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FORECASTING WITH PERIODIC MODELS:

A comparison with time invariant coefficient models

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ABSTRACT

Working with seventeen *UK* macroeconomic variables, characterized as periodically integrated in Franses and Romijn(1993), we have found that unconstrained periodic models do not beat time invariant alternatives in forecasting, even when cointegrating relationships among the seasons are taken into account. However, when appropriately constrained, the forecasting performance of periodic models can be much better than that of non-periodic models. Homogeneity restrictions among some seasons seem to be very important in that respect, which motivates us to propose a switching procedure between a periodic model and a non-periodic univariate *AR* as a representation of the behaviour of these variables. Once season homogeneity is taken into account, incorporating the cointegrating relationships among the seasons through periodic error correction models achieves a substantial additional forecasting improvement.

RESUMEN

Utilizando 17 variables trimestrales macroeconómicas del Reino Unido, caracterizadas por Franses y Romijn (1993) como periódicamente integradas, hemos encontrado que modelos períodicos no restringidos no prevén mejor que modelos univariantes. En ausencia de otro tipo de restricciones, cuando sólo se tienen en cuenta explicitamente las relaciones de cointegración entre trimestres, tampoco se mejoran la previsiones de los modelos univariantes. Sin embargo, cuando los modelos períodicos se restringen adecuadamente, su capacidad predictiva mejora notablemente y el resultado negativo anterior se invierte. Las restricciones de homogeneidad en el comportamiento de los trimestres parecen ser cruciales en este sentido. Este hecho nos ha motivado a proponer la combinación de modelos, períodicos y no períodicos, como una mejor representación del comportamiento de estas variables. Una vez se han incorporado las restricciones de homogeneidad, encontramos que la incorporación de las relaciones de cointegración a través de los modelos períodicos de corrección de error mejora adicionalmente la calidad de las previsiones.

Key words: Seasonality, periodic models, unit root polynomials.
JEL Classification: C32, C52

1. INTRODUCTION

Modelling quarterly macroeconomic *UK* time series Osborn et al.(1988), Osborn and Smith(1989) and Franses and Romijn(1993) have presented empirical evidence against models with parameters that do not vary over the seasons. These authors conclude that in a number of cases, seasonal behaviour is too complex to be captured by standard time invariant coefficient models, and that periodic models can be more appropriate.

Once a time series has been detected to show a periodic behaviour [see Lütkepohl(1993, Section 12.3), Franses and Romijn(1993), Franses(1994), Boswijk and Franses(1995,1996) and Flores and Novales(1996) for useful tests] some important empirical questions arise: Will forecasts improve using a model with parameters that vary over the seasons, as advanced by Tiao and Grupe(1980)? Will this improvement justify the cost of carefully elaborating a periodic model? When short run forecasting is the objective, is there any gain in forecasting accuracy by explicitly considering the possible cointegrating relationships among the seasons?

Using a set of seventeen *UK* quarterly macroeconomic variables, already analyzed in Osborn (1990) and characterized in Franses and Romijn (1993) as periodically integrated, we investigate these questions. We carry out a forecasting competition between several models in two groups: a) time invariant, univariate *ARIMA* models with different unit root filters, and b) periodic models with different types of restrictions. Our approach in this paper is to discuss whether periodic structures are more adequate representations of seasonality on the basis of the forecasting competition between them. We are also interested on identifying the types of restrictions that may lead to a significant improvement in forecasting performance.

In the class of univariate, single-equation models, some issues related to the appropriate degree of differencing remain controversial. Osborn(1990) discussed, for this same data set, the type of unit root filters that would be necessary to achieve stationarity. She warned that the full $(1-B)(1-B^4)$ filter that is suggested by the standard Box-Jenkins(1976) specification tools for most variables in this data set might lead to overdifferencing, with a possible efficiency loss in estimation. We start by analyzing whether the choice between these two unit root filter alternatives significantly affects the forecasting performance of time invariant models. If that were the case, the choice of filter might bias the results of our study.

To compare with nonperiodic specifications, we use a variety of periodic models, from the simpler *PAR*(1), to the more complex *periodic error correction model*, defined in Franses and Romijn(1993).

Unconstrained periodic models turn out not to produce much better forecasting results than non-periodic alternatives but we find evidence that, when appropriately constrained, forecasts of time series which have been detected to be periodic can significantly improve using periodic models. In particular, equality of coefficients among some seasons as well as cointegration constraints seem to be potentially very effective.

The remaining of the paper is organized as follows. Section 2 presents the periodic models we consider and their different representations. In Section 3 the forecasting competition between alternative representations of seasonality on the sample of seventeen quarterly *UK* variables is carried out. Section 4 concludes.

2. PERIODIC AUTOREGRESSIVE REPRESENTATIONS

Let X_t denote a quarterly time series. A *periodic autoregressive process of order h, PAR(h)*, can be represented:

$$X_t = \sum_{s=1}^{4} \mu_s D_{st} + \sum_{j=1}^{h} \sum_{s=1}^{4} \phi_{js} D_{st} X_{t-j} + \epsilon_t \tag{1}$$

where ϵ_t follows a white noise process, although possibly with season specific variances. D_{st} is a dummy variable for quarter s, being equal to 1 when X_t is an observation from that quarter and being 0 otherwise. Index t varies from 1 to $4N$, N being the number of years.

Let x_T be the 4×1 vector of quarters in a year: $x_T = (X_{4T-3}, X_{4T-2}, X_{4T-1}, X_{4T})$, $T=1,2,\ldots,N$. Each component in x_T is the annual time series of data for a given quarter. Consider the following $VAR(p)$ process for x_T:

$$x_T = \delta + \Phi_1 x_{T-1} + \Phi_2 x_{T-2} + \ldots + \Phi_p x_{T-p} + a_T \tag{2}$$

where a_T is a 4×1 vector white noise process with variance-covariance matrix Σ.

The diagonalization of Σ, $A_0 \Sigma A_0' = \Lambda$, where A_0 is a lower triangular matrix with ones in the main diagonal, is uniquely defined and is consistent with interpreting the contemporaneous correlation between a_{iT} and a_{jT} $(1\leq i < j \leq 4)$, two any components of a_T, as intra-year effects from quarter i to quarter j. Residuals can be orthogonalized by premultiplying (2) by A_0:

$$A_0 x_T = \mu + A_1 x_{T-1} + A_2 x_{T-2} + \ldots + A_p x_{T-p} + u_T \tag{3}$$

where $A_j = A_0\Phi_j$ for $j=1,2,\ldots,p$, $\mu = A_0\delta$ and $u_T = A_0 a_T$, with $Var(u_T) = \Lambda$, diagonal. We call (3) an *orthogonalized VAR* model, and refer to it as *OVAR(p)*.

