $u \in \mathbb{R}^n$ and $U \in \mathbb{R}^{n\times k}$ satisfying (6) *and* (7) *respectively.*

We note that (5)-(7) may be written as a multiload-type problem (3), where the loading scenarios have a new interpretation, see [4]. Thus, in order to construct robust structures in this continuous model, it is not necessary to consider explicitly all the loading scenarios but a good representation of them, according to the covariance matrix $P^t P$.

3.1 Minimization stochastic problem including variance

In this section we deal with the case of $\beta \neq 0$ in the formulation of $(\mathcal{D};\mathbb{P};\alpha;\beta)$. Besides we recall the following result from the formulation of the stochastic model $(\mathcal{D};\mathbb{P};\alpha;\beta)$ as a mathematical programming problem (see [4]). We give the proof here only for completeness

Theorem 3.2 (Alvarez, Carrasco [4])**.** *Suppose* $\xi \sim \mathcal{N}_n(0, PP^T)$*, i.e., the distribution of* ξ *is a n-multivariate normal with mean vector* 0 *and covariance matrix* PP^T*. Taking for simplicity,* $\alpha = 0$ *and* $\beta = 1$*, we have that* $(D_{Var}; \mathbb{P}; \alpha; \beta)$ *is given by*

$$\min_{\lambda \in \Delta_m} \left\{ \frac{1}{2} \operatorname{Tr}(P^T U)^2 + f^T U U^T f \mid K(\lambda) u = f,\ K(\lambda) U = P \right\}.$$

It is worth pointing out the high nonlinearity of this problem, thus it would be interesting to develop a primal-dual formulation of $(\mathcal{D}_{Var}; \mathbb{P}; \alpha; \beta)$ which allows us to implement efficient numerical resolution methods for this alternative.

Proof. We will use the two technical lemmas below whose proof can be found in [4].

Let $\lambda \in \Delta_m$ be a feasible for $(\mathcal{D}; \mathbb{P}; \alpha; \beta)$, i.e. $\mathbb{P}(f + \xi \in V_\lambda) = 1$, where $V_\lambda = \operatorname{Im} K(\lambda)$. By Lemma 3.1 we have that $f \in V_\lambda$ and $\xi \in V_\lambda$ $\mathbb{P}$- a.s., then, there exists u such that $K(\lambda)u = f$ and a measurable function $x \colon \Omega \to \mathbb{R}^n$ such that $K(\lambda)x = \xi$ $\mathbb{P}$-a.s. The existence of such function is ensured by classical results on measurable selections (see for instance [26, Ch. 14]). Then $\Psi(\xi(\omega), \lambda) = \frac{1}{2}(f + \xi(\omega))^T (u + x(\omega))$ for $\mathbb{P}$-a.e. $\omega \in \Omega$. By Lemma 3.2, we get

$$\mathbb{E}_\xi[\Psi(\xi, \lambda)] = \frac{1}{2} f^T u + \frac{1}{2} \operatorname{Tr}(P^T U),$$
$$\operatorname{Var}_\xi[\Psi(\xi, \lambda)] = \frac{1}{2} \operatorname{Tr}(P^T U)^2 + f^T U U^T f.$$

where U is such that $K(\lambda)U = P$ and the proof is fulfilled. □

Lemma 3.1. *Let* V *be a nonempty vector subspace of* $\mathbb{R}^n$*. Under the assumptions of Theorem* 3.1*, we have:*
(i) $\mathbb{P}\{f + \xi \in V\} = 1$ *iff* $f \in V$ *and* $\mathbb{P}\{\xi \in V\} = 1$.
(ii) $\mathbb{P}\{\xi \in V\} = 1$ *iff the columns of* P *are vectors in* V*.*

