Decision making and Atanassov's approach to fuzzy sets

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Abstract

In this paper we stress the role that decision making, and therefore an underlying binary (bipolar) approach, plays in Atanassov's *intuitionistic* view of fuzzy sets. Moreover, we point out that such a bipolar view can be found in classical multicriteria models,

Keywords: Multicriteria Decision Making, Atanassov's Intuitionistic Fuzzy Sets.

1 Introduction

Atanassov's intuitionistic fuzzy sets [3] (see also [4, 5, 6, 11, 31]) has received great attention from the scientific community, showing its practical efficiency when applied to quite a number of problems. Although such a model has been also subject to a serious controversy because of the improper *intuitionistic* term and because of its equivalence to already existing models (see [32] and [13, 15], but also [7, 8]), we think with [20] that there are certain specific differences in conception that, when included in the model, can explain such a confusing situation. In particular, in [20] it is shown how those *equivalent* models become different when the underlying structured is taken into account within a classification framework (see [28] but also [1, 2]). It is also interesting to have a look to [12], where several formal generalizations of fuzzy sets [34] are reviewed together with some applications, as alternative models to be tried in those places Atanassov's model has been successful, or not successful. The conflict about the name remains, and some researchers propose to talk about *bipolar* fuzzy sets instead of Atanassov's *intuitionistic* fuzzy sets (see, e.g., [10, 15, 16, 35]).

Atanassov's model [3] was originally defined as a particular type-2 fuzzy set (see [14, 17, 18]) $\mu : \mathcal{X} \to [0,1]^3$ where X represents the family of objects under consideration, and the images represents the degrees of *membership*, *non-membership* and *indeterminacy* for each object $x \in X$. According to Atanassov's initial proposal (but see [20]), these three values should sum up to 1, for all $x \in X$. A binary or *bipolar* argument is clear from that *membership* and *non-membership* (see also [20]).

Next section 2 will be devoted to explain how keeping decision making as the final objective of mathematical modeling may suggest a wrong methodology in such a modeling process, suggesting that this criticism may apply to Atanassov's proposal, too. Section 3 of this paper will be devoted to analyze some implications in multicriteria decision making, pointing out the potential interest of alternative models to Atanassov's approach in decision making processes. Section 4 will be devoted to illustrate this particular issue with an example, in order to show the different stages of a decision process such an evaluation models can impact. A final section 5 will connect the previous discussion into a more general setting in Science.

2 Bipolarity and decision making

Sometimes it has been argued that nothing can be said about, for example, *tallness* without referring to *shortness*. But notice that *shortness* is not the negation of *tallness*, but is antonym (see, e.g., [33]). As pointed out in [20], this should be the framework in Atanassov's model if a classification view is kept (see also [2]).

The main argument in [21] was that final decision is sometimes misleading decision making researchers, and that certain granted *status quo* use to introduce a hidden binary *contamination* of the proposed mathematical model. Such a contamination comes mainly because of the basic information, the *data*, or because of the objective. Although we think with [27, 29] that the true objective of a mathematical model should be to help decision maker to understand reality and design new approaches to the problem (decision making *aiding*), it is true that a relevant number of researchers have been stressing in the past that the main objective of the mathematical effort should be instead to support the final decision, not the decision process itself. These researchers will claim that those final decision are the only *observable* data (we shall address the data issue in the next section). Although we think that mathematical models should stay at a decision aid stage, never telling decision maker what decision maker should do, at this point what we simply want to stress that once we assume that the final (observable) decision is our objective, such an *observability* argument introduces a confusion between the inner *decision* we make and the *act* produced by such a decision (see [22] for a deeper discussion on this issue). It is also interesting to remind that Medicine has proved that the part of the human brain in charge of final decision is different from the part of the brain in charge of its rational analysis [9].

In fact, each act we make is always crisp, although its description is sometimes fuzzy (mainly because they use to be explained in terms of words). So, if we focus our study on acts (not their description but the acts themselves), our study may deal only with crisp information, and the temptation is clear: classify them in terms of crisp subsets. Of course, each one of these crisp subsets can be alternatively described in terms of its complementary. But it must be noticed that, in this crisp context, a subset and its complementary have exactly the same information. One can be deduced from the other, and no additional information is gained when the complementary is given. The useful information (in case a bipolar scheme is pursued) should be by means of the antonym concept. In some way, when the universe of potential acts is considered and an experiment is run, such a universe of potential acts is naturally divided into those acts that happened and those acts that did not happen. Such a bipolar approach in terms of a concept and its complementary may be natural within a crisp context, but it is not informative.

