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TIME VARYING TERM PREMIA AND RISK: THE CASE OF THE SPANISH INTERBANK MONEY MARKET

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ABSTRACT

This paper examines some standard procedures, in the term structure of interest rates, for evaluating the importance of risk in explaining time varying term premia. It highlightes their shortcomings and proposes an alternative VARMA model based approach for dealing with this problem. This procedure is illustrated with the analysis of risk, measured as proposed by Luce (1980), in explaining the behavior of two important term premia in the Spanish interbank money market.

RESUMEN

En este trabajo se examinan algunos procedimientos estandar utilizados para evaluar la importancia del riesgo en la explicación del comportamiento de las primas por plazo dentro de la estructura temporal de tipos de interés. Se ponen de manifiesto sus limitaciones y se propone un procedimiento alternativo basado en la utilización de modelos VARMA. Este procedimiento se ilustra con una evaluación de la importancia del riesgo, medido como en Luce (1980), en el comportamiento de dos importantes primas por plazo dentro del mercado interbancario español.

KEY WORDS: *Determination of Interest rates; Term Structure of Interest Rates, Multiple Time Series Models, Finantial Markets and Macroeconomy.*

I. INTRODUCTION

The great amount of empirical evidence against constant term premia in the analysis of the term structure of interest rates [see for instance Shiller and McCulloch (1990) for a recent survey] has lead to investigate the importance of such term premia, their properties and their possible determinants. Some works in this line are: Nelson (1972), Modigliani and Shiller (1973), Fama (1976a and 1976b), Mishkin (1982), Shiller, Campbell and Shoenholtz (1983), Jones and Roley (1983), Keim and Stambaugh (1986), Campbell (1987), Engle, Lilien and Robins (1987), Taylor (1992) and Freixas and Novales (1992) among others.

The standard solution to the problem of estimating and finding determinants of a term premium embodies both a definition and a static behavioral equation for it. By combining the definition along with the behavioral equation it is possible to estimate the term premium as well as to evaluate the importance of its possible determinants.

As shown in Flores (1995a and 1995b) this type of analysis is not the more appropriate when there are variables, in the set of the term premium determinants, that are dynamically related to interest rates. In this case, the premium can be represented as a weighted sum of past and current one step ahead forecast errors associated to all variables in the information set, implying that a static behavioral equation for the premium will lead to its inadequate estimation as well as to errors in evaluating the relative importance of its determinants.

In this paper we propose an alternative procedure for evaluating the importance of risk or any other particular variable, that on a priori grounds is assumed to be a determinant of the term premium behaviour. This procedure explicitly accounts for the likely presence of dynamic relationships among all variables in the information set, including both term premia determinants and interest rates.

As an illustration we use the family of risk measures proposed by Luce (1980) and studied by Granger and Ding (1993 and 1994); we investigate their relevance in explaining the behavior of some important term premia in the Spanish interbank money market.

The remaining of the paper is organized as follows. Section II states the relationship between term premium and one step ahead forecasts errors associated to all variables in the information set. Using this relationship, Section III describes the procedure for evaluating the importance of a particular variable in explaining the term premium behaviour. In Section IV and following the procedure suggested in Section III, the importance of risk in determining two important term premia in the Spanish interbank market is analyzed, i.e.: the premium included in the 30 days rate versus 15 days rate and the premium in 15 days rate versus 7 days rate. Section V summarizes the most important conclusions.

II. TERM PREMIA AND ONE-STEP AHEAD FORECAST ERRORS

To simplify the exposition, let us assume the existence of two assets, A and B. Maturities are one and two periods respectively, with r_t and R_t being their yearly continuous interest rates. A standard definition of the term premium implicit in B with regard to A, see Shiller and McCulloch (1990) is:

$$\begin{aligned} \pi_t^{2,1} &= 2R_t - r_t - E_t(r_{t+1}) \\ &= f_{t,t+1} - E_t(r_{t+1}) \end{aligned} \qquad (1)$$

where $f_{t,t+1}$ is the forward rate and $E_t(.)$ means the conditional expectation based on information at time t.

The standard method for estimating $\pi_t^{2,1}$ operates as follows. Under the hypothesis that agents' expectations are rational, the relevant parameters of a behavioral equation for the term premium can be estimated by using one the following models:

a) Jones and Roley (1983)

$$\begin{aligned} 2R_t - r_{t+1} &= \pi_t^{2,1} + \beta r_t + \epsilon_{t+1} \\ \pi_t^{2,1} &= X_t' \alpha \end{aligned} \qquad (2)$$

where α is a vector of parameters and X_t' is a row vector of explanatory variables including: U.S. six-month Treasury bill yield, unemployment rate, risk, U.S. Treasury bill supplies and foreign holdings of U.S. Treasury securities. In this formulation β=1 and the significance of the parameter associated to risk in α are hypotheses to be tested.

b) Engle, Lilien and Robins (1987)

$$\begin{aligned} f_{t,t+1} - r_{t+1} &= \pi_t^{2,1} + \epsilon_{t+1} \\ \pi_t^{2,1} &= \alpha_1 + \alpha_2 \ln(h_{t+1}) + \alpha_3 (R_t - r_t) \\ h_{t+1}^2 &= \beta_1 + \beta_2 \sum_{i=1}^{p} \omega_i \epsilon_{t+1-i}^2 \end{aligned} \qquad (3)$$

where $\ln(h_{t+1})$ is a proxy for risk, defined as the logarithm of the error term conditional standard deviation.

c) Freixas and Novales (1992)

$$\begin{aligned} r_{t+1} - r_t &= \pi_t^{2,1} + \beta (f_{t,t+1} - r_t) + \epsilon_{t+1} \\ \pi_t^{2,1} &= \alpha_1 + \alpha_2 v_t \end{aligned} \qquad (4)$$

where v_t is the short term interest rate volatility, defined as in Fama(1976).

In these three cases, a consistent estimate of $\pi_t^{2,1}$ can be obtained by estimating any of the following vectors of parameters: $(\beta\ \alpha)$, $(\alpha_1\ \alpha_2\ \alpha_3\ \beta_1\ \beta_2\ \omega_1\ ...\ \omega_p)$ or $(\beta\ \alpha_1\ \alpha_2)$. While this approach avoids computing $E_t(r_{t+1})$ in the process of estimating $\pi_t^{2,1}$, it introduces a new and arbitrary element, i.e. the unidirectional static behavioral equation for $\pi_t^{2,1}$ that excludes any dynamic between the variables in the vector of determinants and interest rates. The importance of risk is related to the significance of its parameter in the behavioral equation for the premium.

Now, assume that the vector of variables defining the information set follows a general non-stationary VARMA process. For simplicity, let assume that the information set held by the agents contains the present and past values of a 4×1 vector, z_t, of variables which includes: a short term interest rate, r_t, a long term interest rate, R_t, and two variables related to r_t and R_t, namely, x_t and y_t. The VARMA process for z_t can be represented as:

$$z_t = \Psi(B)\, e_t \tag{5}$$

where e_t is a vector of independent, identically and normally distributed random variables, with contemporaneous covariance matrix Σ and $\Psi(B)$ being an infinite order polynomial matrix in B, the back-shift operator, normalized so that $\Psi(0)$=I. Hence, the generic element for $\Psi(B)$ takes the form:

$$\begin{aligned} \Psi_{i,j}(B) &= 1 + \psi_{ij,1}B + \psi_{ij,2}B^2 + \psi_{ij,3}B^3 + \dots \qquad \text{for } i=j \\ &= \psi_{ij,1}B + \psi_{ij,2}B^2 + \psi_{ij,3}B^3 + \dots \qquad \text{for } i\neq j \end{aligned} \tag{6}$$

If the variables in z_t are integrated of order 1 with no cointegrating relationships, $\Psi(B)$ can be factorized as:

$$\begin{aligned} \Psi(B) &= D^{-1}\, \Psi^*(B) \\ D^{-1} &= \nabla^{-1}\, I_{(4x4)} \\ \Psi^*(B) &= \Phi^{-1}(B)\, \Theta(B) \end{aligned} \tag{7}$$

where the roots of $|\Phi(B)|=0$ and $|\Theta(B)|=0$ lie outside the unit circle. In this case a VARMA model for ∇z_t can be obtained following Jenkins and Alavi (1981) or Tiao and Box (1981).

