

Continuous vs discrete-time bioeconomic models: Seasonal fisheries*

Sturla Kvamsdal^a, Jose M. Maroto^{b, †}, Manuel Moran^c, Leif K. Sandal^d

^aCentre for Fisheries Economics, Institute for Research in Economics and Business
Administration, 5045 Bergen, Norway.

^bDepartment of Estadística e Investigación Operativa II. Universidad Complutense, 28223
Madrid, Spain.

^cDepartment of Fundamentos del Análisis Económico I. Universidad Complutense, 28223
Madrid, Spain.

^dDepartment of Business and Management Science, Norwegian School of Economics, 5045
Bergen, Norway.

[†]Corresponding author. PHONE: +34913942898, FAX: +34913942388.

E-mail addresses: Sturla.Kvamsdal@nhh.no (Kvamsdal), maroto@ccee.ucm.es (Maroto),
mmoranca@ccee.ucm.es (Moran), leif.sandal@nhh.no (Sandal).

*This research has been supported by a grant from Iceland, Liechtenstein and Norway through the EEA Financial Mechanism. Operated by Universidad Complutense de Madrid. Ref: 021-ABEL-CM-2013.

*Part of the computations of this research was performed in EOLO, the HPC of Climate Change of the International Campus of Excellence of Moncloa (Universidad Complutense de Madrid), funded by MECD and MICINN.

Abstract

We develop a discretization method of continuous-time bioeconomic models which consists of two steps: first we estimate a proper growth function for the continuous-time model through the ensemble Kalman Filter. Then we use the Runge-Kutta method to analyze the optimal management of seasonal fisheries in a discrete-time setting. We analyze both the case of quarterly harvest and the case of monthly harvest, and we compare these to the case of annually harvest. We find that seasonal harvesting is a win-win optimal solution with higher harvest, higher optimal steady state equilibrium, and higher economic value. We also demonstrate that the discretization method overcomes the economic and biological weakness and preserves the strengths of both continuous and discrete-time bioeconomic models.

Keywords: Bioeconomic modeling; Runge-Kutta; Seasonal fisheries; Seasonal harvesting; Kalman-filter; North-East Arctic cod.

Introduction

There is a fundamental choice to be made when developing a bioeconomic model: discrete or continuous-time modeling.

Most of the literature on bioeconomic modeling of fisheries uses both discrete-time (DM) and continuous-time bioeconomic models (CM) indistinctly without a clear biological and/or economic justification. Even, in some cases, in the literature on population dynamics the choice between both options is a matter of individual taste where DM are generally preferred by biologist while CM are generally preferred by mathematicians (Delahay et al., 2009).

However, it is not obvious how DM and CM are related to each other, and consequently this is not a trivial choice, especially in fisheries economics, since methodologies for discrete-time (modeling with difference equations) and continuous-time (modeling with differential equations) are completely different, and consequently the policy advice provided by them can also be different, with significant implications for sustainability of fish stocks.

CM are based upon the assumption that both biological processes, such as growth, and human activity, such as harvesting, are taking place continuously, while in DM, they are taking place at discrete-time steps (usually annual).

CM have proved to be useful for analytical purposes (Clark, 2010). Moreover, as showed below, CM may be useful as a starting point and reference for DM, despite the difficulty in estimation they present. However, CM are unable to encompass delay effects, which are commonplace in real world fisheries, due to the fact that the response of the fish stocks to external factors, such as harvesting, is instantaneous. Moreover, both biological processes, such as spawning, and human activity, such as harvesting, are seasonal rather than continuous over time (Clark, 2010; Bjørndal and Munro, 2012). In addition, data are usually available on an annual basis. For these reasons, DM are oftenly used in the optimal management of commercial fisheries. Indeed, a digression on DM is in order (Bjørndal and Munro, 2012).

A logistic natural growth function (LG) of the stock is frequently used in both DM and CM. However, in DM, the LG does not properly reflect the natural behavior (without harvest) of fish stocks due to the drastic population decline exhibited by LG at high population sizes which could only be explained by catastrophic biological events. While it is not a problem for CM, errors in mathematical modeling of the natural growth function are frequently found in the literature on bioeconomic modeling of fisheries in a continuous-time setting. In particular, most of the natural growth functions used in CM, which inserted into differential equations, are, however, often estimated in discrete time, which uses difference equations (e.g. Agnarsson et al., 2008, for the cod fisheries), despite the well known fact that the dynamical properties of discrete and continuous-time population dynamics are entirely different. Indeed, it is well known that the discrete-time homologue of the continuous-time LG is not the discrete-time LG but the Beverton–Holt growth function which is non-decreasing at high population sizes.

From an economic point of view, DM neglect the change in the stock during harvesting when considering that the harvest costs depend on the stock at the start of the year, and consequently DM may be incurring in systematic errors by underestimating the harvest costs. A continuous cost function is the appropriate cost function when the harvesting process is dependent on the stock density, and the fraction of the stock harvested within a season is significant (Maroto et al., 2012).

When considering increments in time of one year, DM models also neglect seasonal variations in harvesting due to strong seasonal variations over the year in fundamental biological and economic parameters, such as growth, prices, and costs of harvesting, which are commonplace in real world fisheries.

CM also neglect seasonal harvest when considering time-independent optimal feedback policies. Thus, both DM and CM are not able to cope with the complex phenomenon of seasonality in fisheries.

The aim of this paper is to develop a discretization method of CM (DCM), based on the fourth order Runge-Kutta method (RKM), which allows us to construct a bridge between CM and DM by overcoming the biological and economic weakness and by preserving the strengths of both approaches. Specifically, in the DCM developed in this paper, a CM is thought of as the limiting case of a DM in which the interval between times Δt in the discrete time frame $t + \Delta t$ becomes vanishingly small ($\Delta t \rightarrow 0$). This allows for increments in time less than one year ($\Delta t \leq 1$), and consequently it allows us to analyze seasonal fisheries. This is carried out using the RKM which is one of the well known robust numerical methods used in temporal discretization for the approximation of solutions of differential equations (e.g. Press et al., 2007).

Using the North-East Arctic cod stock (NEAC) by way of illustration, the main contributions of this article include: First, in contrast to the LG used in CM, which is often estimated in discrete-time, a proper continuous-time LG, which is a differential equation, is estimated by using data assimilation methods. In particular, we use the ensemble Kalman filter (EnK) to show that the proper LG estimated in continuous-time is quite different to that estimated in discrete-time. In both settings, a modified logistic growth function is estimated for the NEAC but with different parameters estimates.

Second, using the RKM, a proper discrete-time LG is obtained by using an appropriate discretization of the LG estimated above in a continuous-time setting. In contrast to the LG estimated in DM, we show that the discretized LG obtained by the DCM is non-decreasing at high population sizes by properly reflecting the natural behavior (without harvest) of fish stocks.

It should be noted that, although the DCM is applied to a LG in this paper, it can be used for any natural growth function described by a differential equation.

Third, we show that if DM do not take into account the discretized LG obtained by the DCM then such an approach would lead to erroneous (suboptimal) policy advice with the consequent implications for sustainability of fish stocks. Consequently, we show that the DM obtained by the DCM is the appropriate model for management advice.

Fourth, we show that, at least for the NEAC fishery, seasonal harvesting is a win-win optimal solution. In particular, we analyze both the case of quarterly harvest and the case of monthly harvest, and we compare these to the case of annually harvest. We show that the combined effects of the actualization of the growth rates of the stock, decrease of costs of harvesting, and more frequent discounting, which take place if the stock is seasonally harvested (quarterly or monthly), give rise to higher harvest, higher steady state equilibrium, and higher economic value.

Background

The Gordon-Schaefer model (GSM) is one of the most common models used in the literature on bioeconomic modeling of fisheries, assuming constant prices and costs linear in the harvest rate which implies net revenues linear in harvest. In the GSM is also assumed that the natural growth function of the fish stock is described by a LG.

