Angular shifts of paraxial beams by refraction in a plane dielectric/dielectric interface

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Abstract

The longitudinal and transverse angular shifts in the refraction of a paraxial beam are calculated by using the plane-wave decomposition of the amplitude of the electric field distribution of the incident beam. The transmission coefficients are expanded into powers of the spatial frequencies. In this paper these spatial frequencies need to be within the paraxial approach around the main direction of propagation of the beam. The beam is characterized by the moments of the square of the modulus of the angular spectrum of the electric field. To compute them, it is necessary to calculate how the spatial frequencies of the beam change along the refraction. The state of polarization of the beam is also included in the analysis. Numerical results are obtained to show the dependence of the angular shifts on the polarization's state and the symmetry of the beam.

1 Introduction

The reflection of a beam by a plane interface has been deeply studied in the past from different points of views.[1–5] Very recently, the study of the Goss-Hänchen effect associated with total internal reflection, and the linear shifts for refraction have been described.[6,7] On the other hand, the change in the characteristics of reflected beams have been analyzed by using the moment characterization of beams.[8–10] The results of these papers show how it can be found a longitudinal and a transverse angular shifts in the reflection of beams by a dielectric/dielectric, and metal/dielectric, plane interface. However, as far as we know, the transmittance of beams through an interface has not deserved a similar attention in the literature. Fedoseyev[7] has contributed to this topic using a different approach than ours. As the Fedoseyev's paper points out, the results are in good agreement with those obtained for reflected beams using the transformation of the moments through the interface.[10]

In this paper we show a method to account for the lateral (out-of-plane of incidence) and longitudinal (in-plane of incidence) angular shifts appearing when a paraxial beam crosses a plane interface between two dielectric materials. The beam will be described as a vector complex funcion, $\vec{\Psi}(x, y)$, at a given z plane. A paraxial beam propagates around a main direction. Its transversal distribution defines its angular extent. When the beam incides onto a plane interface between two dielectric media, part of it will be refracted and propagated in the second medium. The amplitude of the electric field will change accordingly to the transmission coefficients of the interface.[11] These transmission coefficients depend on the state of polarization and the angle of incidence. The transverse finite size of the beam produces a distribution of angles of incidence. Therefore, to properly account for the refraction of the beam, it will be necessary to evaluate the transmissivity for each angle of incidence involved in the refraction. To do this, we will make use of the plane-wave decomposition of the incident beam.[12,13] Once we have a plane-wave set it is possible to apply the transmission coefficient to each one of the components. The transmitted plane wave will be defined by its amplitude, calculated with the transmission coefficients, and by its direction of propagation, calculated by means of the vectorial form of the Snell law. The characteristic parameters of the angular spectrum of the transmitted beam (including the definitions of the angular shifts) depend on the moments of this angular spectrum of the electric field distribution. Therefore, the transformation of the moments of this angular spectrum across the interface is needed to properly characterize the beam. This transformation is obtained by using a power expansion of the transmission coefficients. This power expansion makes possible to relate the moments through the interface. This is an important advantage because the measurement and calculation of the moments is elsewhere used in the characterization of laser beams.[14] Besides, the power expansion can be upgraded introducing higher-order coefficients to provide better accuracy. In this paper we have performed this calculation until second order. Then, we still have geometric meanings for the moments, and their combinations until 4th. order, involved in the calculation: direction of propagation of the beam and shifts, divergence, angular asymmetry, and angular uniformity. Once the moments are transformed, it is possible to calculate the angular parameters. In this paper we focus our attention on the first order moments related with the angular shifts. The appearance of angular shifts, both in the longitudinal and transversal directions, reveals an effect that could be of use for those cases requiring high accuracy in the determination of the pointing of the beam. The second order moments, clearly related with the angular divergence of the beam, can be also calculated from the results obtained in this paper.[14,15]

Following the previous reasoning, in section 2 we describe how to transform the amplitude of the electric field of the beam, including the change of coordinates across the interface and its influence on the change of spatial frequencies. Section 3 is devoted to the transformation of the moments of the beam. The results of this transformation is used to find the angular characterization of the transmitted beam and the angular shifts in section 4. In this section we also included some numerical simulations made to prove the feasibility of the method. Finally, in section 5 we summarize the main conclusions of the paper.

2 The refraction of the electric field distribution

A given beam can be described, within the paraxial approach, by the transverse distribution of the electric field vector, $\vec{E}$. If we assume that the beam propagates along the Z-axis, the electric field at a given z plane is given by $\vec{\Psi}(x, y)$ (we have dropped the time dependence because we are primarily interested in the spatial and angular characterization). This vector distribution can be expanded as a sum of plane waves whose respectives amplitudes are given by the Fourier transform of the field , $\vec{\Phi}(\xi, \eta)$. Within the paraxial approach the electric field vector lies on a plane perpendicular to the direction of propagation, i.e., we can neglect the longitudinal component of the electric field at a given point of the wavefront.

The components of the plane-wave decomposition of the incident beam propagating within a homogeneus dielectric is as follows,

$$\vec{\Phi}(\xi, \eta, z) = \vec{\Phi}(\xi, \eta) \exp\left\{ i\frac{2\pi}{\lambda} z[1 - \frac{1}{2}\lambda^2(\xi^2 + \eta^2)] \right\}, \tag{1}$$

where λ is the wavelength within the homogeneous dielectric media, $\lambda =$

λ_0/n, being n the index of refraction of the first medium. The direction of propagation for each one of the plane waves is given by the unitary vector

$$\widehat{u} = (\lambda\xi, \lambda\eta, \sqrt{1 - (\lambda^2\xi^2 + \lambda^2\eta^2)}). \qquad (2)$$

This vector is expressed within a coordinate system where the axis Z coincide with the axis of propagation of the beam, and the plane of incidence is XZ (see Figure 1). In the following analysis we will use the square modulus of the angular spectrum of the electric field to calculate the angular parameters of the beam. This angular spectrum is the Fourier Transform of the complex amplitude of the electric field of the beam. Therefore, the modulus of this Fourier Transform contains information about the modulus and the phase of the beam. Then, although the angular characterization requires only the use of the squared modulus of the angular spectrum, this modulus depends not only on the intensity distribution of the beam but also on the wavefront represented in the phase of the beam.