Any *PAR* model can be written as a restricted *OVAR*. For example, a *PAR*(1) process can be written as an orthogonalized *VAR*(1) with the following matrix structure:

$$A_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix} \quad A_1 = \begin{bmatrix} 0 & 0 & 0 & \phi_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

The opposite proposition is also true: an unrestricted *OVAR(p)* is just a restricted *PAR(h)* with $h \leq (p+1)s-1$ [see Tiao and Grupe(1980), p.367 for both statements]. Hence, the *OVAR(p)* and the *PAR(h)* can be considered as equivalent representations of a same process. In what follows we will refer to both of them as *periodic models*.

In a further step of the modelling process we might want to consider the likely presence of cointegrating relationships among seasons. Under the assumptions that (1) all seasons are $I(1)$ variables and (2) their levels are generated by a *VAR(p)* process, the number and type of cointegration relationships can be investigated with the techniques developed in Johansen and Juselius(1990). The presence of cointegrating relationships leads to the error correction model:

$$\nabla x_T = \delta - B\alpha' x_{T-1} + \Gamma_1 \nabla x_{T-1} + \Gamma_2 \nabla x_{T-2} + \ldots + \Gamma_{p-1} \nabla x_{T-p+1} + a_T \tag{5}$$

where the rows of the rx4 matrix α' are the r cointegrating vectors, and $Var(a_T) = \Sigma$.

Again, diagonalizing Σ, an orthogonalized error correction model is obtained:

$$A_0 \nabla x_T = \mu - (A_0 B)\alpha' x_{T-1} + A_0 \Gamma_1 \nabla x_{T-1} + A_0 \Gamma_2 \nabla x_{T-2} + \ldots + A_0 \Gamma_{p-1} \nabla x_{T-p+1} + u_T \tag{6}$$

which is called by Franses and Romijn(1993) the *periodic error correction model (PECM)*. Non-significant coefficients in (6) can be set to zero, which can safely be done on the basis of their t-statistics, leading to what we call the *constrained periodic error correction model (CPECM)*.

Periodic models (3) and (6) are not fully comparable. While (3) relies just on a decision on the autoregression order p, (6) additionally requires determining the number and type of cointegrating relationships, implying a higher specification cost. Theoretically, the more elaborate models should produce better forecasts, but it is not obvious that this will be the case in practice. One of the objectives of this paper is to evaluate whether the gain in forecasting performance exists and is enough to offset the higher specification costs.

3. FORECASTING PERFORMANCE OF ALTERNATIVE REPRESENTATIONS OF SEASONALITY

3.a The forecasting exercise: Description of competing models.

We compare the forecasting performance of time varying versus time invariant representations of seasonality on a data set of quarterly *UK* macroeconomic variables, already analyzed in Osborn(1990) and Franses and Romijn(1993), selecting among them the seventeen variables that the latter authors characterized as being periodically integrated[1].

In the class of univariate models, the appropriate order of differencing is far from obvious. On the one hand, Osborn(1990) relied on unit root tests to propose specific unit root filters for this same set of variables. In relation to them, standard tools recommended by Box and Jenkins(1976) lead to a higher order of differencing[2]. It has been suggested [Osborn(1990)] that this apparent overdifferencing may lead to an efficiency loss, and a possible deterioration of forecasting ability. Before we compare the forecasting performance of time varying versus time invariant models, we want to test whether the strategy used to select the number and type of unit roots in univariate models has, in fact, any significant influence on forecasting performance. The methodology proposed by Box and Jenkins(1976) to identify *ARMA* structures lead to different specifications, depending on the unit root filters applied. Under those suggested in Osborn(1990) we obtain, in our data set, pure *AR* structures, while the alternative filters mentioned in footnote 2 lead to mixed *ARMA* models. This is to be expected, since in the standard practice of *ARIMA* modelling, overdifferencing usually requires compensating moving average terms. Thus, the two non-periodic models whose forecasting results we compare with those from periodic alternatives are: *a)* *AR* models estimated with Osborn unit root filters, and *b)* *ARMA* models obtained with the unit root filters suggested by Box and Jenkins(1976) methodology.

Additionally, we consider four periodic models: (1) unrestricted *OVAR(p)* models on the levels of the seasons (quarters), (2) constrained *OVAR(p)* models where all parameters with a *t*-statistic lower than 1 have been removed[3], to which we refer as *COVAR* models, (3) periodic autoregressive models of order 1, *PAR*(1), (4) periodic error correction models (*PECM*) where cointegration relationships among the quarters have been explicitly considered, and (5) periodic error correction models where coefficients in lagged differenced quarters with a *t*-statistic lower than 1 have been removed; we label these *CPECM*.

OVAR models are simple to elaborate, but they are possibly overparameterized. *PAR*(1) and *COVAR* models incorporate less parameters, but they may be misspecified: We have reached the *COVAR* specification by first choosing from the outset a minimum level of one for the *individual t*-ratios. This was chosen with the aim to be conservative in a context where the actual significance level is unknown. We then *simultaneously* removed all coefficients which did not fulfill this criterion. In addition, the strategy lacks a rigurous justification since nonstationarity of the seasons produces a non-standard distribution for the *t*-ratios. Misspecification in *PAR*(1) models may come from being too simple to capture all the dynamics of the seasonal characteristics of the time series. With independence of these possible sources of misspecification, these three models share a quite low specification cost, which is why we consider them in our forecasting exercise.

Since the seasons are likely to be cointegrated [see Franses and Romijn(1993)], considering error correction models is an attempt to gain efficiency. As we mentioned in Section 2, they should also be expected to have a better forecasting performance. Our consideration of *PECM* models tries to check whether that is the case. Once the cointegrating relationships among the seasons are introduced, there may be non-significant coefficients. In theory, their removal should increase efficiency and improve forecasts. Whether or not this is the case in practice is the reason why we consider *CPECM* models. As in *COVAR* models, we again use an automatic rule to remove coefficients, although the *t*-distribution is well justified in this case.

