Crisp Acts, Fuzzy Decisions

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Abstract

Verdegay's paper 'Problemas de decisión en ambiente difuso', published in *Trabajos de Estadística e Investigación Operativa*, was one of the first *fuzzy* papers appeared in the official Journal of the national Society of Statistics and Operational Research (S.E.I.O.). In his paper, Prof. Verdegay pointed out that a problem stated in fuzzy terms should allow a fuzzy solution. On the basis of such a simple argument we shall develope the right way we think Bayesian criticism to Fuzzy Sets should be understood. There is no room in the Bayesian model for fuzzy decisions but for acts, and acts are always crisp.

Key words: Fuzzy Sets Theory, Decision Making, Probability Theory.

1 Introduction.

When one of the authors was asked to review Verdegay's TEIO paper [49], before re-reading it he thought it should be pointed out its relevance in the history of Fuzzy Sets in Spain (we are talking about a paper written in 1981). Devoting some short words about the scientific success of the *fuzzy* research team at the University of Granada during the last 15 years would be enough. Such a research team at Granada has indeed shown a high scientific prestige in the international *fuzzy* community. In fact, a couple of papers by Prof. Delgado and Prof. Vila [51,52] plus another one of Prof. Verdegay himself [50] almost simultaneously appeared in *Fuzzy Sets and Systems*, meaning in this way the second research group in Spain getting a paper published in the official Journal of the International Fuzzy Systems Association (I.F.S.A.). The first paper by a research team in Spain being published in *Fuzzy Sets and Systems* had been written by Profs. Ollero and Freire [32], from the University of Sevilla.

In particular, Verdegay's TEIO paper [49] is searching for fuzzy solutions within problems which have been fuzzily stated. This key idea has indeed played a main role in the past research at Granada University. Many Ph.D. thesis and papers later on developed in Granada have been deeply influenced by this simple argument. Unfortunately for the scientific community, this seminal paper did not reach the right publicity in its moment, and its relevance has to be indirectly measured by means of all those papers appeared in international journals inspired on it. Classical crisp approaches can be considered in fuzzy problems by applying them to every crisp α-cut (representation of fuzzy sets by means of α-cuts had been already obtained by Negoitia and Ralescu [28]). Fuzzy problems can be approached by analyzing all α-cuts (see [11] for an interesting short overview by Delgado-Verdegay-Vila, in spanish).

Moreover, Verdegay's TEIO paper [49] was being published in the official journal of the national Society of Statistics and Operational Research. In fact, this is the second *fuzzy* paper published in *Trabajos de*

Estadística e Investigación Operativa (the first *fuzzy* paper that appeared in the S.E.I.O. journal had been written by Prof. Pardo [33]). Two famous books written in spanish language by Azorin [1] and Trillas [47] had been just recently published. By then, it was notorious that some main probability researchers were strongly skeptic about the relevance of Zadeh's proposal [54,55] as an alternative model for uncertainty.

We shall focus our comments on this issue, hoping the reader will get a better understanding of the relevance of Verdegay's claim for fuzzy solutions in fuzzy problems. This paper is partially a consequence of all those long conversations around late beers the authors had while one of them was visiting the University of California at Berkeley under the Berkeley Initiative in Soft Computing (B.I.S.C.) seminar headed by Prof. Zadeh and the Industrial Engineering and O.R. Department.

2 The Fuzzy-Bayesian controversy.

From a historical point of view, the controversy between Fuzzy Sets and Probability Theory comes from the very beginning of the history of Fuzzy Set Theory. Dubois and Prade [13] refer to a very short paper by Loginov [25], submitted for publication as early as December 24 (1965), where a particular classification problem was considered in order to show that Zadeh membership function may be treated as a conditional probability function. Since then, Fuzzy Set Theory has been the focus of a sometimes strong mathematical controversy. No matter fuzziness claims that uncertainty is not always of a probabilistic nature, articles against the fuzzy approach are still published from time to time. After thirty years since the seminal paper of Zadeh [54] the relationship between fuzziness and probability still deserves the attention of Dubois and Prade [13] in a general talk during the FUZZ-IEEE'93 Conference. A special issue of the IEEE Transactions on Fuzzy Systems -vol. 2(1), February 1994- has been recently devoted to the *Fuzziness versus Probability* controversy. And Zadeh himself made a reference to this issue in his University of Oviedo *Honoris Causa* dissertation (December 1, 1995). The title of his previous talk at the Faculty of Sciences had been a significative "Probability Theory and Fuzzy Logic are complementary rather than competitive" (November 30, 1995).

Main criticism to Fuzzy Set Theory use to come from the Bayesian field, so we shall focus our attention on the Bayesian point of view. We shall try to clarify what we personally think should be the right way of focusing the Fuzzy-Bayesian controversy, without considering possible formal links or bridges by means of more general theories (see, e.g., Dubois-Prade [12]). In particular, we shall not consider Dempster-Shafer's Belief Functions (see, e.g., Shafer [39], the special issue of the International Journal of Approximate Reasoning -vol. 4(5/6), Sept/Nov 1990- with its rejoinder Shafer [44], or the recent volume [53]).

