Proton decay in a nucleus: Effects of the nuclear surface

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In this paper, a previously published method to deal with proton decay in nuclear matter is generalized in order to account for effects coming from the nuclear surface. The treatment is applied to nucleon decay in ^{16}O and ^{56}Fe.

I. INTRODUCTION

In this paper, a method to deal with proton decay in nuclear matter, previously published,[1,2] is generalized in order to include effects arising from the nuclear surface.

Although several theoretical estimates[1-4] indicate the (eventual) proton decay width in a nucleus to be rather similar to that of an isolated proton, things are different regarding the spectra of the emitted particles (both leptons and mesons). Hence, as experiments are planned[5] for searching for proton decay mainly in complex nuclei (^{16}O, ^{56}Fe), we consider it worthwhile to make a detailed study of nuclear effects on the corresponding decay properties.

Our treatment is based upon the method of many-body Green's functions, which when applied to a spatially homogeneous system possesses several outstanding advantages. Thus, for instance, one can use rigorous analytic properties and sum up infinite sets of diagrams, which turns out to be quite relevant to dealing with the propagation of emitted pions in nuclear matter due to their strong interaction with nucleons.[1,2] However, once surface effects are accounted for, the spatial homegeneity is obviously missing and therefore the appropriate treatment becomes considerably more complicated. Fortunately enough, a great simplification is achieved provided one sticks to "first-order" surface effects. In other words, any quantity of interest, W (i.e., the total width, lepton spectrum, etc.), is expanded as

$$W \simeq W_V + W_S\left(\frac{S}{V}\right) , \qquad (1.1)$$

where W_V represents the corresponding quantity for a homogeneous system while the second term in the right-hand side of (1.1) yields the first-order surface correction. Notice that such a term depends on the size of the surface but it does not on the particular shape of the latter. It goes without saying that curvature effects have been omitted in (1.1). Similar expansions appear in different physical problems (see, for instance, Ref. 6). Now if curvature effects are disregarded, then we can choose a convenient surface shape; for our present purposes this turns out to be the planar one. In practice, the planar approximation will allow us to work within practically the same precision as in the case of nuclear matter, although obviously the numerical computations are now harder.

II. NUCLEAR EFFECTS ON PROTON DECAY IN A FINITE NUCLEUS

It is worth remarking that to study nuclear effects on proton (or neutron) decay in a nucleus one does not need a precise knowledge of the correct (if any) grand unification theory (GUT) or supersymmetric grand unification theory (SUSY GUT). In fact, as shown in Ref. 2, we can start from an effective coupling such as

$$V_p(x) = \bar{l}(x) F \psi(x) , \qquad (2.1)$$

where $l(x)$ is the lepton field and $\psi(x)$ is an "effective" nucleon field.[2] F is a matrix which, because of symmetry reasons, must be a linear combination of the identity and γ_5 matrices.

Equation (2.1) gives the so-called "pole term" in nucleon decay. Now in the nonrelativistic limit for the nucleon it is necessary to include terms where effective meson fields explicitly appear. In the case of scalar and pseudoscalar mesons, and omitting for simplicity the isospin dependence, the corresponding ("spectator") term is written as

$$V_s(x) = \bar{l}(x) F' \psi(x) \phi(x) , \qquad (2.2)$$

$\phi(x)$ being the meson field and F' is again a linear combination of the identity and γ_5 matrices.

For brevity, we will present here in detail only the treatment of the pole term. At the end of this paper we will somewhat comment about spectator and interference terms. Furthermore, for concreteness, we shall specialize to the case of decay into a positron. We shall compute the decay probability for a nucleus of mass number A into a positron of three-momentum $\mathbf{k}$ ($k \equiv |\mathbf{k}|$) and any hadronic state. In the nonrelativistic approximation for nucleons in the nucleus, that probability reads (cf. Ref. 2)

$$R(k) = C \int \frac{d^2\hat{\mathbf{k}}}{(2\pi)^3} \mathbf{k}^2 \, \mathrm{Im} \int d^3\mathbf{r}_1 \, d^3\mathbf{r}_2 \, e^{-i\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)} G(k^0, \mathbf{r}_2, \mathbf{r}_1) \qquad \left(\hat{\mathbf{k}} \equiv \frac{\mathbf{k}}{k}\right) , \qquad (2.3)$$

where G is the Green's function for a proton in the nuclear ground state $|A\rangle$

$$G(k^0, \mathbf{r}_1, \mathbf{r}_2) \equiv -i \int dt_2 \, e^{ik^0(t_2-t_1)} \times \langle A | T\psi(t_1;\mathbf{r}_1|\psi^\dagger(t_2;\mathbf{r}_2)|A\rangle , \qquad (2.4)$$

and C is a constant which value depends on the specific GUT being considered: If we set $F=a+b\gamma_5$ then $C=|a|^2+|b|^2$.

