Quantifying uncertainties in seismic tomography

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Outline

- Introduction
- The reflection tomography problem
- Uncertainty analysis on the solution model
- Global inversion
- Conclusions

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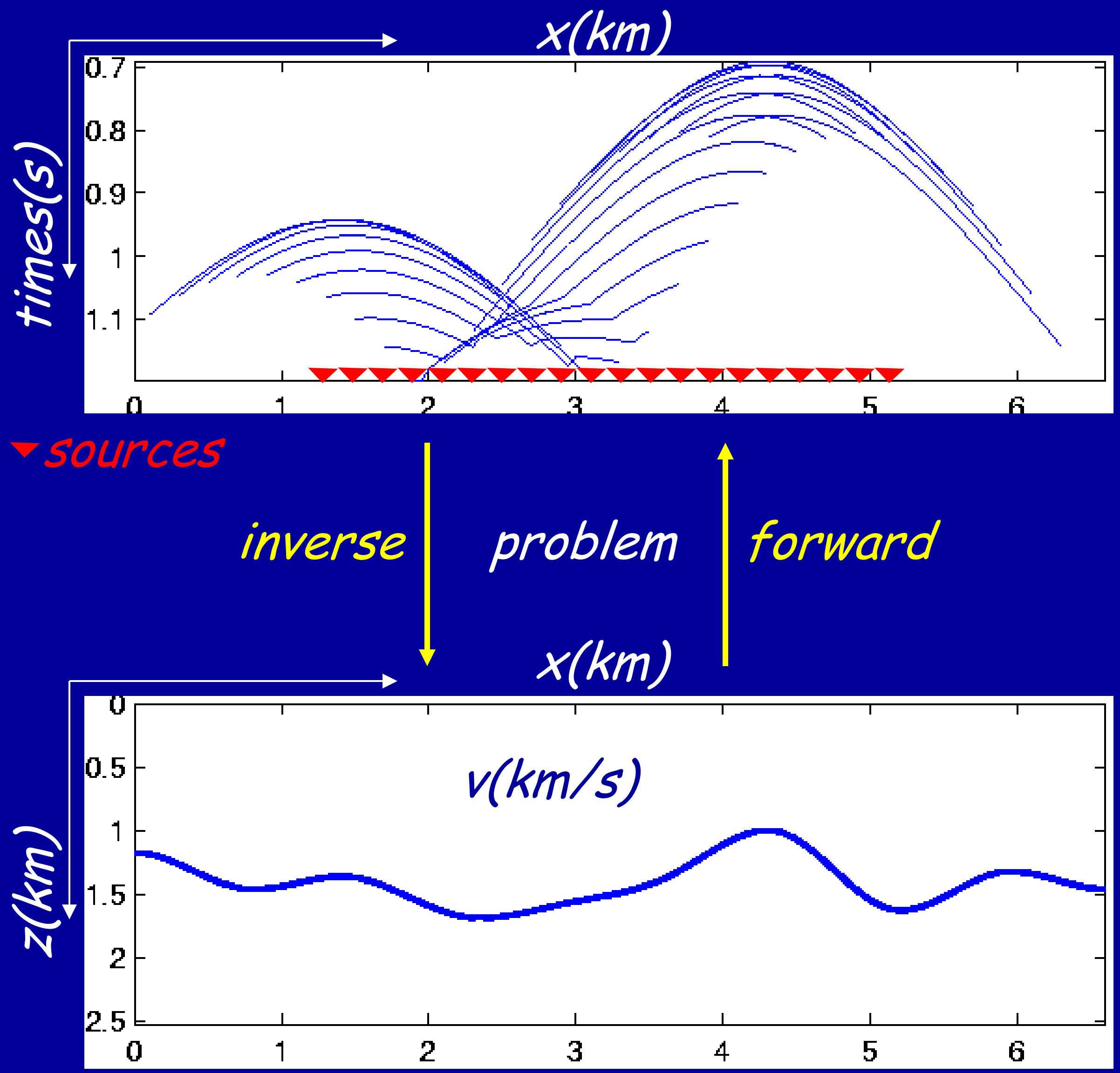
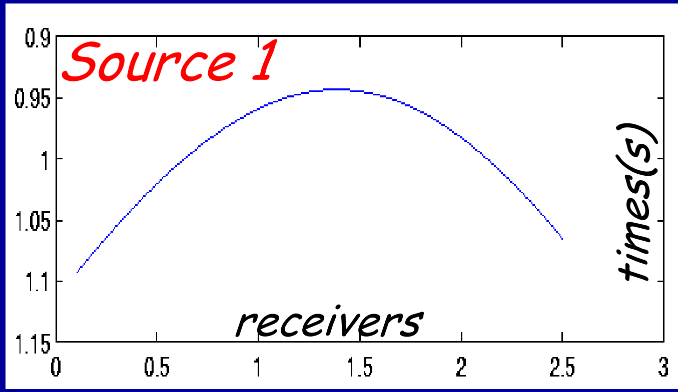
The reflection tomography problem