Once the beam has been decomposed into plane waves, we must transmit each one of them. To do this we need to deal with two problems: how is the change in the direction of propagation, and how the amplitude of each plane wave changes .

2.1 The refraction of the beam.

The change in the square of the electric field of a plane wave is given by:

$$|\vec{\Phi}^t|^2 = |t_{\parallel}|^2|\vec{\Phi}^i_{\parallel}|^2 + |t_{\perp}|^2|\vec{\Phi}^i_{\perp}|^2, \qquad (3)$$

where $\|$ means parallel component to the incidence plane, and $\perp$ means perpendicular component to it.

The direction of propagation is given by the Snell law , whose vectorial form is:[16]

$$(\vec{k} - \vec{k'}) \cdot \vec{r} = 0 \Rightarrow n\widehat{u} - n'\widehat{u'} = m\widehat{N}, \tag{4}$$

where n and n' are the indices of refraction of the involved media, $\widehat{u}$ and $\widehat{u'}$ are the unitary vectors along the directions of propagation of the incident and refracted waves, respectively, $\widehat{N}$ is the unitary vector normal to the dielectric interface (this vector points towards the second medium), and $m = n\cos\varepsilon - n'\cos\varepsilon'$, where ε and ε' are the angles of incidence and refraction for each plane wave.

To better explain the results it is very convenient to set another coordinate system after crossing the interface. We will adopt a frame whose Z' axis coincides with the propagation's direction of the refracted beam's center, i.e., the refracted chief ray of the incident beam (see figure 1). Then, the angular shifts are defined as the departure of its actual direction from this reference direction. The transformation of the reference axis, $(X, Y, Z) \rightarrow (X', Y', Z')$, is given by means of a rotation of angle $\varepsilon_0 - \varepsilon'_0$:

$$R_y(\varepsilon_0 - \varepsilon'_0) = \begin{pmatrix} \cos(\varepsilon_0 - \varepsilon'_0) & 0 & -\sin(\varepsilon_0 - \varepsilon'_0) \\ 0 & 1 & 0 \\ \sin(\varepsilon_0 - \varepsilon'_0) & 0 & \cos(\varepsilon_0 - \varepsilon'_0) \end{pmatrix} \tag{5}$$

after this rotation the transmitted vector is :

$$\widehat{u'}_{\rm rot} = \begin{pmatrix} u'_x \cos(\varepsilon_0 - \varepsilon'_0) - u'_z \sin(\varepsilon_0 - \varepsilon'_0) \\ u'_y \\ u'_x \sin(\varepsilon_0 - \varepsilon'_0) + u'_z \cos(\varepsilon_0 - \varepsilon'_0) \end{pmatrix}. \tag{6}$$

The angles of divergence of the beam are considered paraxial, i.e., the beam propagates with an angular spectrum with paraxial extension around the center of the beam. This center of the beam can be defined as the center of gravity of the angular spectrum of the incident beam. If we differenciate the Snell law, $n \sin\varepsilon = n' \sin\varepsilon'$, around the incidence angle of the center of the beam ε_0, we find the following relation between the increments of the angles,

$$\Delta\varepsilon' = \frac{n \cos\varepsilon_0}{n' \cos\varepsilon'_0} \Delta\varepsilon. \tag{7}$$

This equation can be expressed in terms of the spatial frequencies ξ and η. After taking the first order linear partial derivatives of the functions that evaluate the angles of incidence and refraction in terms of the spatial frequency, $\varepsilon = \varepsilon(\xi, \eta)$ and $\varepsilon' = \varepsilon'(\xi', \eta')$ it is possible to find the following relation between the variation of the spatial frequencies

$$\Delta\xi' = \frac{\cos\varepsilon_0}{\cos\varepsilon'_0} \Delta\xi \text{ , and } \Delta\eta' = \Delta\eta. \tag{8}$$

These two relations can be integrated to obtain the relation between the spatial frequencies as,

$$\xi' = \frac{\cos\varepsilon_0}{\cos\varepsilon'_0} \xi = s\xi, \tag{9}$$

$$\eta' = \eta. \tag{10}$$

This linear transformation of coordinates has to be used for the calculation of the moments of the distribution at both sides of the interface. Summarizing these results, we find that the spatial frequencies along the transversal direction remain unchanged, and the spatial frequencies along the longitudinal direction are affected by a scale factor:

$$s = \cos\varepsilon_0 / \cos\varepsilon_0'. \tag{11}$$

This scale factor is quite important to assure that the beam remains paraxial after refraction. The parameter s is lower than one for any incidence angle when the index of refraction of the second medium is larger than the index of refraction of the first medium ($n < n'$). However, if the index of refraction of the first medium is larger than the index of the second (internal refraction for a glass/air interface, $n > n'$) then this scale factor becomes very large for angles close to the limit angle. Then, although the incident beam can be paraxial, this paraxiallity may be broken for the refracted beam near the limit angle. This condition will be discussed in section 4 when analyzing the results provided by the numerical simulation. This analysis based in the Snell law is necessary to compute the moments of the refracted field in terms of the moments of the incident field.