If there are "r" cointegration relationships in z_t, the above factorization does not exist. In that case it is possible to define a new 4×1 vector, z^*_t, whose elements are: "r" cointegrating relationships and "4-r" first differenced, independent linear combinations of elements in z_t. Then, a VARMA process for z_t can be obtained from the VARMA process for z^*_t. Note that:

$$(I - D_1 B)\, P_0\, z_t = z_t^* \tag{8}$$

where D_1 is a 4×4 matrix with all elements 0 except the last "4-r" of the main diagonal which take the value 1. P_0 is a 4×4 non-singular matrix, whose "r" first rows correspond to "r" cointegration

vectors, and the "4-r" remaining rows correspond to the vectors defining the "4-r" linear combinations mentioned above.

In both cases, the expression relating the term premium $\pi_t^{2,1}$ to the variables in z_t can be obtained from (1) and (5) as:

$$\pi_t^{2,1} = S(B)\, e_t \tag{9}$$

where

$$e_t = [e_{xt}\; e_{rt}\; e_{Rt}\; e_{yt}]' \tag{10}$$

is the error vector and S(B) is the polynomial row vector

$$S(B) = [S_x(B)\; S_r(B)\; S_R(B)\; S_y(B)] \tag{11}$$

with

$$\begin{aligned} S_x(B) &= [2B\Psi_{3,1}(B) - B\Psi_{2,1}(B) - \Psi_{2,1}(B)]\, B^{-1} \\ S_r(B) &= [2B\Psi_{3,2}(B) - B\Psi_{2,2}(B) - \Psi_{2,2}(B) + 1]\, B^{-1} \\ S_R(B) &= [2B\Psi_{3,3}(B) - B\Psi_{2,3}(B) - \Psi_{2,3}(B)]\, B^{-1} \\ S_y(B) &= [2B\Psi_{3,4}(B) - B\Psi_{2,4}(B) - \Psi_{2,4}(B)]\, B^{-1} \end{aligned} \tag{12}$$

See Appendix A for mathematical details.

Using (5) and (9), $\pi_t^{2,1}$ can also be represented as:

$$\pi_t^{2,1} = S(B)\, \Psi^{-1}(B)\, z_t \tag{13}$$

Equation (9) relates $\pi_t^{2,1}$ to current and past one-step-ahead forecasting errors, corresponding to all variables in z_t. These errors have associated a lag structure in $\pi_t^{2,1}$ given by the components of S(B). Note from (7) that the absence of cointegration implies that the elements of S(B) share a factor ∇^{-1}, i.e. $\pi_t^{2,1}$ will be an I(1) variable[1].

This representation is of particular interest because it gives an intuitive interpretation of the term premium, relating its size to the present and past forecasting errors of the variables in the information set. On one hand, present innovations indicate agents' reactions (changes in the premium) to current events. On the other hand, the presence of past forecasting errors indicates that agents do not adjust immediately their premia. Also, the presence of these past errors implies that term premia will not follow white noise processes.

Equation (13) relates $\pi_t^{2,1}$ to z_t. It is clear that if no cancellations occur in the vector $S(B)\Psi^{-1}(B)$, $\pi_t^{2,1}$ will depend on present and past values of all variables in z_t. Thus, by assuming that $\pi_t^{2,1}$ is a linear and static function of some z_t components, many a priori zero constraints on the components of $S(B)\Psi^{-1}(B)$ must hold.

The above discussion highlights the shortcomings of the mentioned standard procedures for estimating term premia. Such procedeure not only may lead to inadequate estimations of the term premia but its use might bias the importance of risk role. These limitations can be overcame by estimating the term premium as follows:

1) Obtain the number of cointegration relationships in z_t.
2) Specify a VARMA model for either z_t or z_t^*, depending on the number of cointegrating relationships.
3) Compute S(B) and estimate $\pi_t^{2,1}$ using (9) or (13).

This procedure has the additional advantage that embodies the standard approach as a particular case. Note that (13) might degenerate to:

$$\pi_t^{2,1} = S(0)\ \Psi^{-1}(0)\ z_t \tag{14}$$

implying a purely static relationship, as in (2), (3) or (4).

III. IMPORTANCE OF RISK IN TERM PREMIA BEHAVIOR

Let's assume that we are interested on the relative importance of risk (y_t) in determining the premium. From (9) and (11), $S_y(B) = 0$ indicates that one step ahead errors associated to y_t

have not a direct contribution to the behaviour of $\pi_t^{2,1}$. In other words, y_t has not forecasting power on the remaining variables in z_t, i.e. y_t does not Granger cause x_t, r_t or R_t. Nevertheless, it is still possible for y_t to have explanatory power on $\pi_t^{2,1}$. Different from zero contemporaneous correlations of e_{xt}, e_{rt} or e_{Rt} with e_{yt} can be seen as y_t having explanatory power on the term premium behaviour. In this case equation (9) can be decomposed in two terms:

$$\pi_t^{2,1} = C_{1,t} + C_{2,t} \tag{15}$$

where $C_{1,t}$ is the contribution of y_t to $\pi_t^{2,1}$ and $C_{2,t}$ is a remainder term, where:

$$\begin{aligned} C_{1,t} &= S(B)\ \beta\ e_{yt} \\ C_{2,t} &= S(B)\ e_t^* \end{aligned} \tag{16}$$

with ß being the 4×1 vector of coefficients:

$$\beta = \begin{pmatrix} \beta_x \\ \beta_r \\ \beta_R \\ 1 \end{pmatrix} \tag{17}$$

in the regression model:

$$e_t = \beta\ e_{yt} + e_t^* \tag{18}$$

In the statationary case (i.e.: when interest rates are cointegrated) the ratio of variances: variance of $C_{1,t}$ / variance of $\pi_t^{2,1}$, will give a measure of the importance of y_t. Note that e_{yt} is orthogonal to the variables in e_t^* and the variance decomposition will be informative.

Thus, the procedure for investigating the importance of a particular variable in the behaviour of a term premium can be summarized as follows:

1) Elaborate a VARMA model for the vector without the variable of interest (y_t). This will be the constrained model.
2) Compute $\pi_t^{2,1}$ associated to model in 1).
3) Elaborate a VARMA model for the vector containing the variable of interest

It will be called the extended model.

4) Investigate the forecasting abilities of the extended model i.e., perform an out of sample forecasting competition between the extended and constrained models.

5) Compute $\pi_t^{2,1}$ associated to the extended model and compare it with that obtained from the constrained one. Statistical properties of differences between premia will be very informative; for instance if the additional variable is not relevant, both models would yield the same term premium, i.e., differences would behave as a white noise process.

6) Compute $C_{1,t}$ and its contribution to the variance of $\pi_t^{2,1}$.

IV. RISK AND THE SPANISH INTERBANK MARKET

The presence of term premia in the interbank market has been justified in terms of liquidity preference and hedging behaviour. See for instance Hurn, Moody and Muscatelly (1995). In that market the rationale for liquidity premia might arise from bank's desire to ensure that they can meet cash requirements without undue recourse to short-term borrowing at unpredictable or penal rates. Also, term premia might arise from hedging behaviour, with banks seeking to match their maturities structure of their assets and liabilities to hedge against short-term interest rate movements; sudden changes in the government's policy stance might increase uncertainty regarding interest rates at the short-end of the interbank interest rate spectrum.