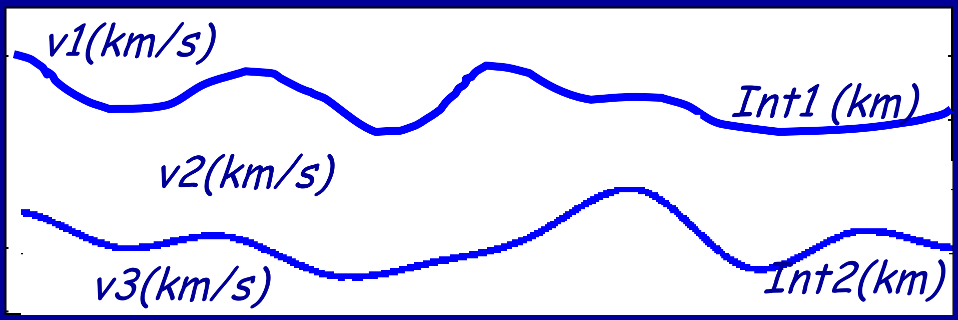




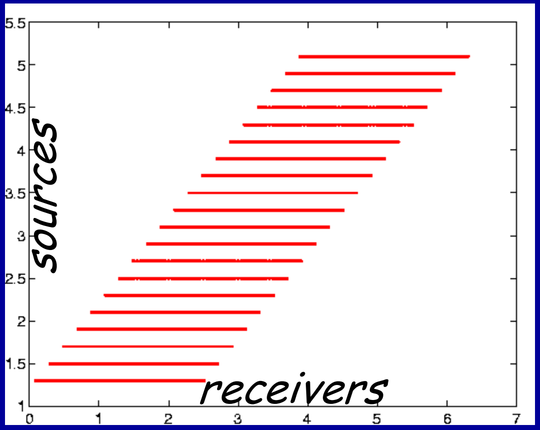
- Forward problem: ray tracing
- Inverse problem: minimizing the misfits between observed and calculated traveltimes

Modelisation

- Modelisation:
  - Model: 2D model parameterization based on b-spline functions
    - interfaces: *z(x), x(z)*
    - velocities: *v(x), v(x)+k.z*



  - Acquisition survey:
    - sources: $S=(x_s,0)$
    - receivers: $R=(x_r,0)$



- Data: traveltimes modeled by the forward problem based on a ray tracing

Ray tracing algorithm

- Ray tracing for specified ray signature :
  - source (*S*) and receiver (*R*) fixed
  - ray signature known (signature = reflectors where the waves reflect)

→ Fermat's principle:

- analytic traveltime formula within layer (*P* = impact point of the ray)

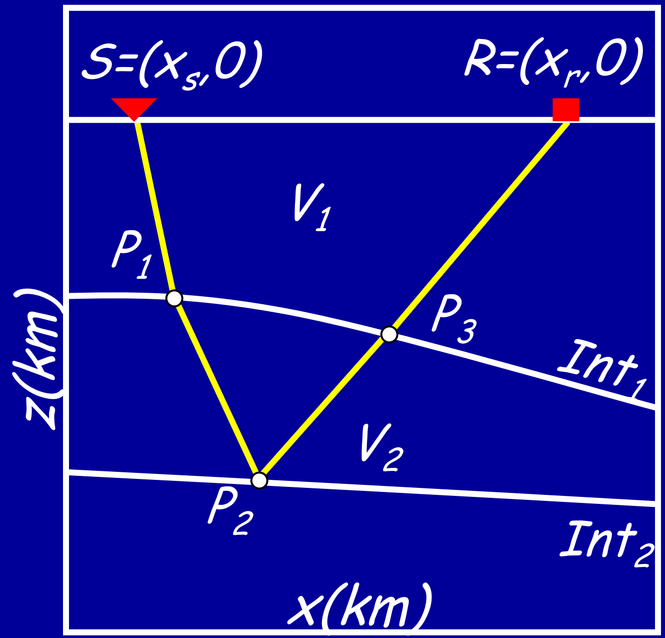
$$t = t(x_s, x_r) = \int_C \frac{ds}{v} = \int_S^P \frac{ds}{v} + \int_P^R \frac{ds}{v}$$

- Fermat's principle: (*C* = trajectory between *S* and *R*)

$$\frac{\delta t}{\delta C} = 0 \quad \Longrightarrow \quad \frac{\delta t}{\delta P} = 0$$ (in particular)

Ray tracing: an optimization problem

A ray is a trajectory that satisfies Fermat 's principle for a given signature



$$t = \underset{P_1,P_2,P_3}{\text{minimize}}\ t(S,R) \implies t = t(P_1,P_2,P_3)$$