Lemma 3.2. *Let* $A \in \mathbb{R}^{n \times n}$ *be a symmetric matrix. Under the assumptions of Theorem* 3.1*, if* $\mathbb{P}\{\xi \in \operatorname{Im} A\} = 1$ *and* $x \colon \Omega \to \mathbb{R}^n$ *is a measurable function satisfying* $Ax(\omega) = \xi(\omega)$ *for -a.e.* $\omega \in \Omega$*. Then* $\mathbb{E}(\xi^T x) = \operatorname{Tr}(P^T U)$*, where* $U \in \mathbb{R}^{n \times k}$ *is any matrix satisfying* $AU = P$*. Furthermore, if we suppose that* $\xi \sim \mathcal{N}_n(0, PP^T)$ *then* $\operatorname{Var}(\xi^T x) = 2 \operatorname{Tr}((A\Gamma)^2) + 4\mu^T A \Gamma A^T \mu$ *(see [27]).*

4 Numerical study

4.1 Numerical problem

In order undertake a numerical study into the interest of the formulation $(D_{Var}; \mathbb{P}; \alpha; \beta)$ presented in Section 3 and the choice of the parameters (α, β), we consider a 3-D benchmark structure (related to [23]) composed of a set of 3×3×3 nodes in which all of them are connected with bars. This structure is depicted by Figure 1-Left. A main vertical load f is applied to a particular node.

This problem can be solved by using the dual approach presented in [3, 1]. The obtained solution, depicted by Figure 1-Right, is resilient to f but is highly unstable in cases of small perturbations of this load. Thus, as we are interested in generating resilient structures to small perturbations of f, we consider a

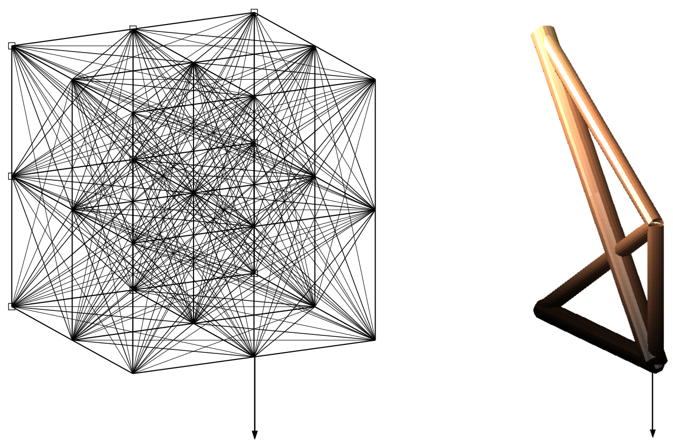

Figure 1: (**Left**) Considered 3D-benchmark structure and (**Right**) associated single-load solution.

random load ξ of law $\mathcal{N}_n(0, PP^T)$ applied to the same node as f and situated in the plane orthogonal to f.

This problem is solved considering both the formulation $(D_{Var}; \mathbb{P}; \alpha; \beta)$ and $S \in \mathbb{N}$ set of values $(\alpha, \beta) \in \Sigma = \{(\alpha_1, \beta_1), ..., (\alpha_S, \beta_S)\}$. In order to solve $(D_{Var}; \mathbb{P}; \alpha; \beta)$, which seems to be a non-convex problem (the cost function associated to the problem appears to have various local minima, see Figure 2), we use a particular global optimization algorithm based on the steepest descent algorithm, where the initial condition is generated using the secant method [25]. A complete description and validation of this algorithm can be found in the following literature [20, 18, 13, 22, 21, 16, 17]. The obtained solutions are denoted by $\lambda_{(\alpha,\beta)}$. It is interesting to notice that for $(D_{Var}; \mathbb{P}; 1; 0)$, $\lambda_{(1,0)}$ the solution is the same to the one obtained using the dual approach described in [12].

In order to have a qualitative comparison of those structures $(\lambda_{(\alpha,\beta)})_{(\alpha,\beta)\in\Sigma}$, we analyze their robustness when they are submitted to random loads, their topology and their compliance iso-contours in function of loads.