2.2 The change in the amplitude of the electric field

The electric field vector for any incoming component of the plane wave decomposition can be written as,

$$\begin{aligned}\vec{\Phi}^i(\xi,\eta) &= \Phi_x^i(\xi,\eta)\widehat{u}_x + \Phi_y^i(\xi,\eta)\widehat{u}_y \\ &= \Phi_{\|}^i(\xi,\eta)\widehat{u}_{\|} + \Phi_{\perp}^i(\xi,\eta)\widehat{u}_{\perp}. \end{aligned} \tag{12}$$

In this equation $\widehat{u}_{\|}$ and $\widehat{u}_{\perp}$ are two unitary vectors along the parallel and perpendicular directions respectively. On the other hand, $\widehat{u}_x$ and $\widehat{u}_y$ are the unitary vectors along the axes of reference of the transversal plane XY. In general $\widehat{u}_x$ and $\widehat{u}_{\|}$ do not coincide. For every plane wave of the angular spectrum it is possible to define a local incidence plane. The relation between the parallel and perpendicular direction of the chief ray and the parallel and perpendicular direction in the local reference plane is given by a rotation characterized by an angle β that can be calculated as[10]

$$\beta = -\tan^{-1}\left[\frac{-\lambda\eta\cos\varepsilon_0}{\lambda\xi\cos\varepsilon_0 - \sin\varepsilon_0\sqrt{1-(\lambda\xi)^2-(\lambda\eta)^2}}\right]. \tag{13}$$

The transmitted angular spectrum is then calculated by locally rotating the incident angular spectrum to meet the actual, local, parallel and perpendicular directions. After this rotation we apply the transmission coefficients, and finally the rotation is undone to retrieve the correct orientation. This transformation can be written in matricial form as follows,

$$\begin{pmatrix}\Phi_x^t \\ \Phi_y^t\end{pmatrix} = \begin{pmatrix}\cos\beta & -\sin\beta \\ \sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}t_{\|} & 0 \\ 0 & t_{\perp}\end{pmatrix}\begin{pmatrix}\cos\beta & \sin\beta \\ -\sin\beta & \cos\beta\end{pmatrix}\begin{pmatrix}\Phi_x^i \\ \Phi_y^i\end{pmatrix}$$

$$= \begin{pmatrix} t_{\|} \cos^2 \beta + t_{\perp} \sin^2 \beta & (t_{\|} - t_{\perp}) \sin \beta \cos \beta \\ (t_{\|} - t_{\perp}) \sin \beta \cos \beta & t_{\|} \sin^2 \beta + t_{\perp} \cos^2 \beta \end{pmatrix}. \tag{14}$$

The last 2×2 matrix of the previous equation can be denoted as the transmission matrix.

As we stated at the beginning of this paper, this analysis is valid for beams whose angular extension is within the paraxial range. This condition is also related with the transversality condition for beams. This means that once the paraxial condition can be applied then the component of the electric field along the propagation direction at a given point of a curved wavefront can be neglected. Therefore, within the paraxial approach, the electric vector lies on the transversal plane XY.

3 Transformation of the moments

To evaluate the angular shifts of the refracted beam in terms of the symmetry of the incoming beam, we will make use of the moments of the transmitted beams. These moments are given by

$$\mu^t_{i,j} = \iint |\Phi^t(\xi', \eta')|^2 \xi'^i \eta'^j d\xi' d\eta', \tag{15}$$

where the square of the amplitude is expressed in terms of the variables in the second medium, (ξ', η'). For the reflected beam it was possible to use a simple identification of the variables because of the choice of the coordinated system. For the refracted beam the transformation of coordinates needs a more detailed analysis based in equations (9) and (10).

The calculus of the moments uses the squared amplitude of the transmitted beam, $|\Phi^t|^2$. This magnitude can be written in terms of the squared amplitude of the incoming beam by using equation (14). The result is

$$\begin{aligned} |\Phi^t|^2 &= \Big[\left(|t_\parallel|^2\cos^2\beta + |t_\perp|^2\sin^2\beta\right)|\chi_0|^2 \\ &\quad + \left(|t_\parallel|^2\sin^2\beta + |t_\perp|^2\cos^2\beta\right) \\ &\quad + 2\left(|t_\parallel|^2 - |t_\perp|^2\right)\sin\beta\cos\beta|\chi_0|\cos\alpha_0\Big]|\Phi_y^i|^2, \end{aligned} \tag{16}$$

where we have written the spatial distribution of the incident beam in terms of the square of its component along the axis Y, by using the parameter χ. This parameter carries the information about the polarization of the beam, and it is defined as

$$\chi(\xi,\eta) = \frac{\Phi_x^i(\xi,\eta)}{\Phi_y^i(\xi,\eta)}. \tag{17}$$

This parameter is complex in nature and will be expressed as $\chi = |\chi|e^{i\alpha}$. Within the paraxial approach, as Nasalski states,[5] the polarization ratio can be considered constant for the whole beam, $\chi(\xi,\eta) \approx \chi_0$. This $\chi_0 = |\chi_0|e^{i\alpha_0}$ is the value that has been used in equation (16).

Equation (16) can be expanded in powers of the spatial frequencies by using the Taylor series of $t_\parallel$, $t_\perp$, $\cos^2\beta$, $\sin^2\beta$, and $\sin\beta\cos\beta$. The expansion of the transmission coefficient is given as

$$t(\xi,\eta) = t_0^{0,0} + t_0^{1,0}(\xi) + \frac{1}{2}\left[t_0^{2,0}(\xi)^2 + t_0^{0,2}(\eta)^2\right] + \cdots \tag{18}$$

where the subscripts $_0$ means that the expresion must be evaluated a $\xi = \eta = 0$, and the superscripts indicates the order of the derivation with respect to the variables (ξ, η). In this equation we have made use that the coefficients $t_0^{(0,1)} = t_0^{(1,1)} = 0$, both for the parallel and the perpendicular transmission coefficients. The expansions of the trigonometric functions appearing in equation (16) are

$$\cos^2 \beta = 1 - 2\Lambda^2 \eta^2 \tag{19}$$

$$\sin^2 \beta = 2\Lambda^2 \eta^2 \tag{20}$$

$$\sin \beta \cos \beta = -\Lambda \eta, \tag{21}$$

where $\Lambda = \lambda / \tan \varepsilon_0$, and $\lambda = \lambda_0 / n$, being λ_0 the wavelength in the vacuum and n the index of refraction of the first medium. The expansion of the trigonometric expressions has been done up to the second order because in the following analysis we will using the expansion of the relation between the moments of the incoming and refracted beams until this second order.