In both cases risk coming from uncertainty on future rates is behind the existence of term premia. The importance of risk in explaining the behaviour of time varying term premia has been studied by many authors. Modigliani and Shiller (1973) or Shiller, Campbell and Schoenholtz (1982) used a moving standard deviation of interest rates as a proxy for the level of uncertainty (volatility), Engel, Lilien and Robins (1987) used an ARCH model in order to represent the time varying variance of interest rates, Miskin (1982), Jones and Roley (1983) and Freixas and Novales (1992) uses the volatility definition proposed in Fama (1976a and 1976b). Very often such proxies for time varying risk became statistically significant.

In this Section we investigate the importance of risk in determining the behaviour of term premia in the Spanish interbank market. This analysis departs from those mentioned above because: (1) We allow for all kind of feedback relationships among the variables in the information set. (2) The information set has been made explicit and may include more than one measure of risk. (3) Finally, as in Flores (1995a and 1995b) we carry out this analysis by using a particular expectations generating mechanism (EGM), i.e.: a VARMA model.

Expectations, generated with this kind of EGM, will take into account any dynamic relationship that might be present among the variables in the information set. In particular, interest rates at different maturities might be dynamically related, with longer term interest rates having relevant information in forecasting shorter term rates [see for instance Hall, Anderson and Granger (1992)]. In that case more than a source of risk (uncertainty) could be relevant in explaining term premia, that is, not only uncertainty levels on shorter term interest rates might be relevant but also uncertainty levels on longer term interest rates.

On the other hand, there are not reasons for avoiding the possibility that such a risk measures could be dynamically related among them as well as with other variables such as interest rates, in which case, interest rates forecasts might be improved by incorporating risk measures to the information set.

In such a framework the importance of risk on any term premium can be evaluated by studying the premium variations when risk is excluded from the information set. If none lagged risk measure has relevant information, term premia computed with or without risk in the information set will not differ. Nevertheless, risk might be still important if high contemporaneous correlations between risk and interest rates do exit and they are interpreted as the former having instantaneous explanatory power on the later.

If risk helps to forecast interest rates, it becomes not only an important variable but a crucial one. The exclusion of risk from the information set will lead to biases in computing any premium.

Data and variables

We will assume that agents information set includes the present and past of the following interest rates:

$$
\begin{aligned}
r30_t &\equiv \frac{360}{30}\ln\left(1+\frac{30}{360}s30_t\right)\\
r15_t &\equiv \frac{360}{15}\ln\left(1+\frac{15}{360}s15_t\right)\\
r7_t &\equiv \frac{360}{7}\ln\left(1+\frac{7}{360}s7_t\right)\\
r1_t &\equiv \frac{360}{1}\ln\left(1+\frac{1}{360}s1_t\right)
\end{aligned}
\qquad (19)
$$

where $s30_t$, $s15_t$, $s7_t$ and $s1_t$ are 360 days basis, simple interest rates, corresponding to 30, 15, 7 and 1 days to maturity, of the Spanish interbank money market. Variables $r30_t$, $r15_t$, $r7_t$ and $r1_t$ are the continuously compounded N-day yield to maturity, N=30,15,7 and 1 days.

The sample size was 276 weekly observations, from 4-1-89 to 4-30-94, but all models have been elaborated with the first 251 observations, leaving the last 55 for out of sample forecasting exercises. Series have been elaborated from Banco de España daily time series, taking as the representative rate for the week that corresponding to wednesday. This choice minimizes the number of extreme observations.

Along with mentioned interest rates, it is assumed that agents perceive risk through the following variables:

$$
\begin{aligned}
v3015_t &\equiv |r30_t-r15_t|^{0.5}\\
v157_t &\equiv |r15_t-r7_t|^{0.5}\\
v71_t &\equiv |r7_t-r1_t|^{0.5}\\
v11_t &\equiv |r1_t-r1_{t-1}|^{0.5}
\end{aligned}
\qquad (20)
$$

These variables belong to the family of measures of risk proposed by Luce (1980). This

family has the following general representation:

$$|R-m|^{\theta} \qquad (21)$$

where R is the return, m is the mean of R, θ is a parameter at the choice of the individual investor and bars indicate absolute values. In this analysis θ has been set to 0.5 because is just for this particular value of θ that the corresponding risk measure seems to be normally distributed. Normality will be a very convenient feature if maximum likelihood estimation procedures need to be used. That could be the case if mixed VARMA models are allowed to serve as expectation generating mechanisms.

This family of risk measures has two nice characteristics: (1) They are very easy to compute and (2) they do not depend, as in the case of ARCH based volatility measures, of a particular data generating process (DGP) specification.

It is important to mention that we will use spreads instead of rate levels. The reason is that integration and cointegration analyses [using Johansen's (1988) trace test] on our sample, indicate that interest rates are I(1) variables but spreads are I(0) i.e., rates are cointegrated with cointegrating vectors of the type (1 -1). This result is also found by Hall, Anderson and Granger (1992) using US Treasury bills interest rates and by Hurn, Moody and Muscatelli (1995) using interest rates from London interbank market. Thus we decided to measure risk accordingly.

In order to minimize the effects of extreme values (some times greater than 5 standard deviations) and before using interest rates for elaborating spreads and risk measures, interest rate levels have been purged from mentioned extreme values. We have used Box and Tiao (1975) intervention analysis. In all cases outliers have been modeled just with one parameter (ω_0) associated to an impulse type variable.

In section 1 of Appendix B the reader can find graphs for the levels of interest rates (purged of outliers), spreads and risk measures. Also its includes a table with the results of Johansen's trace test and a table with the extreme values dates, estimates for ω_0 and their corresponding standard deviations in parentheses.

Working plan and VARMA models

Our first objective is to know how many risk measures (if any) among $v3017_t$, $v157_t$, $v71_t$ and $v11_t$, are needed to capture the effects of risk on interest rates and premia behaviour. Once fulfilled this objective, the next step will be to evaluate the importance of risk in term premia behaviour.

Then, we will proceed according to the following plan:

1) Elaborate a VARMA model for the vector of 3 spreads plus ∇r_{1t}. Note that all risk measures have been excluded.
2) Compute term premia ($\pi_t^{30,15}$ and $\pi_t^{15,7}$) based on previous model.
3) Elaborate 4 alternative VARMA models each of them containing the variables in 1) plus a measure of risk.
4) If necessary, incorporate to models in 3) a second, a third and a forth measure of risk.
5) Compare the forecasting performance of model in 1) with that of models in 3) and 4).
6) Select the best model, i.e., the model with a higher forecasting performance and risk measure(s) showing higher contemporaneous correlations with spreads.
7) Compute term premia associated to the selected model and evaluate the importance of risk according to the procedure outlined in Section III.

Section 2 of Appendix B contains (Tables 4-5) univariate ARMA models for spreads and risk measures. Also it contains Table 6, with the estimated VARMA models for the vectors of variables[2]:

i) Model M1: $(s3015_t\ s157_t\ s71_t\ s11_t)'$
ii) Model M21: $(s3015_t\ s157_t\ s71_t\ s11_t\ v11_t)'$
iii) Model M22: $(s3015_t\ s157_t\ s71_t\ s11_t\ v3015_t)'$
iv) Model M23: $(s3015_t\ s157_t\ s71_t\ s11_t\ v157_t)'$
v) Model M24: $(s3015_t\ s157_t\ s71_t\ s11_t\ v71_t)'$
vi) Model M25: $(s3015_t\ s157_t\ s71_t\ s11_t\ v11_t\ v3015_t\ v157_t)'$
vii) Model M26: $(s3015_t\ s157_t\ s71_t\ s11_t\ v3015_t\ v157_t)'$

Where $s3015_t$, $s157_t$, $s71_t$ and $s11_t$ are:

$$s3015_t \equiv r30_t - r15_t$$
$$s157_t \equiv r15_t - r7_t$$
$$s71_t \equiv r7_t - r1_t$$
$$s11_t \equiv r1_t - r1_{t-1}$$

Finaly, Table 7 contains the contemporaneous residual correlation matrices associated to those VARMA models.