More precisely, to study the robustness of each structure $\lambda_{(\alpha,\beta)}$, we consider the random variable $\Psi_{(\alpha,\beta)} = \Psi(\xi, \lambda_{(\alpha,\beta)})$ and we approximate its density function $\rho_{\Psi_{(\alpha,\beta)}}$ using a Monte-Carlo approach [19] (i.e. generating $M \in \mathbb{N}$ values of ξ). Then, we compute the minimum, maximum and average values of $\rho_{\Psi_{(\alpha,\beta)}}$ and a particular risk measure of $\Psi_{\alpha,\beta}$. *Risk measures* on $L^\infty(\Omega, \mathcal{A}, \mathbb{P})$ are mapping $\varpi : L^\infty(\Omega, \mathcal{A}, \mathbb{P}) \to \mathbb{R}$ where $(\Omega, \mathcal{A}, \mathbb{P})$ is a probability space (a complete presentation can be found in [6]). They are used in various areas, such as financial analysis [19], in order to study the value of the worst case scenarios (in our case, the random loads which generate the highest compliances of the structure). Here we focus on a particular and popular risk measure called the Coherent-Value at Risk (*C-VaR*) [5], defined as:

$$\text{C-VaR}_\gamma(X) = \frac{1}{\gamma}\int_0^\gamma \inf[Z| \int_0^Z \rho_X(x)dx > (1-p)]dp \tag{8}$$

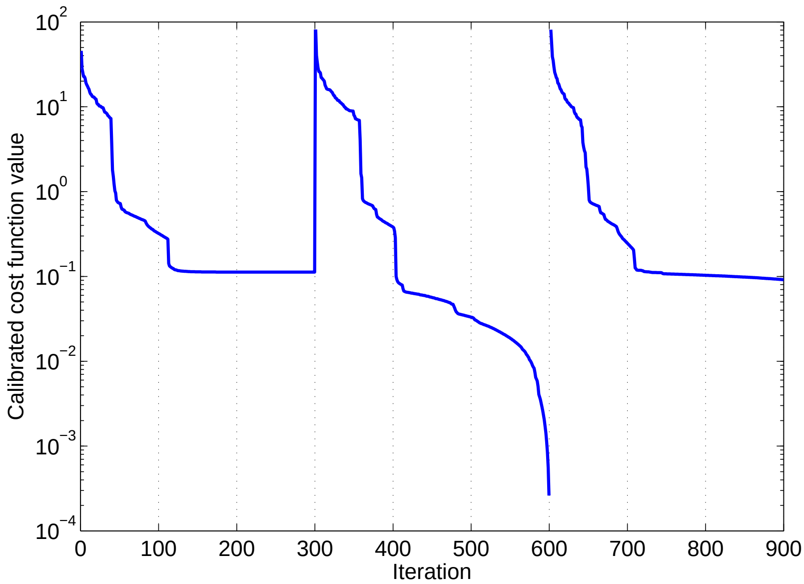

Figure 2: Part of the convergence history obtained from the considered global optimization algorithm used to solve problem $(D_{Var}; \mathbb{P}; (1, 0.25))$. The value of the cost function to minimize is calibrated in order to obtain 0 as the minimum value. As we can observe, the steepest descent algorithm visits various local attraction basins (i.e. the cost function seems to have several local minima).

where γ is a percentile, $X \in L^\infty(\Omega, \Im, \mathbb{P})$ and ρ_X is the density function of X. The C-VaR$_\gamma$ corresponds to the average value of the worst γ % case scenarios of X.

Finally, to study the compliance iso-contours of a structure $\lambda_{(\alpha,\beta)}$ in function of the loads, we consider the function $J_{\lambda_{(\alpha,\beta)}} : \mathbb{R}^3 \to \mathbb{R}$ defined by:

$$J_{\lambda_{(\alpha,\beta)}}(x_1, x_2, x_3) = \frac{1}{2} \bar{f}^T(x_1, x_2, x_3) u \tag{9}$$

where $\bar{f}(x_1, x_2, x_3) = x_1 f + x_2 d^1 + x_3 d^2$, and where d^1 and d^2 are two orthogonal loads situated in the plane orthogonal to the main load, as well as $u \in \mathbb{R}^n$ is a solution of $K(\lambda_{(\alpha,\beta)})u = \bar{f}$. Then, fixing $r \in \mathbb{R}$, we compute numerically one iso-contour of this function considering $J_{\lambda_{(\alpha,\beta)}}(x) = r$.