Univariate *AR* and *ARMA* models have been estimated using the Marquardt algorithm with backforecasting as described in Box and Jenkins(1976). *OVAR* and *COVAR*, as well as *PECM* and *CPECM* models, have been treated as a seemingly unrelated set of equations and jointly estimated by generalized least squares. This procedure takes into account the fact that, under misspecification of the *PAR* order, there might arise non-zero off-diagonal elements in the residual covariance matrix. We estimated *PECM* and *CPECM* models by a two-stage procedure: cointegrating relationships among the seasons[4] were first estimated using Johansen's method. Then, the system of equations for the seasons, incorporating the estimated cointegrating relations, were jointly estimated as a set of

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[1] Our Data Appendix contains a brief description of the variables. See Osborn(1990) for more details on the data set. All variables are in logs, except the exchange rate and the yield on Treasury bills.

[2] In most variables in our data set, simple and partial autocorrelation functions, together with graphs of the differenced variables suggest a $\nabla\nabla^4$ filter. Columns 6 and 7 in Table 1 in Section 3.b contain detailed information on this issue.

[3] But always maintaining those of a *PAR*(1) structure, even if their *t*-statistics were below one.

[4] The number of cointegrating relations was taken from Franses and Romijn (1993).

seemingly unrelated regressions[5].

All models were estimated leaving outside the sample the last four years, i.e., 16 forecasting points, which were later used to evaluate the forecasting performance of the different models. Models were reestimated with each new data point, to obtain each the 16 one-step-ahead forecasts[6]. Root mean square errors (*RMSE*) from one step ahead *recursive* forecast errors for each variable, were computed[7]: (1) for each quarter over the period of four years, i.e., four different *RMSE*'s corresponding to the first, second, third and fourth quarters, and (2) a single *RMSE* over the 16 quarters.

3.b Comparing between non periodic alternatives

After implementing unit root tests at the regular and seasonal frequencies, Osborn(1990) proposed the filters shown in the second column of Table 1. Given the long-standing tradition on (possibly overdifferenced) *ARIMA* modelling of seasonal time series, it seems worthwhile to explore how possibly overdifferenced structures perform in forecasting, relative to those obtained using Osborn(1990) unit root filters.

(INSERT TABLE 1 HERE)

RMSE columns in Table 1 allow for comparing the two time invariant *ARIMA* specifications: the pure autoregressive (*AR*) model that is achieved once the unit root filters in Osborn(1990) are imposed [left panel], and an alternative specification reached using the Box and Jenkins(1976) methodology [right panel], which turned out to lead to mixed models in most cases. The *AR* models contain two autoregressive polynomials, at the regular and seasonal frequencies. We have found second order polynomials to be quite common in our data set [see columns 3 and 4]. Interest rates seem to follow a random walk, since no autoregression was needed at either frequency. No autoregression was needed at the regular frequency for imports (*RIMPORTS*), workforce (*WORKFOR*)

[5] Estimated models are available from the authors upon request.

[6] Given the short number of years in our data set we do not consider advisable to analyze forecasting performance at horizons beyond one.

[7] We use standard root mean square errors as the single criterion for forecast comparison in order to maintain a sensible volume of results as well as simplify their interpretation. Checking whether qualitative results depend on the criterion used is an interesting issue for further research, but is beyond the scope of this paper.

and Treasury bill interest rates (*TBILLYLD*). On the other hand, columns 6 and 7 show that once the double differencing has been applied[8], there is almost no need for autoregressive polynomials, short moving average terms capturing the remaining stochastic structure. That leads to more parsimonious representations than reached with the lower order of differencing proposed by Osborn(1990).

Even though *RMSE* values for these two non-periodic alternatives are not very different, *ARMA* models perform slightly better than pure *AR* specifications for all variables, except for *RCONS, RNONDUR, RGVCONS* and *RINVPUB*, in all cases by a very small margin and, more clearly, for *M4COR* and *EXRATE*. Thus, the possible *overdifferencing* in *ARMA* models does not seem to lead, in general, to a deterioration of forecasting performance. This result suggests that even though we may lack enough sample information to discriminate between alternative specifications based on different unit root filters, this ambiguity on the order of differencing has a minor impact on forecasting performance in our data set. In consistency with previous research, we maintain Osborn(1992) differences and forecasts from the resulting *AR* specification for comparison with those we will derive from periodic models.

3.c Comparing periodic with non-periodic models

Table 2 summarizes the basic results of the forecasting comparison between non-periodic and periodic models. Next to the variable name, column 2 shows the percent *RMSE* associated to non-periodic *AR* models, calculated over the 16 quarters. Below that, we show quarter specific *RMSE* values. Columns 3 to 7 present similar information for the periodic models. Numbers in parentheses in columns 3 and 6 indicate the order p of each *VAR* and the number r of cointegrating relationships, respectively. The order p of the VAR was chosen on the basis of the standard likelihood ratio test that incorporates Sims'(1980) correction, making sure that there was no evidence of autocorrelation or dynamic residual cross-correlations. It turned out to be 1 for most variables, corresponding to a maximum *PAR* order of 7; just in four cases we got an order of 2, which corresponds to a maximum *PAR* order of 11. The likelihood ratio statistic described in Appendix 1 to test between a *PAR*(1) and a higher order *PAR(h)* model, shows that a *PAR*(1) model may be adequate in just 5 of 17 cases (*RCONS, REXPORTS, RIMPORTS, M4COR* and *EXRATE*), and that a higher order *PAR(h)* model should be preferred for the remaining 12 variables.

(INSERT TABLE 2 HERE)

[8] A regular and a seasonal difference were taken in all cases except *TBILLYLD* and *EXRATE*, for which no seasonal differences were needed.

As mentioned in footnote 4, the number r of cointegration relationships among the quarters was taken from Franses and Romijn(1993). They detected a single cointegration relationship in three cases, two relationships in ten cases, and three cointegration relationships in the remaining 4 variables.