From models M21, M22, M23 and M24 it can be seen that only $v11_t$, $v3015_t$ and $v157_t$ show significant coefficients. Also, while the contemporaneous residual correlation matrices associated to models M22 and M23 show important correlations of $v3015_t$ and $v157_t$ with spreads, neither for model M21 nor model M24 residual correlation matrices show important correlations between measures of risk and spreads.

As $v11_t$, $v3015_t$ and $v157_t$ are likely to have the most important contribution to term premia behaviour we decided to build the model M25, where those three measures of risk were included. Finally, $v11_t$ was removed from the vector of variables, leading to model M26, due to its weak effects on spreads and ∇r_{1t}.

Table 1 show the results of an out of sample forecasting exercise. As mentioned above, it was carried out using the last 55 observations.

Table 1

Forecasting Exercise: Theil's U coefficient

	M1	M21	M22	M23	M24	M25	M26
$E_t(r_{7t+1})$	0.0217	0.0219	0.0217	0.0215	0.0218	0.0213	0.0213
$E_t(r_{15t+2})$	0.0287	0.0292	0.0288	0.0282	0.0289	0.0283	0.0282

Taking as a bench mark the performance of model M1, measured as the value of the Theil's U statistic, model M26 show the gratest forecasting improvements in forecasting short term interest rates. Note that shorter term interest rates are the relevant variables to be forecasted when $\pi_t^{30,15}$ and $\pi_t^{15,7}$ need to be determined. These results indicate that risk as measured by $v11_t$ has a small contribution once $v3015_t$ and $v157_t$ have been included in the information set.

Risk and Premia Estimation

Section 3 of Appendix B contains graphs of $\pi_t^{30,15}$ and $\pi_t^{15,7}$ computed for both models M1 and M26, that is without and with measures of risk included in the information set. Also that section includes graphs, tables with descriptive statistics and univariate models for the differences between premia (those computed with M1 - those computed with M26).

As we can see from Tabe 8 in Appendix B, $\pi_t^{30,15}$ is the most affected by omitting risk, its mean becomes underestimated. Also the second moments are affected, the differences from computing it with and without risk show autocorrelation. In the case of $\pi_t^{15,7}$ the mean is not affected but differences seem to be autocorrelated, see Table 9 in Appendix B.

Then, risk is important in explaining the behavior of term premia. The share of risk in the total variance for $\pi_t^{30,15}$ is 16%, while in the case of $\pi_t^{15,7}$ is about 14 %. See Appendix C for more details on these computations.

V. CONCLUDING REMARKS

Time varying risk has been one of the most important explanatory variables of time varying term premia, in the term structure of interest rates.

The standard approach for evaluating the importance of risk has relied on static behavioral equations for term premia. In this paper we show that term premia are made of present and past one step ahead forecast errors associated to all variables in the information set, including risk. Unless very particular cancellations occur, a static behavioral equation for a term premia will not

be a correct specification and it will lead to wrong estimates of the importance of risk.

In this paper we propose a method for evaluating the importance of risk, which explicitly takes into account the presence of dynamic relationships among all variables in the information set. This method uses different assumptions about the information set and VARMA models as expectations generating mechanisms. It analyzes the forecasting performance of all competing VARMA models and studies the statistical properties of term premia associated to them.

In studying the importance of risk for time varying term premia in the Spanish interbank market, we have shown that interest rates and risk measures, those proposed by Luce (1980) and Granger and Ding (1992), are dynamically related. These measures have useful information in forecasting interest rates. An immediate implication of this result is that risk is indeed an important variable in explaining the behavior of term premia in the Spanish interbank market. Thus, our results do not differ from those obtained for example by Modigliani and Shiller (1973), Fama (1976a and 1976b), Mishkin (1982), Shiller, Campbell and Shoenholtz (1983), Jones and Roley (1983), Engle, Lilien and Robins (1987) or Freixas and Novales (1992). All of them find the coefficient associated to the proxy variable for risk to be significant.

More than a measure for risk seem to be necessary for capturing most of risk effects. Also they help to improve the forecasting performance of the expectation generating mechanism.

APPENDIX A

Weekly data imply that if the maturity of short term interest rate is grater than a week we will have overlapping samples, for example 30 days versus 15 days. In this case $E_t(r_{t+1})$ in the expression (1) is the expected 15-days yield to maturity in the week t+2 evaluated in the week t. Therefore, expression (12) is only valid for the non-overlapping case. In this appendix we will give a more general case.

Consider the vector $z_t = (x_t, r_t, R_t, y_t)'$ which follows the process:

$$\begin{pmatrix} x_t \\ r_t \\ R_t \\ y_t \end{pmatrix} = \begin{pmatrix} \Psi_{1,1}(B) & \Psi_{1,2}(B) & \Psi_{1,3}(B) & \Psi_{1,4}(B) \\ \Psi_{2,1}(B) & \Psi_{2,2}(B) & \Psi_{2,3}(B) & \Psi_{2,4}(B) \\ \Psi_{3,1}(B) & \Psi_{3,2}(B) & \Psi_{3,3}(B) & \Psi_{3,4}(B) \\ \Psi_{4,1}(B) & \Psi_{4,2}(B) & \Psi_{4,3}(B) & \Psi_{4,4}(B) \end{pmatrix} \begin{pmatrix} e_{xt} \\ e_{rt} \\ e_{Rt} \\ e_{yt} \end{pmatrix} \qquad \text{(A1)}$$

1. Overlapping case:

If we compare term 2K versus term K, K being an integer > 1, the term premium is:

$$\pi_t^{2K,K} = 2R_t - r_t - E_t(r_{t+K}) \qquad \text{(A2)}$$

From (A1):

$$2R_t = 2\Psi_{3,1}(B)e_{xt} + 2\Psi_{3,2}(B)e_{rt} + 2\Psi_{3,3}(B)e_{Rt} + 2\Psi_{3,4}(B)e_{yt}$$

$$r_t = \Psi_{2,1}(B)e_{xt} + \Psi_{2,2}(B)e_{rt} + \Psi_{2,3}(B)e_{Rt} + \Psi_{2,4}(B)e_{yt}$$

$$E_t(r_{t+K}) = \left(\Psi_{2,1}(B) - \sum_{i=1}^{K-1}\psi_{2,1,i}B^i\right)B^{-K}e_{xt} + \left(\Psi_{2,2}(B) - \left(1 + \sum_{i=1}^{K-1}\psi_{2,2,i}B^i\right)\right)B^{-K}e_{rt} +$$
$$+ \left(\Psi_{2,3}(B) - \sum_{i=1}^{K-1}\psi_{2,3,i}B^i\right)B^{-K}e_{Rt} + \left(\Psi_{2,4}(B) - \sum_{i=1}^{K-1}\psi_{2,4,i}B^i\right)B^{-K}e_{yt}$$

Then:

$$\pi_t^{2K,K} = S(B)\,e_t \qquad \text{(A3)}$$

where

$$S(B) = \left[S_x(B), S_r(B), S_R(B), S_y(B)\right]$$

with:

$$S_x(B) = \left(2B^K\Psi_{3,1}(B) - B^K\Psi_{2,1}(B) - \left(\Psi_{2,1}(B) - \sum_{i=1}^{K-1}\psi_{2,1,i}B^i\right)\right)B^{-K}$$

$$S_r(B) = \left(2B^K\Psi_{3,2}(B) - B^K\Psi_{2,2}(B) - \left(\Psi_{2,2}(B) - \left(1 + \sum_{i=1}^{K-1}\psi_{2,2,i}B^i\right)\right)\right)B^{-K}$$

$$S_R(B) = \left(2B^K\Psi_{3,3}(B) - B^K\Psi_{2,3}(B) - \left(\Psi_{2,3}(B) - \sum_{i=1}^{K-1}\psi_{2,3,i}B^i\right)\right)B^{-K}$$

$$S_y(B) = \left(2B^K\Psi_{3,4}(B) - B^K\Psi_{2,4}(B) - \left(\Psi_{2,4}(B) - \sum_{i=1}^{K-1}\psi_{2,4,i}B^i\right)\right)B^{-K}$$