3 The Bayesian Model.

Some Bayesian theorists claim that Fuzzy Sets and Possibility Theory are superfluous and that Probability is all that is needed. In order to get this position rightly understood, in this section a schematic exposition of the Bayesian point of view will be exposed on the basis of an overview of some key papers devoted to the Fuzzy-Bayesian controversy: Cheesman [6], French [16], Laviolette-Seaman [22], Lindley [23], Natvig [27] and Stallings [46] (we recommend Bernardo-Smith [4] for a detailed exposition of the Bayesian Theory).

3.1 Subjectiveness.

Bayesian Theory use to be presented as a *subjective* theory for Probability, opposed to the *frequentist* approach. In fact, some arguments initially coming from the fuzzy field did apply only to a frequentist approach to probability. From a frequentist point of view, it is assumed the existence of probabilities as something defining reality and how real events are connected. Apart from some consistency problems (how can we use arguments based upon *infinite* or even just *long* sequences of *identical* experiments?), it is widely accepted that the ability for modeling uncertainty just within a pure frequentist approach is quite limited. The relevance of this distinction about the meaning of *probability* was made patent very soon (see, e.g., Stallings [46]).

A similar argument appears in the paper that Natvig [27] publishes in *Fuzzy Sets and Systems* with the declared objective of starting a discussion that should bring more light into the controversy. Natvig's main objective is to stress that a possibility distribution "at least in some applications can be interpreted as a family of probabilities (possible subjective)", pointing out that the difference between Fuzzy Set Theory and Probability Theory "is not always clear". In particular, Natvig writes that "a lot of work in Fuzzy Set Theory seems to be motivated by the inadequacy of a frequentistic interpretation of probability when dealing with a series of applications, however not taking into account the development of Bayesian statistics and the subjectivistic interpretation of probability".

It is of key importance that the term *subjective* is rightly understood within the Bayesian framework. Bayesian Theory pursues in fact objectivity. Only that portion of reality that can be checked should be modeled. The assumption of a *probability* associated to a dice, for example, is something that can not be directly checked; what we can check is our own opinion about symmetries on a dice, not the dice itself. In this sense, Bayesians claim that thinking in probability as a limit of frequencies is not *operational*. A dice will be considered a *fair* dice by the decision maker simply because the decision maker accepts certain dice bets as fair bets, not the other way round. If we want to build up a model in order to deal with uncertainty, we have to use only available information, not assumptions. On one hand, such information contains -of course- standard data, that is, the list of results of previous experiments (since the number of previous experiments is always finite, Bayesian Theory should focus its attention on the treatment of finite data). On the other hand, previous information refers to personal experience or intuition (which will be modeled in terms of *prior* probability distributions).

Inclusion of such a *prior* information must not make us think that Bayesian Theory is *not objective*. This would be the case if those *priors* are defined without justification. Personal experience and opinions may not be considered as *data*. Perhaps they can not be directly known. But we do know about the decisions the decision maker makes on the basis of them. We can look at his/her *acts*. Acts will allow Bayesian Theory the required operationality characteristic. Bayesian Theory indeed deals with *personal* activities, but it tries in fact to be *objective* in every aspect.

All those points were exposed in a key paper of Lindley [23]. In addition, he showed that if the scores are additive for different events and if the person chooses admissible values only, then there exists a known transform to values which are probabilities. His result generalizes previous results given in De Finetti [10] and Savage [38]. Hence, it is concluded by Lindley that only probability is a sensible description of uncertainty: "The message -Lindley says- is essentially that only probabilistic descriptions of uncertainty are reasonable". The paper has an added interest because it contains a discussion by Zadeh, among others. In his short comment, Zadeh closes the discussion pointing out Lindley's key assumption on additivity. Lindley answers back that such an assumption is not essential. But the most important idea pointed out by Lindley is that Bayesian approach defines a measure of uncertainty that is "operationally testable". His article explicitly contains the claim that the only way of dealing with uncertainty is (Bayesian) probability. In fact, the controversy should not be understood as fuzziness versus probability. There is no specific battle between Bayesian Theory and Fuzzy Sets. The controversy is Bayesian approach versus any other model for uncertainty.

3.2 Consistent Decision Making.

Related to Lindley's approach are Cox's papers [8,9]. Cox's work shows how basic rules of probability follow from consistence with Boolean algebra, where a key role is played by the basic Aristotelian assumption which classifies every assertion in one and only one of two classes: true and false (Zadeh [56]). Therefore, from this binary logic we reach probability, and from probability we'll fall into the Bayesian model. That's the underlying Bayesian argument in Cheeseman [6]. Cox recognizes that reality needs a more complex model than a *binary* model, but he claims that "any assertion, however ambiguous, can be constructed as a disjunction of unambiguous assertions."

Cox [9] arguments should not be properly taken in order to justify any claim on *crispness* of real world. "Probability Theory does not require clarity in the real world, it only requires clarity in our representation of information about the real world", writes Cheeseman [7]. The model -and for a Bayesian it is a decision making model- is the one that has to be crisp.