4.2 Numerical results

We set $f = (0, 0, 1)$, $P = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (both in space coordinate), $\Sigma = \{(1, 0), (1, .25), (1, .5), (1, .75), (1, 1), (.75, 1), (.5, 1), (.25, 1), (0, 1)\}$, $M = 10^6$ and $\gamma = 0.01$ (this level is often used as it reduces the impact of too extreme scenarios [19]).

α	1	1	1	1	1	0.75	0.5	0.25	0
β	0	0.25	0.5	0.75	1	1	1	1	1
Av. Comp.	32.2	34.0	36.3	38.3	40.2	42.6	47.7	56.3	8e+7
E. Comp.	28.1	31.3	33.7	35.8	37.8	40.2	45.3	54.1	8e+7
C-VaR$_{0.01}$	54.3	49.6	50.3	51.3	52.9	54.7	59	66.5	8e+7
Vari.	55	30.7	24.2	20.9	18.7	16.7	13.8	10.8	1.8

Table 1: Summary of the results obtained for each set of parameters (α, β): Average compliance (**Av. Comp.**), Expected compliance (**E. Comp.**), Coherent Value at Risk (**C-VaR**$_{0.01}$) and Variance (**Vari.**) of the structure $\lambda_{(\alpha,\beta)}$. The double bar represents changes in the topology (with apparition/vanishing of bars).

All results are reported in Table 1. The different topology configurations and their compliance iso-contour in function of the loads, obtained considering $r = 0.1$, are presented by Figures 3-4. A boxplot [28] representation of the densities $\rho_{\Psi_{(\alpha,\beta)}}$ is presented in Figure 5.

As we can observe in Table 1 and Figure 5, the solution $\lambda_{(1,0)}$ is less resilient to perturbations of the main load, considering the 0.01 % worst case scenarios, than the solutions $\lambda_{(1,0.25)}$ up to $\lambda_{(1,1)}$. This is confirmed taking into account the compliance of the worst case scenario which is more important for $\lambda_{(1,0)}$ than for all other structures, except $\lambda_{(0,1)}$. Furthermore, the variance of the compliance decreases with the increase of β. This is intuitive as this value is controlled by our optimization problem. However, the average and the expected compliance values raise with the increase of the β proportion. For high values of the β proportion, the structure becomes meaningfully less resilient to the main load.

Figures 3-4 and Table 1 point out that the topology of the structure changes with the evolution of the coefficients α and β. There are four topology configurations. In fact, it seems to have a mass transfer phenomenon, which occurs when β is rising from the upper to the lower bars. The final topology, with two main bars, tends to confirm this situation. This is intuitive as those two bars provide a good resistance to horizontal loads and thus to the perturbations of the main load. In counterpart, they are not resilient to the main load. As we can observe in Figures 3-4, the iso-contours are ellipses which tend to straighten and flatten in the orthogonal plane of the main load f as the β proportion increases. This geometrical evolution confirms that the structure becomes more robust for the horizontal perturbations of the main load but, more fragile to the main load. Currently, considering the discrete model used during this work, our understanding of this mass transfer phenomenon is limited. In Section 5, we present some issues for this limitation.

From previous results, we can deduce that considering the formulation $(D_{Var}$ $; \mathbb{P}; \alpha; \beta)$ with a reasonable proportion of β, it can help to generate more robust structures for the perturbations of the main load. For example, in our particular case, $(\alpha, \beta) = (0.25, 1)$ is a good compromise as it generates the lowest risk measure C-VaR$_{0.01}$, reduces the variance by 45 % and has a compliance value (31) close to the best one (28).