After some algebra using these previous expansions, and considering the change of coordinates, we can obtain a relation for the moments of the transmitted beam in terms of the moments of the incident beam. This relation is as follows,

$$\mu^t_{j,k} = \frac{1}{|\chi_0|^2 + 1} \sum_{l+m=0}^{N} a_{l,m} \mu^i_{j+l,k+m}, \tag{22}$$

being N the order of the expansion. In this equation we have used that $|\Phi^i_y|^2 = (|\chi_0|^2 + 1)|\Phi^i|^2$. The level of accuracy is better as the order of the expansion is higher. In the rest of the paper we will restrict ourselves to the second order. This is done to handle the moments of the angular spectrum incoming beam until third order when calculating the angular shifts. If we were interested in the calculation of the divergence of the transmitted beam, the involved moments would be those until fourth order. These first four order of moments of the angular spectrum still have geometrical meanings related with the angular location, extension, symmetry and shape of the beam.

The coefficients $a_{l,m}$ are given by the following equations until second order ($N = 2$)

$$a_{0,0} = |t_{\parallel}^{00}|^2|\chi_0|^2 + |t_{\perp}^{00}|^2, \tag{23}$$

$$a_{1,0} = \left(t_{\parallel}^{00}t_{\parallel}^{10*} + t_{\parallel}^{00*}t_{\parallel}^{10}\right)|\chi_0|^2 + \left(t_{\perp}^{00}t_{\perp}^{10*} + t_{\perp}^{00*}t_{\perp}^{10}\right) \tag{24}$$

$$a_{0,1} = -2\Lambda|\chi_0|\cos\alpha_0\left(|t_{\parallel}^{00}|^2 - |t_{\perp}^{00}|^2\right) \tag{25}$$

$$a_{2,0} = \frac{1}{2}\left[\left(t_{\parallel}^{00}t_{\parallel}^{20*} + t_{\parallel}^{00*}t_{\parallel}^{20}\right)|\chi_0|^2 + \left(t_{\perp}^{00}t_{\perp}^{20*} + t_{\perp}^{00*}t_{\perp}^{20}\right)\right] + \left(|t_{\parallel}^{10}|^2|\chi_0|^2 + |t_{\perp}^{10}|^2\right) \tag{26}$$

$$a_{1,1} = -2\Lambda|\chi_0|\cos\alpha_0\left[\left(t_{\parallel}^{00}t_{\parallel}^{10*} + t_{\parallel}^{00*}t_{\parallel}^{10}\right) - \left(t_{\perp}^{00}t_{\perp}^{10*} + t_{\perp}^{00*}t_{\perp}^{10}\right)\right] \tag{27}$$

$$a_{0,2} = \frac{1}{2}\left[\left(t_{\parallel}^{00}t_{\parallel}^{02*} + t_{\parallel}^{00*}t_{\parallel}^{02}\right)|\chi_0|^2 + \left(t_{\perp}^{00}t_{\perp}^{02*} + t_{\perp}^{00*}t_{\perp}^{02}\right)\right] + 2\Lambda^2\left(|t_{\parallel}^{00}|^2 - |t_{\perp}^{00}|^2\right)\left(1 - |\chi_0|^2\right). \tag{28}$$

The coefficient $a_{0,0}$ is related with the transmissivity across the plane interface. The transmission coefficients previously described relate the electric field amplitude across the interface. The relation between the flux of energy before and after the interface is given by the squared modulus of the corresponding transmission coefficients multiplied by the following obliquity factor[11]

$$K = \frac{n'}{n}\frac{\cos\varepsilon'}{\cos\varepsilon}, \tag{29}$$

where the angles of incidence would be different for each component of the plane-wave spectrum. In the zero order approximation we can state that $\varepsilon = \varepsilon_0$, and $\varepsilon' = \varepsilon'_0$. In Figure 2 we have represented the values of the $a_{i,j}$ coefficients. Instead of plotting the value of $a_{0,0}$ we show the transmissivity for both polarizations ($\|$ and $\perp$) as, $T = K_0 a_{0,0}$, assuming $K_0 = n' \cos\varepsilon'_0 / n \cos\varepsilon_0$ calculated for the zero order approximation. The result is in total coincidence with the transmissivity obtained for a dielectric/dielectric plane interface.[11]

4 Angular shifts

After obtaining these previous relations between the moments of the incident beam and the moments of the transmitted beam we are in good conditions to calculate the angular shifts produced in the refraction of a beam. The definition of the angular direction of propagation of the transmitted beam is given by the following equations,[15]

$$\theta_x^t = \lambda' \mu_{1,0}^t / \mu_{0,0}^t, \tag{30}$$

$$\theta_y^t = \lambda' \mu_{0,1}^t / \mu_{0,0}^t, \tag{31}$$

where $\lambda' = \lambda_0/n'$, being λ_0 the wavelength in the vacuum. The zeroth and the first order moments involved in these relations can be calculated by using equation (22) until a certain degree of the expansion. On the other hand, the choice of the reference system for the incident and the transmitted beam makes possible to assure that $\mu^i_{1,0} = \mu^i_{0,1} = 0$. Then, the parameters describing the angular extension of the beam within the XYZ frame are

$$\theta^i_{xx}{}^2 = 4\lambda^2[\mu^i_{2,0}/\mu^i_{0,0}], \tag{32}$$

$$\theta^i_{xy}{}^2 = 4\lambda^2[\mu^i_{1,1}/\mu^i_{0,0}], \tag{33}$$

$$\theta^i_{yy}{}^2 = 4\lambda^2[\mu^i_{0,2}/\mu^i_{0,0}], \tag{34}$$

where $\lambda = \lambda_0/n$. By using these relations it is possible to obtain the angular shifts of the transmitted beam. In the zero order approach the angular shifts are also zero, as the angular departures with respect to the Z axis for the incoming beam. The solutions for the first order approach are

$$\theta^t_x(1) = \frac{n^2}{4n'\lambda_0 a_{0,0}}\left(a_{1,0}\theta^i_{xx}{}^2 + a_{0,1}\theta^i_{xy}{}^2\right) \tag{35}$$

$$\theta^t_y(1) = \frac{n^2}{4n'\lambda_0 a_{0,0}}\left(a_{1,0}\theta^i_{xy}{}^2 + a_{0,1}\theta^i_{yy}{}^2\right). \tag{36}$$