The reflection tomography problem: an inverse problem

- Search a model which
  - fits traveltime data for given uncertainties on the data
  - and fits a priori information
- The least square method

$$\text{minimize } (T(m) - T^{obs})^T C_D^{-1} (T(m) - T^{obs}) + (m - \bar{m})^T C_m^{-1} (m - \bar{m})$$

with C_D the a priori covariance operator in the data space

C_m the a priori covariance operator in the model space

This classical approach give the estimate m^{est}

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Uncertainty analysis on the solution model

- Linearized approach:
  - Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial t_1}{\partial v} & \frac{\partial t_1}{\partial z_1} & \cdots & \frac{\partial t_1}{\partial z_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial t_{nt}}{\partial v} & \frac{\partial t_{nt}}{\partial z_1} & \cdots & \frac{\partial t_{nt}}{\partial z_n} \end{pmatrix}$$

  - Acceptable models = $\left\{ m^{est} + \delta m \text{ with: } J\delta m \text{ small in the error bar} \atop \delta m \text{ in the model space.} \right\}$

- Motivations:
  - find error bars on the model parameters

Uncertainty analysis: two approaches

- We propose two methods to access the uncertainties
  - Linear programming method
  - Classical stochastic approach

Linear programming method

(Dantzig, 1963)

- Solve the linear programming problem

$$\min_{m} c^T m$$ *Under the constraints* $$t^{obs} - \delta t \leq Jm \leq t^{obs} + \delta t$$ $$\bar{m} - \delta m \leq m \leq \bar{m} + \delta m$$

where:

– $\delta t = 0.003 (s)$,

– $\delta m = (\delta m_v, \delta m_z)$ avec $\delta m_v = min(|v_m - 0.8|, |3 - v_m|)$ et $\delta m_z = \delta z/2$

Stochastic approach

(Franklin, 1970)

- Solve the stochastic inverse problem

$$m^{est} = \bar{m} + (C_m^{-1} + J^T C_D^{-1} J)^{-1} J^T C_D^{-1} (T^{obs} - J\bar{m})$$

$$C_m^{apost} = (C_m^{-1} + J^T C_D^{-1} J)^{-1}$$

where:

$$C_D = \mathrm{diag}_{1 \le i \le n_t} \left(\frac{\delta t_i^2}{3} \right) \qquad C_m = \mathrm{diag}_{1 \le i \le n_m} \left(\frac{\delta m_i^2}{3} \right)$$

with

$\delta t = 0.003(s)$,

$\delta m = (\delta m_v, \delta m_z)$ avec $\delta m_v = \min(|v_m - 0.8|, |3 - v_m|)$ et $\delta m_z = \delta z/2$