2. Non-overlapping case (K=1):

The expresión (A2) reduces to:

$$\pi_t^{2,1} = 2R_t - r_t - E_t(r_{t+1}) \qquad \text{(A5)}$$

with:

$$E_t(r_{t+1}) = \Psi_{2,1}(B)B^{-1}e_{xt} + (\Psi_{2,2}(B)-1)B^{-1}e_{r,t} + \Psi_{2,3}(B)B^{-1}e_{Rt} + \Psi_{2,4}(B)B^{-1}e_{yt}$$

and the term premium can be represented as in (A3) with:

$$S_x(B)=[2B\Psi_{3,1}(B)-B\Psi_{2,1}(B)-\Psi_{2,1}(B)]B^{-1}$$

$$S_r(B)=[2B\Psi_{3,2}(B)-B\Psi_{2,2}(B)-\Psi_{2,2}(B)+1]B^{-1}$$

$$S_R(B)=[2B\Psi_{3,3}(B)-B\Psi_{2,3}(B)-\Psi_{2,3}(B)]B^{-1}$$

$$S_y(B)=[2B\Psi_{3,4}(B)-B\Psi_{2,4}(B)-\Psi_{2,4}(B)]B^{-1}$$

Table 2 Johansen's Trace Test						
Vector	H_0	p=2		p=9		C.V. (99%)
		Trace	Vector	Trace	Vector	
$(r_{30}\ r_{15})'$	1	0.30	1.00	0.02	1.00	13
	2	33.12	-0.98	33.69	-1.00	24.6
$(r_{15}\ r_7)'$	1	0.96	1.00	0.03	1.00	13
	2	102.05	-0.99	40.77	-0.99	24.6
$(r_7\ r_1)'$	1	0.8	1.00	0.27	1.00	13
	2	102.34	-1.00	44.06	-1.01	24.6

Note: A constant term has been included in all regresions. The null hypotheses are: at most one cointegration relationship and at most two cointegration relationships. C.V. are the critical values in Osterwald-Lenum (1992).

Table 3
Intervention Analysis

Date	s11	s71	s157	s3015	Date	s11	s71	s157	s3015
2/1989		0.0031 (0.0008)		0.0045 (0.0008)	40/1992		0.0036 (0.0008)	0.0022 (0.0005)	0.0046 (0.0008)
4/1989				0.0035 (0.0008)	47/1992	0.008 (0.0021)			
5/1989		0.0078 (0.0008)	0.0028 (0.0005)	0.0032 (0.0008)	48/1992	0.019 (0.0028)	-0.013 (0.0008)	-0.0017 (0.0005)	
6/1989	0.123 (0.0026)				49/1992	-0.016 (0.0027)	0.0046 (0.0008)		
7/1989				0.0014 (0.0005)	51/1992		0.0028 (0.0005)		0.0018 (0.0005)
8/1989				0.0014 (0.0005)	1/1993	0.01 (0.0021)	0.0019 (0.0005)	-0.0028 (0.0005)	-0.0017 (0.0005)
9/1989		0.0028 (0.0005)	0.0019 (0.0005)		2/1993	-0.008 (0.0021)			
12/1989				0.0029 (0.0008)	4/1993	-0.009 (0.0021)			
15/1989		0.0022 (0.0005)			8/1993	0.045 (0.0027)	-0.014 (0.0008)	0.021 (0.0005)	-0.0077 (0.0008)
17/1989				0.0041 (0.0008)	9/1993	-0.014 (0.0027)		0.003 (0.0005)	
18/1989	0.006 (0.0021)				10/1993				-0.0022 (0.0005)
25/1989		0.0022 (0.0005)			12/1993		0.0026 (0.0005)		
26/1989	0.013 (0.0026)		-0.0018 (0.0005)		13/1993	-0.019 (0.0026)			
28/1989	-0.011 (0.0022)	0.004 (0.0008)	0.0032 (0.0005)		15/1993		0.0029 (0.0005)	-0.0013 (0.00005)	0.005 (0.0008)
29/1989	0.007 (0.0021)				17/1993	0.0225 (0.0026)	0.0024 (0.0005)	-0.0016 (0.0005)	
31/1989		0.0032 (0.0008)			19/1993	0.0165 (0.0027)		0.0015 (0.0005)	-0.0036 (0.0008)
1/1991		-0.005 (0.0008)			20/1993	-0.05 (0.0027)		-0.0023 (0.0005)	-0.0016 (0.0005)
3/1991			0.0015 (0.0005)		21/1993	-0.008 (0.0021)			
11/1991				-0.002 (0.0005)	22/1993		0.0027 (0.0005)		
12/1991	-0.009 (0.0021)				25/1993				0.0026 (0.0008)
43/1991			0.0017 (0.0005)		30/1993	0.019 (0.0027)	0.0024 (0.0005)		-0.0051 (0.0008)
16/1992		0.0051 (0.0008)			31/1993	-0.025 (0.0027)		0.0017 (0.0005)	
37/1992			0.0018 (0.0005)		32/1993		0.0023 (0.0008)		0.0029 (0.0005)
38/1992			0.0036 (0.0005)	0.0039 (0.0008)	41/1993		-0.005 (0.0008)		
39/1992		0.0045 (0.0008)	0.005 (0.0005)		42/1993	-0.01 (0.0021)			
					45/1993			0.0036 (0.0005)	

Standard errors in parentheses. Parameter estimates (ω_0) corresponds to an impulse type variable.

Section 2

Table 4
Univariate Models for Spreads

$s_{it}=\phi s_{it-1}+\varepsilon_t-\theta\varepsilon_{t-1}$

	ϕ	θ	$\hat{\sigma}(*100)$	Q(20)
s3015	0.96 (0.02)	0.73 (0.05)	0.055	17.41
s157	0.94 (0.03)	0.64 (0.06)	0.047	27.83
s71	0.91 (.06)	0.81 (0.09)	0.059	16.98
s11	0.073 (0.3)	0.27 (0.2)	0.2	20.3

$\hat{\sigma}$ is the residual standard error. Q(20) is the Ljung-Box statistic with 20 degrees of freedom. Standard errors in parentheses. Non significant constant terms were removed.

Table 5
Univariate Models for the Risk Measures

$v_{it}=\mu+\phi_1 v_{it-1}+\phi_2 v_{it-2}+\varepsilon_t$

	μ	ϕ_1	ϕ_2	$\hat{\sigma}$	Q(20)
v3015	0.0196 (0.001)	0.231 (0.06)	---	0.011	19.8
v157	0.018 (0.001)	0.297 (0.06)	0.148 (0.06)	0.01	17.59
v71	0.017 (0.007)	---	---	0.011	18.92
v11	0.031 (0.002)	0.233 (0.065)	---	0.021	12.29

$\hat{\sigma}$ is the residual standard error. Q(20) is the Ljung-Box statistic with 20 degrees of freedom. Standard errors in parentheses.