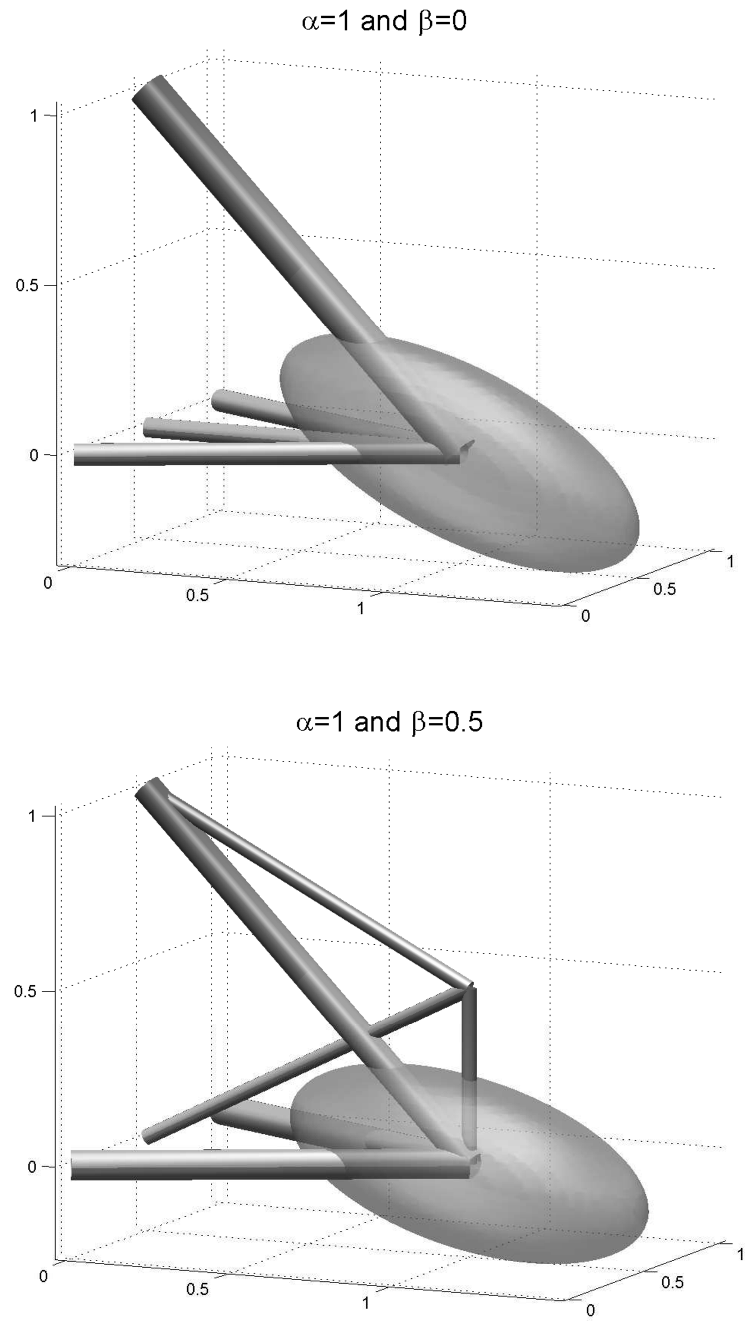

Figure 3: Topology and compliance iso-contour in function of the loads of the structures $\lambda_{\alpha,\beta}$ with $((\alpha, \beta))$ set to (**Top**)(1,0), (**Bottom**) (1,0.5).