It is interesting to note that in some particular cases, precisely for those truly crisp concepts, perception of the antonym and perception of the complementary are the same (see [25]).

A bipolar classification in terms of a concept and its antonym can be nevertheless informative within a fuzzy context. But it does not seem so natural within a fuzzy context. Talking about *height*, for example, in no way restricts our mind into only two classes like *tallness* and *shortness*: there are many alternative fuzzy concepts related to *height*.

Notice that the same argument applies to facts (again, not to their description but the facts themselves). Fuzziness in an experiment or observation appears when those acts or facts need to be described in linguistic terms, which is the standard vehicle for communication among people (sometimes the information we exchange is crisp, of course).

3 Bipolarity in multicriteria decision making

In this section we shall show how this *intuitionistic* or bipolar argument has been present is classical multicriteria decision making models (such a polarity may be considered unnatural in a decision making aid approach).

A number of multicriteria decision making tools are based upon the binary direct comparison between every possible pair of alternatives. Given a finite family of crisp alternatives X, a fuzzy weak preference relation is a mapping $\mu : X \times X \rightarrow [0,1]$ that assigns to each pair of alternatives $(x, y) \in X \times X$ a value $\mu(x, y)$ the degree to which alternative x is better or equivalent to y.

The classical ELECTRE model [27], for example, assumes basic data in terms of a family of fuzzy preference relations for each criteria. Being $\mathcal{C}$ a finite family of criteria describing the main features of each alternative $x \in X$, for each criteria $i \in \mathcal{C}$ it is assumed in [27] (see also [30, 26]) the existence of a fuzzy preference relation

$$\mu_i : X \times X \rightarrow [0,1]$$

such that $\mu_i(x, y)$ represents the degree of intensity to which alternative x in weakly better than alternative y whenever we restrict comparison to criteria i. Based on such an information about the decision maker preferences decomposed by criteria, an *outranking* relation [27] is defined over the cartesian product of all alternatives. And together with such an *outranking* relation, it is also defined a *veto* fuzzy set, in such a way that from both intensities a fuzzy set of non-dominated alternatives is defined. This fuzzy set should hopefully point out one solution or a small set of solutions.

Note that the meaning of such an *outranking* relation seems to be close in meaning to a strict preference of each alternative x over each alternative y, and that *veto* meaning seems to be related to the opposite strict preference. But in no way outranking is the *negation* of veto.

The point we want to stress is that such a model [27] acknowledges the interest of introducing *dual* concepts in order to analyze how good a solution is. It is not simply the strict preference of x over y in opposition to the strict preference of alternative y over x, but how an alternative x is supported in opposition to the intensity to which an alternative is unacceptable. We can not make a decision without taking into account *pro* arguments and *contra* arguments at the same time. A *contra* argument is not simply the logical *negation* of a *pro* argument. In this sense, see an interesting bipolar approach to Roy's concordance and discordance in [24].

4 About the knowledge process

In this section we shall illustrate with an example the different stages of a knowledge process, each one subject in principle to a crisp, fuzzy, bipolar or even a more general information support. If Atanassov's bipolar model has been successful at any stage of a decision making process, any alternative extension of fuzzy sets (see, e.g., [12, 20]) are good candidates to improve those results, introducing a more accurate representation of each possible description or valuation.

Imagine for example that your kid is sick in bed, and that he ask for a bottle filled with *warm* water for his cold feet. We more or less will proceed following the following stages:

1. Design an *observational system* for reality, if there is an alternative to the existing one (perhaps a controlled experiment), in order to get data (reality is never what we observe, in the same way that what we see is not the reality itself but the processed information obtained from our eyes). In our case we may decide to fill up a bottle in our kitchen, mixing water from the cold tab water at 10 Celsius degrees and the hot tab water at 70 Celsius degrees. This observational system (by means of those two particular tabs) implies a binary perception of reality. But no one outside this observational system will claim that water can be only obtained by means of those two tabs in our kitchen, and no one outside the previous observational system thinks that *cold* means 10 Celsius degrees or that *not cold* means 70 Celsius degrees.

