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A GENERALIZED LEAST SQUARES ESTIMATION METHOD FOR VECTOR MOVING AVERAGE MODELS

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ABSTRACT

A new GLS procedure for estimating VMA models is proposed. Its main feature is to consider explicitly the stochastic structure of the approximation errors arising when lagged VMA innovations are replaced with lagged residuals from a long VAR.

RESUMEN

Se propone un nuevo método lineal para la estimación de modelos VMA. Este método tiene como característica principal, la de considerar explicitamente la estructura estocástica de los errores de aproximación que se cometen al sustituir las innovaciones del VMA por residuos obtenidos a partir de la estimación de un VAR de orden elevado.

JEL classification: C32

1. INTRODUCTION

Moving average terms in multivariate models complicate specification and estimation procedures. Thus, while it is recognized that in some situations pure VMA or mixed VARMA models can produce more efficient forecasts than an appropriate finite order VAR approximation, the fact is that VAR models have dominated the empirical work.

Simplifying the task of elaborating VMA and VARMA models has been the goal of many authors, see for instance: Hannan and Rissanen (1982), Spliid (1983) and Koreisha and Pukkila (1989) among others. These authors have developed linear estimation procedures with some desirable features:

i) They are easy to implement. Most of them only require a standard least squares (LS) routine.

ii) They are fast. Either no iterations or just a few are needed for obtaining accurate estimates, comparable with that of maximum likelihood (ML) methods. Further, for the univariate case Koreisha and Pukkila (1990) have found that their generalized least squares (GLS) procedure: (1) yields accurate estimations even when short samples are used, (2) seldom generates non-invertible or non-stationary situations, and (3) performs better than ML when a pure moving average (MA) processes is needed to fit a short sample.

iii) Fast estimation procedures have proved to be quite effective in detecting non-zero parameters.

iv) The use of these estimates in order to initialize more efficient estimation procedures as exact ML, reduces the number of iterations needed for convergence.

In this paper a new linear method for estimating pure VMA models is proposed[1]. We use an idea introduced by Koreisha and Pukkila (1990) in the univariate context, i.e. that there is an approximation error when replacing, in the original VARMA model, lagged innovations with lagged residuals from a long VAR. But instead of using their white noise assumption for the approximation error, we derive its exact stochastic structure, which depends on "L", the order of the long VAR, as well as on "q", the order of the VMA model. We show how the VMA(q+L-1) structure of the epproximation errors will induce a VMA(2q+L) structure in the noise of the transformed model, instead of a VMA(q), as implied by Koreisha and Pukkila's (1990) assumption. Our method not only generalized those proposed in Koreisha and Pukkila (1989) but also that in Koreisha and Pukkila (1990) for univariate models.

The paper is organized as follows. Section 2 describes our proposed GLS approach to estimating VMA processes. Section 3 presents our simulation results. Finally, Section 4 concludes.

2. A NEW GLS APPROACH TO ESTIMATING VMA MODELS

Consider the k×1 vector z_t of time series following the invertible VMA process:

$$z_t = \theta_q(B)\, a_t \qquad (1)$$

t= 1, 2, ... N, where $\theta_q(B) = I - \theta_1 B - \ldots - \theta_q B^q$ is a k×k finite order (q) polynomial matrix

[1] The extension to mixed VARMA models is in progress.

in the lag operator B, with the roots of $|\theta(B)| = 0$ lying all outside the unit circle. The k×1 vector a_t is assumed to follow a white noise process with covariance matrix Σ_a.