Under the limitations of our analysis, which is based on the sample of 17 variables, and uses just *RMSE*'s for one-step ahead predictions over a four years horizon, when the non-periodic *AR* specification is compared with any of the five periodic models, the following results hold [Compare column 2 with columns 3-7 of Table 2]:

1. The time invariant *AR* model produces the lowest *RMSE* over the full year, i.e., computed for the 16 quarters, in 9 of the 17 variables, while *OVAR*, *COVAR*, *PAR*(1), *PECM* and *CPECM* produce the lowest *RMSE* in just one, three, zero, two and two cases, respectively. Compared to each of these periodic specifications, the *AR* model dominates for 14, 12, 14, 12 and 11 of the variables in the sample.
2. We are specially interested in analyzing quarter specific forecast errors to gain some insight into the seasonal characteristics of a given variable. Even if we used a narrower criterion to select from a set of models the one that produces not only the lowest *RMSE* over the full year but also the best forecasts in at least two quarters, the same results as in 1) would hold. So, contrary to what might be expected given the periodic nature of these variables, periodic models do not predict better than non-periodic models. Again in contradiction with a reasonable intuition, incorporating the cointegrating relationships among the seasons into periodic specifications does not seem to improve forecasting performance. Attempts to reduce the number of parameters using some *ad-hoc* statistical rules does not seem to be of much help either.
3. However, for each variable, there is no single specification, periodic or non-periodic, that dominates, in terms of forecasting performance, over all quarters. That means, that periodic specifications have some potential ability to produce better forecasts than nonperiodic models, since for each variable, there is always at least one quarter for which the non-periodic *AR* is beaten by the simple, unrestricted *OVAR* model[9]. This represents additional evidence to that in Franses and Romijn(1993) on the periodic behaviour of this set of variables.

Comparing the forecasting results in Table 2 for each variable and quarter shows that, in most

[9] Third and fourth quarters sem to be particularly prone to show a behavior different form the rest.

cases, season heterogeneity seems to be concentrated in one or at most two quarters, those in which periodic models dominate *AR*. In these conditions, an unrestricted periodic model may be a too general representation of seasonality. Lack of precision in estimates because of overparameterization may explain that such specifications do not produce better forecasts than time invariant models. Incorporating cointegration restrictions, even if accurately estimated, does not seem to significantly reduce the inefficiency of estimates and consequently, the resulting models are still dominated, in terms of forecasting, by time invariant alternatives.

These considerations suggest that a particular forecasting method based on switching between a periodic and a non-periodic model for different quarters might provide forecasts with lower *RMSE* than either one[10]. It would be interesting to check whether in such a forecasting approach, the supposed efficiency gain from incorporating cointegration restrictions helps to produce better forecasts.

We have performed a preliminary analysis of this suggestion, although in a very favourable position, since we have used realized forecast errors to guide our selection of models. Left panel in Table 3 shows percent *RMSE*'s obtained by combining the non-periodic *AR* model with every periodic alternative, labelled *AR+OVAR*, *AR+COVAR*, *AR+PAR*(1), *AR+PECM* and *AR+CPECM*. For each quarter, we chose the forecasts produced by the specification that performed best. The right panel shows the maximum[11] percent reduction in *RMSE* that could be obtained, relative to forecasts from the non-periodic *AR* model. *RMSE* reductions can be quite important, as shown in columns 7-11 of Table 3 which clearly reflect our previous comment on Table 2 that for most variables, season heterogeneity seems to be limited to one or at most two quarters.

(INSERT TABLE 2 HERE)

[10] Switching between models can be interpreted as using a restricted periodic model: for instance, if just one quarter is better predicted from the periodic specification, the switching strategy would amount to having a periodic model where three quarters behave identically, with parameters equal to those in the univariate *AR* model, while the remaining quarter shows a different behaviour. At this preliminary stage we have not estimated the *VAR* constrained so that some seasons behave identically, as implied by the switching procedure.

[11] Being an *ex-post* exercise that uses non-statistical information, is not subject to any uncertainty. This is similar to assuming that a criterion for switching between models was available from the outset that led to selecting the model that *actually* performed best. Hence, these are the *maximum* forecast gains that could have possibly been achieved.

3.d Comparing different switching combinations

Contrary to the results for individual models in Table 2, cointegration restrictions seem to be important in these mixtures: in 12 of the 17 variables, combinations with periodic error correction models produce a lower *RMSE* than the combination *AR+OVAR* model, being the differences between them very important in some cases. Once the cointegrating relationships among seasons are taken into account, further reductions in *RMSE* can be attained by removing nonsignificant coefficients, but these seem to be of minor importance. A reduction in *RMSE* of 11% in *RINVPUB* 7% in *M4COR*, and 6% in *RNONDUR* are the most important.

A lower specification cost alternative exists: If we remove from an *OVAR* the apparently less significant coeficients, even in a somewhat *ad-hoc* fashion, as we did to get the *COVAR* model, and the resulting specification is combined with an *AR* model, the forecasting ability is improved in a fair number of cases (11 out of 17). Hence, an *OVAR* model, with some criterion to reduce the number of coeficients, seems to perform well in a switching procedure with the time invariant *AR*.

On the other hand, the combination of the *PAR*(1) and the univariate *AR* does not work as well, beating the combination of *OVAR* and *AR* in just 6 of the 17 variables. These results support the finding in Section 3.c that *PAR*(1) models seem to be too simple to capture the periodic characteristics of this set of variables. A similar result is obtained in Franses and Paap(1994).

Summarizing, these results suggest that having identified a periodic behaviour in a given time series, a periodic model might provide better forecasts than a non periodic alternative. Nevertheless, to obtain significant gains in accuracy with respect to simpler non periodic specifications, it will be necessary to constrain the periodic model (allowing for identical behaviour among some quarters). Sizeable improvements from this option can still be obtained incorporating cointegrating relationships among seasons and zero constraints on statistically non-significant coefficients.

Proceeding with our discussion in Section 3.c, this finding shows that once season homogeneity is properly taken into account[12], tighter estimates can be obtained, clearly improving forecasting performance. It is in this more restrictive setup, that cointegration constraints produce an additional gain in efficiency, enough to produce still better forecasts. Periodic error correction models with some homogeneity restrictions among seasons seem to be adequate representations of seasonality, on which some additional improvements can be achieved removing nonsignificant coefficients, if any.

We are currently working on the design of a complete procedure for selecting a *best* periodic and a *best* non-periodic model, as well as for switching between them for each specific quarter. The limitations we have pointed out for the previous exercise just allow for interpreting the reported results with a lot of caution, and just as *upper bounds* on the gain on forecasting performance associated to *the best* possible combination of models.