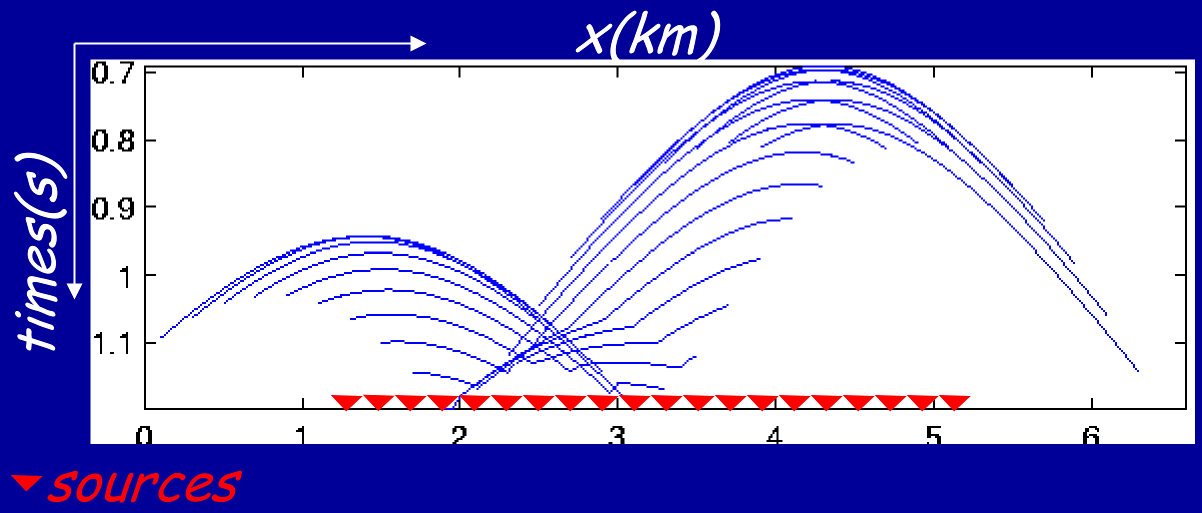
Stochastic approach

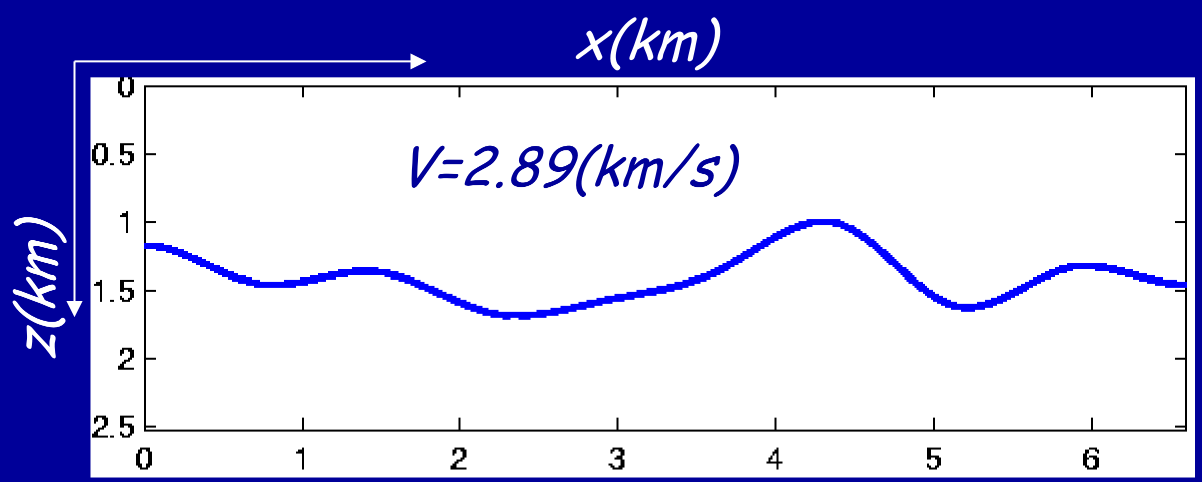
- Linearized framework: analysis of the a posteriori covariance matrix

$$C_m^{apost} = (C_m^{-1} + J^T C_D^{-1} J)^{-1}$$

  - $\sqrt{(C_m^{apost})_{ii}}$ uncertainties on the inverted parameters
  - $(C_m^{apost})_{ij,i \neq j}$ correlation between the uncertainties

Application on a 2D synthetic model





- Acquisition survey:
  - 20 sources (Δxs=200m)
  - 24x20 receivers (offset=50m)
- data:
  - 980 traveltimes data
  - uncertainty 3ms
- model:
  - 1 layer
  - 10 interface parameters
  - 1 constant velocity

Uncertainty analysis on the solution model

Linear programming

VELOCITY

INTERFACE

v	26.40 m/s
p1	374.99 m
p2	113.93 m
p3	17.05 m
p4	42.72 m
p5	19.68 m
p6	24.75 m
p7	14.81 m
p8	193.19 m
p9	374.99 m
p10	374.99 m

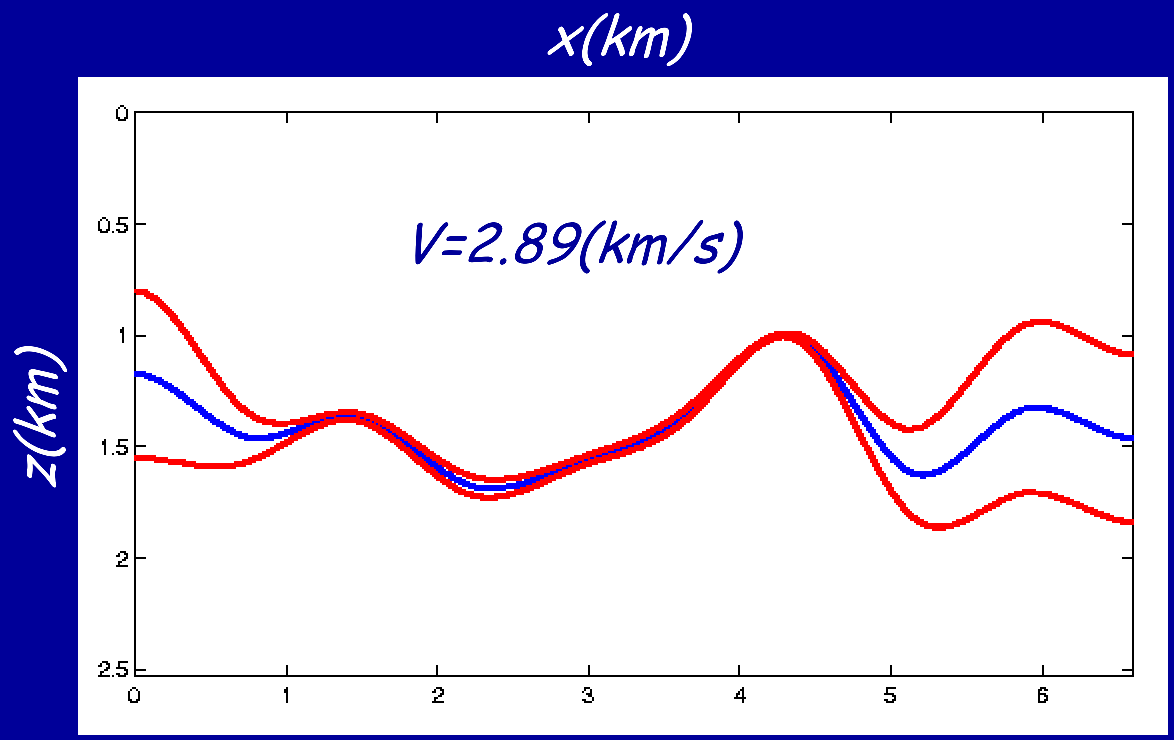
Boundaries of the model

Velocity:

V=2.89(km/s)

Uncertainty analysis on the solution model

Linear programming



Uncertainty analysis on the solution model

Stochastic inverse

VELOCITY

INTERFACE

v	3.3 m/s
p1	304.3 m
p2	69.2 m
p3	1.9 m
p4	6.9 m
p5	2.3 m
p6	3.2 m
p7	1.8 m
p8	167.7 m
p9	579.8 m
p10	645.4 m

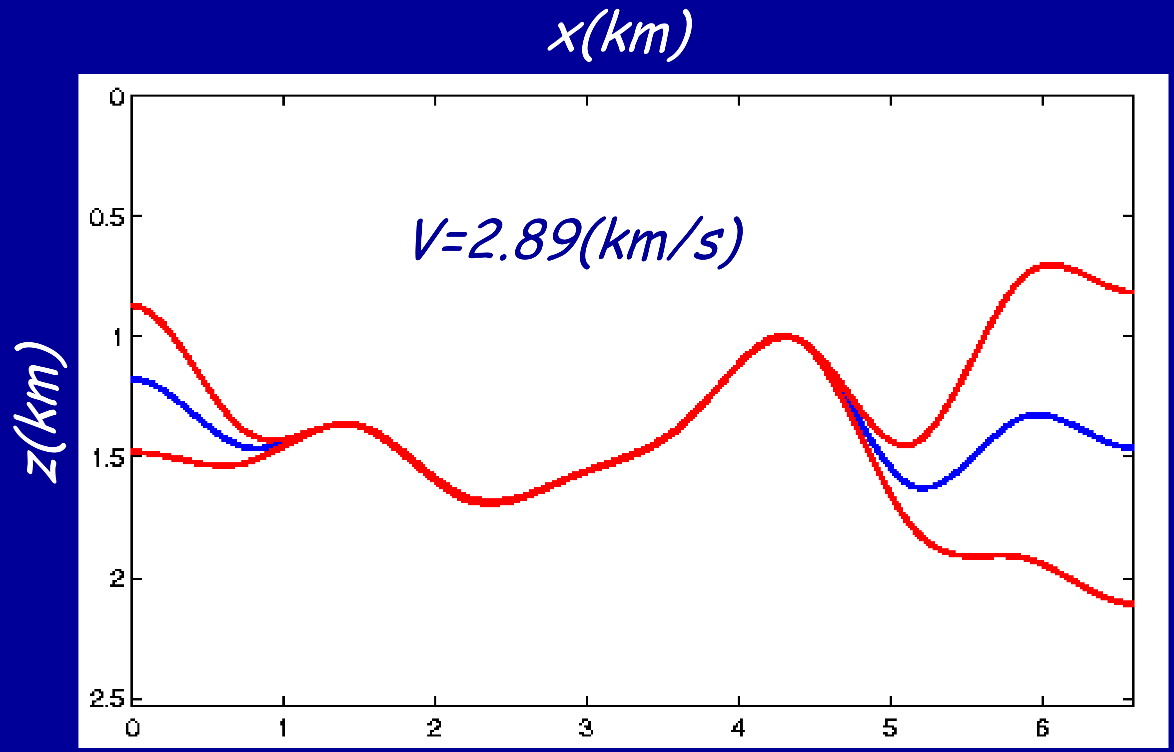
Boundaries of the model

Velocity:

V=2.89(km/s)