What Bayesians really need is a *crisp* space of actions. They do not care about what *probability* means in the real world. There is no such a *probabilistic uncertainty.* As pointed out by Wilson in its comments to Laviolette-Seaman's paper [22], probability does not even exist in our head, but "in the records of the gambles" we have agreed to accept in the past -along with those gambles that can be derived from the mathematical axioms, of course-. Bayesians do not care if concepts are *crisp* or *fuzzy.* They do not like arguments like "the world is fuzzy, therefore our mathematics should also be fuzzy" (see French [16]). We do not know about arguments, we just know about our own acts, they claim ("By their behavior ye shall know them", Lindley graphically quotes). Crispness assumption in the Bayesian model applies to *acts*, and the Bayesian model refers just to *acts.* The set of acts must be clearly well defined. Acts are always crisp. Only occurrence of (crisp) acts can be checked. This is a key Bayesian argument.

So, when French [16] focuses his criticism on fuzzy decision analysis, being this a particular field someone could think that his comments may not be so relevant within the fuzzy controversy. Decision Making plays in the Bayesian model a key role. In fact, many people refer to *Bayesian Probability Theory* just as *(Bayesian) Decision Theory.* If a mathematical model does not give any help to decision makers in order to explore and clarify their own preferences and beliefs, such a model is useless from a pure Bayesian point of view. French's main objective is to show his concerns on how fuzzy mathematics can help decision makers. Hence, in French's paper we can find sentences like "surely I will admit that such a class ('short men') does not have the sharp boundary demanded by classical Set Theory, but then the class of short men is a concept of absolute unimportance to me". A pure Bayesian does not care about the existence of any *probability* or *uncertainty* outside the a decision-maker's mind. "Probability does not exist" wrote De Finetti [10]. Probability is just a measure of uncertainty in each decision-maker's mind, explaining his/her behavior, if *consistent*.

Some more criticisms do appear in French [16]. Among them, an important criticism is on how come membership functions are commensurate in Bellman-Giertz [3] ("what's the meaning of 'equal shortness and fatness'?"). Such a problem is related with the numerical assignment and points out a *scale* problem within membership functions. Although Goguen [18,19] suggested that fuzziness can be explained just in terms of a qualitative relation (a partial order, not necessarily complete) between the different degrees of membership, French rightly claims for a way of checking if a certain curve matches or not the decision-maker subjective attitude, and building consistent operations between degrees of membership. We absolutely need some method allowing introspection in order to reconcile inconsistencies.

Anyway, being *consistent* in our decision making processes represents another key argument in the Bayesian model. Some basic Bayesian assumption can be critizied, but it is not possible to make rational decisions without making such assumptions, points out Cheeseman [6]. Like Stallings [46], Cheeseman's approach (see also [7]) is quite *competitive*, listing superior features of the Bayesian approach against fuzzy sets, both in calculi rules and their real meaning. "All reasoning under uncertainty can be fully captured, and captured correctly, by probability... there is no need to introduce a new theory", Cheeseman writes.

Bayesian probability is a prescriptive theory, not a descriptive one. Bayesians do not pursue explaining how human beings think, but how to model consistent decision making. If you have been coherent in your behavior, your preferences can be modeled as if you had a Bayesian mind. From a Bayesian point of view, fuzziness should be undestood just as uncertainty about meaning. Such an uncertainty can be perfectly represented as a probability distribution defined on the family of all possible intended meanings (Cheeseman [6]). Ocurrence of these *intended meanings* should be testable. Therefore, each one of these intended meanings are *crisp*.

Although the Bayesian model is talking about a *subjective* probability, such a subjective probability is not arbitrary at all. Bayesian Theory indeed models *personal* but *consistent* behavior, which refers to decision making about crisp acts. Moreover, it is understood as an operational concept: it can be measured by means of bets.

3.3 Operationality.

The recent paper of Laviolette-Seaman [22] gives a Bayesian answer to what they consider the five basic arguments of advocates of Fuzzy Sets. But their main argument is the above *operationality* characteristic of the Bayesian approach. Following De Finetti [10], Laviolette and Seaman point out from the beginning that "every definition must always be based upon a criterion which allows measurement". They reproduce an indeed clear paragraph from De Finetti [10]: "That definitions should be operational is one of the fundamental needs of science, which has to work with notions of ascertained validity, in a pragmatic sense, and which must not run the risk of taking as concepts illusory combinations of words of a metaphysical character". The lack of an operational definition of Fuzzy Sets allowing measurement of membership function appears as their main criticism. No theory is useful if not supplemented by a method of measurement. Among discussants to Laviolette-Seaman's paper [22], Dubois-Prade and Hisdal cite some papers addressing this topic (including Hisdal's TEE model [20]), but Dubois-Prade recognize that more work on measurement theoretic foundations of fuzzy sets is needed (see the work being developed by Prof. Turksen and his colleagues [29,30,31,48] for a measurement theoretic approach to Fuzzy Sets).

Bayesian Decision Making allows measurement of *subjective* opinions (*degrees of belief*) by means of *acts* and *bets*. These bets play in the Bayesian model the role of *laboratory experiments*. Consistency of decision maker can also be checked by means of bets. A Bayesian does not care about *opinions* but about acts. They care about acts themselves. They are modeling consistent behavior. Then they conclude that our behavior -if consistent- can be solely modeled as if we had in our mind a probabilistic distribution on the set of acts, as if we had a *probabilistic opinion*.