Again we should point out that relative nuclear effects are independent of C. Also it is worth noticing that an expression quite similar to (2.3) would be obtained for nucleon decay into any kind of antineutrinos. Even for decay into a muon, one does not expect any appreciable variation from (2.3) since the emitted muons would be very relativistic for practically all the decay spectrum.

On the other hand, nuclear effects on exclusive mesonic field states certainly depend on the considered state. Thus we have already commented about the importance of such effects on pionic channels. Also they should be quite significant for decays into ρ and ω mesons (notice, however, that their corresponding branching ratios are small according to almost all the models). Now the situation would be different for decays into K^0 and K^+ mesons which, although having a small branching ratio in the minimal SU(5) GUT, are rather important in certain supersymmetric models.[7] Because K^0,K^+ nucleon are "exotic" channels, these mesons should be little effected by the nuclear medium and hence small nuclear effects are expected on those particular channels.

Consequently, we will restrict our study to decays into pions, which are dominant not only in the minimal SU(5) GUT (indeed, already practically ruled out) but also in many others, including some supersymmetric (supergravity) models.[8]

As already said, we will use the "planar" approximation to compute the Green's function defined in (2.4). Setting the z axis normal to the surface, the system becomes homogeneous in both x and y directions, then being useful to introduce their corresponding conjugate momenta. In particular, this allows writing the decay probability per unit nuclear surface as [cf. with (2.3)]

$$\frac{R(k)}{s}=C\int d^2\hat{\mathbf{k}}\,\mathbf{k}^2\int dz_1\,dz_2\cos[k_z(z_2-z_1)]\times\mathrm{Im}G(k^0,\bar{\mathrm{k}},z_2,z_1)\ , \tag{2.5}$$

where $G(k^0,\bar{\mathrm{k}},z_2,z_1)$ is defined in a mixed representation, i.e., bimomentum $\bar{\mathrm{k}}\equiv(k_x,k_y)$ and coordinate z

$$G(k^0,\bar{\mathrm{k}},z_2,z_1)=\int\frac{d^2\bar{\mathrm{k}}}{(2\pi)^2}e^{i\bar{\mathrm{k}}\cdot(\bar{\mathrm{r}}_2-\bar{\mathrm{r}}_1)}G(k^0,\bar{\mathrm{r}}_2,\bar{\mathrm{r}}_1)\ , \tag{2.6}$$

where $\bar{\mathrm{r}}_i\equiv(x_i,y_i)$.

Consequently, the only remaining spatial dependence is on coordinate z. This simplifies the expressions appearing in the computation of G, which now look simpler than those arising in a purely spatial representation, especially concerning their spin structure (we deal with zero-spin nuclei). Even so, the remaining spatial dependence still prevents us from explicitly summing up infinite sets of diagrams. Moreover, it makes the corresponding numerical computations extraordinarily difficult, because some z-dependent terms can have fast oscillations.

To solve those problems we will introduce a complete set of functions of the z coordinate (for instance, a basis of plane waves with appropriate periodic conditions) and write any function $f(z_1,z_2)$ as

$$f(z_1,z_2)=\sum_{\alpha\beta}f_{\alpha\beta}\phi_\alpha(z_1)\phi^*_\beta(z_2) \tag{2.7}$$

$$\left[f_{\alpha\beta}\equiv\int dz_1dz_2\phi^*_\alpha(z_1)\phi_\beta(z_2)f(z_1,z_2)\right]\ .$$

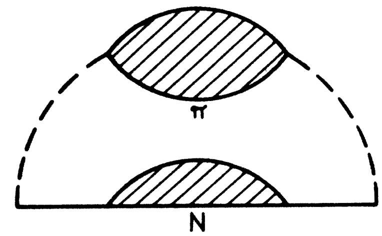

FIG. 1. Type of diagrams considered in the computation of the imaginary part of the proton Green's function (see text).

By using such a representation, it is possible to completely eliminate all the spatial dependence and formally sum up an infinite set of diagrams.

Now our main task is computing the imaginary part of the Green's function or, equivalently, the proper nucleon self-energy in the nucleus. Because of some general properties of many-body G functions, it turns out that a detailed knowledge of the real part of this self-energy is not necessary (see also Ref. 2). Hence, we have considered the significant contributions to the real part which come from the mean nuclear potential and from trivial off-shell effects.[1]

As for the imaginary part we have considered the family of diagrams generically shown in Fig. 1, wherein the self-energy insertion in the nucleon propagator accounts for the effect of the mean nuclear potential. Also, it is supposed that an insertion of the right polarization is made in the pion propagator. This allows using the analyticity properties coming from the Lehmann representation which greatly simplifies the treatment. Concretely, we have computed the N- and Δ-hole diagrams renormalized by the nuclear mean potential (a Saxon-Wood type of potential has been used[9]). Moreover, the effects of the finite Δ width have been also taken into account.