Two useful alternative representations of process (1) for the whole sample are:

Alternative Representation 1:

$$\mathrm{vec}(Z_N) = D_{\theta,N}\mathrm{vec}(A_N) + G_{q,N}\mathrm{vec}(A^*) \qquad (2)$$

Alternative Representation 2:

$$\mathrm{vec}(Z_N) = [A' \otimes I_k]\ \mathrm{vec}(\Theta) + \mathrm{vec}(A_N) \qquad (3)$$

where:

$$Z_N = [z_1\ z_2\ \ldots\ z_N]_{(k\times N)} \qquad (4)$$

$$A_N = [a_1\ a_2\ \ldots\ a_N]_{(k\times N)} \qquad (5)$$

$$A^* = [a_{-q+1}\ a_{-q+2}\ \ldots\ a_0]_{(k\times q)} \qquad (6)$$

$$D_{\theta,N} = \begin{bmatrix} I_k & 0 & 0 & .. & 0 & 0 & .. & 0 & 0 & 0 \\ -\theta_1 & I_k & 0 & .. & 0 & 0 & .. & 0 & 0 & 0 \\ -\theta_2 & -\theta_1 & I_k & .. & 0 & 0 & .. & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ -\theta_q & -\theta_{q-1} & -\theta_{q-2} & .. & I_k & 0 & .. & 0 & 0 & 0 \\ 0 & -\theta_q & -\theta_{q-1} & .. & -\theta_1 & I_k & .. & 0 & 0 & 0 \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ 0 & 0 & 0 & .. & -\theta_{q-1} & -\theta_{q-2} & .. & I_k & 0 & 0 \\ 0 & 0 & 0 & .. & -\theta_q & -\theta_{q-1} & .. & -\theta_1 & I_k & 0 \\ 0 & 0 & 0 & .. & 0 & -\theta_q & .. & -\theta_2 & -\theta_1 & I_k \end{bmatrix}_{(Nk\times Nk)} \qquad (7)$$

$$\Theta = [-\theta_1\ -\theta_2\ \ldots\ -\theta_q]_{(k\times qk)} \qquad (8)$$

Provided that a finite order VAR(L) is an appropriate approximation to (1), consistent estimates of the elements in A can be obtained from residuals ($\hat{a}_t$) in the regression:

These estimates can be used for estimating vec(Θ) in (3). Using either the first iteration of Spliid's (1983) method or the extension of Hannan and Rissanen (1982) procedure, proposed by Koreisha and Pukkila (1989), consistent estimates of parameters and

$$A = \begin{bmatrix} a_0 & a_1 & \cdots & a_{N-1} \\ a_{-1} & a_0 & \cdots & a_{N-2} \\ .. & .. & ... & .. \\ a_{-q+1} & a_{-q+2} & \cdots & a_{N-q} \end{bmatrix}_{(qk \times N)} \quad (9)$$

$$G_{\theta,N} = \begin{bmatrix} -\theta_q & -\theta_{q-1} & .. & .. & -\theta_2 & -\theta_1 \\ 0 & -\theta_q & .. & .. & -\theta_3 & -\theta_2 \\ .. & .. & .. & .. & .. & .. \\ 0 & 0 & .. & .. & -\theta_q & -\theta_{q-1} \\ 0 & 0 & .. & .. & 0 & -\theta_q \\ 0 & 0 & .. & .. & 0 & 0 \\ .. & .. & .. & .. & .. & .. \\ 0 & 0 & .. & .. & 0 & 0 \end{bmatrix}_{(kN \times qk)} \quad (10)$$

$$z_t = \sum_{j=1}^{L} \pi_j z_{t-j} + u_t \quad (11)$$

their associated standard errors can be obtained by applying LS to the model:

$$vec(Z_N) = [\hat{U}' \otimes I_k] \, vec(\Theta) + vec(A_N) \quad (12)$$

where

$$\hat{U} = \begin{bmatrix} \hat{u}_0 & \hat{u}_1 & \cdots & \hat{u}_{N-1} \\ \hat{u}_{-1} & \hat{u}_0 & \cdots & \hat{u}_{N-2} \\ .. & .. & ... & .. \\ \hat{u}_{-q+1} & \hat{u}_{-q+2} & \cdots & \hat{u}_{N-q} \end{bmatrix}_{(qk \times N)} \quad (13)$$