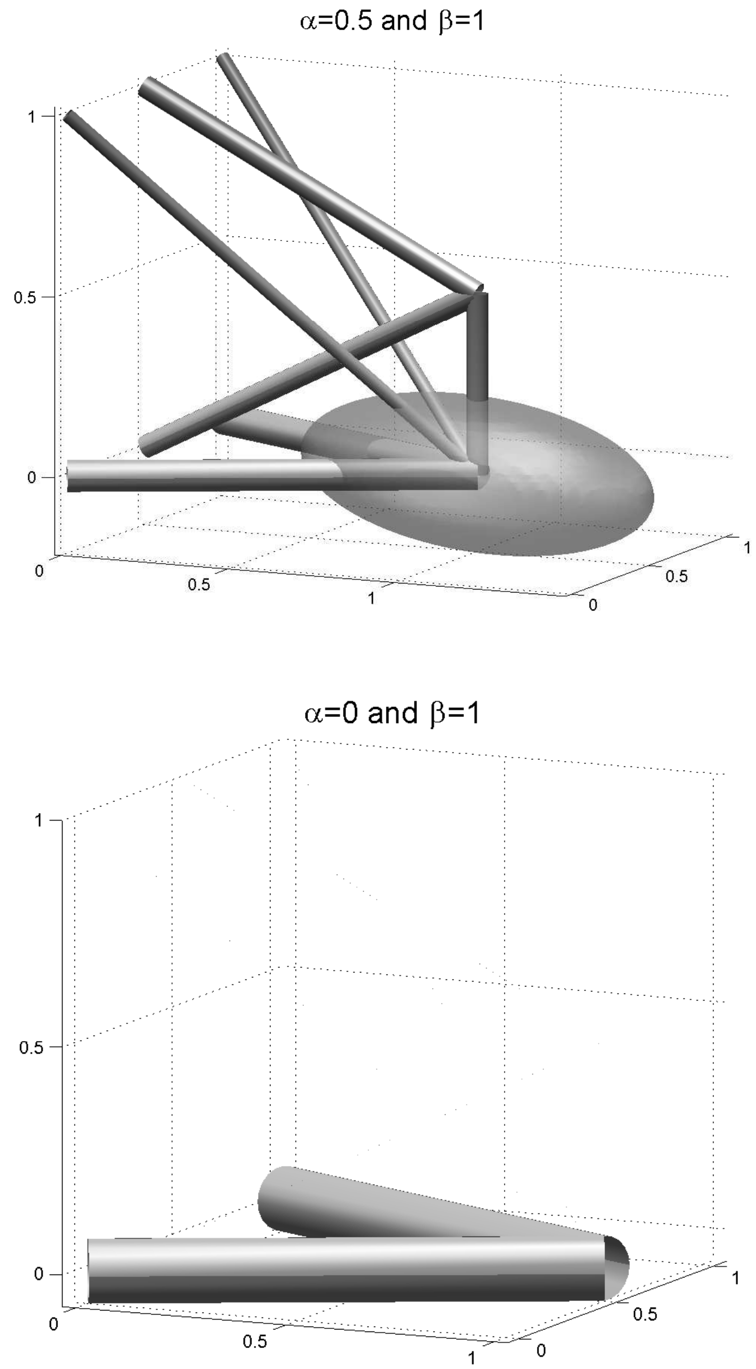

Figure 4: Topology and compliance iso-contour in function of the loads of the structures $\lambda_{\alpha,\beta}$ with $((\alpha,\beta))$ set to (**Top**) (0.5,1) and (**Bottom**) (0,1).