Seasonality has been analyzed in the literature on bioeconomic modeling of open access fisheries by using the GSM in a continuous-time setting. In Flaaten (1983), the GSM is used to analyze the optimal harvest with seasonal growth by assuming a time-dependent intrinsic growth rate $r(t)$ in the LG (cyclical growth rates). Such growth pattern is frequently observed in plankton preying species like capelin and herring (see also Anderson and Seijo, 2010, for seasonality in recruitment in age structured models). In that paper, the optimal harvest is derived to be seasonal with shorter fishing season than would occur under an open access

fishery. In Ardito et al. (1993), a two-dimension dynamical system with periodic coefficients is used to analyze both biological and economic periodic fluctuations in open access fisheries. In their analysis, they also use the GSM but with time-dependent carrying capacity $K(t)$ in the LG. They show that there exists a unique T -periodic solution of the logistic differential equation which plays the role of the carrying capacity. In Kennedy and Hannesson (2006), the GSM is used to analyze whether rents are maximized or dissipated in open access fisheries within seasonal harvesting. They show that in many cases rent maximization is the more plausible outcome.

It should be noted that, in optimally managed fisheries, which is the framework adopted in this paper, instead of an open access fishery, the optimal harvest can be obtained by solving an optimal control problem which, in the case of the GSM, is a linear, non-autonomous optimal control problem. In this setting, the optimal harvest is time-dependent (open-loop policies), and it is well known (Clark, 2010) that it is optimal to approach the steady state as quickly as possible (bang-bang solution).

However, it is also well known (Clark, 2010) that the GSM is based on highly simplified economic and biological conditions. In real word fisheries, prices may depend on the level of supply and the harvesting costs may depend on stock levels, which implies net revenues non-linear in harvest due to, for example, a downward sloping demand (market power) or increasing marginal costs. Under non-linearity in harvest, a feedback rather than an open-loop policy is generally adopted. By a feedback model is meant that the optimal control (harvest) is a direct function of the state variable (stock) and is not found by forecasting (time-independent harvest). In this setting, an optimal feedback policy, as opposed to the bang-bang solution, is characterized by an asymptotic approach to steady state (Sandal and Steinshamn, 2001).

The DCM developed herein is substantially different from the method used in the above literature. In particular, the DM obtained by the DCM is a more general model for the optimal management of seasonal fisheries. Taking as a starting point a feedback CM, as described above, and using the EnK, the DCM includes both general specifications of demand and cost relationships, and population dynamics properly estimated in a continuous-time setting. Moreover, constructing a bridge between CM and DM, the DCM allows us to analyze, in a discrete-time setting, not only strong seasonal variations over the year in biological and economic parameters, as described above, but also seasonal regulatory measures in order to avoid the collapse of fish stocks and to reduce negative socio-economic impacts of seasonality. As pointed out by Bjørndal and Munro (2012), it is often not only necessary, but also appropriate to turn to discrete-time models, especially when using empirical models. Thus, the DCM provides a theoretically-grounded, practical starting point for seasonal fisheries management.

Discretization method of continuous-time bioeconomic models (DCM)

In this section we describe the DCM which takes as a starting point a CM given by:

$$\begin{aligned} \max_h \int_0^{\infty} e^{-\beta t} \Pi(h, x) dt \\ \text{s.t. } \dot{x}(t) = F_c(x) - h \\ x(0) = x_0, \end{aligned} \quad (1)$$

where x represents the fish stock biomass, h the harvest rate, Π net revenues, β the discount rate, and $\dot{x}(t) = dx / dt$ the population dynamics where $F_c(x)$ represents the natural growth function.

The DCM consists of several stages:

i) The natural growth function

$$\dot{x}(t) = F_c(x), \quad (2)$$

is properly estimated in a continuous-time setting by using, for instance, data assimilation methods. In particular, we use the EnK.

ii) The natural growth function estimated in *i)* is discretized by using the fourth order Runge-Kutta method (RKM). The RKM is one of the well known robust numerical methods used in temporal discretization for the approximation of solutions of differential equations. A brief description of the Runge-Kutta algorithm is given in the Appendix.

Given a temporal interval $[t, t + \Delta t]$, the RKM (see Appendix) allows us to obtain the stock value at period $t + \Delta t$, $x_{t+\Delta t}$, as a function of the stock value at period t

$$x_{t+\Delta t} = f_{RK}(\Delta t) := f_{RK}(x_t), \quad (3)$$

where $f_{RK}(x_t)$ is the proper discrete-time approximation, for incremental time Δt , derived from the continuous-time growth function $F_C(x)$, as defined in (2). In this way, the discrete-time approximation of the continuous-time population dynamics, as defined in (1), is given by:

$$x_{t+\Delta t} = f_{RK}(x_t) - h_t, \quad (4)$$

where h_t represents harvest quantity at the end of period t .

It should be noted that, equation (4) can be rewritten as $h_t = f_{RK}(x_t) - x_{t+\Delta t}$. This implies a constraint on the escapement (stock after harvesting) which we call y . This constraint in the optimization problem described below is given by:

$$y \leq f_{RK}(x), \quad (5)$$

which implies $h = f_{RK}(x) - y \geq 0$.

iii) The net revenue function from the fishery $\Pi(h, x)$, as defined in (1), is discretized by considering the temporal interval $[t, t + \Delta t]$, as described in *ii)*. In particular, a generic net revenue function

$$\Pi(h, x) = p(h)h - C(h, x), \quad (6)$$

which is formulated in a continuous-time setting (h represents the harvest rate), and where both the inverse demand function $p(h)$ and the cost function $C(h,x)$ have been estimated on an annual basis, is reformulated to contemplate the temporal interval $[t, t + \Delta t]$ under consideration. In this way, as described below, the discrete-time approximation of the net revenue function, as defined in (6), will be given by:

$$\Pi(h_t, x_t, \Delta t) = p(h_t, \Delta t)h_t - C(h_t, x_t, \Delta t), \quad (7)$$

where h_t represents harvest quantity at the end of period t , as described in (4).

It should be noted that, using (4), the right-hand term in equation (7) can be rewritten as

$$\Pi(x_t, x_{t+\Delta t}, \Delta t).$$

The DCM described above allows us to obtain a discrete-time approximation of the CM, as defined in (1). In particular, the DM obtained by the DCM is given by:

$$\begin{aligned} \max_{\{x_{t+\Delta t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^{\Delta t} \Pi(x_t, x_{t+\Delta t}, \Delta t) \\ \text{s.t. } 0 \leq x_{t+\Delta t} \leq f_{RK}(x_t), \quad t = 0, 1, \dots, \\ x_0 > 0 \text{ given, } \Pi(x_t, x_{t+\Delta t}, \Delta t) \geq 0, \quad t = 0, 1, \dots, \end{aligned} \quad (8)$$

where the objective functional is the present value of net revenues from the fishery, Π is as defined in (7), $\delta \in (0,1)$ is an annual discount factor, and x_0 is the initial stock level. If $f_{RK}(x_t)$ is as defined in (4), then $f_{RK}(x_t) - x_{t+\Delta t} = h_t$ in problem (8).

Using the dynamic programming approach (Stokey et al., 1989), we can define the following Bellman equation associated with (8):

$$V(x) = \max_{0 \leq y \leq f_{RK}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)], \quad (9)$$

where the constraint on the escapement is as defined in (5).

The DCM described above allows us to construct a bridge between CM and DM due to the fact that, a CM, as defined in (1), may be thought of as the limiting case of the DM, as defined in (9), in which the interval between times Δt in the discrete-time frame $t + \Delta t$ becomes

vanishingly small. In other words, the DM, as defined in (9), converges to the CM, as defined in (1), when $\Delta t \rightarrow 0$.

North-East Arctic cod fishery

In order to make the numerical experiments realistic while keeping their scope of application wide enough, we take as our starting point the North-East Arctic cod (NEAC) which is one of the most important species in Norwegian fisheries.

There are strong seasonal fluctuations in abundance of NEAC due to its migration pattern. In particular, Maturing cod migrate to the Norwegian coast to spawn and back to the Barents Sea after spawning. 80% of the NEAC is harvested during the winter in the area where the stock has gathered and migrated from the Barents Sea to spawn (Hermansen and Dreyer, 2010).