2. Design an *informational system* that will allow us to manipulate data according to a certain logic (to be chosen) and process those data into information (in the same way as our brain builds up a continuous perception from a finite family of images obtained from our eyes). Such an informational system includes knowledge about the mechanical behavior of both tabs and the physical laws ruling the mixture of liquids. Of course this informational system is being limited by the previous observational system (we know for example that we shall never be able to obtain mixed water outside the range 10-70 Celsius degrees).

3. Design an *analytic tool* in order to process the previous information, understand the specific characteristics of our problem and be able

to make inferences (in our case, we need to measure temperature, of course, but we should also evaluate the specific characteristic of the bottle, how sick is the kid and even the personality of the kid, for example).

4. Design a *decision tool* which allows to establish goodness criteria or comparison between different actions (taking into account, for example, that too hot water is dangerous and too cold water is disgusting). Consistency use to be a key requirement at this stage.

5. Design a *decision generator*, that will generate a final decision accordingly to the previous decision analysis, perhaps taking into account our own intuition and any other available information or external advise. For example, we may decide to fill up the bottle half with cold water and half of hot water, so final temperature should be around 40 degrees (generation of an act may require a random component). But this decision stage should not be confused with the previous supporting stage, neither the initial global analysis, and neither to the next stage with the execution of such a final decision into an action. This stage can include the creation of new alternatives.

6. Execute decision: the particular act, being a consequence of the previous final decision, depends on the particular size and shape of the bottles, the mechanical tools we may have at hand to fill up the bottle, our personal abilities manipulating the bottle and even some contextual or random events (that loud heavy rock loved by our sick kid can reduce our personal abilities, for example).

7. Design an *evaluation tool* in order to check results accordingly to final objectives (including health of the kid, of course, but perhaps kid's opinion, too) and decide about a revision of the whole process.

It may happen, like most probably in the above example, that each stage has been previously assumed by the decision maker because of tradition. It is then important to realize that such *a priori* assumptions may represent a serious problem in more complex problems, when the decision maker is not conscious of key assumption, like for example the binary conception of the observational analysis or the binary logic ruling the informational system, as pointed out in [19].

In general, we can say that data are obtained from reality by means of a observational system. Data are processed by means of a logic in order to produce information and allow inferences. Information should be globally analyzed by means of several tools (decomposition, aggregation, graphical representation, etc.) Information suffers a second analysis focussed on decision (such analysis depends on the type of decision we are looking for, meanwhile the previous information analysis pursues a better knowledge of the system under study). Then a decision can be made (outside any previous analytic tool), and depending on the circumstances, an act is produced, hopefully being consistent with such a decision (the final act it is not the decision itself: first, such an act is rarely fully described when decision is made and details are left to the very last moment, and second, its execution may depend on a random or uncontrolled context). Still, such an act and its consequences need to be described and evaluated according the previously defined objective.

But of course the declared objective does not affect only to the evaluation stage. The whole process is obviously under the influence of such an objective. For example, the data produced by the observational system should be consistent with the informational system that will manipulate those data (the observational system of course introduces a clear restriction on the informational system, but the logic ruling the informational system introduces also restrictions on the design of the observational system). Analogously, the creation of a new alternative may imply to go back one stage, and a surprising result in the evaluation stage can make us return to re-design the informational system or even the observational system. The above knowledge stages are not independent, and the process itself is neither unidirectional.

5 Conclusions

In this paper we have pointed out a possible double *contamination* of the original Atanassov's model, coming from a natural binary crisp approach to decision making: on the one hand, because of the description in terms of two concepts; and on the other hand, because of the confusion between complementary and antonym (which makes Atanassov's indeterminacy perhaps meaningless). A first consequence on multicriteria analysis is considered, suggesting a systematic application of alternative extensions of fuzzy sets to those problems where a bipolar approach has been successful or failed. Moreover, an example is presented in order to get a better understanding of the different stages within the knowledge process where those alternative extensions should be tried. In particular, we want to stress again that classical experiments use to assume in fact an observational system that implies a crisp logic, which may remain hidden to many scientists if they are not conscious of such an traditional assumption (see again [19]).

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