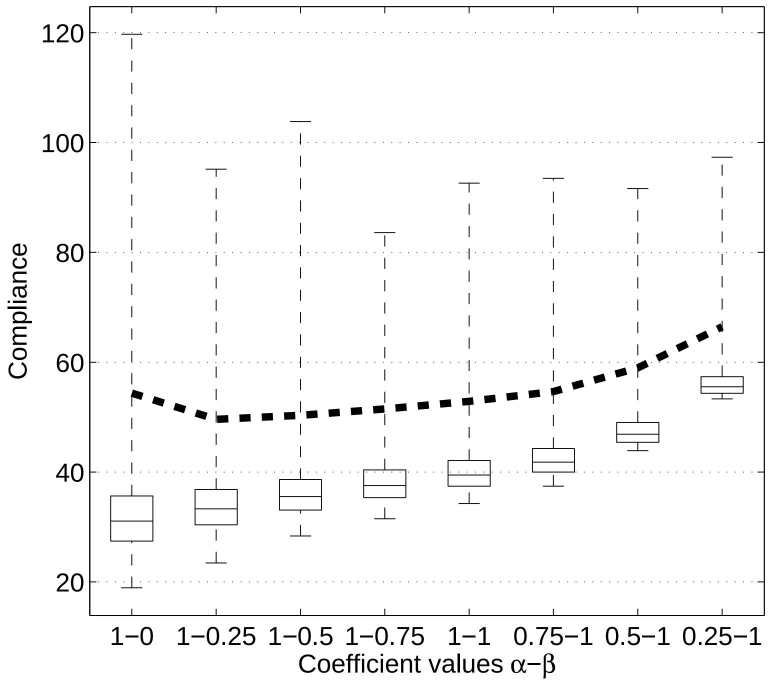

Figure 5: Boxplot representation of the density function of the compliance of the solutions $(\lambda_{(\alpha,\beta)})_{(\alpha,\beta)\in\Sigma-(0,1)}$. C-Var (**- -**) is also reported.

5 Conclusions

A new variance-expected compliance formulation has been validated numerically on a 3-D benchmark test. This new formulation allows us to generate more resilient structures to perturbations of the main load. However, a good choice of the balance between compliance variance and expectancy weights should be chosen in order to avoid fragile structures to the main load.

After this paper, we intend to generalize the formulation and theorems to a continuous model (see [10]), and carry out new numerical tests.

Acknowledgments

This work was carried out thanks to the financial support of the Chilean "Centro de Modelamiento Matemático of the Universidad de Chile"; the Spanish "Ministry of Education and Science" under the projects No. MTM2007-64540; "Ingenio Matemática (i-MATH)" No. CSD2006-00032 (Consolider-Ingenio 2010); the "Dirección General de Universidades e Investigación de la Consejería de Educación de la Comunidad de Madrid"; and the "Universidad Complutense de Madrid" in Spain, under the project No. CCG07-UCM/ESP-2787. The first author was partially supported by the FONDECYT grant 3080037. Finally, we would like to thank Felipe Alvarez for useful discussions of this subject.

References

[1] W. Achtziger. Topology optimization of discrete structures: an introduction in view of computational and nonsmooth aspects. In *Topology optimization in structural mechanics*, volume 374 of *CISM Courses and Lectures*, pages 57–100. Springer, Vienna, 1997.

[2] W. Achtziger. Multiple-load truss topology and sizing optimization: some properties of minimax compliance. *J. Optim. Theory Appl.*, 98(2):255–280, 1998.

[3] W. Achtziger, M. Bendsøe, A. Ben-Tal, and J. Zowe. Equivalent displacement based formulations for maximum strength truss topology design. *Impact Comput. Sci. Engrg.*, 4(4):315–345, 1992.

[4] F. Alvarez and M. Carrasco. Minimization of the expected compliance as an alternative approach to multiload truss optimization. *Struct. Multidiscip. Optim.*, 29(6):470–476, 2005.

[5] P. Artzner, D. Delbaen, J.M. Eber, and D. Heath. Thinking coherently. *Risk*, 10:68–71, 1997.

[6] P. Artzner, D. Delbaen, J.M. Eber, and D. Heath. Coherent measures of risk. *Risk*, 1998.

[7] A. Ben-Tal and M.P. Bendsøe. A new method for optimal truss topology design. *SIAM J. Optim.*, 3(2):322–358, 1993.

[8] A. Ben-Tal and A. Nemirovski. Robust truss topology design via semidefinite programming. *SIAM J. Optim.*, 7(4):991–1016, 1997.

[9] A. Ben-Tal and M. Zibulevsky. Penalty/barrier multiplier methods for convex programming problems. *SIAM J. Optim.*, 7(2):347–366, 1997.

[10] M. P. Bendsøe and O. Sigmund. *Topology optimization*. Springer-Verlag, Berlin, 2003. Theory, methods and applications.

[11] D. P. Bertsekas. *Nonlinear Programming: 2nd Edition*. Athena Scientific, 1995, (2nd Edition, 1999).