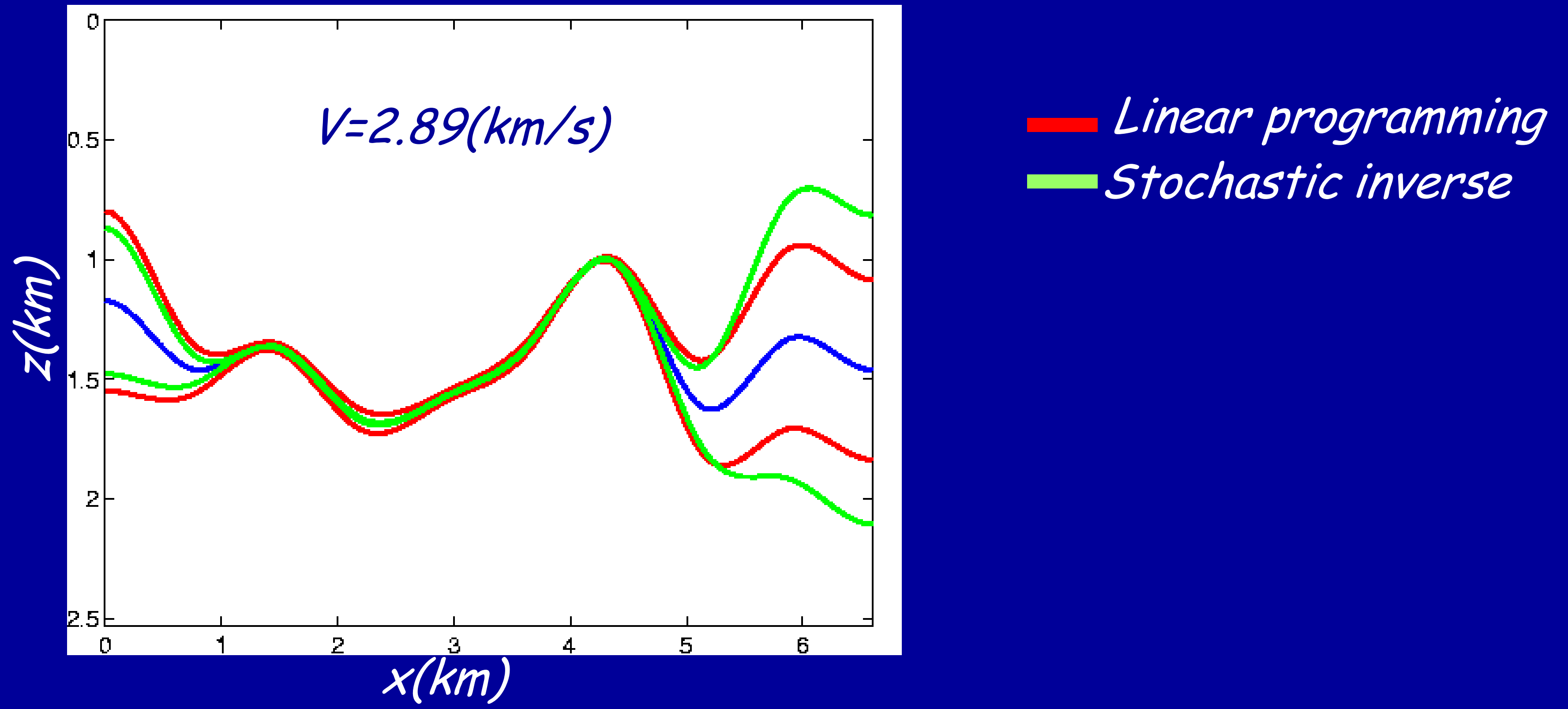
Uncertainty analysis on the solution model

Stochastic inverse



Comparison of the two approaches

- Results obtained by the two methods are similar



Comparison of the two approaches

- Results obtained by the two methods are similar

- Linear programming method is more expensive but as informative as classical stochastic approach

- However, these two approaches may furnish uncertainties on the model parameters : error bars