Alternatively, Koreisha and Pukkila (1989) propose the Double Regression (DR) method, as LS to the model:

$$vec(Z_N) - vec(\hat{U}_N) = [\hat{U}' \otimes I_k] \, vec(\Theta) + vec(E_N) \quad (14)$$

where

$$\hat{U}_N = [\hat{u}_1 \; \hat{u}_2 \; \cdots \; \hat{u}_N]_{(k \times N)} \quad (15)$$

Approximate standard errors for DR estimates can be calculated with the same formula used in calculating standard errors in (12), see Koreisha and Pukkila (1989):

$$\hat{Cov}(vec(\hat{\theta})) = [\, (\hat{U}\hat{U}')^{-1} \otimes \hat{\Sigma}_a \,] \quad (16)$$

Note that none of above methods take into account the fact that an approximation error will be committed when replacing lagged innovations a_{t-j} with lagged residuals $\hat{u}_{t-j}$. Noises in transformed models (12) and (14) are implicitly assumed to be white noise processes. We will show that approximation errors [noise in (14)] will follow a VMA(L+q-1) structure, implying a VMA(2q+L) process for noise in (12).

Process (1) will have an infinite VAR representation:

$$z_t = \sum_{j=1}^{\infty} \pi_j z_{t-j} + a_t \quad (17)$$

LS to (11) gives:

$$z_t = \sum_{j=1}^{L} \hat{\pi}_j z_{t-j} + \hat{u}_t \quad (18)$$

By subtracting (17) from (18):

$$a_t = \hat{u}_t + S_{1t} - S_{2t} = \hat{u}_t + \epsilon_t \quad (19)$$

where

$$S_{1t} = \sum_{i=1}^{L} \hat{\pi}_i z_{t-i}$$

$$S_{2t} = \sum_{i=1}^{\infty} \pi_i z_{t-i} \quad (20)$$

$$\epsilon_t = S_{1t} - S_{2t}$$

In Koreisha and Pukkila (1989), ϵ_t is assumed to be zero. In Koreisha and Pukkila (1990), ϵ_t is assumed to be a white noise vector .

S_{1t} can be expressed as:

$$S_{1t} = \hat{\Pi} R_\theta A_{t-1,t-(q+L)} \quad (21)$$

where

$$\hat{\Pi} = [\hat{\pi}_1 \; \hat{\pi}_2 \; \cdots \; \hat{\pi}_L] \quad (22)$$

$$R_\theta = \begin{bmatrix} I & -\theta_1 & -\theta_2 & \dots & -\theta_q & 0 & 0 & 0 \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & I & -\theta_1 & \dots & -\theta_{q-1} & -\theta_q & 0 & 0 \dots & 0 & 0 & 0 & \dots & 0 \\ : & : & : & ::: & : & : & : & : ::: & : & : & : & ::: & : \\ : & : & : & ::: & : & : & : & : ::: & : & : & : & ::: & : \\ 0 & 0 & 0 & ::: & I & -\theta_1 & : & : ::: & -\theta_q & 0 & 0 & ::: & 0 \\ 0 & 0 & 0 & ::: & 0 & I & -\theta_1 & : ::: & -\theta_{q-1} & -\theta_q & 0 & ::: & 0 \\ : & : & : & ::: & : & : & : & : ::: & : & : & : & ::: & : \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \dots & -\theta_1 & -\theta_2 & : & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \dots & I & -\theta_1 & -\theta_2 & \dots & -\theta_q \end{bmatrix}_{(Lk \times k(L+q))} \quad (23)$$

$$A_{t-1,t-(q+L)} = \begin{pmatrix} a_{t-1} \\ a_{t-2} \\ :: \\ :: \\ a_{t-(q+L)} \end{pmatrix}_{(k(q+L)\times 1)} \quad (24)$$

or

$$S_{1t} = \psi_1^{s1} a_{t-1} + \psi_2^{s1} a_{t-2} + \dots + \psi_{L+q}^{s1} a_{t-(L+q)} \quad (25)$$

where

$$\hat{\Pi} R_\theta = [\ \psi_1^{s1}\ \psi_2^{s1} \dots \psi_{L+q}^{s1}\ ]_{(k \times k(L+q))} \quad (26)$$