[12] M. Carrasco. *Diseño óptimo de estructuras reticulares en elasticidad lineal vía teoría de la dualidad. Estudio teórico y numérico*. PhD thesis, Universidad de Chile, 2003. Engineering Degree Thesis.

[13] L. Debiane, B. Ivorra, B. Mohammadi, F. Nicoud, A. Ern, T. Poinsot, and H. Pitsch. A low-complexity global optimization algorithm for temperature and pollution control in flames with complex chemistry. *International Journal of Computational Fluid Dynamics*, 20(2):93–98, 2006, DOI: 10.1080/10618560600771758.

[14] W. Dorn, R. Gomory, and M. Greenberg. Automatic design of optimal structures. *J mechanique*, 3:25–52, 1964.

[15] A. Evgrafov, M. Patriksson, and J. Petersson. Stochastic structural topology optimization: existence of solutions and sensitivity analyses. *ZAMM Z. Angew. Math. Mech.*, 83(7):479–492, 2003.

[16] D.E. Hertzog, B. Ivorra, B. Mohammadi, O. Bakajin, and J.G. Santiago. Optimization of a microfluidic mixer for studying protein folding kinetics. *Analytical chemistry*, 78(13):4299–4306, 2006, DOI: 10.1021/ac051903j.

[17] D. Isebe, F. Bouchette, P. Azerad, B. Ivorra, and B. Mohammadi. Optimal shape design of coastal structures. *International Journal of Numerical Method in Engineering*, 74(8):1262–1277, 2008, DOI: 10.1002/nme.2209.

[18] B. Ivorra, B. Mohammadi, L. Dumas, O. Durand, and P. Redont. Semi-Deterministic vs. Genetic Algorithms for Global Optimization of Multichannel Optical Filters. *International Journal of Computational Science for Engineering*, 2(3):170–178, 2006, DOI: 10.1504/IJCSE.2006.012769.

[19] B. Ivorra, B. Mohammadi, and A.M. Ramos. Optimization strategies in credit portfolio management. *Journal Of Global Optimization*, Accepted, in Early view, to be published, DOI: 10.1007/s10898-007-9221-6.

[20] B. Ivorra, B. Mohammadi, and A.M. Ramos. Optimizing Initial Guesses to Improve Global Minimization. *Journal Of Global Optimization*, submitted.

[21] B. Ivorra, B. Mohammadi, D.E. Santiago, and J.G. Hertzog. Semi-deterministic and genetic algorithms for global optimization of microfluidic protein folding devices. *International Journal of Numerical Method in Engineering*, 66(2):319–333, 2006, DOI: 10.1002/nme.1562.

[22] B. Ivorra, A.M. Ramos, and B. Mohammadi. Semideterministic global optimization method: Application to a control problem of the burgers equation. *Journal of Optimization Theory and Applications*, 135(3):549–561, 2007, DOI: 10.1007/s10957-007-9251-8.

[23] F. Jarre, M. Kočvara, and J. Zowe. Optimal truss design by interior-point methods. *SIAM J. Optim.*, 8(4):1084–1107 (electronic), 1998.

[24] K. Marti and G. Stöckl. Optimal (topology) design under stochastic uncertainty. *Safety and Reliability,(eds. Schueller, G.I., Kafka, P.)*, 2:1597–1602, 1999. Rotterdam-Brookfield: A.A. Balkema.

[25] B. Mohammadi and J-H. Saiac. *Pratique de la simulation numérique.* Dunod, 2002.

[26] R. Tyrrell Rockafellar and Roger J.-B. Wets. *Variational analysis*, volume 317 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences].* Springer-Verlag, Berlin, 1998.

[27] G. A. F. Seber. *Linear regression analysis.* John Wiley & Sons, New York-London-Sydney, 1977. Wiley Series in Probability and Mathematical Statistics.

[28] J.W. Tukey. *Exploratory Data Analysis.* Addison-Wesley, Reading, MA., 1977.