Table 6
VARMA Models

M1

$$\begin{pmatrix} s3015_t \\ s157_t \\ s71_t \\ s11_t \end{pmatrix} = \begin{pmatrix} .96 & - & - & - \\ - & .96 & - & - \\ - & - & .88 & - \\ - & - & - & -.44 \end{pmatrix} \begin{pmatrix} s3015_{t-1} \\ s157_{t-1} \\ s71_{t-1} \\ s11_{t-1} \end{pmatrix} + \begin{pmatrix} a3015_t \\ a157_t \\ a71_t \\ a11_t \end{pmatrix} - \begin{pmatrix} .78 & -.17 & - & - \\ -.12 & .81 & - & - \\ -.1 & - & .81 & - \\ -.43 & - & - & -.32 \end{pmatrix} \begin{pmatrix} a3015_{t-1} \\ a157_{t-1} \\ a71_{t-1} \\ a11_{t-1} \end{pmatrix} - \begin{pmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ -.61 & - & - & - \end{pmatrix} \begin{pmatrix} a3015_{t-2} \\ a157_{t-2} \\ a71_{t-2} \\ a11_{t-2} \end{pmatrix}$$

$$\begin{pmatrix} .02 & - & - & - \\ - & .014 & - & - \\ - & - & .07 & - \\ - & - & - & .17 \end{pmatrix} \quad \begin{pmatrix} .05 & -.045 & - & - \\ .035 & .035 & - & - \\ .053 & - & .09 & - \\ .26 & - & - & .18 \end{pmatrix} \quad \begin{pmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ -.25 & - & - & - \end{pmatrix}$$

M21

$$\begin{pmatrix} s3015_t \\ s157_t \\ s71_t \\ s11_t \\ v11_t \end{pmatrix} = \begin{pmatrix} - \\ - \\ - \\ - \\ .022 \end{pmatrix} + \begin{pmatrix} .96 & - & - & - & - \\ - & .96 & - & - & - \\ - & - & .83 & - & - \\ - & - & - & -.45 & - \\ - & - & - & - & .23 \end{pmatrix} \begin{pmatrix} s3015_{t-1} \\ s157_{t-1} \\ s71_{t-1} \\ s11_{t-1} \\ v11_{t-1} \end{pmatrix} + \begin{pmatrix} a3015_t \\ a157_t \\ a71_t \\ a11_t \\ av11_t \end{pmatrix} - \begin{pmatrix} .78 & -.16 & - & - & .002 \\ -.12 & .76 & - & - & - \\ -.1 & - & .74 & - & - \\ -.47 & - & - & -.34 & - \\ - & - & - & - & - \end{pmatrix} \begin{pmatrix} a3015_{t-1} \\ a157_{t-1} \\ a71_{t-1} \\ a11_{t-1} \\ av11_{t-1} \end{pmatrix} - \begin{pmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ -.61 & - & - & - & - \\ - & - & - & - & - \end{pmatrix} \begin{pmatrix} a3015_{t-2} \\ a157_{t-2} \\ a71_{t-2} \\ a11_{t-2} \\ av11_{t-2} \end{pmatrix}$$

$$\begin{pmatrix} - \\ - \\ - \\ - \\ .002 \end{pmatrix} \begin{pmatrix} .019 & - & - & - & - \\ - & .018 & - & - & - \\ - & - & .093 & - & - \\ - & - & - & .16 & - \\ - & - & - & - & .065 \end{pmatrix} \quad \begin{pmatrix} .05 & .047 & - & - & .0008 \\ .04 & .05 & - & - & - \\ .058 & - & .11 & - & - \\ .27 & - & - & .18 & - \\ - & - & - & - & - \end{pmatrix} \quad \begin{pmatrix} - & - & - & - & - \\ - & - & - & - & - \\ - & - & - & - & - \\ .25 & - & - & - & - \\ - & - & - & - & - \end{pmatrix}$$

M24

$$\begin{pmatrix}s3015_t\\ s157_t\\ s71_t\\ s11_t\\ v71_t\end{pmatrix}=\begin{pmatrix}-\\-\\-\\-\\.017\end{pmatrix}+\begin{pmatrix}.95&-&-&-&-\\-&.96&-&-&-\\-&-&.84&-&-\\-&-&-&-.44&-\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}s3015_{t-1}\\ s157_{t-1}\\ s71_{t-1}\\ s11_{t-1}\\ v71_{t-1}\end{pmatrix}+\begin{pmatrix}a3015_t\\ a157_t\\ a71_t\\ a11_t\\ av71_t\end{pmatrix}-\begin{pmatrix}.77&-.16&-&-&-\\-.12&.8&-&-&-\\-.10&-&.75&-&-\\-.43&-&-&-.32&-\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-1}\\ a157_{t-1}\\ a71_{t-1}\\ a11_{t-1}\\ av71_{t-1}\end{pmatrix}-\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-.62&-&-&-&-\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-2}\\ a157_{t-2}\\ a71_{t-2}\\ a11_{t-2}\\ av71_{t-2}\end{pmatrix}$$

$$\begin{pmatrix}-\\-\\-\\-\\.001\end{pmatrix}\quad\begin{pmatrix}.019&-&-&-&-\\-&.015&-&-&-\\-&-&.09&-&-\\-&-&-&.16&-\\-&-&-&-&-\end{pmatrix}\qquad\qquad\begin{pmatrix}.05&.046&-&-&-\\.036&.044&-&-&-\\.055&-&.11&-&-\\.27&-&-&.18&-\\-&-&-&-&-\end{pmatrix}\qquad\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\.25&-&-&-&-\\-&-&-&-&-\end{pmatrix}$$

M25

$$\begin{pmatrix}s3015_t\\ s157_t\\ s71_t\\ s11_t\\ v11_t\\ v3015_t\\ s157_t\end{pmatrix}=\begin{pmatrix}-\\-\\-\\-\\.024\\.015\\.012\end{pmatrix}+\begin{pmatrix}.89&-&-&-&-&-&-\\-&.85&-&-&-&-&-\\-&-&.9&-&-&-&-\\-&-&-&-.17&-&-&-\\-&-&-&-&.18&-&-\\-&-&-&-&-&.19&-\\-&-&-&-&-&-&.23\end{pmatrix}\begin{pmatrix}s3015_{t-1}\\ s157_{t-1}\\ s71_{t-1}\\ s11_{t-1}\\ v11_{t-1}\\ v3015_{t-1}\\ v157_{t-1}\end{pmatrix}+\begin{pmatrix}-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&.08\end{pmatrix}\begin{pmatrix}s3015_{t-2}\\ s157_{t-2}\\ s71_{t-2}\\ s11_{t-2}\\ v11_{t-2}\\ v3015_{t-2}\\ s157_{t-2}\end{pmatrix}+\begin{pmatrix}a3015_t\\ a157_t\\ a71_t\\ a11_t\\ av11_t\\ av3015_t\\ a157_t\end{pmatrix}-\begin{pmatrix}.66&-.16&-&-&.001&.004&-\\-.11&.6&-&-&-&-&-\\-.07&-&.81&-&-&-&.005\\-.62&-&-&-.02&-&-&.025\\-&-&-&-&-&-&.34\\-&-&-&-&-&-&-.18\\-&-&-&-&-&-.19&-\end{pmatrix}\begin{pmatrix}a3015_{t-1}\\ a157_{t-1}\\ a71_{t-1}\\ a11_{t-1}\\ av11_{t-1}\\ av3015_{t-1}\\ av157_{t-1}\end{pmatrix}-\begin{pmatrix}-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-.66&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-2}\\ a157_{t-2}\\ a71_{t-2}\\ a11_{t-2}\\ av11_{t-2}\\ av3015_{t-2}\\ av157_{t-2}\end{pmatrix}$$

$$\begin{pmatrix}-\\-\\-\\-\\.002\\.001\\.001\end{pmatrix}\begin{pmatrix}.028&-&-&-&-&-&-\\-&.033&-&-&-&-&-\\-&-&.063&-&-&-&-\\-&-&-&.19&-&-&-\\-&-&-&-&.064&-&-\\-&-&-&-&-&.054&-\\-&-&-&-&-&-&.052\end{pmatrix}\qquad\begin{pmatrix}-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&.05\end{pmatrix}\qquad\begin{pmatrix}0.06&.049&-&-&.0008&.002&-\\.041&.067&-&-&-&-&-\\.05&-&.09&-&-&-&.002\\.26&-&-&.2&-&-&.014\\-&-&-&-&-&-&.14\\-&-&-&-&-&-&.06\\-&-&-&-&-&.05&-\end{pmatrix}\qquad\begin{pmatrix}-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\.27&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\\-&-&-&-&-&-&-\end{pmatrix}$$