[12] Even though we do it in an informal way, through our proposed switching procedure.

4. CONCLUSIONS

Recent studies have shown that the seasonal behaviour of quarterly economic time series can be more complicated than reflected in standard seasonal *ARIMA* models, and also that periodic models can be useful tools for capturing such a behaviour.

Working with seventeen *UK* macroeconomic variables, found to be periodically integrated in Franses and Romijn(1993), simple unconstrained periodic models do not beat time invariant alternatives in forecasting, even when cointegrating relationships among the seasons are taken into account. However, when appropriately constrained, the forecasting performance of periodic models can be much better that of non periodic models. Homogeneity restrictions among some seasons seem to be the most important in that respect, which has led us to proposing a switching procedure between a periodic model and a non-periodic univariate *AR*.

We have also found that once season homogeneity is taken into account, incorporating the cointegrating relationships among the seasons through the corresponding periodic error correction models achieves a substantial additional forecasting improvement which, enough to compensate for the higher specification cost.

We are currently undergoing research on accurate and efficient strategies for switching between periodic and nonperiodic structures. More experiments with different sets of real and simulated variables and different forecast horizons should be carried out to analyze the robustness of our results. Before that is done, our results should be taken as preliminary and interpreted with caution.

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DATA APPENDIX

RGDP:	Gross Domestic product at 1985 prices. 1955.1 - 1988.4
RCONS:	Total personal expenditure on goods and services at 1985 prices. 1956.1 - 1988.4
RCONSDUR:	Personal expenditure on durable goods at 1985 prices. 1955.1-1988.4
RNONDUR:	Personal expenditure on non-durable goods and services at 1985 prices.1955.1 1988.4
RGVCONS:	Total government final consumption at 1985 prices. 1955.1 -1988.4
RINVPRIV:	Gross fixed capital formation on the private sector at 1985 prices. 1962.1 - 1988.4
RINVPUB:	Gross fixed capital formation of the public sector at 1985 prices. 1962.1 - 1988.4
REXPORTS:	Exports of goods and services at 1985 prices. 1955.1 - 1988.4
RIMPORTS:	Imports of goods and services at 1985 prices. 1955.1 - 1988.4
RADJFC:	Factor cost adjustment (taxes on expenditure less subsidies) at 1985 prices. 1955.1 - 1988.4
RPDY:	Real personal disposable income at 1985 prices. 1955.1 - 1988.4
WORKFOR:	Workforce. 1955.1 - 1988.4
PROD:	Productivity (GDP at 1985 prices/employment). 1955.1 - 1988.4
M0COR:	Stock of narrow money. 1969.3 - 1988.4
M4COR:	Stock of broad money. 1963.1 - 1988.4
TBILLYLD:	Percentage yield on Treasury bills. 1963.1 - 1988.4
EXRATE:	Sterling exchange rate against US dollar. 1973.1 - 1988.4

All variables are in logs except the sterling/US$ exchange rate (EXRATE) and the yield on Treasury bills (TBILLYLD). For more details about the data set, the interested reader can refer to Osborn(1990).

APPENDIX 1

The *PAR*(1) model is the most widely used periodic process, since it is quite easy to elaborate. Although useful in many cases, it might however be too simple to incorporate the dynamic structure that can be present in a seasonal variable. Thus, it seems reasonable to test it against higher order *PAR* models.

This comparison can be done in two stages: (1) conducting a specification test to choose an order p for the *VAR* representation for the vector of seasons. That implies a *maximum order* for an equivalent *PAR(h)*, $h > 1$, model, on which (2) test the constraints implied by the *PAR*(1) on the orthogonalized *VAR(p)*.

We test for the constraints implied by the *PAR*(1) model on a higher order *PAR(h)* using a likelihood ratio test that incorporates Sims'(1980) correction:

$$LR = n\,[\ln|\Omega| - \ln|\Sigma|]$$

where n denotes the number of effective years in estimation and Ω is the covariance matrix of the intra-year residuals in the *PAR*(1) model. Σ is the covariance matrix in the orthogonalized *VAR(p)*. This likelihood ratio test statistic is in the spirit of the one proposed in Flores and Novales(1996) to test for time invariant coefficients in periodic models for nonstationary variables.

The statistic follows a χ^2 asymptotic distribution [see Lütkepohl(1993, Section 12.3), Boswijk and Franses(1996) and Flores and Novales (1996)] with J degrees of freedom, J being the number of constraints, i.e., the number of estimated parameters in the orthogonalized *VAR* minus the number of parameters in the *PAR*(1) model.

For our sample of variables, Table A.1 contains the results of this test. Only in 5 cases is the *PAR*(1) not rejected: *RCONS, REXPORTS, RIMPORTS, M4COR* and *EXRATE.*

TABLE 1

FORECAST RESULTS FROM TIME INVARIANT, UNIVARIATE MODELS

Variable	Models obtained applying Osborn(1990) unit root filters $AR(p)(P)_4$				$ARIMA(p,d,q)(P,D,Q)_4$ models		
	Filter	p	P	RMSE	(p,d,q)	(P,D,Q)	RMSE
RGDP	$(1-B^4)$	2	2	1.44	(0,1,1)	(0,1,1)	1.24
RCONS	$(1-B^4)$	2	2	1.09	(0,1,1)	(0,1,1)	1.10
RCONSDUR	$(1-B^4)$	2	2	4.18	(0,1,1)	(0,1,1)	3.90
RNONDUR	$(1-B^4)$	1	2	0.85	(0,1,0)	(0,1,1)	0.87
RGVCONS	$(1-B^3)$	4	2	1.72	(0,1,1)	(0,1,1)	1.78
REXPORTS	$(1-B)$	1	3	2.80	(0,1,1)	(0,1,1)	2.76
RIMPORTS	$(1-B)$	0	2	3.63	(0,1,1)	(0,1,1)	3.41
RADJFC	$(1-B)$	2	2	2.26	(0,1,1)	(0,1,2)	1.70
RPDY	$(1-B)$	1	2	1.06	(0,1,1)	(1,1,1)	1.04
WORKFOR	$(1-B)$	0	2	0.30	(0,1,0)	(1,1,1)	0.24
PROD	$(1-B^4)$	2	2	1.29	(0,1,1)	(0,1,1)	1.20
RINVPRIV	$(1-B)$	1	2	6.12	(0,1,1)	(0,1,2)	5.27
RINVPUB	$(1-B^4)$	2	2	10.17	(0,1,1)	(2,1,0)	10.24
M0COR	$(1-B)$	2	4	0.69	(2,1,0)	(0,1,1)	0.64
M4COR	$(1-B)^3$	4	2	0.80	(1,1,0)	(0,1,1)	0.96
TBILLYLD	$(1-B)$	0	0	1.47	(0,1,0)	(0,0,0)	1.47
EXRATE	$(1-B^3)$	4	1	0.09	(1,1,0)	(0,0,0)	0.09