III. RESULTS AND CONCLUSIONS

Our results are expressed in the way given by (1.1). By taking the ratio S/V to refer to a spherical nucleus of mass number A, we compute the normalized positron spectrum r defined as

$$r(k)=R(k)/(A\Gamma)\ , \tag{3.1}$$

where $R(k)$ is that previously introduced and Γ is the decay width for an isolated nucleon $\Gamma=\frac{1}{2}(\Gamma_p+\Gamma_n)$ (the meaning of Γ_n is obvious). Consequently, we will write

$$r(k)=r_V(k)+r_S(k)A^{-1/3}\ . \tag{3.2}$$

Notice that function $r_V(k)$ is that corresponding to nuclear matter.[2] In Fig. 2, we plot $r(k)$ for $A=16$ (O) and 56 (Fe). The limit $A\to\infty$ (nuclear matter) is also shown. ($NN\pi$ and $N\Delta\pi$ vertices have been taken from the cloudy bag model, see Ref. 2.) Notice that nuclear effects are certainly significant in what concerns the positron spectrum. Also the effects coming from the nuclear surface and the finite Δ width are not negligible (cf. Ref. 2). Concerning the total decay width we obtain, respectively, 1.08, 1.15, and 1.25 times the decay width in free space.

In a similar way as in Ref. 2, we can make an estimation

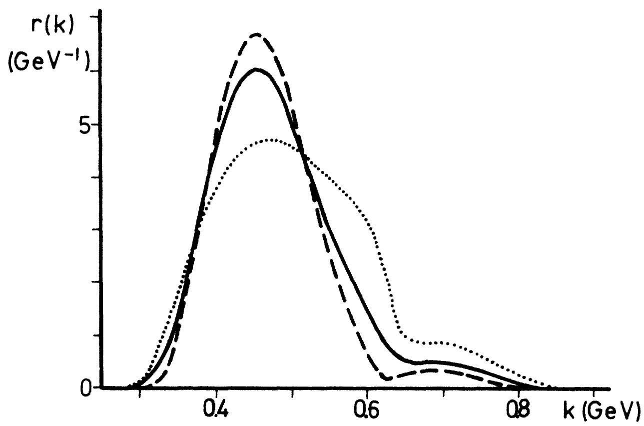

FIG. 2. Normalized lepton spectrum for ^{16}O (dashed curve), ^{56}Fe (solid curve), and nuclear matter (dotted curve).

of the characteristics of the final hadronic states in spite of having performed an inclusive computation (i.e., a sum over all hadronic final states). This can be achieved by studying the pion propagator in the nucleus.[2] To be concrete, let us consider the ^{56}Fe case. The hadronic spectrum can be divided into three different regions (recall that only the decay into a pion is considered). The first one, correlated with positron energies of $\sim 0.43 \pm 0.10$ GeV, corresponds to a pion traversing the nucleus almost freely. Then, one expects the pion and positron to be emitted practically back to back from the nucleus for positrons of the above-mentioned energy, which constitute about 60% of the whole energy spectrum. A second part corresponding to a positron energy of $\sim 0.55 \pm 0.10$ GeV is related to pions quickly absorbed after being emitted and which yield nuclear Δ excitations. The Δ decays thereupon by emitting a pion which is no longer back to back with the primary positron. This kind of process has a relative probability of $\sim 30\%$. Finally, with a probability of $\sim 10\%$, positrons of $\sim 0.75 \pm 0.10$ GeV are emitted together with fast nucleons ejected from the nucleus without meson creation (cf. Ref. 2).

Very similar results, except for some insignificant details, are obtained in the case of spectator and interference terms as also happened in the case of nuclear matter.[2]

Finally, we would like to comment about the planar approximation introduced here. It could be asked why a more conventional approach such as, for instance, an expansion in spherical harmonic-oscillator eigenfunctions has not been used. Obviously, the latter approach would have the advantage of directly dealing with curvature effects; however, it presents very serious inconveniences, the principal one being that harmonic-oscillator functions are appropriate for treating nuclear bound states, while in the present case highly excited states do appear which can have momenta of the order of the nucleon mass. This fact, together with some technical complications arising from the large number of partial waves involved, makes such an expansion rather useless. On the other hand, our method has the advantage that, to deal with surface effects, no further approximation besides the planar one must be introduced, with the obvious exception of those made in numerical computation which are easily controllable.[10]

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