The NEAC has been analyzed extensively in the literature on fisheries economics by using CM, as defined in (1) (e.g. Arnason et al., 2004; Agnarsson et al., 2008). In this article we look upon the NEAC as an illustration. In this sense, we do not try to analyze the current fishery policy, but analyze the effects of seasonal harvesting.

Using the EnK described above, the natural growth function for the NEAC, as defined in (2), is estimated in a continuous-time setting. In particular, we estimate a modified logistic function

$$F_C(x) = r_C x^2 \left(1 - \frac{x}{K_C}\right), \quad (10)$$

where growth is measured in 1,000 tons, $r_C = 0.00045371$ is the intrinsic growth rate, and $K_C = 3,703$ (1,000 tons) is the carrying capacity of the environment.

Thus, in contrast to the standard CM, the EnK allows us to properly estimate the growth function of the species in a continuous-time setting, which is the starting point of DCM described above (see *i*) in the previous section).

Using annual data, a generic net revenue function, as defined in (6), from the NEAC fishery has been estimated by Agnarsson et al. (2008)

$$\Pi(h, x) = p(h)h - C(h, x), \quad (11)$$

where $p(h) = (p_1 - p_3h)$, $p_1 = 12.65$ and $p_3 = 0.00839$; $C(x, h) = c \frac{h^{1.1}}{x}$, $c = 5,848.1$. Stock x and

harvest h are measured in 1,000 tons, net revenue $\Pi(h, x)$ is measured in million NOK, and prices are measured in NOK/kg.

Using the DCM (see *ii* and *iii* above), the discrete-time approximation, as defined in (7), of the net revenue function (11) is given by:

$$\begin{aligned} \Pi(x_t, x_{t+\Delta t}, \Delta t) &= (p_1 - p_3(h_t / \Delta t))h_t - c\Delta t \frac{(h_t / \Delta t)^{1.1}}{x_t} = \\ &= (p_1 - p_3((f_{RK}(x_t) - x_{t+\Delta t})\Delta t^{-1}))(f_{RK}(x_t) - x_{t+\Delta t}) - c\Delta t^{-0.1} \frac{(f_{RK}(x_t) - x_{t+\Delta t})^{1.1}}{x_t}, \end{aligned} \quad (12)$$

where $h_t = f_{RK}(x_t) - x_{t+\Delta t}$, as defined in (8), and $f_{RK}(x_t)$ is the discrete-time approximation, as defined in (3), of the continuous-time growth function $F_C(x)$, as defined in (10).

Using (12), the Bellman equation, as defined in (9), for the NEAC fishery is given by:

$$V(x) = \max_{0 \leq y \leq f_{RK}(x)} [\Pi(x, y, \Delta t) + \delta^{\Delta t} V(y)]. \quad (13)$$

Thus, the DCM allows us to construct a bridge between CM and DM for the NEAC fishery.

Biological and economic weakness of DM

In this section we describe how the DCM allows us to overcome biological and economic weakness of DM.

All data in the numerical experiments described below were performed using code written in standard FORTRAN.

Biological weakness

In DM, the population dynamics of fish stocks is described by the difference equation

$$x_{t+1} = x_t + F_D(x_t) - h_t = f_D(x_t) - h_t, \quad (14)$$

$$F_D(x_t) = r_D x_t^\alpha \left(1 - \frac{x_t}{K_D}\right),$$

where x_t , $F_D(x_t)$, and h_t are stock size, natural growth, and harvest¹, respectively, in period (year) t .

In most of the literature on fisheries economics, a discrete-time LG, as described by $F_D(x_t)$ in (14), is frequently estimated. An illustration of this is the case of the NEAC, where $r_D=0.000665$, $\alpha=2$ (modified LG), and $K_D=2,473$ (1,000 tons) were estimated in Agnarsson et al. (2008).

It should be noted that, as expected, the above parameter estimations are very different from those obtained in a continuous-time setting (see $F_C(x)$, as defined in (10)).

Figure 1 represents the growth function of the NEAC $f_D(x_t) = x_t + F_D(x_t)$, as defined in (14), estimated by Agnarsson et al. (2008), and the growth dynamics from a high initial stock value $x_0^D = 3,375$ (1,000 tons), $x_0^D > K_D = 2,473$, (discontinuous line). We can observe in this figure the drastic population decline exhibited by $f_D(x_t)$ at high population sizes which could only be explained by catastrophic biological events. For example, we can see in this figure that, after only one year, there is a drastic decrease, of about 85%, in population where the extremely low stock value $x_{t+1} = x_1 = 500$ is obtained from the high initial stock level $x_t = x_0^D = 3,375$.

We can also see in Figure 1 that the growth dynamics from the initial stock level $x_t = x_0^D$ is obtained through the path $x_0^D \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow x_4 \rightarrow x_5 \rightarrow K_D$. This means that the drastic population decline is followed by five growth periods until the biological equilibrium K_D is achieved.

Figure 1 also represents the discrete-time approximation $f_{RK}(x_t)$, as defined in (3) for $\Delta_t=1$, of the continuous-time LG of the NEAC $F_C(x)$, as described in (10), and the growth dynamics

from a high initial stock value $x_0^{RK} = 4,700$ (1,000 tons), $x_0^{RK} > K_C = 3,703$, (discontinuous line). We can observe in this figure that $f_{RK}(x_t)$ is a non-decreasing growth function that reflects a gradual approximation to K_C from high initial stock values $x_0^{RK} > K_C$. We can also see in this figure that K_C is achieved in a shorter period of time. In contrast to standard DM, this behavior seems to properly reflect the natural behavior of fish stocks at high population sizes.

Figure 2 represents the optimal policy function OP_D and the optimal policy dynamics which correspond to the solution of problem (13) with the growth function of the NEAC $f_D(x_t)$, as defined in (14) (see Figure 1), for a discount factor $\delta = 0.95$, and for $\Delta t = 1$. This is the case of standard DM.

Figure 2 also represents the optimal policy function OP_{RK} and the optimal policy dynamics which correspond to the solution of problem (13) with the discrete-time approximation $f_{RK}(x_t)$ for the NEAC, as defined in (3) for $\Delta t = 1$ (see Figure 1), for a discount factor $\delta = 0.95$. This is the case of the DCM for $\Delta t = 1$.

We can see in Figure 2 that, in both cases, the optimal policy function represents the optimal stock level in the next period (year), x_{t+1}^* (after harvesting), as a function of the current stock level, x_t (1,000 tons). For example, in the case of standard DM, the optimal stock level, $x_{t+1}^* = x_1^* = 1,480$, is obtained from the initial stock level $x_t = x_0^D = 2,950$. In this way, the optimal policy dynamics from the initial stock level, $x_t = x_0^D$, is obtained through the optimal path $x_0^D \rightarrow x_1^* \rightarrow x_2^* \rightarrow x_3^* \rightarrow x_D^*$. This means that the solution converges to an optimal steady-state equilibrium (SS) $x_D^* = 2,173$.

We can observe in Figure 2 that the solution obtained in the case of standard DM is very different from that obtained by the DCM, although in both cases, the solution converges to an

optimal SS, $x_D^* = 2,173$ and $x_{RK}^* = 3,065$, respectively. In particular, we can see in this figure that, in the case of the DCM, the resource is preserved at higher stock levels than in the case of standard DM, $x_{RK}^* > x_D^*$ (about 29% greater).

We can also see in Figure 2 that, in the case of standard DM, the transition phase towards the SS from high initial stock values $x_0^D = 2,950 > K_D = 2,473$, is very different from that obtained by the DCM from high initial stock values $x_0^{RK} = 4,900 > K_C = 3,703$. In particular, we can see in this figure that, in the case of standard DM, the SS $x_D^* = 2,173$ is achieved from its left hand side, while in the case of the DCM, $x_{RK}^* = 3,065$ is achieved from its right hand side which implies that x_{RK}^* is gradually achieved in a shorter period of time. The reason for this is that the optimal policy functions, OP_D and OP_{RK} , preserve the properties of the growth functions described above, $f_D(x_t)$ and $f_{RK}(x_t)$, respectively (see Figure 1).