Note: 1) All *RMSE* are in percent terms, except in *TBILLYLD* and *EXRATE.*
2) p : non-seasonal *AR* order; P : seasonal *AR* order; q : order of non-seasonal *MA* term; Q : order of seasonal *MA* term; d : number of non-seasonal differences; D : number of seasonal differences.

TABLE 2

ROOT MEAN SQUARE ERRORS OF FORECASTS FOR NONPERIODIC AND PERIODIC MODELS

	NON PERIODIC MODELS	PERIODIC MODELS				
Variable	AR	OVAR(p)	COVAR	PAR(1)	PECM(r)	CPECM
RGDP	1.44	1.61(1)	1.50	2.09	1.56(2)	1.62
Q1	0.72	1.77	1.67	2.62	1.84	1.84
Q2	0.98	1.56	1.40	2.42	1.56	1.65
Q3	2.01	1.15	1.26	1.19	0.92	0.89
Q4	1.68	1.85	1.63	1.83	1.75	1.90
RCONS	1.09	1.26(1)	1.21	1.15	1.48(2)	1.57
Q1	0.87	0.92	0.74	0.74	0.98	0.64
Q2	1.06	1.54	1.71	1.59	2.05	1.72
Q3	1.59	1.57	1.29	1.29	1.75	2.33
Q4	0.60	0.84	0.82	0.74	0.71	1.05
RCONSDUR	4.18	5.98(2)	5.21	6.17	4.45(2)	5.71
Q1	2.54	8.72	7.72	7.42	5.24	5.06
Q2	3.91	6.33	3.55	7.41	5.61	4.01
Q3	6.78	3.82	2.10	5.85	3.71	8.72
Q4	1.43	3.53	5.66	2.87	2.57	3.54
RNONDUR	0.85	1.18(1)	1.14	1.07	1.16(2)	1.03
Q1	0.73	1.07	1.09	1.09	0.74	0.68
Q2	0.82	1.33	1.23	1.22	1.63	1.48
Q3	1.08	0.82	0.83	0.94	0.86	0.83
Q4	0.74	1.40	1.33	1.01	1.21	0.96
RGVCONS	1.72	1.89(1)	1.65	2.34	1.73(1)	1.67
Q1	2.20	3.29	2.66	2.59	2.89	2.79
Q2	1.66	1.52	1.18	3.75	1.69	1.49
Q3	1.72	1.04	1.46	1.00	0.40	0.28
Q4	1.15	0.36	0.56	0.38	0.76	1.02
REXPORTS	2.80	3.85(1)	3.30	3.75	3.18(3)	2.94
Q1	3.70	4.18	2.81	2.81	3.96	3.40
Q2	3.24	3.08	3.47	5.13	2.06	1.87
Q3	1.95	3.36	2.61	2.64	2.64	2.94
Q4	1.85	4.59	4.09	3.89	3.68	3.31
RIMPORTS	3.63	3.98(1)	3.65	3.70	5.63(2)	4.22
Q1	3.07	3.34	3.54	3.54	2.17	3.24
Q2	5.30	3.57	3.89	3.89	2.92	3.74
Q3	3.83	4.82	3.93	4.37	6.12	5.73
Q4	0.76	4.04	3.19	2.84	8.73	3.72
RADJFC	2.26	2.02(2)	2.12	3.43	2.19(2)	2.50
Q1	1.38	1.80	1.41	4.27	1.60	1.60
Q2	2.65	1.93	1.83	3.96	2.57	2.72
Q3	2.05	2.84	3.10	3.52	2.84	3.07
Q4	2.69	1.11	1.72	0.93	1.41	2.38

TABLE 2 (CONT.)

Variable	AR	VAR	CVAR	PAR1	PECM	PCECM
RPDY	1.06	1.35(2)	1.17	1.79	0.81(2)	0.87
Q1	0.58	0.46	0.44	0.75	0.43	0.51
Q2	1.19	1.97	1.86	2.77	0.89	1.04
Q3	0.77	1.57	1.13	1.96	1.02	1.01
Q4	1.47	0.86	0.75	0.85	0.79	0.82
WORKFOR	0.30	0.34(1)	0.38	0.42	0.45(2)	0.45
Q1	0.30	0.17	0.26	0.52	0.27	0.33
Q2	0.27	0.34	0.30	0.29	0.65	0.62
Q3	0.32	0.43	0.42	0.38	0.27	0.38
Q4	0.31	0.38	0.49	0.47	0.50	0.42
PROD	1.29	1.59(1)	1.48	2.01	1.82(2)	1.81
Q1	1.01	1.44	1.46	2.30	2.61	2.66
Q2	0.63	1.74	1.82	2.61	1.78	1.82
Q3	1.57	1.54	1.11	1.11	0.93	0.98
Q4	1.67	1.61	1.44	1.69	1.53	1.35
RINVPRIV	6.12	5.39(1)	4.88	5.85	4.67(2)	5.63
Q1	7.87	6.93	6.61	6.24	5.17	6.79
Q2	8.01	6.29	5.50	8.25	4.28	6.55
Q3	4.19	4.42	4.24	5.22	5.04	4.00
Q4	2.50	2.99	1.83	1.67	4.11	4.68
RINVPUB	10.17	11.0(1)	10.20	18.31	8.13(1)	7.20
Q1	7.92	12.05	12.19	13.09	8.44	2.32
Q2	14.46	12.09	12.33	32.87	10.50	10.66
Q3	10.78	12.90	10.30	8.84	6.83	7.05
Q4	5.10	5.12	3.01	3.22	6.01	6.22
M0COR	0.69	0.69(2)	0.61	0.64	0.64(3)	0.77
Q1	0.65	0.77	0.69	0.71	0.91	0.98
Q2	0.47	0.23	0.42	0.54	0.21	0.75
Q3	0.99	0.92	0.79	0.61	0.75	0.80
Q4	0.55	0.65	0.48	0.70	0.45	0.45
M4COR	0.80	0.92(1)	0.91	0.81	1.11(3)	0.71
Q1	0.85	0.92	0.97	0.74	1.00	0.93
Q2	0.19	0.76	0.82	0.81	1.71	0.64
Q3	0.97	1.30	1.23	1.06	0.67	0.42
Q4	0.94	0.55	0.47	0.56	0.75	0.77
TBILLYLD	1.47	1.80(1)	1.43	1.44	2.17(3)	2.31
Q1	1.92	2.61	2.09	2.09	2.72	2.59
Q2	0.90	1.44	0.85	0.85	2.50	2.46
Q3	1.49	1.51	1.30	1.30	1.73	2.53
Q4	1.37	1.36	1.18	1.23	1.48	1.45
EXRATE	0.09	0.11(1)	0.09	0.09	0.12(1)	0.19
Q1	0.08	0.09	0.09	0.09	0.12	0.30
Q2	0.09	0.08	0.09	0.09	0.14	0.16
Q3	0.10	0.11	0.09	0.08	0.13	0.16
Q4	0.08	0.14	0.10	0.11	0.06	0.06