Bets are indeed another characteristic piece of the Bayesian model. Bets allow measurement of individual acts and help probability to be *operationally* defined. How come an individual reaches to a decision is not observable: final acts (decisions) are the only observable information.

Of course there are some problems with bets. In fact, bets different from $(1/2, 1/2)$ lotteries are in practice difficult to be understood by regular decision makers. Intuition basically works only with *symmetries* (see Mendel [26]). Moreover, using bets may make also difficult to talk about Bayesian probability without its accompanying Utility Theory. Somewhere Prof. Barlow [2] writes that an expert being a coherent Bayesian "means that, in principle, each has an utility function and each has reached his/her decision (...) by maximizing expected utility". Finally, as Lindley [23] points out, "bets are usually supposed to be for small stakes so that utility is locally linear". Not being involved with utility considerations when assessing Bayesian probabilities is not so easy. In fact, utility can be defined as a probability that equates a gamble with a sure consequence (see Lindley [24]). Lindley [24] is of course aware of the difficulties and complexity of assessing prior probabilities and likelihoods (notice they are both subjective in the Bayesian sense). There is also work to be done on probability measurement.

A good survey of measurement-theoretic foundations of probability is Fishburn [14]. Fishburn points out, for example, that "comparative probability axioms are usually thought of as criteria of consistency and coherence for a person's attitudes toward uncertainty". These axioms of subjective probability refer to assumed properties of a binary relation *is more probable than* on a set of propositions, and under these axioms (asymmetry, non-triviality, non-negativity, monotonicity, inclusion monotonicity, transitivity, additivity, complementarity and comparability) we get the uniqueness of the probabilistic representation (see Fishburn [14]).

3.4 The Controversy.

Bayesian Theory is justified as an operational consistent decision-based measure of personal uncertainty. From a Bayesian point of view it has no sense to think about *probability* if there is no decision maker, if there is no decision problem stated in terms of *crisp* actions. What De Finetti [10], Savage [38] and Lindley [23] are showing is that any *consistent* set of actions could have been modeled in terms of these *personal* probabilities (see also Lindley [24] for alternative references of axiom systems leading to the inevitability of probability). "It is not being claimed here -Cheeseman [6] writes- that probability represents the way people think". Bayesian rationality is about acts, no matter which previous arguments are supporting them. A Bayesian claims just for people to be consistent in their acts.

The only thing a Bayesian will be worried about is rationality in personal decision making (acts) procedures. What they claim is that if you want your acts to be consistent and not be fooled with Ducht Books, then your acts should be modeled according to the probability model. Modeling personal acts is the only realistic goal. Bayesians will have no problems in agreeing with Klir [21] on the existence of different kinds of uncertainties. But a Bayesian would say it has no sense to consider uncertainties that have not been operationally defined, and only acts are operational. It is a waste of time to consider the existence of different kinds of uncertainties (or even more general models), since there is only one consistent and operational measure of personal uncertainty.

For a Bayesian, being OPERATIONAL means that only ACTS should be modeled. In case YOU have been CONSISTENT in your own acts, these acts can be solely modeled according to the Bayesian model. For sure you have been acting as if you had in mind a lottery decision making procedure.

4 Discussion.

The classical example of *tallness* as a fuzzy set may lead to some misunderstanding. A Bayesian approach where the degree of membership is understood as a likelihood indeed captures a particular uncertainty about *tallness*. A likelihood function seems fully appropriate in a conversation where the word *tall* is used in order to guess the particular *height* of John. In this case, the vague concept *tall* is used to carry the lack of a desirable exact estimation about the crisp property *height*. The distance may not allow to get a good enough estimation of John's *height*, but in comparison with other people one may perceive he is tall. Analogously, if somebody tells to one of us that *John is tall*, one may immediately wonder what his exact *height* is. In both cases, one is guessing the exact *height* just on the basis of such a fuzzy information *tall*, as perceived by oneself or as intended by others. There is nothing fuzzy about *height*.

But it may be the case that the required representation of the crisp attribute is not clear or it is too complex to be formalized, as for many composite concepts (see Saaty [36]). Being complex, an alternative fuzzy approach may be justified at least due to a low *operativeness* of the Bayesian approach. (even if we accept Cox's argument so that any problem can be approached by a binary model, it does not assure that such a binary model is computationally operative, and some more operative approximation is still desirable). A key characteristic with *tallness* is that *height* can be represented in the real line. An analogous representation for more complex concepts may be in general far from being clear. We may not be even able to guess a descriptive family of crisp attributes. Talking about a 'beautiful landscape' does not suggest guessing any particular landscape or assigning any probability distribution on all possible landscapes. Sometimes we do argue directly with fuzzy concepts without even intending any exact representation upon which probability distributions can be defined. *Beautifulness* is difficult to model according to a probabilistic uncertainty model on the basis of an observable and measurable representation as *tallness* is in terms of *height*. Again, a pure Bayesian will be interested only in those acts associated to *beautifulness*. For example, answers to bets based upon the place you would like to spend the next one-weekend holiday.