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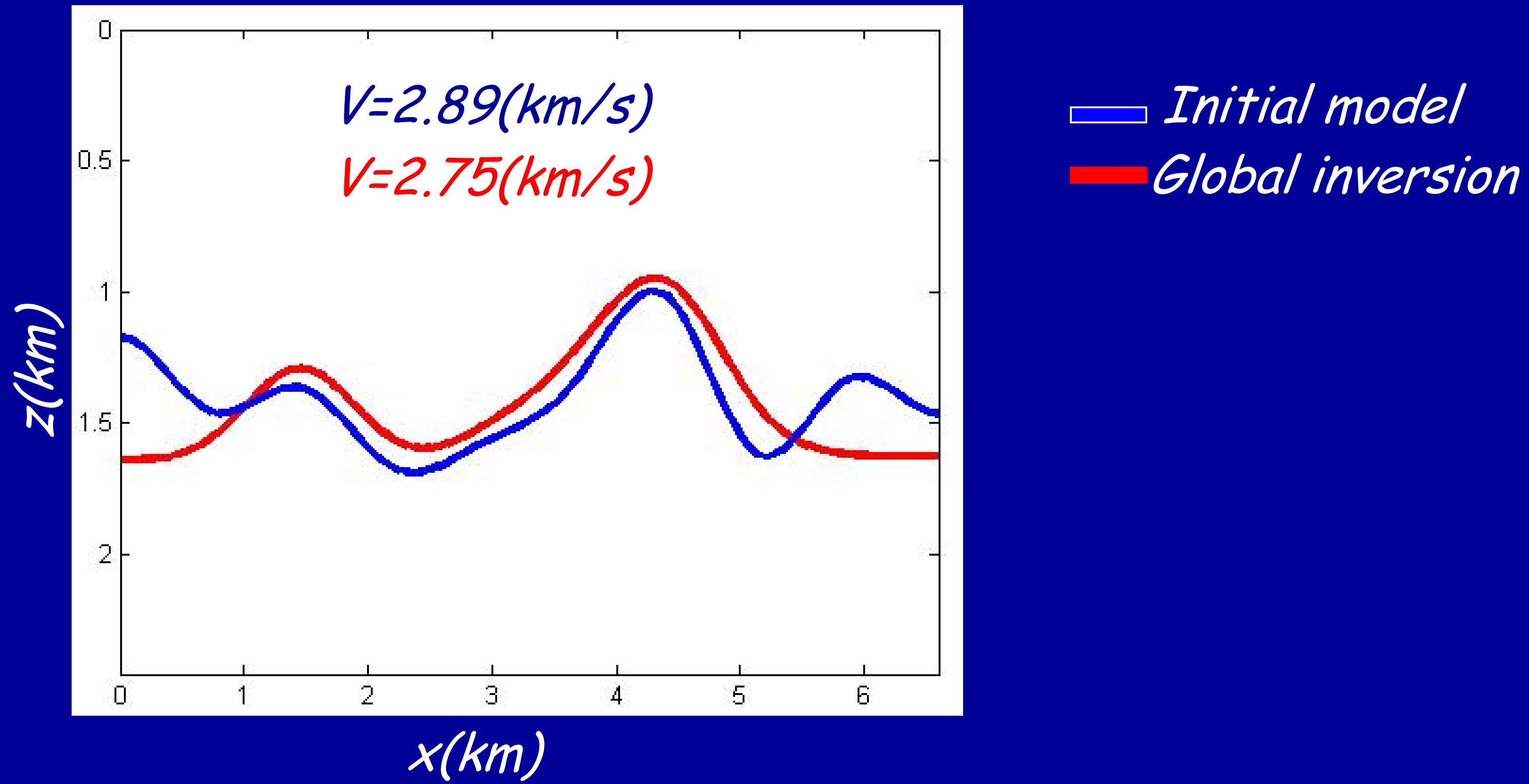
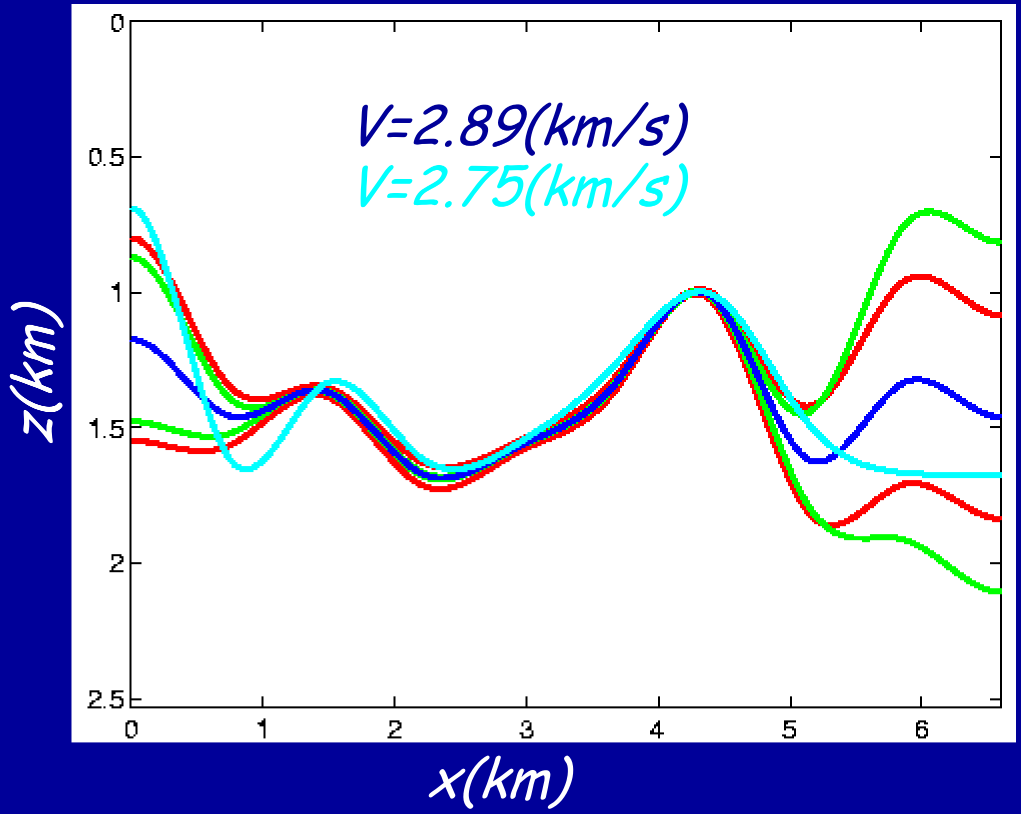
Global inversion