M22

$$\begin{pmatrix}s3015_t\\ s157_t\\ s71_t\\ s11_t\\ v3015_t\end{pmatrix}=\begin{pmatrix}-\\-\\-\\-\\.015\end{pmatrix}+\begin{pmatrix}.93&-&-&-&-\\-&.92&-&-&-\\-&-&.8&-&-\\-&-&-&-.44&-\\-&-&-&-&.22\end{pmatrix}\begin{pmatrix}s3015_{t-1}\\ s157_{t-1}\\ s71_{t-1}\\ s11_{t-1}\\ v3015_{t-1}\end{pmatrix}+\begin{pmatrix}a3015_t\\ a157_t\\ a71_t\\ a11_t\\ av3015_t\end{pmatrix}-\begin{pmatrix}.7&-.11&-&-&.004\\-.12&.71&-&-&-\\-.07&-&.69&-&-\\-.43&-&-&-.33&-\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-1}\\ a157_{t-1}\\ a71_{t-1}\\ a11_{t-1}\\ av3015_{t-1}\end{pmatrix}-\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-.61&-&-&-&-\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-2}\\ a157_{t-2}\\ a71_{t-2}\\ a11_{t-2}\\ av3015_{t-2}\end{pmatrix}$$

$$\begin{pmatrix}-\\-\\-\\-\\.001\end{pmatrix}\quad\begin{pmatrix}.024&-&-&-&-\\-&.023&-&-&-\\-&-&.12&-&-\\-&-&-&.17&-\\-&-&-&-&.05\end{pmatrix}\qquad\qquad\begin{pmatrix}.06&.042&-&-&.002\\.044&.056&-&-&-\\.058&-&.14&-&-\\.27&-&-&.18&-\\-&-&-&-&-\end{pmatrix}\qquad\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\.25&-&-&-&-\\-&-&-&-&-\end{pmatrix}$$

M23

$$\begin{pmatrix}s3015_t\\ s157_t\\ s71_t\\ s11_t\\ v157_t\end{pmatrix}=\begin{pmatrix}-\\-\\-\\-\\.01\end{pmatrix}+\begin{pmatrix}.9&-&-&-&-\\-&.86&-&-&-\\-&-&.9&-&-\\-&-&-&-.19&-\\-&-&-&-&.3\end{pmatrix}\begin{pmatrix}s3015_{t-1}\\ s157_{t-1}\\ s71_{t-1}\\ s11_{t-1}\\ v157_{t-1}\end{pmatrix}+\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&.12\end{pmatrix}\begin{pmatrix}s3015_{t-2}\\ s157_{t-2}\\ s71_{t-2}\\ s11_{t-2}\\ v157_{t-2}\end{pmatrix}+\begin{pmatrix}a3015_t\\ a157_t\\ a71_t\\ a11_t\\ av157_t\end{pmatrix}-\begin{pmatrix}.74&-.17&-&-&-\\-.07&.59&-&-&-\\-.05&-&.8&-&.004\\-.66&-&-&-.03&.03\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-1}\\ a157_{t-1}\\ a71_{t-1}\\ a11_{t-1}\\ av157_{t-1}\end{pmatrix}-\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-.74&-&-&-&-\\-&-&-&-&-\end{pmatrix}\begin{pmatrix}a3015_{t-2}\\ a157_{t-2}\\ a71_{t-2}\\ a11_{t-2}\\ av157_{t-2}\end{pmatrix}$$

$$\begin{pmatrix}-\\-\\-\\-\\.001\end{pmatrix}\quad\begin{pmatrix}.029&-&-&-&-\\-&.035&-&-&-\\-&-&.07&-&-\\-&-&-&.17&-\\-&-&-&-&.05\end{pmatrix}\qquad\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&.054\end{pmatrix}\qquad\begin{pmatrix}.059&.057&-&-&-\\.038&.068&-&-&-\\.048&-&.092&-&.002\\.26&-&-&.2&.013\\-&-&-&-&-\end{pmatrix}\qquad\begin{pmatrix}-&-&-&-&-\\-&-&-&-&-\\-&-&-&-&-\\.25&-&-&-&-\\-&-&-&-&-\end{pmatrix}$$

M26

$$\begin{pmatrix} s3015_t \\ s157_t \\ s71_t \\ s11_{4t} \\ v3015_t \\ v157_t \end{pmatrix} = \begin{pmatrix} - \\ - \\ - \\ - \\ .015 \\ .012 \end{pmatrix} + \begin{pmatrix} .88 & - & - & - & - & - \\ - & .85 & - & - & - & - \\ - & - & .9 & - & - & - \\ - & - & - & -.2 & - & - \\ - & - & - & - & .18 & - \\ - & - & - & - & - & .23 \end{pmatrix} \begin{pmatrix} s3015_{t-1} \\ s157_{t-1} \\ s71_{t-1} \\ s11_{t-1} \\ v3015_{t-1} \\ v157_{t-1} \end{pmatrix} + \begin{pmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & .08 \end{pmatrix} \begin{pmatrix} s3015_{t-2} \\ s157_{t-2} \\ s71_{t-2} \\ s11_{t-2} \\ v3015_{t-2} \\ v157_{t-2} \end{pmatrix} + \begin{pmatrix} a3015_t \\ a157_t \\ a71_t \\ a11_t \\ av3015_t \\ a157_t \end{pmatrix} - \begin{pmatrix} .63 & -.16 & - & - & .005 & - \\ -.11 & .59 & - & - & - & - \\ -.06 & - & .8 & - & - & .005 \\ -.6 & - & - & -.05 & - & .025 \\ - & - & - & - & - & -.19 \\ - & - & - & - & -.2 & - \end{pmatrix} \begin{pmatrix} a3015_{t-1} \\ a157_{t-1} \\ a71_{t-1} \\ a11_{t-1} \\ av3015_{t-1} \\ av157_{t-1} \end{pmatrix} - \begin{pmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ -.67 & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{pmatrix} \begin{pmatrix} a3015_{t-2} \\ a157_{t-2} \\ a71_{t-2} \\ a11_{t-2} \\ av3015_{t-2} \\ a157_{t-2} \end{pmatrix}$$

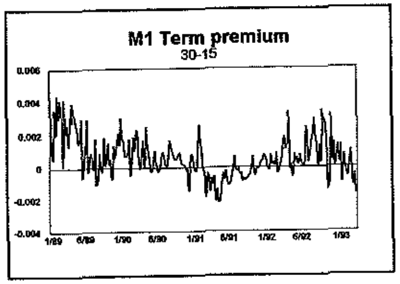
$$\begin{pmatrix} - \\ - \\ - \\ - \\ .002 \\ .001 \end{pmatrix} \begin{pmatrix} .028 & - & - & - & - & - \\ - & .033 & - & - & - & - \\ - & - & .066 & - & - & - \\ - & - & - & .17 & - & - \\ - & - & - & - & .053 & - \\ - & - & - & - & - & .05 \end{pmatrix} \qquad \begin{pmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & .05 \end{pmatrix} \qquad \begin{pmatrix} .063 & .05 & - & - & .002 & - \\ .042 & .068 & - & - & - & - \\ .048 & - & .09 & - & - & .002 \\ .26 & - & - & .2 & - & .014 \\ - & - & - & - & - & .06 \\ - & - & - & - & .05 & - \end{pmatrix} \qquad \begin{pmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ .27 & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{pmatrix}$$