Note: 1) All *RMSE*'s are in percent terms, except those for *TBILLYLD* ands *EXRATE*.
2) p: order of *VAR*; r: number of cointegrating relationships among the seasons.
3) The first line for each variable contains *RMSE*'s over 16 quarters. Lines to the right of *Qn* contain *RMSE*'s for quarter n, $n = 1,2,3,4$.

TABLE 3

ROOT MEAN SQUARE ERRORS OF FORECASTS FOR COMBINATIONS OF MODELS

Variable	RMSE FOR COMBINED MODELS: PERIODIC + AR					PERCENT RMSE REDUCTIONS RELATIVE TO NON PERIODIC AR MODELS				
	OVAR+AR	COVAR+AR	PAR(1)+AR	PECM+AR	CPECM+AR	OVAR+AR	COVAR+AR	PAR(1)+AR	PECM+AR	CPECM+AR
RGDP	1.19	1.20	1.20	1.13	1.13	17.89	17.17	17.21	21.44	21.86
Q1	0.72	0.72	0.72	0.72	0.72	0.00	0.00	0.00	0.00	0.00
Q2	0.98	0.98	0.98	0.98	0.98	0.00	0.00	0.00	0.00	0.00
Q3	1.15	1.26	1.19	0.92	0.89	42.79	37.31	40.80	54.23	55.72
Q4	1.68	1.63	1.68	1.68	1.68	0.00	2.98	0.00	0.00	0.00
RCONS	1.08	0.96	0.96	1.09	1.05	0.66	11.97	11.97	0.00	3.71
Q1	0.87	0.74	0.74	0.87	0.64	0.00	14.94	14.94	0.00	26.44
Q2	1.06	1.06	1.06	1.06	1.06	0.00	0.00	0.00	0.00	0.00
Q3	1.57	1.29	1.29	1.59	1.59	1.26	18.87	18.87	0.00	0.00
Q4	0.60	0.60	0.60	0.60	0.60	0.00	0.00	0.00	0.00	0.00
RCONSDUR	3.10	2.53	3.81	3.06	4.18	25.83	39.53	8.81	26.63	0.00
Q1	2.54	2.54	2.54	2.54	2.54	0.00	0.00	0.00	0.00	0.00
Q2	3.91	3.55	3.91	3.91	3.91	0.00	9.21	0.00	0.00	0.00
Q3	3.82	2.10	5.85	3.71	6.78	43.66	69.03	13.72	45.28	0.00
Q4	1.43	1.43	1.43	1.43	1.43	0.00	0.00	0.00	0.00	0.00
RNONDUR	0.78	0.78	0.81	0.79	0.77	8.85	8.54	4.97	7.60	9.87
Q1	0.73	0.73	0.73	0.73	0.68	0.00	0.00	0.00	0.00	6.85
Q2	0.82	0.82	0.82	0.82	0.82	0.00	0.00	0.00	0.00	0.00
Q3	0.82	0.83	0.94	0.86	0.83	24.07	23.15	12.96	20.37	23.15
Q4	0.74	0.74	0.74	0.74	0.74	0.00	0.00	0.00	0.00	0.00
RGVCONS	1.45	1.47	1.48	1.44	1.43	16.09	14.52	14.21	16.24	17.01
Q1	2.20	2.20	2.20	2.20	2.20	0.00	0.00	0.00	0.00	0.00
Q2	1.52	1.18	1.66	1.66	1.49	8.43	28.92	0.00	0.00	10.24
Q3	1.04	1.46	1.00	0.40	0.28	39.53	15.12	41.86	76.74	83.72
Q4	0.36	0.56	0.38	0.76	1.02	68.70	51.30	66.96	33.91	11.30
REXPORTS	2.76	2.53	2.53	2.51	2.36	1.62	9.69	9.69	10.51	15.78
Q1	3.70	2.81	2.81	3.70	3.40	0.00	24.05	24.05	0.00	8.11
Q2	3.08	3.24	3.24	2.06	1.87	4.94	0.00	0.00	36.42	42.28
Q3	1.95	1.95	1.95	1.95	1.95	0.00	0.00	0.00	0.00	0.00
Q4	1.85	1.85	1.85	1.85	1.85	0.00	0.00	0.00	0.00	0.00
RIMPORTS	3.06	3.15	3.15	2.67	3.11	15.79	13.14	13.14	26.53	14.40
Q1	3.07	3.07	3.07	2.17	3.07	0.00	0.00	0.00	29.32	0.00
Q2	3.57	3.89	3.89	2.92	3.74	32.64	26.60	26.60	44.91	29.43
Q3	3.83	3.83	3.83	3.83	3.83	0.00	0.00	0.00	0.00	0.00
Q4	0.76	0.76	0.76	0.76	0.76	0.00	0.00	0.00	0.00	0.00
RADJFC	1.66	1.76	1.87	1.92	2.17	26.29	21.93	17.11	15.04	3.94
Q1	1.38	1.38	1.38	1.38	1.38	0.00	0.00	0.00	0.00	0.00
Q2	1.93	1.83	2.65	2.57	2.65	27.17	30.94	0.00	3.02	0.00
Q3	2.05	2.05	2.05	2.05	2.05	0.00	0.00	0.00	0.00	0.00
Q4	1.11	1.72	0.93	1.41	2.38	58.74	36.06	65.43	47.58	11.52

TABLE 3 (CONT.)