From (1) S_{2t} can be expressed as:

$$S_{2t} = E_{t-1}(z_t) = -\theta_1 a_{t-1} - \theta_2 a_{t-2} - \dots - \theta_q a_{t-q} \quad (27)$$

Then

$$\epsilon_t = \psi_1^{\epsilon} a_{t-1} + \psi_2^{\epsilon} a_{t-2} + \dots + \psi_{L+q}^{\epsilon} a_{t-(L+q)} \quad (28)$$

where

$$[\psi_1^{\epsilon}\ \psi_2^{\epsilon} \dots \psi_{L+q}^{\epsilon}] = [\ \psi_1^{s1}\ \psi_2^{s1} \dots \psi_{L+q}^{s1}\ ] - [-\theta_1 \dots -\theta_q\ 0 \dots 0] \quad (29)$$

Note that:

$$\psi_k^{\epsilon} = 0 \quad \forall\ k, \quad \text{if } \pi_j = \hat{\pi}_j \quad \forall\ j \quad (30)$$

In that case $\epsilon_t = 0$ and $a_t = \hat{u}_t$.

In general, finite samples estimation errors will make weights π_j to differ from their LS estimates, and therefore they will induce in ϵ_t the VMA(L+q-1) structure (28).

Taking into account (28), process (1) becomes:

$$z_t = -\theta_1 \hat{u}_{t-1} - \theta_2 \hat{u}_{t-2} - \dots - \theta_q \hat{u}_{t-q} + \eta_t$$
$$\eta_t = a_t - \psi_1 a_{t-1} - \psi_2 a_{t-2} - \dots - \psi_{2q+L} a_{t-(2q+L)} \quad (31)$$

where

$$(-\psi_1 B - \dots - \psi_{2q+L} B^{2q+L}) = (-\theta_1 B - \dots - \theta_q B^q)(\psi_1^{\epsilon} B + \dots + \psi_{q+L}^{\epsilon} B^{q+L}) \quad (32)$$

Representation 2 for (31) is:

$$\text{vec}(Z_N) = (\hat{U}' \otimes I_k)\ \text{vec}(\Theta) + \text{vec}(H_N)$$
$$\text{vec}(H_N) = D_{\psi,N}\ \text{vec}(A_N) + G_{\psi,N}\ \text{vec}(A^{**}) \quad (33)$$

with

$$H_N = [\ \eta_1\ \eta_2 \dots \eta_N\ ]_{(k \times N)}$$
$$A^{**} = [a_{-2q+L+1} \dots a_0\ ]_{(k \times 2q+L)} \quad (34)$$

Model (33) suggests the feasible GLS estimator for vec(Θ):

$$\text{vec}(\hat{\Theta}) = [(\hat{U} \otimes I_k)\hat{\Omega}^{-1}(\hat{U}' \otimes I_k)]^{-1}\ (\hat{U} \otimes I_k)\hat{\Omega}^{-1}\text{vec}(Z_N) \quad (35)$$

whose covariance matrix can be computed as:

$$\hat{\text{Cov}}(\text{vec}(\hat{\Theta})) = [(\hat{U} \otimes I_k)\hat{\Omega}^{-1}(\hat{U}' \otimes I_k)]^{-1} \quad (36)$$

where

$$\hat{\Omega} = (\hat{G}_{\psi,N}\ \hat{D}_{\psi,N})\ (I_{N+2q+L} \otimes \hat{\Sigma}_a)\ (\hat{G}_{\psi,N}\ \hat{D}_{\psi,N})' \quad (37)$$

An initial estimation of vec(Θ), needed to evaluate D and G, can be obtained from LS to (33). An estimation of Σ_a can be obtained as:

$$\hat{\Sigma}_a = \frac{\hat{U}_N \hat{U}_N'}{N} \qquad (38)$$

or from:

$$\text{vec}(\hat{\Sigma}_{H_a}) = (I + \hat{\Psi}_1 \otimes \hat{\Psi}_1' + \ldots + \hat{\Psi}_{2q+l} \otimes \hat{\Psi}_{2q+l}') \text{ vec}(\hat{\Sigma}_a) \qquad (39)$$

once LS has been applied to model (33).