Thus, the above results provide an example of two well-established discrete-time bioeconomic models, which however give rise to quite different optimal policies for the same species, with the consequent uncertainty on what is the appropriate model for management advice.

According to the arguments provided above on the growth function², we suggest that the DCM is the most appropriate one³.

Economic weakness

In DM, the cost function, as defined in (11) and (12), is given by:

$$C(x_t, h_t) = c \frac{h_t^\beta}{x_t}. \quad (15)$$

It should be noted in (15) that DM neglect the change in the stock during harvesting when considering that the harvest costs depend on the stock at the start of the year x_t , and consequently there exists an underestimation of the costs.

As showed in Maroto et al. (2012), a continuous cost function is the appropriate cost function when the harvesting process is dependent on the stock density, and the fraction of the stock

harvested within a season is significant. This can be contemplated by the DCM by considering in (12) the following continuous cost function⁴:

$$C(h, x) = \int_t^{t+\Delta t} \frac{ch^\alpha}{x} dt. \quad (16)$$

Thus, according to the arguments provided above, the DCM allows us to overcome the biological and economic weakness of DM.

Seasonal fisheries

The DCM allows for increments in time less than one year which, as described above, is the standard time interval, $\Delta t = 1$, considered in DM. Specifically, if the one-year interval is divided in N periods, then this can be done by defining $\Delta t = \frac{1}{N} \leq 1$ in (3) and (9). In this way, the DCM allows us to analyze the phenomenon of seasonality in fisheries for different values of $\Delta t < 1$. In particular, taking the NEAC fishery as an illustration, in this section we analyze both the case of quarterly harvest, $\Delta t = 0.25$ ($N=4$), and the case of monthly harvest, $\Delta t = 0.083$ ($N=12$), and we compare these to the case of annually harvest, $\Delta t = 1$ ($N=1$).

Numerical analysis

Figure 3 represents the optimal harvest H^* (1,000 tons) which corresponds to the solution of (13) with the discrete-time approximation $f_{RK}(x_t)$ for the NEAC, as defined in (3), for a discount factor $\delta = 0.95$, and for different values of Δt . In particular, this figure represents the annual optimal harvest if the stock is: annually harvested, $AH(x_t)$, quarterly harvested, $AQH(x_t)$, and monthly harvested $AMH(x_t)$. In all cases, x_t (1,000 tons) represents the stock value at the beginning of the year.

It should be noted that, while $AH(x_t)$ can be obtained directly by solving (13) with the discrete-time approximation $f_{RK}(x_t)$, for $\Delta t = 1$, a previous step is needed to obtain both $AQH(x_t)$ and $AMH(x_t)$. For example, in the case of $AQH(x_t)$, the optimal harvest in any

quarter Q_i of the year, $Q_i H(x_t)$, must first be obtained by solving (13) with the discrete-time approximation $f_{RK}(x_t)$, for $\Delta t = 0.25$. Once $Q_i H(x_t)$ is obtained for each stock value x_t at the beginning of the quarter Q_i , the optimal quarterly harvest path over the year is given by:

$$Q_1 H(x_t) = f_{RK}(x_t) - y_1 \rightarrow Q_2 H(y_1) = f_{RK}(y_1) - y_2 \rightarrow Q_3 H(y_2) = f_{RK}(y_2) - y_3 \rightarrow Q_4 H(y_3) = f_{RK}(y_3) - y_4, \quad (17)$$

where y_1 is the optimal stock level at the beginning of the second quarter (after harvesting) if x_t is the stock level at the beginning of the year, y_i ; $i=2,3$, is the optimal stock level at the beginning of the quarter Q_{i+1} if y_{i-1} is the optimal stock level at the beginning of the quarter Q_i , and y_4 is the optimal stock level at the beginning of the next year if x_t is the stock level at the beginning of the current year. As described below, the latter is the annual optimal policy function if the stock is quarterly harvested. In this way, $AQH(x_t)$ can be obtained for each stock level x_t at the beginning of the year

$$AQH(x_t) = Q_1 H(x_t) + Q_2 H(y_1) + Q_3 H(y_2) + Q_4 H(y_3). \quad (18)$$

Based on the above, $AMH(x_t)$ can be obtained for each stock level x_t at the beginning of the year

$$AMH(x_t) = M_1 H(x_t) + M_2 H(y_1) + \dots + M_{12} H(y_{11}), \quad (19)$$

where $M_i H(\cdot)$ is the optimal harvest in any month M_i of the year.

We can observe in Figure 3 that, in all cases, the optimal harvest follow the same pattern as that obtained in the CM described in Agnarsson et al. (2008), which consists of harvest moratorium at low stock levels, $x \leq x^m = 900$, with a gradual increase in harvest at high enough stock values, $x \geq x^m$. It seems to confirm the robustness of the DCM due to the fact that the discrete-time solution converges to that obtained in continuous-time if $\Delta t \rightarrow 0$. Moreover, we can also see in Figure 3 that AQH is similar to AMH which implies that, at least for this fishery, there is a fast convergence.

We can also observe in Figure 3 that both AQH and AMH are greater than AH, specially at high enough stock values, a somewhat counterintuitive result which can be explained by the actualization of the growth rates (AGR) of the resource which takes place if the stock is more frequently harvested, on a monthly or quarterly basis, instead of waiting for growth the whole year, as is the case of annual harvest. If the stock is more frequently harvested then the AGR gives rise to greater growth rates of the resource which in turn implies higher annual harvest. Figure 4 represents the optimal policy function and the optimal policy dynamics which correspond to the solution of (13) with the discrete-time approximation $f_{RK}(x_t)$ for the NEAC, for a discount factor $\delta = 0.95$, and for different values of Δt . In particular, this figure represents the annual optimal policy functions, AP and APQ, if the stock is annually and quarterly harvested, respectively. In both cases, the annual optimal policy function represents the optimal stock level in the next period (year), x_{t+1}^* (after optimal harvesting), as a function of the stock level at the beginning of the current year x_t (1,000 tons), where the optimal harvest is represented in Figure 3, AH and AQH, respectively.

We can see in Figure 4 that, if the stock is annually harvested, then the optimal stock level $x_{t+1}^* = x_1^*$, is obtained from the initial stock level $x_t = x_0 = 1,000$. In this way, the annual optimal policy dynamics (AD) from the initial stock level $x_t = x_0$ is obtained through the optimal path

$$x_0 \rightarrow x_1^* \rightarrow x_2^* \rightarrow x_3^* \rightarrow x_4^* \rightarrow \dots \rightarrow x_A^*. \quad (20)$$

This means that the solution converges to an SS $x_A^* = 3,065$.

Figure 4 also represents the quarterly optimal policy function (QP) which represents the optimal stock level in the next quarter (after harvesting, as defined in (17)) as a function of the stock level in the current quarter. We can see in Figure 4 that, as defined in (17) and (18), if

the stock is quarterly harvested, then the quarterly optimal policy dynamics (QD) from the initial stock level $x_t = x_0 = 1,000$ is obtained through the optimal path

$$x_0 \rightarrow y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow \dots \rightarrow x_Q^*. \quad (21)$$

This means that the solution converges to an SS $x_Q^* = 3,266$.

Figure 4 also represents the annual optimal policy function if the stock is quarterly harvested (APQ), as defined in (17) and (18). For example, y_4 is the optimal stock level at the beginning of the next year if x_0 is the stock level at the beginning of the current year. This figure also represents the annual optimal policy function if the stock is monthly harvested (APM), as defined in (19), which is similar to APQ due to the fact that, as described above (see Figure 3), AMH is similar to AQH.

We can see in Figure 4 that, if the stock is more frequently harvested, on a monthly or quarterly basis, then the resource is preserved at higher stock values than if the stock is annually harvested, despite the lower annual harvest in this last case (see Figure 3). In particular, we can see in Figure 4 that $x_Q^* = 3266$ is about 6% greater than $x_A^* = 3065$. This somewhat counterintuitive result can also be explained by the AGR, as explained above.