The dependence on the divergences of the beam in this equations is in agreement with equation (21) of Fedoseyev paper.[7] A similar agreement with the equivalent first-order shifts obtained in the case of the reflexion (equation (34) of Ref. (10)) was also pointed out by Fedoseyev. The second-order approach is described by

$$\theta_x^t(2) = \frac{\lambda_0}{n'}\left[\frac{a_{1,0}{\theta_{xx}^i}^2 + a_{0,1}{\theta_{xy}^i}^2 + \frac{n^2}{4\lambda_0^2}\left(a_{2,0}\frac{\mu_{3,0}^i}{\mu_{0,0}^i} + a_{1,1}\frac{\mu_{2,1}^i}{\mu_{0,0}^i} + a_{0,2}\frac{\mu_{1,2}^i}{\mu_{0,0}^i}\right)}{a_{0,0}\frac{n^2}{4\lambda_0^2} + a_{2,0}{\theta_{xx}^i}^2 + a_{1,1}{\theta_{xy}^i}^2 + a_{0,2}{\theta_{yy}^i}^2}\right], \tag{37}$$

$$\theta_y^t(2) = \frac{\lambda_0}{n'}\left[\frac{a_{1,0}{\theta_{xy}^i}^2 + a_{0,1}{\theta_{yy}^i}^2 + \frac{n^2}{4\lambda_0^2}\left(a_{2,0}\frac{\mu_{2,1}^i}{\mu_{0,0}^i} + a_{1,1}\frac{\mu_{1,2}^i}{\mu_{0,0}^i} + a_{0,2}\frac{\mu_{0,3}^i}{\mu_{0,0}^i}\right)}{a_{0,0}\frac{n^2}{4\lambda_0^2} + a_{2,0}{\theta_{xx}^i}^2 + a_{1,1}{\theta_{xy}^i}^2 + a_{0,2}{\theta_{yy}^i}^2}\right] \tag{38}$$

4.1 Numerical calculation

In this subsection we evaluate the angular shifts obtained along the incidence plane, θ_x^i, and along the transverse plane, θ_y^i, for several beams. These shifts depend on the characteristics of the incoming beam, the ratio between the index of refraction at both sides of the interface, and the angle of incidence. Although the numerical simulations carried out in this paper only deal with Gaussian and Hermite-Gaussian beams, any other more general amplitude distribution could be treated by using the corresponding moments of the square of the angular spectrum of the electric field of a generalized thre-dimensional beam (assuming the beam to remain paraxial). The effects that we have found are similar to those observed in the analysis of the reflected light from a plane interface.[9,10] A customized beam (defining the complex amplitude distribution and the state of polarization) could be fabricated to properly compensate the variation of the transmission coefficients and to provide null shifts. In the case of the longitudinal shift, due to the monotonic decreasing character of the transmission coefficients, such a "zero-shift" beam would be asymmetric with respect to the incidence angle defined by the center of the beam. These "zero-shift" beams are very particular distributions that only would apply for a certain value of the incidence angle and for a given configuration of materials. On the other hand, because of the power expansion in which is based this paper, the "zero-shift" character should be related with a given order of the expansion. This means that a "zero-shift" beam in the first order approach may have a non-null shift at higher order. This is also applied to the case of the calculation of any other shifts, where as higher is the order of the expansion as better and more accurate is the result. Because of the previous reasons, these "zero-shift" beams will not be considered in the further examples and discussions.

Longitudinal shifts are always present (except for the previously mentioned "zero-shift" beams). It occurs even for a circular Gaussian beam as we will see in the next examples. This is a consequence of the variation of the transmission coefficients within the angular extension of the incident beam. Transverse shifts are enhanced when the symmetry of the beam is decoupled with respect to the symmetry of the incidence. This means that if the two halfs of the angular spectrum at each side of the principal plane of incidence defined by the center of the incoming beam, are not symmetric, then the transmission coefficients will enhance this asymmetry and provide a non-null transverse shift.

Figure 3 compares the angular shifts calculated at the first and second-order approach (equations (35)-(38)) for an Hermite-Gaussian beam. The beam is elliptic and the axis of the ellipse are oriented at 45° with respect to the plane of incidence. The order of the Hermite-Gaussian beam is 3 along the longest axis (in length) and 1 along the shortest axis. The divergences are 20 mrad and 10 mrad respectively. We can see that both solutions differ when approaching an angle of incidence of 90°. In this region, the first-order approach diverges.