Table 7
Residual Correlation Matrices

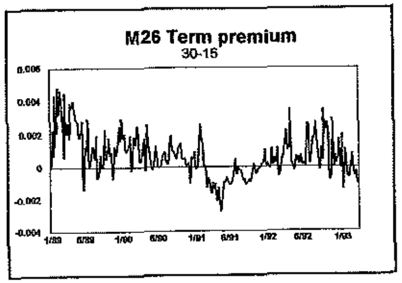
	M1		
a3015	1.00		
a157	0.18 1.00		
a71	0.33 0.23 1.00		
a11	-0.27 -0.22 -0.26 1.00		
	M21		**M22**
a3015	1.00	**a3015**	1.00
a157	0.19 1.00	**a157**	0.17 1.00
a71	0.32 0.23 1.00	**a71**	0.30 0.24 1.00
a11	-0.28 -0.22 -0.26 1.00	**a11**	-0.27 -0.22 -0.25 1.00
av11	0.05 0.06 0.03 -0.14 1.00	**av3015**	0.66 0.08 0.12 -0.15 1.00
	M23		**M24**
a3015	1.00	**a3015**	1.00
a157	0.19 1.00	**a157**	0.18 1.00
a71	0.32 0.24 1.00	**a71**	0.33 0.23 1.00
a11	-0.28 -0.24 -0.27 1.00	**a11**	-0.27 -0.22 -0.26 1.00
av157	0.15 0.61 0.29 -0.20 1.00	**av71**	0.03 0.18 0.20 -0.03 1.00
	M25		**M26**
a3015	1.00	**a3015**	1.00
a157	0.20 1.00	**a157**	0.20 1.00
a71	0.32 0.24 1.00	**a71**	0.32 0.24 1.00
a11	-0.29 -0.24 -0.27 1.00	**a11**	-0.29-0.24 -0.27 1.00
av11	0.03 0.07 0.06 -0.16 1.00	**av3015**	0.64 0.12 0.16 -0.16 1.00
av3015	0.64 0.12 0.16 -0.15 0.01 1.00	**av157**	0.16 0.61 0.30 -0.20 0.24 1.00
av157	0.17 0.61 0.30 -0.20 0.15 0.24 1.0		

$\pm 2\sqrt{n} = 0.13$

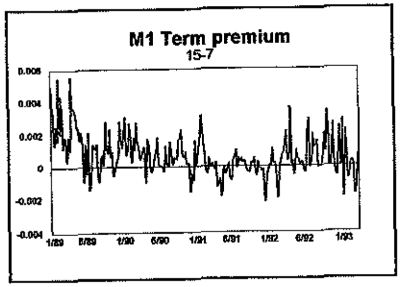
Section 3



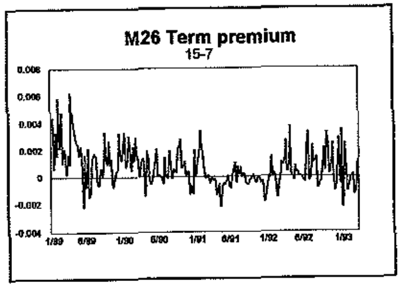
Graph 13



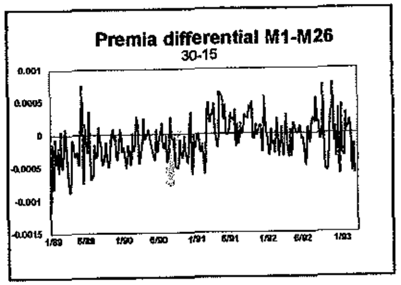
Graph 14



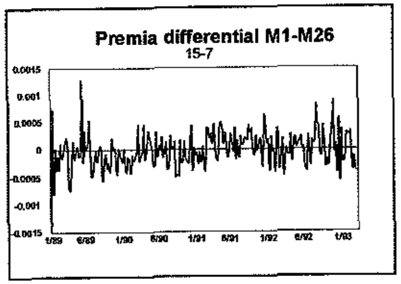
Graph 15



Graph 16



Graph 17



Graph 18

Table 8
M1 vs M26: Difference of Premia
Descriptive Statistics

Term	Mean	Std. Error	Min.	Max.
30-15	-0.076 (-3.53)	0.318	-1.187	0.761
15-7	-0.0011 (-0.54)	0.313	-0.294	0.315

All numbers are multiplied by 10^3. t-statistic in parentheses.

Table 9
M1 vs M26: Difference of Premia
Univariate models

$$Dif_t^{term} = \mu + \phi_1 Dif_{t-1}^{term} + \phi_2 Dif_{t-2}^{term} + u_t$$

Term	μ^*	ϕ_1	ϕ_2	σ_u^*	R^2	Q(20)
30-15	-0.067 (0.032)	0.225 (0.068)	0.144 (0.066)	0.296	0.09	35.3
15-7	...	0.023 (0.064)	0.171 (0.063)	0.295	0.033	24.3

A (*) indicates that numbers in the column are multiplied by 10^3. t-statistic in parentheses. Q(20) is the Ljung-Box statistic with 20degrees of freedom.

APPENDIX C

Using model M26 term premia $\pi_t^{15,7}$ and $\pi_t^{30,15}$ can be expressed as in (9):

$$\pi_t^{15,7}=\frac{-0.656+0.7B+0.52B^2-0.56B^3}{1-1.547B+0.4B^2+0.156B^3}\,es30_t+\frac{2.656-3.28B-0.067B^2+0.779B^3}{1-1.547B+0.4B^2+0.156B^3}\,es15_t+$$

$$+\frac{-1.094+1.41B-0.38B^2}{1-1.751B+0.765B^2}\,es7_t+\frac{-0.756+0.49B+0.165B^2}{1-1.7B-0.185B^2}\,es1_t+\frac{0.03-0.026B-0.01B^2}{1-1.7B-0.185B^2}\,ev15_t$$

$$\pi_t^{30,15}=\frac{0.71-1.61B+1.56B^2-0.92B^3+0.28B^4}{1-2.428B+1.77B^2-0.2B^3-0.138B^4}\,es30_t+\frac{0.072+0.11B-0.79B^2+0.93B^3-0.33B^4}{1-2.428B+1.77B^2-0.2B^3-0.138B^4}\,es15_t+$$

$$+\frac{0.132-0.73B+0.98B^2-0.38B^3}{1-2.632B+2.31B^2-0.674B^3}\,es7_t+\frac{-0.795+0.52B+0.165B^2}{1-1.7B-0.185B^2}\,es1_t+$$

$$+\frac{-0.01B}{1-.881B}\,ev30_t+\frac{0.024-0.022B-0.001B^2}{1-1.7B-0.185B^2}\,ev15_t$$

From (16), the contribution of risk in each is:

$$C_{1t}^{15,7}=\frac{-0.023+0.025B-0.018B^2-0.027B^3}{1-1.547B+0.408B^2+0.156B^3}\,ev30_t+\frac{0.1-0.14B-0.017B^2+0.066B^3}{1-1.547B+0.408B^2+0.156B^3}\,ev15_t$$

$$C_{1t}^{30,15}=\frac{0.025-0.068B+0.07B^2-0.071B^3-0.018B^4}{1-2.428B+1.77B^2-0.2B^3-0.137B^4}\,ev30_t+\frac{0.06-0.14B+0.097B^2-0.015B^3-0.05B^4}{1-2.428B+1.77B^2-0.2B^3-0.137B^4}\,ev15_t$$

where (17) in this case is estimated as:

$$\hat{\beta}=\begin{pmatrix}0.015 & 0.012\\ -0.018 & 0.011\\ -0.017 & -0.019\\ -0.023 & -0.036\\ 1 & 0\\ 0 & 1\end{pmatrix}$$

Ratios $Var(C_{1t}^{15,7})/Var(\pi_t^{15,7}) = 0.14$ and $Var(C_{1t}^{30,15})/Var(\pi_t^{30,15}) = 0.16$, gives the contribution of risk to the variance of each premium.

FOOTNOTES

1. Hall, Anderson and Granger(1992) show that two I(1) interest rates are cointegrated if and only if the term premium is I(0).

2. The SCA Statistical System was used in order to carry out the computations. This software uses an exact maximum likelihood estimation algorithm based on Hillmer and Tiao(1979).

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