3. SIMULATION EXERCISE

Tables 1 and 2 show the simulation results for two vector processes: a VMA(1) and a VMA(2). These models are the same used in Koreisha and Pukkila (1989) for illustrating the properties of their method. As these authors, we simulated 50 realizations for each model. The sample size N was set equal to 100 and the order "L" of the long VAR was set equal to $\sqrt{N}=10$.

Both tables have the same structure, the first pannel shows the mean value of parameter estimates obtained with three different estimation procedures: Our Generalized Hannan Risanen (GHR) procedure, Koreisha and Pukkila's (1989) Double Regression (DR) procedure and Hillmer and Tiao's (1979) Exact Maximum Likelihood estimation procedure. The second pannel shows the mean values of the estimated standard errors associated to each parameter. Finally, the third pannel shows the frequency of significant non-zero parameters (95% confidence) tentatively identified by each method.

Comparative results are similar for both models and can be summarized as follows:

1) All estimation procedures yield similar parameters estimates. This result is the same obtained by Koreisha and Pukkila (1989).

2) Estimated standard errors associated to DR estimates are greater than those associated to GHR or EML methods. This precision gain is behind the differences in the frequencies with which significant non-zero values are identified with the three estimation methods. Those differences are more evident for parameter values under .3. For instance, in the case of the VMA(1) model, the DR method identifies, as being different from zero, the parameter .2 only in 28% of cases while EML does it in 50% and GHR in 62%. On the other hand, both GHR and EML methods lead to overparametrize the model more often than DR.

3) All estimation procedures perform poorly in detecting low parameter values, showing a tendency to conclude that they are not significant, even with moderate sample sizes. This result suggests that removing non significant parameters by blindly using the 95% standard rule could lead to important misspecification errors. We leave for future research the evaluation of the possible consequences of such an identification bias.

[INTRODUCE TABLES 1 AND 2]

4. CONCLUSIONS

In this paper we generalize, for VMA models, the extension of Hannan Rissanen estimation method proposed in Koreisha and Pukkila (1989). The idea is that the differece between innovations associated to a VMA model and residuals from a long VAR(L) will not follow a white noise vector. We derive its exact stochastic structure that is a VMA(q+L-1). Taking into account this result we propose our Generalized Hannan Rissanen (GHR) estimator.

Simulations results indicate that GHR performs better than DR. It increases the precision of parameters estimates and helps to better identify significant non-zero parameters. This feature is particularly important in the case of low parameters values, very difficult to detect even using ML estimation procedures.

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TABLE 1
Summary of simulation results VMA(1), K=5, N=100, 50 replications

$$\theta_1 = \begin{bmatrix} 0 & 0 & 0 & 1.1 & 0 \\ 0 & 0 & 0 & 0 & .2 \\ 0 & 0 & 0 & 0 & 0 \\ -.55 & 0 & 0 & .8 & 0 \\ 0 & 0 & 0 & 0 & .6 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & & & & \\ .2 & 1 & & & \\ 0 & 0 & 1 & & \\ 0 & 0 & .7 & 1 & \\ 0 & 0 & 0 & -.4 & 1 \end{bmatrix}$$

Mean values of the estimated parameters

GHR					DR					EML*				
-0.09	0.03	0.08	1.05	-0.02	-0.05	0.02	0.03	1.06	-0.00	0.01	0.02	-0.06	1.18	0.04
0.01	-0.05	-0.01	-0.01	0.23	-0.00	-0.03	-0.01	-0.01	0.18	-0.03	0.00	-0.04	0.05	0.25
0.00	0.08	-0.07	0.00	0.00	0.03	0.04	-0.03	0.01	0.015	0.01	0.00	0.02	0.01	0.02
-0.56	0.06	0.03	0.73	0.00	-0.50	0.02	0.01	0.77	0.016	-0.56	0.01	-0.02	0.83	0.01
-0.00	0.03	-0.01	0.02	0.51	-0.01	0.03	-0.00	0.02	0.56	-0.03	0.00	0.01	-0.02	0.63