- Global Optimization Method:
  - We choose a classical optimization method (e.g.: Levenberg-Marquardt, Genetic, …)
  - Our algorithm improve the initial condition to this method using Recursive Linear Search.
  - Reference: “Simulation Numérique” Mohammadi B. & Saiac J.H. , Dunod, 2001

Global Inversion Results



Global Inversion Results



Linear programming

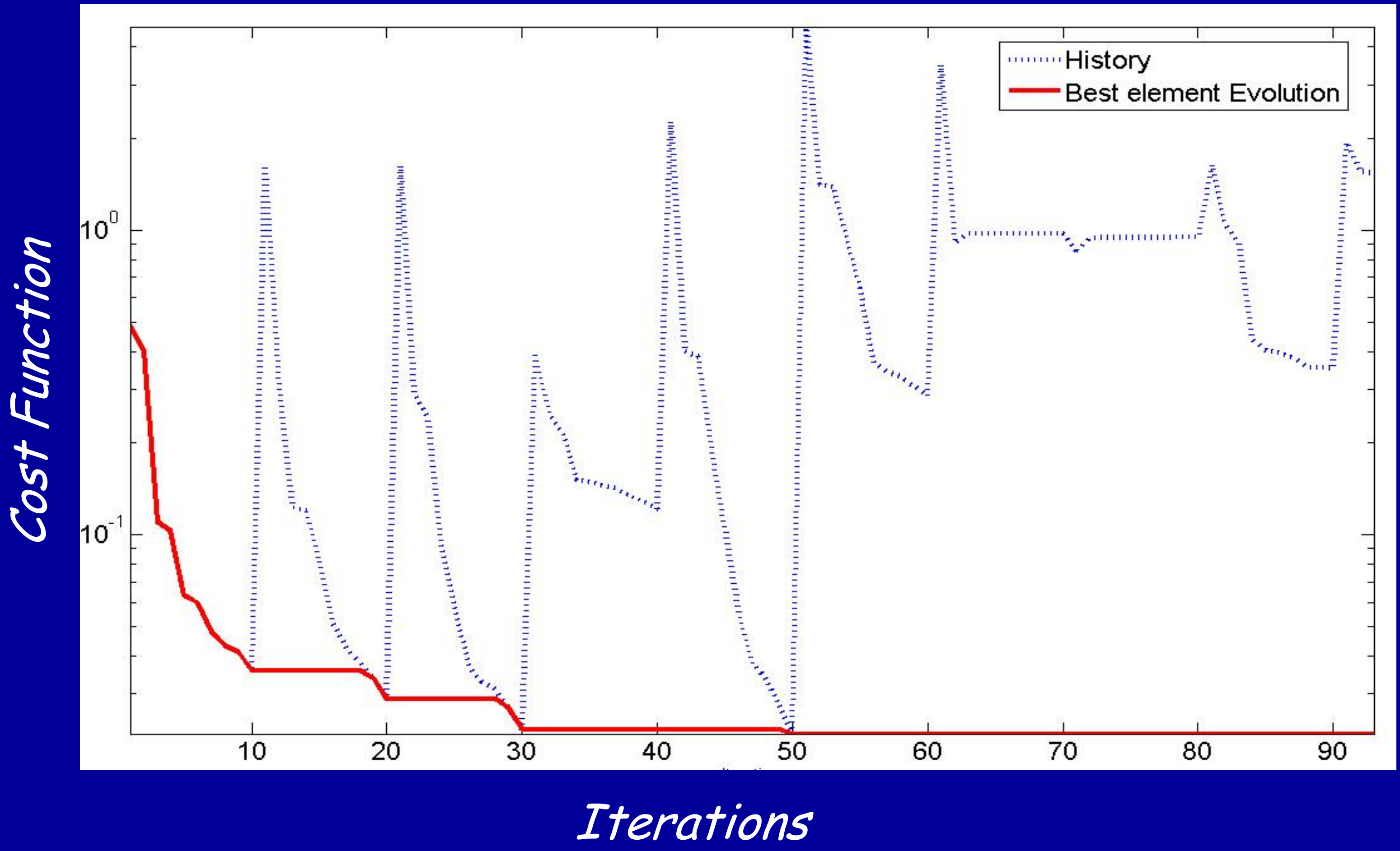
Stochastic inverse

Global inversion

σv_{PL} = +/- 26.40 m/s

σv_{IS} = +/- 3.3 m/s

Global Inversion Convergence



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Conclusions

- We propose two methods to access a posteriori uncertainties:
  - Linear programming method
  - Classical stochastic approach

- The two approaches of uncertainty analysis furnish similar results in the linearized framework .

- These two approaches to quantify uncertainties may be applied to others inverse problems

Conclusions

- Linearized approach explores only the vicinity of the solution model
- Future work: global inversion can allow to overcome the difficulties to quantify uncertainties in the nonlinear case.
- Estimations given by the stochastic inverse approach could (to do) be used as initial iterate in linear programming problems