Bayesian Theory has shown in the past the formal weakness of the Frequentist approach to Probability. If either fuzziness rests on a problem of meaning or on a problem of perception (Saaty [36]), in both cases such a vagueness is highly subjective (they obviously depend upon our past personal learning process and experiences). So, if one has to choose for a probabilistic approach to fuzziness, one is tempted to follow the Bayesian postulates.

But Bayesian Theory is not a so closed theory. It has its own problems and paradoxes. Shafer's review [41] on Savage's *The Foundations of Statistics* [37] provides, together with discussants, and excellent exposition. For example, Bayesian model assumes that every person has always well-defined complete transitive preferences, regardless any difficulty (if you insist not being transitive, that's the end). Comparison between acts may not be clear enough for the decision maker. Strength on comparisons vary and may be so low as incomparability. Decision maker does not always defines a complete order relation on the set of alternatives (see, e.g., Fodor-Rooubens [15] and Roubens-Vincke [34]). Incomparability is not allowed in the Bayesian model. Gambling with bets in search for fair prices may be not accepted after all (see also Shafer [44]). Moreover, the decision must be between clearly defined acts, so consequences can be checked (otherwise operationality is lost). Bets on fuzzy events, as suggested in Smets [45], are not so easy to accept. The bet "you win 10 dollars if the next man enters the room is tall; otherwise, you lose 10 dollars" does not give a rule of what to do if we do not agree on if the man just entering the room is *tall* or *not tall* (agreement will be for sure more difficult to reach if instead of 10 dollars we fix a much higher prize).

Do people have a clear intuition about Probability and Utility so bets can be understood by decision maker? Can these theoretical bets be assimilated to real decision experiences? We do have some kind of intuition about what *frequency* is, always on the basis of some physical symmetry or exchangeability. But being both pure mathematical objects, the fact that Probability and Utility seem to be *practical* concepts does not imply that measurement based upon bets are always as *operational* as they should be.

From our point of view, the main argument is that the concepts of *decision* and *act* should not be confused. Bayesians always talk about acts: they are particular and observable. But usually the observable act is a consequence of a previous inner decision process which is in fact the important human decision (which may be not observable in the Bayesian sense). For example, while walking on Berkeley streets one may decide to help a homeless sitting ahead on the sidewalk. Such a decision is not an act. Giving a particular amount of money is an act, and it depends on many circumstances (some of them are basically random, as the total amount of money each one of us has in our pockets and how it is distributed in bills and coins). Once one has decided to help that homeless, we are faced to the problem of choosing an act according to that decision, between those alternatives crossing our mind while still walking towards him. Once well-defined acts have been chosen among all those at hand developing our previous decision about attitudes is where the Bayesian model works. Acts as alternatives use to appear as those (crisp) alternatives at the end of an inner hierarchical decision process. For example, once one has decided to help that homeless, we may choose giving him some 'spare change', which is neither a well-defined decision (in fact, we introduce our hand in our pocket, pick up at random a couple of coins, and only then, when we give these two coins to that homeless, we have an act). Each step in the inner decision is about a personal attitude which is not well defined. A Bayesian would not take care about those previous non-observable decisions about attitudes (they are not acts, they are not crisp, they are not observable or testable). Of course "telling the observer our decision" is an act (Giles [17]), but it is more a game against the observer, which in that very same moment is no longer an outside observer (the decision making problem should be re-defined).

Our objective may be to model an inner reasoning process, not just modeling observable acts. In this sense it is essential -as claimed in Verdegay's paper [49]- to search for fuzzy solutions to fuzzy decision making problems. It is true that from acts we can try to guess how our mind worked out the final decision. But our inner decision process may not be making use at all of *acts* as pieces of the model, neither well defined consequences. In our homeless example, this is the case only once we are faced to the last step of what to do in order to help him: "how many and which ones from those coins in my pocket I'm gonna give him?"). In fact, we reach to the decision of helping that homeless after clearly ill-defined arguments (and that may include the existence of a God giving us in a future life *ten times* what we are giving now to this homeless). "When asked to make choices, they (people) look for arguments on which to base these choices", says Shafer [41]. Our decision making procedures are most of the time decision processes quite hierarchically structured (in fact, Saaty [36] claims that a fuzzy property is a hierarchical system of properties). Hopefully, this decision process involves in each step less and less abstract alternatives, so pure acts will appear only at its final stage. "The preferences we construct depend on the questions we ask ourselves, and hence the selection of questions is an essential part of the construction" (Shafer [41]). We do not choose among all possible acts we can imagine. We construct decision processes by comparing *attitudes* (*policies*) which could be understood as poorly-defined families of acts; each one of these processes will at the end allow us to define a *nice* set of feasible acts. It's a defuzzifying decision process leading to the construction of an *a priori* not known crisp set of feasible acts as the set of alternatives (see Shafer [41] for the importance of *constructive* methods). Choosing among these *attitudes* or *policies* means we are choosing among different models of actions, not particular acts, and in this context Bayesian Theory has problems in being adequately stated (see Shafer's comments to Cheeseman's paper [7] and his comments on the way Box [5] understands the appropriate on the division of labor between Bayesian and non-Bayesian methods within Statistics). Bayesian Theory will fit perfectly only into the last stage of such a decision process, when the problem has been already stated in terms of (crisp) acts. Acts are the output of an inner decision making process. Decision making, if understood in a wide sense, contains decisions which are not acts. Human key decisions do not use to be acts (the important human decision was not giving two dimes to that homeless, but deciding to help him in some way). Much more important than modeling our acts should be the understanding of our arguments. A pure Bayesian will claim that this approach is not operational.