Figure 5 represents the annual costs C^* (million NOK) incurred in optimal harvesting, $AC(x_t)$ and $ACQ(x_t)$, if the stock is annually and quarterly harvested, respectively. In both cases, x_t (1,000 tons) represents the stock value at the beginning of the year.

It should be noted that $AC(x_t)$ can be obtained directly by using the cost function, as defined in (12), for $\Delta t = 1$, and for $AH(x_t)$, as defined above (see Figure 3)

$$AC(x_t) = c \frac{AH(x_t)^{1.1}}{x_t}. \quad (22)$$

It should be noted also that a previous step is needed to obtain $ACQ(x_t)$. In particular, the quarterly costs incurred in optimal harvesting, $Q_iC(x_t)$, must first be obtained by using both the cost function, as defined in (12), for $\Delta t = 0.25$, and $Q_iH(x_t)$, as defined in (17)

$$Q_iC(x_t) = c(0.25)^{-0.1} \frac{Q_iH(x_t)^{1.1}}{x_t}, \quad (23)$$

where x_t represents the stock value at the beginning of the quarter Q_i . Using both (17) and (23), the quarterly costs path incurred in optimal harvesting over the year is given by:

$$Q_1C(x_t) \rightarrow Q_2C(y_1) \rightarrow Q_3C(y_2) \rightarrow Q_4C(y_3). \quad (24)$$

In this way, $ACQ(x_t)$ can be obtained for each stock level x_t at the beginning of the year⁵

$$ACQ(x_t) = Q_1C(x_t) + Q_2C(y_1) + Q_3C(y_2) + Q_4C(y_3). \quad (25)$$

We can see in Figure 5 that, there exists a stock value $x' \approx x_4^* \approx 2800$, which correspond to x_4^* in Figure 4, such that, for stock values $x_t \leq x'$, the annual costs incurred in optimal harvesting if the stock is quarterly harvested are lower than that obtained if the stock is annually harvested, $ACQ(x_t) \leq AC(x_t)$, while the opposite is the case for high stock values $x_t \geq x'$. The reason for this is that, for stock values $x_t \leq x' \approx x_4^*$ at the beginning of the year, we can see in Figure 4 that $x_t < y_1 < y_2 < y_3 < y_4 < x_{t+1}$, and consequently, $ACQ(x_t)$, as defined in (23) and (25), are lower than $AC(x_t)$, as defined in (22). We can also see in Figure 4 that the opposite is the case for high stock values $x_t \geq x' \approx x_4^*$.

Figure 6 represents the value function V (million NOK), as defined in (13), for a discount factor $\delta = 0.95$, and for different values of Δt . In particular, this figure represents the annual value function if the stock is: annually harvested, $AV(x)$, quarterly harvested, $AVQ(x)$, and monthly harvested, $AVM(x)$. In all cases, x represents the stock value at the beginning of the year.

It should be noted that $AV(x)$ can be obtained directly by solving (13) for $\Delta t = 1$, while $AVQ(x)$ and $AVM(x)$ are given by:

$$AVQ(x) = \Pi(Q_1H(x), x) + \delta^{0.25}\Pi(Q_2H(y_1), y_1) + \delta^{0.5}\Pi(Q_3H(y_2), y_2) + \delta^{0.75}\Pi(Q_4H(y_3), y_3) + \delta AVQ(y_4), \quad (26)$$

$$AVM(x) = \Pi(M_1H(x), x) + \delta^{\frac{1}{12}}\Pi(M_2H(y_1), y_1) + \dots + \delta^{\frac{11}{12}}\Pi(M_{12}H(y_{11}), y_{11}) + \delta AVM(y_{12}),$$

where $Q_iH(\cdot)$ and $M_iH(\cdot)$ have been defined in (17) and (19), respectively.

We can see in Figure 6 that $AVQ(x)$ is similar to $AVM(x)$ due to the fact that, as pointed out above, the annual optimal harvest is similar in both cases, $AQH(x) \approx AMH(x)$ (see Figure 3).

We can also see in Figure 6 that both $AVQ(x)$ and $AVM(x)$ are greater than $AV(x)$. The lower annual costs incurred in optimal harvesting if the stock is quarterly or monthly harvested, as described above, could be an explanation for this counterintuitive result for stock values $x_t \leq x'$. However, based on this argument, the opposite should be the case,

$$AVQ(x) \approx AVM(x) < AV(x), \text{ for stock values } x_t \geq x', \text{ which is not the case (see Figure 6).}$$

This means that, besides the costs, other factors, like the AGR and discounting, influence the value of the problem. In particular, the AGR, as described above, is a kind of compound interest where the effect of a smaller Δt is to increase the growth rate of the species over a unit of time which in turn implies, as described above, both higher annual harvest and higher SS if the stock is seasonally harvested (quarterly and/or monthly). This effect is similar to that obtained in the economic value of the problem if it is more frequently discounted by using $\delta^{\Delta t}$, as described in (26), which implies higher value.

Discussion and conclusions

Knowledge of the relation between CM and DM is crucial in order to avoid biologically and economically meaningless models that can lead to erroneous (suboptimal) policy advice, with the consequent uncertainty regarding the appropriate bioeconomic model which should be used to ensure long term sustainability. DM present biological and economic weakness by not

properly reflecting the natural behavior of fish stocks at high population sizes, and by underestimating the harvest costs, respectively. Errors in mathematical modeling of the population dynamics of fish stocks are frequently found in CM by using as a differential equation a growth function estimated in discrete-time.

In this paper we have developed a DCM which allows us both to overcome the biological and economic weakness of DM, and to properly estimate the population dynamics of fish stocks in a continuous-time setting.

Actual fisheries are managed on an annual basis where the collection and management of the annual biological and fisheries data are used by management agencies, like ICES, to provide annual advice regarding the stock status, reference points, and total allowable catches (TACs). However, the main stakeholders, fish stocks and fishers, do not show an uniform behavior in real world fisheries in which there are strong seasonal variations over the year in biological and economic parameters such as growth, prices, and costs of harvesting which implies seasonal variations in harvesting. An example of this is the NEAC fishery where 80% of the NEAC is harvested during the winter in an area where the stock has gathered and migrated from the Barents Sea to spam. This stock dynamic behavior gives rise to higher catchability, higher prices and lower costs which implies higher concentration of fishing effort (seasonal harvesting) where catch per unit effort (CPUE) of stern trawlers is reduced by 70% from the high to the low season. This pattern is even more pronounced for the coastal fleet (Hermansen and Dreyer, 2010). Both DM and CM are not able to cope with the complex phenomenon of seasonality in fisheries.

In this paper we have shown that the DCM is able to deal with seasonal fisheries. In the case of the NEAC fishery, we have shown that seasonal harvesting is a win-win optimal solution by achieving higher annual harvest, higher steady state equilibrium, and higher economic value. These results can be explained by the combined effects of AGR, decrease of costs of

harvesting, and more frequent discounting which take place if the stock is seasonally harvested (quarterly or monthly).

The DCM developed in this paper has an enormous range of different application possibilities in the optimal management of seasonal fisheries by considering both strong seasonal variations over the year in biological and economic parameters, and seasonal regulatory measures. In the case of the NEAC, the optimal seasonal harvesting pattern could be obtained by the DCM by considering that the stock is quarterly harvested. In this seasonal framework, biological and economic parameters can be different between quarters by reflecting the strong seasonal variations over the year observed in this fishery.

The DCM also allows us to analyze seasonal regulatory measures in order to avoid the collapse of the species and to reduce negative socio-economic impacts of seasonality (e.g. discontinuity in employment). In this sense, seasonal moratoriums and/or seasonal quotas could be analyzed. In the case of the NEAC, the rural community quotas introduced by the Norwegian fisheries authorities in order to reserve quota for fishing in the low season (Hermansen and Dreyer, 2010) could also be analyzed by the DCM.

The above could be carried out by an appropriate extension of the state space of the DCM which gives rise to a non-autonomous problem with periodic solutions. In contrast to the optimal steady state equilibrium obtained in discrete time bioeconomic models on annual basis, a wide variety of optimal policy dynamics, like cyclical harvesting, can be obtained in seasonal fisheries.