In Figure 4 we show the second-order angular shifts (equations (37) and (38)) when the incidence is from air ($n = 1$) to glass ($n' = 1.5$). The beam is a circular Gaussian beam having 20 mrad of divergence. The lines in each plot correspond with different states of polarization. This case is quite interesting because some of the terms appearing in the equations describing the angular shifts vanish because of the symmetry of the beam. The null terms in the second-order equations are: θ^i_{xy}, $\mu^i_{3,0}$, $\mu^i_{2,1}$, $\mu^i_{1,2}$, $\mu^i_{0,3}$. Besides $\theta^i_{xx} = \theta^i_{yy}$. By applying all these previous relations it is easy to check that $\theta^t_x(1) = (n^2 a_{1,0} {\theta^i_{xx}}^2)/(4n'\lambda_0 a_{0,0})$, and $\theta^t_y(1) = (n^2 a_{0,1} {\theta^i_{xx}}^2)/(4n'\lambda_0 a_{0,0})$. For the second order approach the results are also in terms of the divergence of the incident beam:

$$\theta^t_x(2) = \left(\frac{n^2 {\theta^i_{xx}}^2}{4n'\lambda_0}\right) \frac{a_{1,0}}{a_{0,0} + \frac{n^2}{4\lambda_0^2} {\theta^i_{xx}}^2 (a_{2,0} + a_{0,2})} \tag{39}$$

$$\theta^t_y(2) = \left(\frac{n^2 {\theta^i_{xx}}^2}{4n'\lambda_0}\right) \frac{a_{0,1}}{a_{0,0} + \frac{n^2}{4\lambda_0^2} {\theta^i_{xx}}^2 (a_{2,0} + a_{0,2})}. \tag{40}$$

From these previous equation we can see that the difference between the two angular shifts have the same dependence and the difference is in the introduction of the coefficient $a_{1,0}$ for the longitudinal shift and $a_{0,1}$ for the transversal shift. Under the second order approximation, even in this case that deals with a beam having a symmetric amplitude distribution, we find a transverse angular shift that depends on the state of polarization of the beam, defined by the parameter χ_0 (see Eq. (25)). This transverse shift is zero when the beam is linearly polarized along the parallel or the perpendicular directions, as it should be expected from symmety considerations.

In Figure 5 we have plotted the second-order angular shifts for an internal reflection from glass ($n' = 1.5$) to air ($n = 1$). The incoming beams are Gaussian elliptic beams. The orientation of the ellipse is at 45° with respect to the incidence plane. One of the axis of the ellipse has an angular extension of 20 mrad, and the other axis takes the values of 10 mrad, 13 mrad, and 20 mrad. The polarization is parallel for the plots in the top, and perpendicular for the plots in the bottom of the figure. In this case, when analyzing the values of the shift near the limit angle, it is necessary to check before when the scale factor, s, defined in equation (11), produces a component beyond the paraxial approach. The criteria to obtain a valid range has to calculate the angular extension of the plane wave spectrum along the X axis of the input beam. This is given by θ^i_{xx} defined in equation (34). At the same time the relation between the spatial frequencies (equation (9)) can be written in terms of the angle with respect to the main direction of propagation defined by the center of the beam. This relation is $\theta' = (n/n')s\theta$. The value of θ' should remain paraxial for the components of the angular spectrum of the incoming beam. The angular extension along the X direction, of this incoming beam is characterized by θ^i_{xx}. In order to assure that we will properly transform the most of the angular spectrum we will assume that θ may reach a value of $3\theta^i_{xx}$ (this limit can be adapted to the case treated). At the same time we need to stablish a paraxial angle for θ'. Being conservatives in this election we choose $\theta' < 15°$. For the case of the beams treated in figure 5 the maximum value of the scale factor is $s = 4.10$. This scale factor corresponds with an angle of incidence of $\varepsilon_{0\mathrm{parax}} = 39.8°$. The limit angle for this incidence is $\varepsilon_{0\mathrm{limit}} = 41.8°$. Then, the analysis would be valid until an angle of incidence of 39.8°.

5 Conclusion

In this paper we have shown how the moments of the angular spectrum of a beam can be transformed and calculated after refracting through a plane interface between two dielectric media. The method uses the relation between the spatial frequencies at both sides of the interface. To find these relations we have locally analyzed the Snell law under paraxial conditions. This analysis shows a linear relation between the spatial frequencies of the incident and refracted beams, and is valid when the refracted beam maintains paraxiallity. The paraxial condition has been calculated numerically for the case of internal refraction (glass/air interface) showing the range of applicability of the method. By using the transformed moments we calculate the longitudinal and transverse shifts produced in the refraction of paraxial beams. These shifts depends on the polarization status of the incoming beam and on the distribution of its angular spectrum. The case of beams showing a "zero-shift" behavior has been discussed. Except for those rare beams we could conclude that the longitudinal shifts always exist. Besides, the transverse shift is enhanced when the symmetry of the incoming beam is not the same than the symmetry of the refraction, i.e., the incidence plane is not a plane of symmetry for the irradiance of the incoming beam. The ratio between the squared amplitudes of the electric fields at both sides of the interface is given by the coefficient $a_{0,0}$. After introducing the obliquity factor, we retrieved the familiar results of the tranmissivity along the interface provided by the well-known Fresnel equations. The analytical and numerical calculation has been done until second-order in the expansion of the coefficients relating the moments of the angular spectra. In this case, the angular shifts depend on the moments of the beam until 3rd. order. If we were interested in the calculation of the divergence of the refracted beam, the divergence would depend on the moments until 4th. order. These moments are of common use in the characterization of laser beams and corresponds with the direction of propagation, divergence, angular asymmetry, and angular flatness of the beam, respectively from the first to the fourth order. If the moments of the beam are measured or calculated until higher-than-fourth order, then it would be possible to use higher than second order expansions of the transmission coefficients and the trigonometric functions involved in the calculation.

Summarizing, we have presented a method for calculating angular shift for refracted paraxial three-dimensional beams with arbitrary, but constant, polarization. The analytical results and the numerical simulations have shown that, as for the reflected beam, longitudinal and transversal shift are present. The shifts depends on the angle of incidence, the indices of refraction of both media, the amplitude distribution of the electric field of the incoming beam, and the state of polarization. Although both angular shifts could be of importance, the most disturbing one is the transversal shift because it corresponds with a refraction out of the plane of incidence of the center of the incoming beam.