Another example: increasing support for public health is just a promise comming from politicians. The act will only come when the government fixes a particular budget for public health, attached to particular projects. Of course we do ask from politicians to have some coherency between their promises and their acts. But notice that when a politician is being not clear about his/her proposals, it is not only because he/she does not want to be caught in flagrant contradiction, but also because it leaves the door open for new information and even possible agreements with opponents. It's a difficult game, but our (fuzzy) language allows such constructive bargaining games searching for new solutions. We do need poorly-defined concepts in order to reach to agreements in democratic societies. More than the particular voting procedure, it is the consensus process of defining which set of alternatives has to be voted the key process in democracy.

Language is an open system that evolves -like Science- searching for a better explanation of reality. No model -about language or in Science- can fully explain reality. We can try to improve explanations by getting better and wider models, but if not based upon acts, again these models will be considered non operational by Bayesians.

Many Fuzzy theorists share with Frequentists the Platonic way they relate with reality. In fact, many arguments in Fuzzy Sets, as Frequentist Probability, come from the attempt to get mathematical models about the (perhaps subjective, but pre-existent) world. Such a model will most probably help us to make future decisions, but that's not the main issue to be taken into account in order to create a model. Better understanding of reality may be by itself the direct objective of our studies (though if we want to understand reality sure it is because we have some previous decision problem related with that piece of reality). "I want models that represent what I think reality is" Bezdek claims in the Editorial of the special issue of the IEEE Transactions on Fuzzy Systems 2 (1), February 1994. Both Frequentist and many fuzzy models share a *descriptive* representation view, opposite to the *prescriptive* decision making approach of the Bayesian Theory (see comments of Dubois-Prade in Laviolette-Seaman's paper [22]). The problem is again that a Bayesian claims that only *acts* are *operational*, so this is the only thing we should be actually modeling.

Bayesians have a very liberal definition of probability. They maintain that any probability that a rational decision maker comes up with is a valid probability (in particular, De Finetti [10] showed under which conditions -i.e., exchangeability- a Bayesian's opinions would equal a frequentist's). Given the very liberal interpretation of fuzziness, the Fuzzy-Bayesian controversy comes at no surprise.

As pointed out by Shafer [44], probability provides us with a protocol for new information in statistical problems, "but in problems of everyday life, we constantly encounter unexpected information -information for which we have no protocol and no frequency experience." Again, Bayesian do not care about these arguments meanwhile they are non-observable. Since only individual acts are considered observable, Bayesians will have difficulties in modeling nothing except individual decision (act) making problems. It is not only that probability is *personal* and *prescriptive* what characterizes the Bayesian point of view, but the fact that the whole mathematical model is built from individual acts. It is not that Probability Theory provides a good model for *uncertainty*, but that individual acts -if consistent in the Bayesian sense- can be explained as if decision maker was playing lotteries.

The key question of Cheeseman [7] (he is referring to whether probability or logic should be the fundamental language of Artificial Intelligence) "hinges on the question whether there is such a thing as truth (independent of any agent) or only beliefs (of agents) that can change with new information". *Truth* is not an *operational* concept in the Bayesian context.

As Dempster points out in the discussion to Shafer's paper [40], "many statisticians are inclined to reject the conception of probability as a degree of belief, at least in their scientific work. Instead, they prefer to stress an objective basis in an empirical frequency, or possibly in physical symmetry, as of a dice". In fact, Dempster's original work -Shafer [43] writes- was motivated "by the desire to obtain probability judgments based only on sample data, without dependence on prior subjective opinion". For example, non informativeness can be modeled by means of the pair of trivial upper and lower bounds, instead of any particular prior distribution. Shafer [42] asks for an effort to "reformulate the mathematical foundations of probability in a way that incorporates rather than ignores the pioneer's insights into the fundamental role of fair price and repetition." Probabilities, Shafer points out in [41], "should be constructed by examining evidence, not by examining one's attitude toward bets."

Taking the *path of realism* is defined by Roy [35] as "acknowledging that a certain number of objects, about which we can reason objectively, pre-exist 'out there' independently of any research carried out". This is the classical 'scientific' attitude, which is basically *descriptive for discovering*. Roy [35] is talking about the future of Decision Aid Sciences, and distinguishes such a *path of realism* from the *axiomatic path*, which basically looks for norms for prescribing (Roy notices that under this approach it is usually required some statement which uses the path of realism, as the existence of a complete order in the set of betting rates in the mind of a decision maker). Roy [35] claims that Decision-Aid Sciences should be mainly developed within a third path: the *path of constructivism*. This path acknowledges the difficulties in apprehending clearly, and basically pursues to provide the basis for what he calls a *recommendation*. "The concepts, models, procedures and results are here seen as suitable tools for developing convictions and allowing them to evolve" (Roy [35]). Roy's position [35] is indeed related with Shafer's advocate [41] for *constructive* methods.