Finally, we must stress the urgent need for seasonal data and, consequently, for additional research in stock assessments, to better understand the seasonal population dynamics of fish stocks in order to implement seasonality in real world fisheries by ensuring long term sustainability of fish stocks.

Research in these directions is in progress.

Notes

1. In the population dynamics described in (14), harvesting occurs at the end of each period (after growth). However, harvest may occur at the end or start of each period, without affecting the qualitative nature of the model (Bjorndal and Munro, 2012).
2. Further numerical experiments reveal that the solution obtained by the DCM with a standard logistic natural growth function (SLG), as defined in (14) for $\alpha = 1$, follows the same pattern as the estimated modified logistic natural growth function, as defined in (14) for $\alpha = 2$ (see Figure 1). However, in the case of SLG, the drastic population decline occurs at higher stock values due to the absence of depensatory population dynamics at low stock values (convex growth function at low stock levels).
3. One could argue that not only high stock values will be hard to achieve in commercially valuable fish stocks but also that it is inherently impossible to observe stock values greater than the carrying capacity K . However, NEAC is an economically protected stock due to a downward sloping demand (market power), as defined in (12), which implies that the resource is preserved at a high SS x_{RK}^* (see Figure 2). Moreover, there is a high level of uncertainty in K estimates not only in models that apply to only single stocks (Hilborn and Walters, 1992), but also in multispecies models. An example of this last case can be found in Link and Tol (2006) where reductions of up to 50% of the NEAC carrying capacity are analyzed.
4. Further numerical experiments reveal that the solution obtained by the DCM with a continuous cost function, as defined in (16), converges to an optimal steady-state equilibrium which is similar to x_{RK}^* (see Figure 2) but with different transition phase.
5. It should be noted in (25) that the DCM takes into account the change in the stock during seasonal harvesting by avoiding the underestimation of the harvest costs.

Appendix

Taking as a starting point the differential equation $\dot{x}(t) = F_C(x)$, as defined in (2), where x represents the fish stock biomass, and $F_C(x)$ the natural growth function of the species in a continuous-time setting, a brief description of the fourth order Runge-Kutta algorithm (RKA) is as follows (see, e.g., Press et al., 2007, for further details):

For each initial stock value $x = x_t \in X$, where X is the discrete-time state-space, the temporal interval $[t, t + \Delta t]$ is partitioned in M subintervals (number of iterations) where M should be high enough to ensure enough accuracy. Then, the following formula of the RKA is evaluated in each iteration

$$\begin{aligned}
 k_1 &= hF_C(x) \\
 k_2 &= hF_C(x + \frac{k_1}{2}) \\
 k_3 &= hF_C(x + \frac{k_2}{2}) \\
 k_4 &= hF_C(x + k_3) \\
 x &= x + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),
 \end{aligned} \tag{2}$$

where $h = \frac{\Delta t}{M}$ is the step size.

Thus, for each stock value at period t , x_t , the RKA computes an approximate solution for the stock value at period $t + \Delta t$, $x_{t+\Delta t}$, that is

$$x_{t+\Delta t} = f_{RK}(x_t) = x + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

where $f_{RK}(x_t)$ is the discrete-time approximation of the continuous growth function $F_C(x)$, as defined in (2).

References

- Agnarsson, S., Arnason, R., Johannsdottir, K., Ravn-Jonsen, L., Sandal, L.K., Steinsham, S.I., and Vestergaard, N., 2008. Multispecies and Stochastic issues: Comparative Evaluation of the Fisheries Policies in Denmark, Iceland and Norway, TemaNord 2008:540, Nordic Council of Ministers, Copenhagen.
- Anderson, L.G., and Seijo, J.C., 2010. Bioeconomics of Fisheries Management, Wiley-Blackwell, Iowa.
- Ardito, A., Ricciardi, P., and Schiaffino, A., 1993. The effects of seasonal fluctuations on an open access fishery problem. *J. Math. Biol.* 31, 879-890.
- Arnason, R., Sandal, L.K., Steinsham, S.I., and Vestergaard, N., 2004. Optimal Feedback Controls: Comparative Evaluation of the Cod Fisheries in Denmark, Iceland, and Norway. *Amer. J. Agr. Econ.* 86 (2), 531-542.
- Bjørndal, T., and Munro, G.R., 2012. *The Economics and Management of World Fisheries*, Oxford University Press, Oxford.
- Clark, C.W., 2010. *Mathematical Bioeconomics: The Mathematics of Conservation*, John Wiley & Sons, New Jersey.
- Delahay, R.J., Smith, G.C., and Hutchings, M.R., 2009. *Management of Disease in Wild Mammals*, Springer, Tokio.
- Flaaten, O., 1983. The Optimal Harvesting of a Natural Resource with Seasonal Growth. *Can. J. Econ.* 16, 447-462.
- Hermansen, Ø., and Dreyer, B., 2010. Challenging spatial and seasonal distribution of fish landings-The experiences from rural community quotas in Norway. *Mar. Policy* 34, 567-574.
- Hilborn, R., and Walters, C.J., 1992. *Quantitative Fisheries Stock Assessment: Choice, Dynamics, and Uncertainty*, Chapman and Hall, New York.

Kennedy, J., and Hannesson, R., 2006. Within-Season Rents: Maximised or Dissipated in an Open-Access Fishery? *Mar. Resour. Econ.* 21, 251-267.

Link, P.M., and Tol, R.S.J., 2006. Economic impacts of changes in the population dynamics of fish on the fisheries of the Barents Sea. *ICES J. Mar. Sci.* 63, 611-625.

Maroto, J.M., Morán, M., Sandal, L.K., and Steinshamn, S.I., 2012. Potential Collapse in Fisheries with Increasing Returns and Stock-dependent Costs. *Mar. Resour. Econ.* 27, 43-63.

Maroto, J.M., and Morán, M., 2014. Detecting the presence of depensation in collapsed fisheries: The case of the Northern cod stock. *Ecol. Econ.* 97, 101-109.

Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.P., 2007. *Numerical Recipes: The Art of Scientific Computing*, Cambridge University Press, New York.

Sandal, L.K., and Steinshamn, S.I., 2001. A Simplified Approach to Optimal Resource Management. *Nat. Resour. Model.* 14, 419-432.

Stokey, N.L., Lucas, R., and Prescott, E., 1989. *Recursive Methods in Economic Dynamics*, Harvard University Press, Cambridge.