SERIES	OVAR+AR	COVAR+AR	PAR(1)+AR	PECM+AR	CPECM+AR	OVAR	COVAR	PAR(1)	PECM	CPECM
RPDY	0.86	0.83	0.88	0.74	0.81	18.95	21.67	17.49	30.22	23.94
Q1	0.46	0.44	0.58	0.43	0.51	20.69	24.14	0.00	25.86	12.07
Q2	1.19	1.19	1.19	0.89	1.04	0.00	0.00	0.00	25.21	12.61
Q3	0.77	0.77	0.77	0.77	0.77	0.00	0.00	0.00	0.00	0.00
Q4	0.86	0.75	0.85	0.79	0.82	41.50	48.98	42.18	46.26	44.22
WORKFOR	0.27	0.29	0.30	0.28	0.30	8.84	3.15	0.00	6.67	0.00
Q1	0.17	0.26	0.30	0.27	0.30	43.33	13.33	0.00	10.00	0.00
Q2	0.27	0.27	0.27	0.27	0.27	0.00	0.00	0.00	0.00	0.00
Q3	0.32	0.32	0.32	0.27	0.32	0.00	0.00	0.00	15.63	0.00
Q4	0.31	0.31	0.31	0.31	0.31	0.00	0.00	0.00	0.00	0.00
PROD	1.26	1.09	1.17	1.08	1.02	2.20	15.86	9.71	16.75	20.65
Q1	1.01	1.01	1.01	1.01	1.01	0.00	0.00	0.00	0.00	0.00
Q2	0.63	0.63	0.63	0.63	0.63	0.00	0.00	0.00	0.00	0.00
Q3	1.54	1.11	1.11	0.93	0.98	1.91	29.30	29.30	40.76	37.58
Q4	1.61	1.44	1.67	1.53	1.35	3.59	13.77	0.00	8.38	19.16
RINVPRIV	5.28	4.87	5.56	4.15	5.27	13.80	20.46	9.25	32.23	13.85
Q1	6.93	6.61	6.24	5.17	6.79	11.94	16.01	20.71	34.31	13.72
Q2	6.29	5.50	8.01	4.28	6.55	21.47	31.34	0.00	46.57	18.23
Q3	4.19	4.19	4.19	4.19	4.00	0.00	0.00	0.00	0.00	4.53
Q4	2.50	1.83	1.67	2.50	2.50	0.00	26.80	33.20	0.00	0.00
RINVPUB	9.37	9.08	9.49	7.84	6.98	7.91	10.74	6.71	22.98	31.42
Q1	7.92	7.92	7.92	7.92	2.32	0.00	0.00	0.00	0.00	70.71
Q2	12.09	12.33	14.46	10.50	10.66	16.39	14.73	0.00	27.39	26.28
Q3	10.78	10.30	8.84	6.83	7.05	0.00	4.45	18.00	36.64	34.60
Q4	5.10	3.01	3.22	5.10	5.10	0.00	40.98	36.86	0.00	0.00
M0COR	0.64	0.60	0.57	0.55	0.61	8.17	13.13	17.28	20.03	12.17
Q1	0.65	0.65	0.65	0.65	0.65	0.00	0.00	0.00	0.00	0.00
Q2	0.23	0.42	0.47	0.21	0.47	51.06	10.64	0.00	55.32	0.00
Q3	0.92	0.79	0.61	0.75	0.80	7.07	20.20	38.38	24.24	19.19
Q4	0.55	0.48	0.55	0.45	0.45	0.00	12.73	0.00	18.18	18.18
M4COR	0.71	0.69	0.68	0.67	0.62	11.96	13.78	15.64	17.22	23.09
Q1	0.85	0.85	0.74	0.85	0.85	0.00	0.00	12.94	0.00	0.00
Q2	0.19	0.19	0.19	0.19	0.19	0.00	0.00	0.00	0.00	0.00
Q3	0.97	0.97	0.97	0.67	0.42	0.00	0.00	0.00	30.93	56.70
Q4	0.55	0.47	0.56	0.75	0.77	41.49	50.00	40.43	20.21	18.09
TBILLYLD	1.47	1.37	1.38	1.47	1.47	0.28	6.63	5.98	0.00	0.00
Q1	1.93	1.93	1.92	1.93	1.93	0.00	0.00	0.00	0.00	0.00
Q2	0.90	0.85	0.85	0.90	0.90	0.00	5.77	5.71	0.00	0.00
Q3	1.49	1.30	1.30	1.49	1.49	0.00	12.81	12.80	0.00	0.00
Q4	1.36	1.18	1.23	1.37	1.37	1.31	13.71	10.55	0.00	0.00
EXRATE	0.09	0.09	0.08	0.09	0.09	1.75	3.00	7.11	4.21	4.21
Q1	0.08	0.08	0.08	0.08	0.08	0.00	0.00	0.00	0.00	0.00
Q2	0.08	0.09	0.09	0.09	0.09	6.85	4.20	3.87	0.00	0.00
Q3	0.10	0.09	0.08	0.10	0.10	0.00	6.33	21.45	0.00	0.00
Q4	0.08	0.08	0.08	0.06	0.06	0.00	0.00	0.00	21.66	21.66

Note: As in Table 2.

TABLE A.1	
Variable	LIKELIHOOD RATIO TEST (Degress of freedom)
RGDP	22.58 (8)
RCONS	5.72 (4)
RCONSDUR	60.29 (12)
RNONDUR	21.84 (6)
RGVCONS	19.74 (7)
RINVPRIV	24.25 (8)
RINVPUB	37.92 (9)
REXPORTS	4.66 (5)
RIMPORTS	7.23 (5)
RADJFC	66.31 (11)
RPDY	44.90 (14)
WORKFOR	12.00 (5)
PROD	19.72 (5)
M0COR	86.01 (7)
M4COR	6.73 (4)
TBILLYLD	10.35 (4)
EXRATE	5.10 (3)

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