Mean values of the estimated standard errors

(0.09)	(0.08)	(0.13)	(0.14)	(0.10)	(0.11)	(0.10)	(0.16)	(0.18)	(0.13)	(0.06)	(0.06)	(0.09)	(0.10)	(0.07)
(0.08)	(0.08)	(0.12)	(0.13)	(0.10)	(0.11)	(0.11)	(0.17)	(0.18)	(0.13)	(0.10)	(0.10)	(0.16)	(0.17)	(0.12)
(0.08)	(0.08)	(0.12)	(0.13)	(0.09)	(0.11)	(0.11)	(0.16)	(0.18)	(0.13)	(0.07)	(0.08)	(0.12)	(0.13)	(0.09)
(0.08)	(0.08)	(0.13)	(0.14)	(0.10)	(0.11)	(0.10)	(0.16)	(0.18)	(0.13)	(0.04)	(0.04)	(0.07)	(0.07)	(0.05)
(0.08)	(0.08)	(0.12)	(0.13)	(0.09)	(0.11)	(0.10)	(0.16)	(0.18)	(0.13)	(0.07)	(0.07)	(0.11)	(0.12)	(0.08)

Frequency of significative non-zero values (%)

38	36	38	100	26	20	12	18	100	10	14	18	24	100	24
36	44	42	38	62	10	24	22	16	28	10	12	14	12	50
40	38	48	44	44	22	12	14	18	14	18	18	14	14	10
100	32	40	98	32	94	16	12	94	22	100	18	22	100	18
30	46	42	40	92	10	18	16	12	88	16	16	14	18	100

(*) From Koreisha and Pukkila (1989)

TABLE 2
Summary of simulation results VMA(2), K=3, N=100, 50 replications

$$\theta_1 = \begin{bmatrix} .7 & 0 & 0 \\ 0 & 1.25 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \theta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -.75 & 0 \\ 0 & .3 & .6 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 1 & & \\ -.7 & 1 & \\ .4 & 0 & 1 \end{bmatrix}$$

Mean values of the estimated parameters

	GHR			DR			EML*		
	0.64	0.02	-0.03	0.67	-0.01	-0.02	0.74	0.00	0.01
θ_1	0.00	1.14	-0.00	-0.01	1.23	0.01	0.01	1.29	0.00
	0.06	0.07	-0.12	0.03	0.02	-0.06	0.01	0.00	0.00
	0.03	0.02	-0.02	0.04	0.01	-0.03	0.02	0.00	0.03
θ_2	-0.09	-0.83	0.03	-0.05	-0.78	0.02	0.02	0.78	0.01
	0.00	0.31	0.57	0.01	0.30	0.54	0.02	0.28	0.69

Mean values of the estimated standard errors

(0.14)	(0.13)	(0.10)	(0.17)	(0.16)	(0.12)	(0.15)	(0.13)	(0.11)
(0.15)	(0.14)	(0.11)	(0.18)	(0.16)	(0.12)	(0.13)	(0.13)	(0.10)
(0.14)	(0.13)	(0.10)	(0.18)	(0.16)	(0.12)	(0.16)	(0.14)	(0.10)
(0.14)	(0.12)	(0.10)	(0.17)	(0.16)	(0.12)	(0.15)	(0.15)	(0.10)
(0.15)	(0.14)	(0.10)	(0.18)	(0.16)	(0.12)	(0.12)	(0.13)	(0.08)
(0.14)	(0.13)	(0.10)	(0.18)	(0.16)	(0.12)	(0.15)	(0.17)	(0.12)

Frequency of significative non-zero values (%)

92	36	40	96	14	6	100	10	12
40	100	28	6	100	10	28	100	20
34	40	36	12	2	8	6	12	10
18	22	32	10	16	16	14	14	8
34	96	38	28	92	20	20	100	14
44	64	96	18	54	98	14	60	100

(*) From Koreisha and Pukkila (1989)