Figures

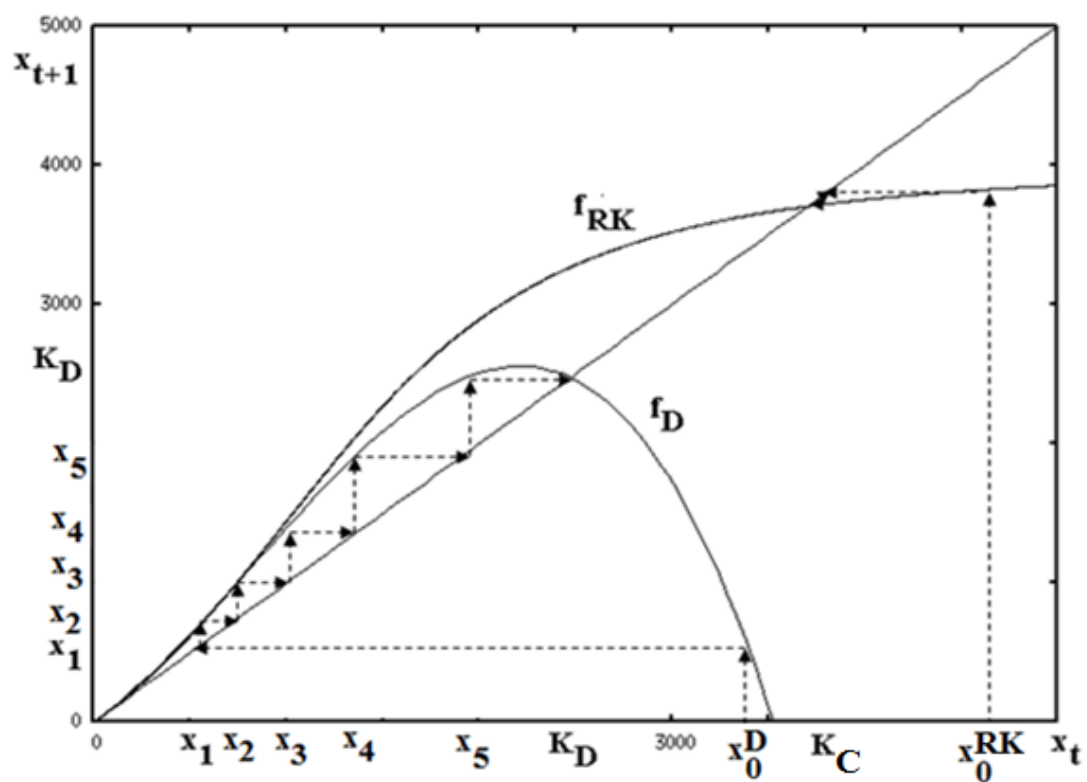


Figure 1.

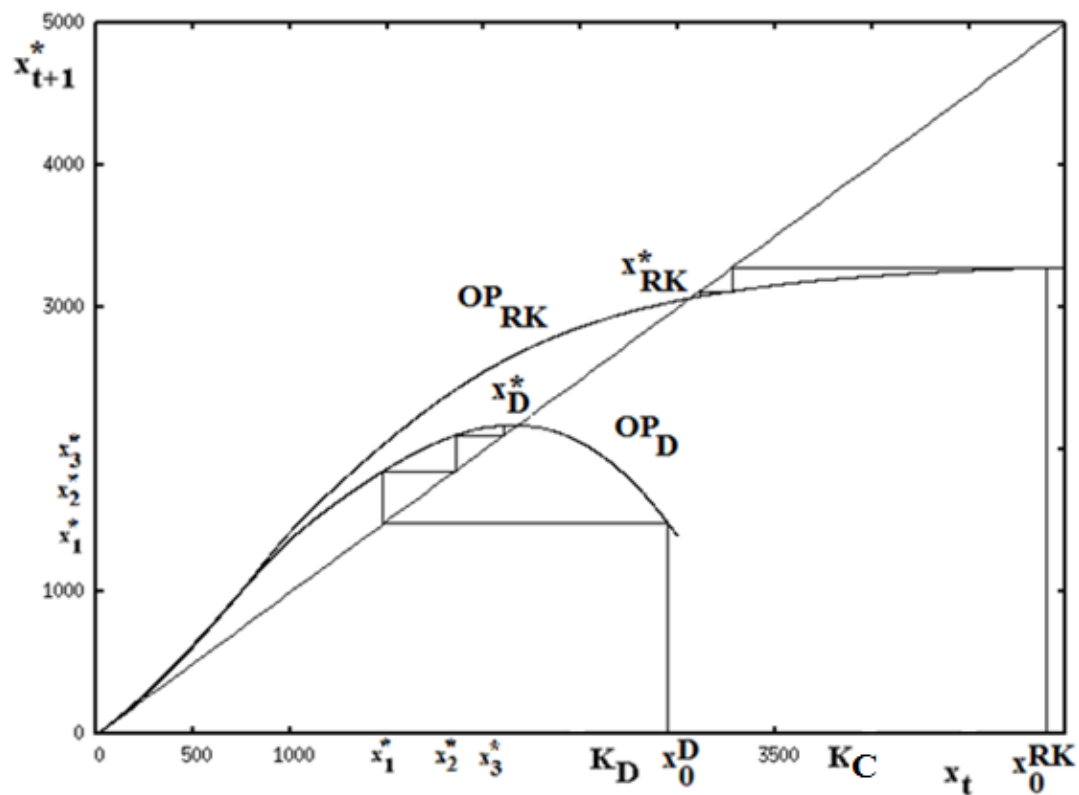


Figure 2.

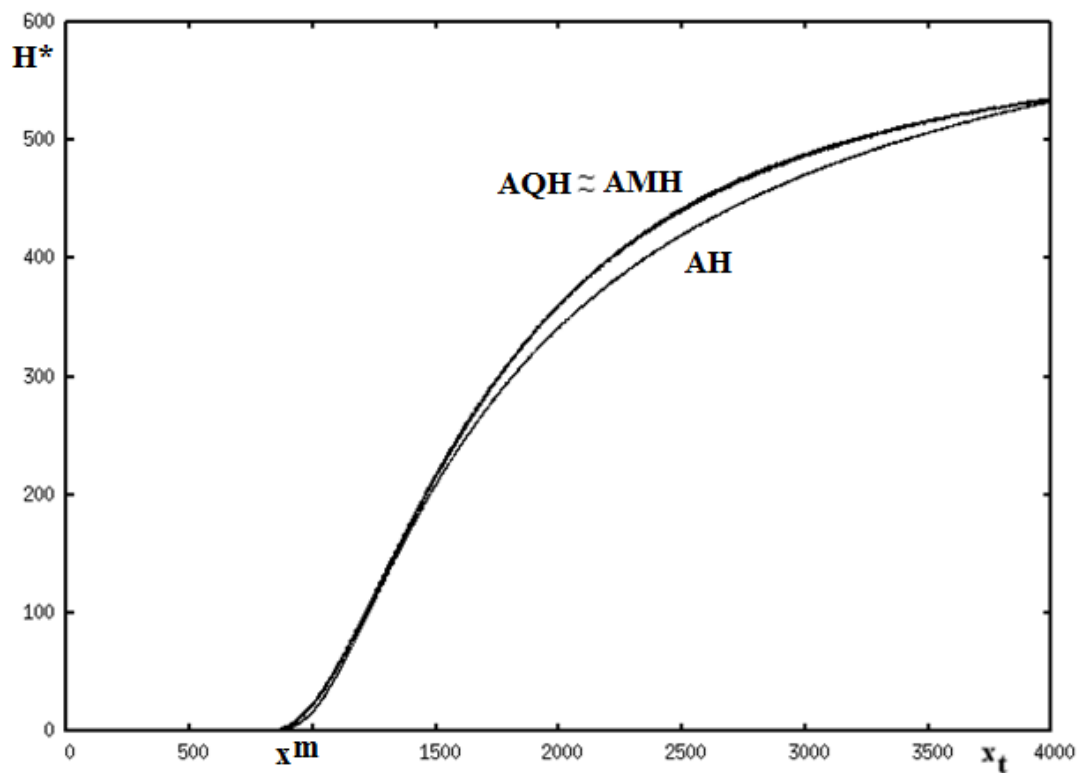


Figure 3.