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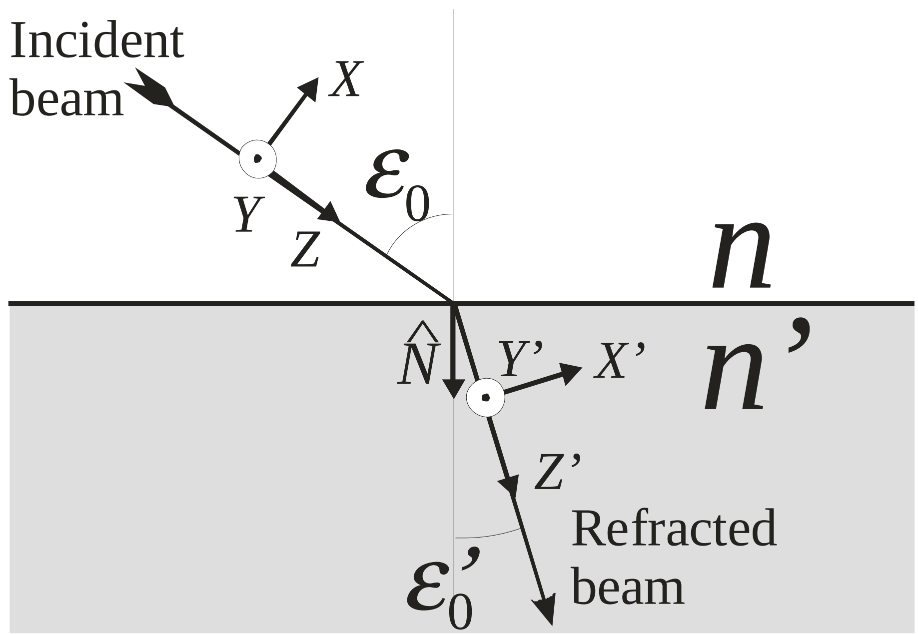

Figure 1: Rotation of the axis to describe the transmitted beam.

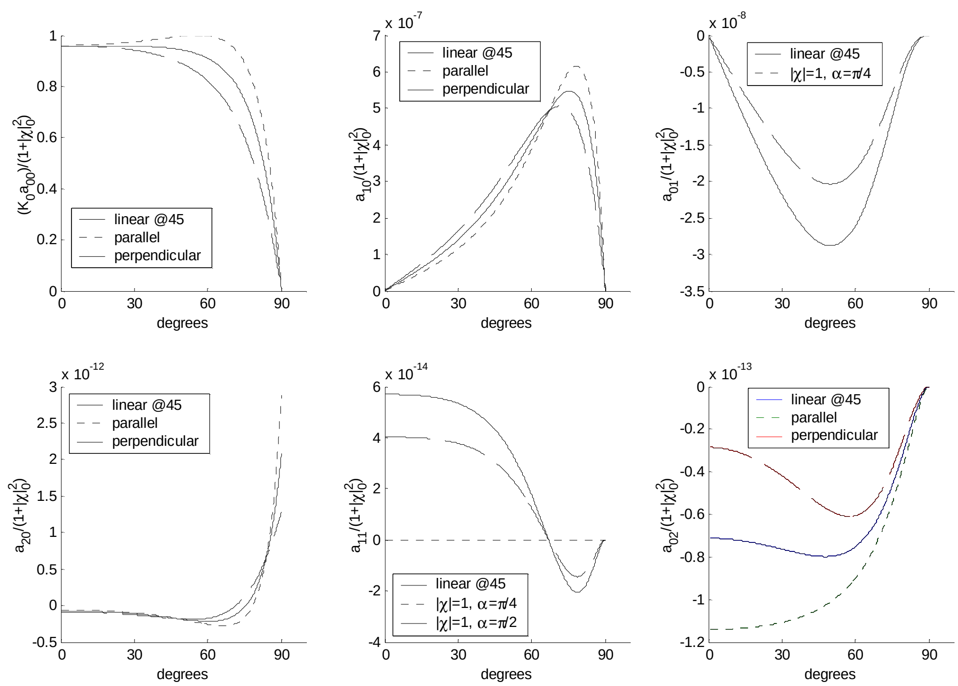

Figure 2: Plot of the $a_{i,j}/(|\chi_0|^2+1)$ coefficients for the case of an incidence air/glass ($n = 1$, $n' = 1.5$). The plot of the $a_{0,0}$ coefficients has been replaced by the value of the transmissivity calculated as $a_{0,0}K_0$, where K_0 is the obliquity factor in the zero order approach.

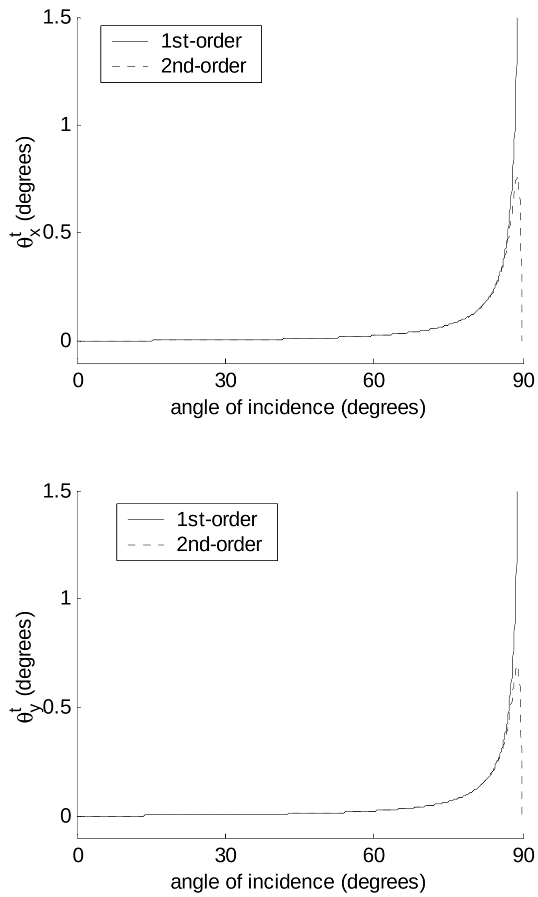

Figure 3: Comparison between the results for the angular shift when the first and the second order or approximation are applied. The beam is an elliptic Hermite-Gaussian beam. The axis of the ellipse are oriented at 45° with respect to the plane of incidence. The divergences along these axis are 20 mrad, and 10 mrad. The order of the Hermite-Gaussian beam is 3 for the largest semiaxis and 1 for the shortest one.