5 Final Comments.

A basic scheme of the Bayesian position should be understood in the following terms: in order to be useful, any theory has to be operational, and in order to be operational such a theory should be based upon observable facts; for a Bayesian, the only observable facts are personal acts, so these personal acts is the only thing we should be modeling; then, the only model that allows a consistent decision making procedure among acts is Probability. That does not mean that any probabilistic uncertainty exists: it just means that if an individual has a consistent behavior, his/her acts can be modeled as if he/she is playing lotteries. Any other approach to uncertainty will be non operational or will lead to non-consistent decision making procedures.

The key point is the Bayesian claim that only acts are observable. Listing advantages and features about particular details is not helping to focus such a deep argument.

Decision Making in a Bayesian context is a limited model. Bayesian procedures can be properly applied only when such a Bayesian Decision Making problem has been stated. It may be the case that such a Bayesian problem is so complex to be described in terms of crisp characteristics or crisp alternatives, that perhaps a hierarchical solution process based upon fuzzy arguments or fuzzy alternatives can be found more operative.

Not every human Decision Making problem is a Bayesian Decision Making problem (alternatives may be not clearly defined in such a way that bets are not fully accepted by the decision maker). Bayesians will still claim these non-Bayesian Decision Making problems are not operational, therefore out of any Bayesian interest.

When Prof. Verdegay was claiming in his TEIO paper [49] for fuzzy solutions to problems fuzzily stated, he was addressing a key piece in the problem of modeling human *constructive* decision making processes.

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References

[1] F. Azorín, Algunas Aplicaciones de los Conjuntos Borrosos a la Estadística. Instituto Nacional de Estadística, Madrid (1979).

[2] R. Barlow, *Combination of expert's opinions based on decision theory.* In: A.P. Basu (Ed.), Reliability and Quality Control. Elsevier Science Publishers, North-Holland (1986).

[3] R. Bellman and M. Giertz, *On the analytic formalism of the theory of fuzzy sets.* Information Sciences 5 (1973), 149–156.

[4] J.M. Bernardo and A.F.M. Smith, Bayesian Theory. Wiley, New York (1994).

[5] G.E.P. Box, *Sampling and Bayes' inference in scientific modeling and robustness (with discussion).* Journal of the Royal Statistical Society (A) 143 (1980), 383–430.

[6] P. Cheeseman, *Probabilistic versus fuzzy reasoning.* In: Uncertainty in Artificial Intelligence. L.N. Kanal and J.F. Lemmer (eds.), Elsevier Science Pub. (1986), 85–102.

[7] P. Cheeseman, *An inquiry into computer understanding (with discussion).* Computational Intelligence 4 (1988), pp. 58–142.

[8] R.T. Cox, *Probability, frequency and reasonable expectation.* American Journal of Physics 14 (1946), pp. 1–13.

[9] R.T. Cox, *Of inference and inquiry - An essay in inductive logic.* In: The Maximum Entropy Formalism, R.D. Levine and M. Tribus, eds. Massachusetts Institute of Technology Press, Cambridge, MA. (1979).

[10] B. De Finetti, Theory of Probability. Wiley, New York (1974).

[11] M. Delgado, J.L. Verdegay and M.A. Vila, *Decisión en ambiente fuzzy.* Arbor CXLVI 573-574 (1993), 109–126.

[12] D. Dubois and H. Prade, *Fuzzy sets, probability and measurement.* European Journal of Operational Research 40 (1989), 135–154.

[13] D. Dubois and H. Prade, *Fuzzy sets and probability: misunderstandings, bridges and gaps.* In: Second IEEE International Conference on Fuzzy Systems (vol. II). IEEE Press (1993), 1059–1068.

[14] P.C. Fishburn, *The axioms of subjective probability.* Statistical Science 1 (1986), 335–358.

[15] J. Fodor and M. Roubens, Fuzzy Preference Modeling and Muticriteria Decision Support. Kluwer, Dordrecht (1994).

[16] S. French, *Fuzzy decision analysis: some criticisms.* TIMS/Studies in the Management Sciences 20 (1984), 29–44.

[17] R. Giles, *The concept of grade of membership.* Fuzzy Sets and Systems 25 (1988), 297–323.

[18] J.A. Goguen, *L-fuzzy sets.* Journal of Mathematical Analysis and Applications 18 (1967), 145–174.

[19] J.A. Goguen, *The logic of inexact concepts.* Syntheses 19 (1968-69), 325–373.

[20] E. Hisdal, *Are grades of membership probabilities?.* Fuzzy Sets and Systems 25 (1988), 325–348.

[21] G.J. Klir, *Is there more to uncertainty than some probability theorists might have us believe?* International Journal of General Systems 15 (1989), 347–378.

[22] M. Laviolette and J. Seaman, *The efficacy of fuzzy representations of uncertainty (with discussion).* IEEE Transaction on Fuzzy Systems 2 (1994), 4–42.