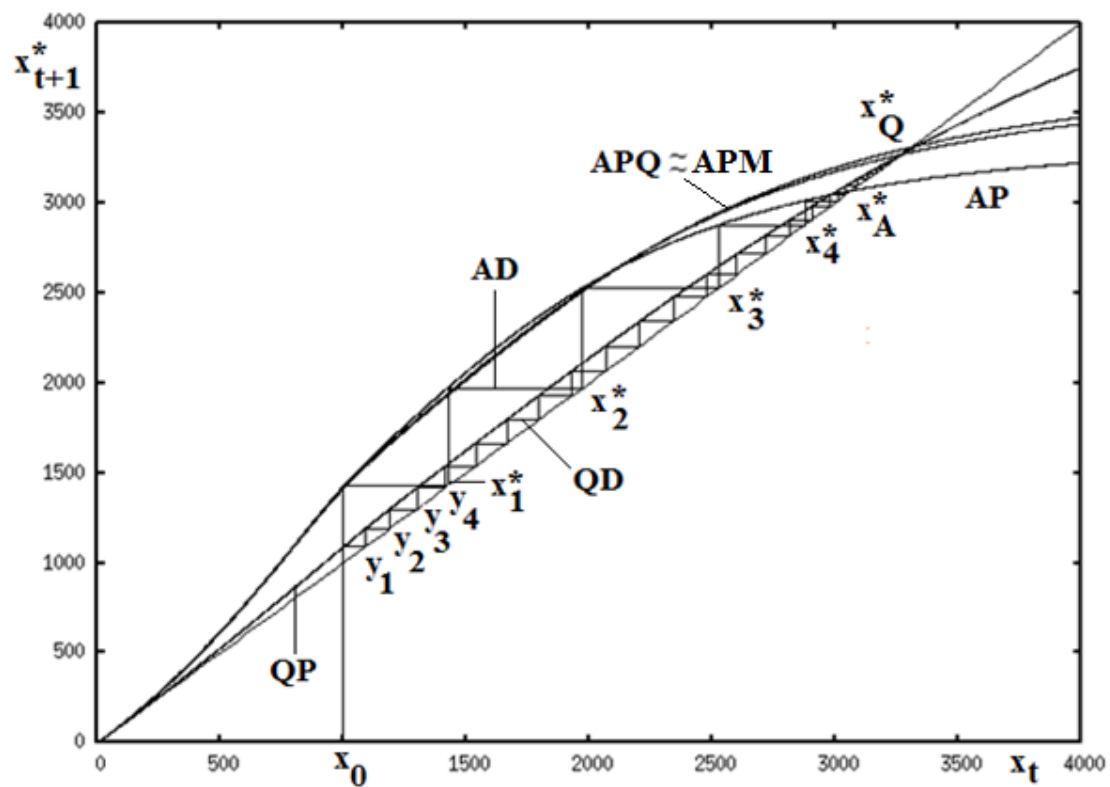


Figure 4.

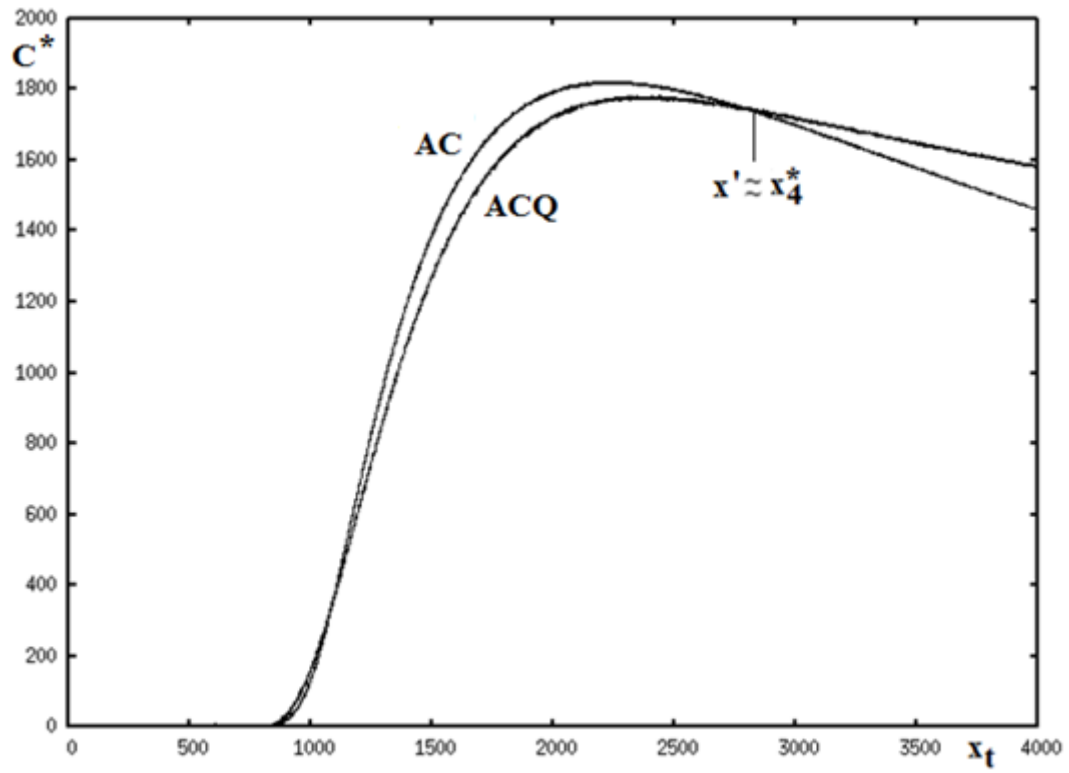


Figure 5.

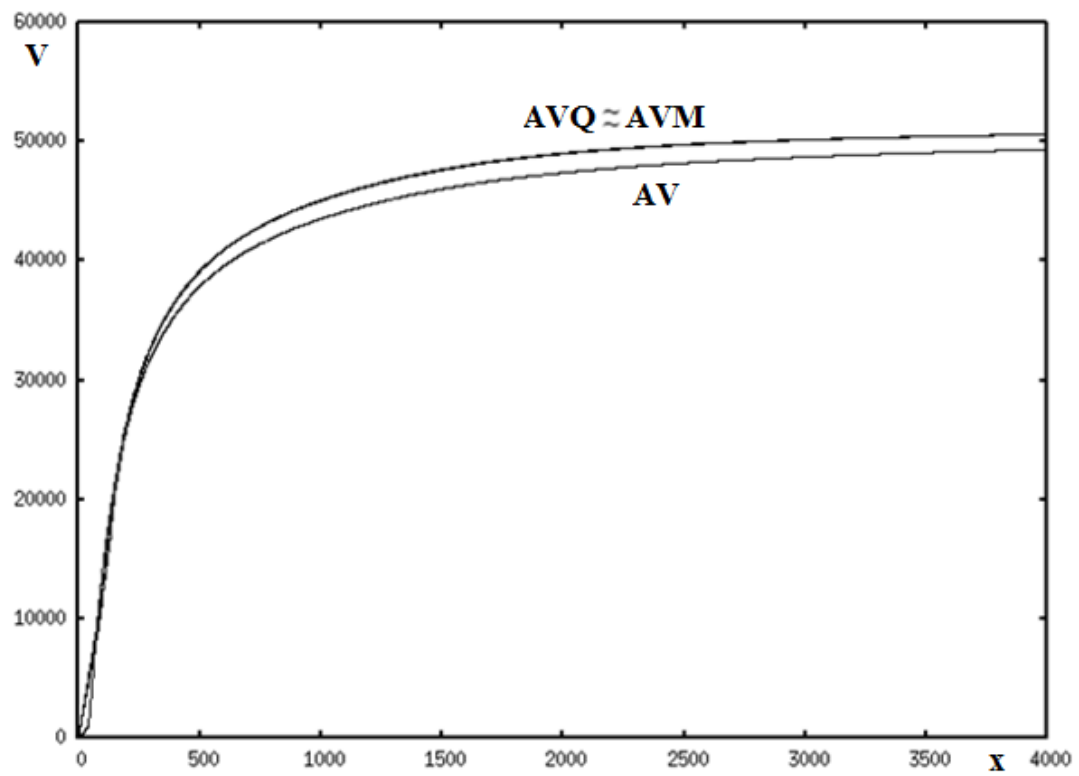


Figure 6.

Figure captions

Figure 1. Growth function $f_D(x_t)$ estimated by Agnarsson et al. (2008), discrete-time approximation $f_{RK}(x_t)$ obtained by the DCM for $\Delta t = 1$, and the 45 degree line.

Figure 2. Optimal policy function and optimal policy dynamics for both the standard DM and the DCM for $\Delta t = 1$.

Figure 3. Annual optimal harvest if the stock is: annually harvested, $AH(x_t)$, quarterly harvested, $AQH(x_t)$, and monthly harvested $AMH(x_t)$.

Figure 4. Optimal policy function and optimal policy dynamics for different values of Δt .

Figure 5. Annual costs incurred in optimal harvesting if the stock is annually harvested, $AC(x_t)$, and quarterly harvested, $ACQ(x_t)$.

Figure 6. Annual value function if the stock is: annually harvested, $AV(x)$, quarterly harvested, $AVQ(x)$, and monthly harvested, $AVM(x)$.