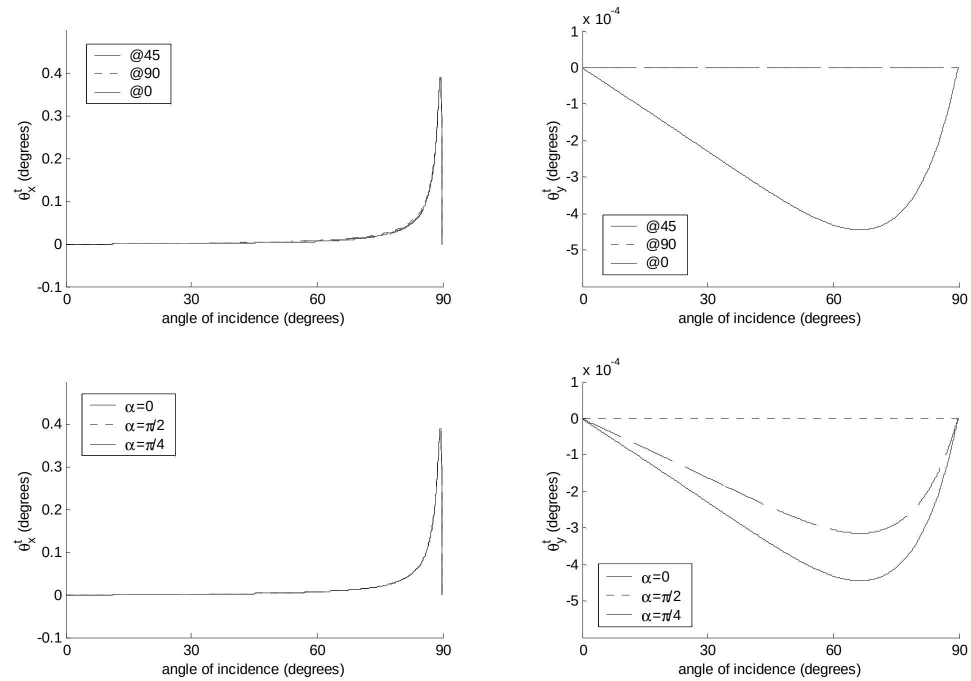

Figure 4: Transverse angular shift for different polarizations when the incidence is air / glass ($n = 1$, $n' = 1.5$). The beam is a circular Gaussian beam having 20 mrad of divergence. The polarization is linear at different azimuths for the figures in the top (0°, parallel; 45°; and 90°, perpendicular). The transverse shift, θ_y^t, is zero when the beam is linearly polarized along the parallel and the perpendicular direction. In the bottom we have again a linear polarization oriented at 45° ($\alpha = 0$), and two states of elliptic polarization (for $\alpha = \pi/2$ we have circular polarization).

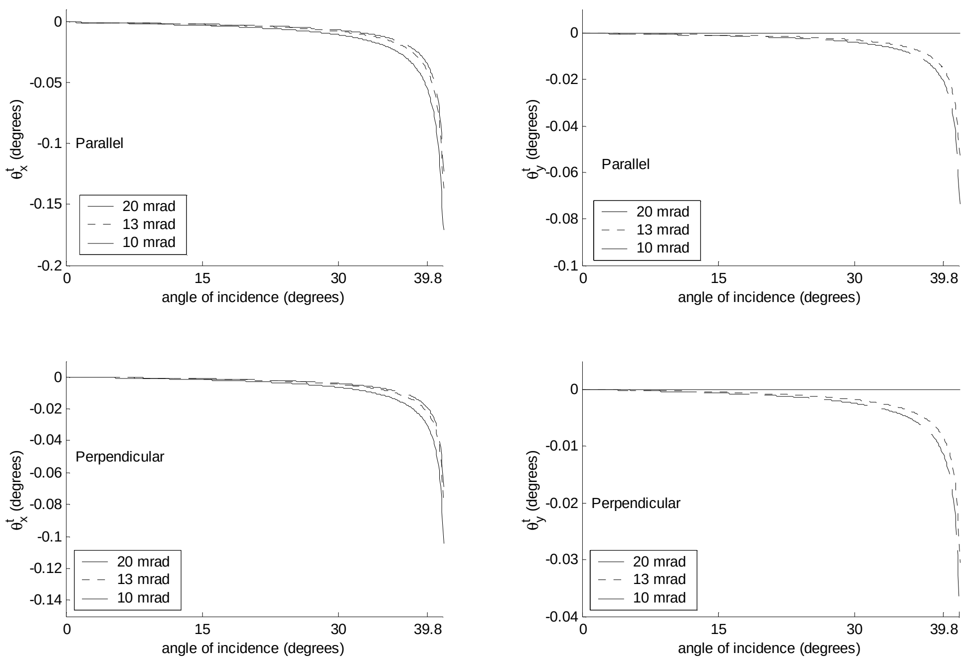

Figure 5: Transverse angular shift for different polarizations when the incidence is glass / air ($n = 1.5$, $n' = 1$). The existence of the limit angle and the possibility of having plane wave components propagating beyond the paraxial range around the chief ray direction, makes the plot valid until $\varepsilon_{\text{parax}} = 39.8°$.