[23] D.V. Lindley, *Scoring rules and the inevitability of probability.* International Statistical Review 50 (1982), 1–26.

[24] D.V. Lindley, *The 1988 Wald memorial lectures: the present position in Bayesian statistics (with discussion).* Statistical Science 5 (1990), 44–89.

[25] V.I. Loginov, *Probability treatment of Zadeh membership functions and their use in pattern recognition.* Engineering Cybernetics 2 (1966), 68–69.

[26] M.B. Mendel, Development of Bayesian Parametric Theory with Applications to Control. Massachusetts Institute of Technology (1989).

[27] B. Natvig, *Possibility versus probability.* Fuzzy Sets and Systems 10 (1983), 31–36.

[28] C.V. Negoitia and D. Ralescu, Applications of Fuzzy Sets to Systems Analysis. Birkhäuser-Verlag, Basel (1975).

[29] A.M. Norwich and I.B. Turksen, Stochastic fuzziness. In: Approximate Reasoning in Decision Analysis. M.M. Gupta and E. Sanchez (eds.), North-Holland (1982), 13–22.

[30] A.M. Norwich and I.B. Turksen, The fundamental measurement of fuzziness. In: Fuzzy Sets and Possibility Theory: Recent Developments. R.R. Yager (ed.), Pergamon Press (1982), 49–60.

[31] A.M. Norwich and I.B. Turksen, A model for the measurement of membership and the consequences of its empirical implementation. Fuzzy Sets and Systems 12 (1984), 1–20.

[32] A. Ollero and E. Freire, *The structure of relations in personnel management.* Fuzzy Sets and Systems 5 (1981), 115–125.

[33] L. Pardo, *Medidas de nitidez para conjuntos difusos.* Trabajos de Estadística e Investigación Operativa 34 (1983), 43–59.

[34] M. Roubens and Ph. Vincke, *Preference Modeling.* Springer-Verlag, Berlin (1985).

[35] B. Roy, *Decision science or decision-aid science.* European Journal of Operational Research 66 (1993), 184–203.

[36] T.L. Saaty, *Exploring the interface between hierarchies, multiple objectives and fuzzy sets.* Fuzzy Sets and Systems 1 (1978), 57–68.

[37] L. Savage, The Foundations of Statistics. Wiley, New York (1954).

[38] L. Savage, Elicitation of personal probabilities and expectations. Journal of the American Statistical Association 66 (1971), 783-801.

[39] G. Shafer, *A mathematical theory of evidence.* Journal of the American Statistical Association 77 (1982), 325-351.

[40] G. Shafer, *Lindley's paradox.* Princeton University Press, Princeton, NJ (1976).

[41] G. Shafer, *Savage revisited (with discussion).* Statistical Science 1 (1986), 463–501.

[42] G. Shafer, *The unity and diversity of probability (with discussion).* Statistical Science 5 (1990), 435–462.

[43] G. Shafer, *Perspectives on the theory and practice of belief functions.* International Journal of Approximate Reasoning 4 (1990), 322–362.

[44] G. Shafer, *Rejoinders to comments on "Perspectives on the theory and practice of belief functions".* International Journal of Approximate Reasoning 6 (1992), 445–480.

[45] P. Smets, *Subjective probability and fuzzy measures.* In: Fuzzy Information and Decision Processes. M.M. Gupta and E. Sanchez (eds.), North-Holland (1982), 87–91.

[46] W. Stallings, *Fuzzy set theory versus bayesian statistics.* IEEE Transaction on Systems, Man and Cybernetics, March (1977), 216–219.

[47] E. Trillas, Conjuntos Borrosos. Vicens-Vives, Barcelona (1980).

[48] I.B. Turksen, *Measurement of membership functions and their acquisition.* Fuzzy Sets and Systems 40 (1991), 5–38.

[49] J.L. Verdegay, *Problemas de decisión en ambiente difuso.* Trabajos de Estadística e Investigación Operativa 34 (1983), 66–78.

[50] J.L. Verdegay, *A dual approach to solve the fuzzy linear programming problem.* Fuzzy Sets and Systems 14 (1984), 131-141.

[51] M.A. Vila and M. Delgado, *Problems of classification in a fuzzy environment.* Fuzzy Sets and Systems 9 (1983), 229–239.

[52] M.A. Vila and M. Delgado, *On medical diagnosis using possibility theory.* Fuzzy Sets and Systems 10 (1983), 211–222.

[53] R.R. Yager, J. Kacprzyk and M. Fedrizzi (Eds.), Advances in the Dempster-Shafer theory of evidence. Wiley (1994).

[54] L.A. Zadeh, *Fuzzy sets* Information and Control 8 (1965), 338–353.

[55] L.A. Zadeh, *Fuzzy sets as a basis for a theory of possibility* Fuzzy Sets and Systems 1 (1978), 3–28.

[56] L.A. Zadeh, *Is probability theory sufficient for dealing with uncertainty in AI: a negative view* In: Uncertainty in Artificial Intelligence. L.N. Kanal and J.F. Lemmer (eds.), Elsevier Science Pub. (1986), 103–116.