Nature of the axial-vector mesons from their N_c behavior within the chiral unitary approach

L. S. Geng, E. Oset

Departamento de Física Teórica e IFIC, Centro Mixto Universidad de Valencia-CSIC

Institutos de Investigación de Paterna, Apdo 22085, 46071, Valencia Spain

J. R. Peláez

Departamento de Física Teórica II, Universidad Complutense de Madrid, 28040 Madrid, Spain

L. Roca

Departamento de Física, Universidad de Murcia, E-30071, Murcia, Spain

Abstract

By describing within the chiral unitary approach the s-wave interaction of the vector meson nonet with the octet of pseudoscalar Goldstone Bosons, we find that the main component of the axial vector mesons $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, $a_1(1260)$, $f_1(1285)$ and the two states associated to the $K_1(1270)$ does not follow the QCD dependence on the number of colors for ordinary $q\bar{q}$ mesons.

Even though QCD is well established as the theory of strong interactions, the fact that it becomes non perturbative in the hadronic regime makes it very complicated to use within the realm of hadron spectroscopy. Many states are easily accommodated within lattice calculations or QCD inspired quark models, but such calculations in terms of fundamental degrees of freedom, i.e., quarks and gluons, are usually troubled with chiral symmetry breaking, physical masses of quarks or Goldstone Bosons and their decay widths. In contrast, many models based on hadronic degrees of freedom cannot extract the quark and gluon content of hadrons without assumptions hard to relate or justify within QCD, and frequently have the composition already built in a priori. Furthermore, all these approaches are complicated by the possible mixture of states with a different nature.

Most of these caveats can be overcome by studying [1, 2] ([3] for a review) the dependence on the number of colors, N_c, of the poles associated to resonances that appear in the unitarized meson-meson scattering amplitudes obtained within a Chiral Effective Theory. See also refs. [4, 5, 6] for works in the meson-baryon sector.

The relevance of the $1/N_c$ expansion [7] is that it provides an analytic approach to QCD in the whole energy region and a *clear identification of $q\bar{q}$ states*, that become bound as $N_c \to \infty$ and whose masses scale as $O(1)$ and their widths as $O(1/N_c)$, without the need for the definition of valence quarks, or QCD inspired potentials. Other hadronic states may show different behavior [8].

The use of an Effective Theory ensures that all degrees of freedom below a certain scale are included consistently with the QCD symmetries. In this respect, chiral symmetry becomes essential, since it is possible to identify the pions, kaons and the eta with the eight Goldstone Bosons (GB) associated with the spontaneous chiral symmetry breaking that is known to exist in QCD, which are separated by a mass gap of the order of $4\pi f_\pi \simeq 1.2$ GeV from all other hadrons. Actually, since light quarks have a tiny mass that breaks chiral symmetry explicitly, the lightest pseudoscalars have a small mass and the mass gap is slightly reduced. Usually $\Lambda \simeq 1$ GeV is taken as the cutoff of the QCD Effective Theory, such that p/Λ, where p is a typical momentum in the theory, is smaller than one. Since these pseudoscalars are GB, not only their self-interactions, but the interaction terms with other fields allowed by chiral symmetry are much reduced. When a well defined power counting exists, it is possible to make a derivative and mass expansion in the Effective Lagrangian and calculate loop corrections whose divergences are absorbed in higher order parameters, that encode the information on Physics beyond the cutoff scale. This is the case of Chiral Perturbation Theory (ChPT) [9]. From QCD it is not possible to calculate the Effective Lagrangian parameters, but at least it is possible to know their dependence on certain QCD parameters. One example of relevance for this work is the leading N_c scaling of the parameters appearing in the effective Lagrangian.

Over the last few years, it has been shown that it is possible to generate heavier resonances not initially present in the Chiral Effective Lagrangian by imposing unitarity on two body scattering amplitudes [10, 11, 12, 13, 14, 3, 15]. The advantage of this approach for spectroscopy is that these resonances are generated from first principles, namely chiral symmetry and unitarity, without any assumption about their existence or their spectroscopic nature. Finally, since the leading QCD N_c scaling of the Effective parameters is known, it is possible to obtain the leading N_c scaling of the mass and width of each generated resonance and check whether or not their leading N_c behavior corresponds to that predicted by QCD for a $q\bar{q}$ state. Note that we are interested in the large N_c expansion close to the physical value of $N_c = 3$, but not in the $N_c \to \infty$ limit. This limit is interesting on its own, but the phenomenology of many hadrons is not well described in the limit or for too large values of N_c. For instance, baryons become infinitely heavy and mesons become strictly bound, like the rho meson that becomes a bound state decoupled from pions as $N_c \to \infty$, which has little to do with the familiar physical rho behavior. Nevertheless, the rho and its contributions to the effective Lagrangian are very well described by the $1/N_c$ expansion evaluated at $N_c = 3$ [1]. Similarly, different components within a mixed state could change their proportions for very large N_c. Since we are interested in the nature of physical states, without altering radically their composition, we will extract their leading $1/N_c$ behavior by studying their N_c dependence not very far from $N_c = 3$.

Paradigmatic examples are the lightest scalar and vector octets, generated with the coupled channel Inverse Amplitude Method (IAM) using one-loop ChPT meson-meson amplitudes [12, 3], which describe data up to $\sqrt{s} \simeq 1.2\,\text{GeV}$. Remarkably, light vectors follow nicely a $q\bar{q}$ behavior, whereas the N_c behavior of the light scalars is at odds with a predominant $q\bar{q}$ nature [1]. This result has been confirmed [2] at two loops for the ρ and $f_0(600)$ mesons, even getting a hint for the latter of a subdominant $q\bar{q}$ component, rising around 1 GeV, most likely due to mixing between light non-$q\bar{q}$ and heavier $q\bar{q}$ states. Scalar states with non-$q\bar{q}$ behavior can be obtained, for instance, from different combination of tetraquarks, [8], including, of course, the "molecular", or $\pi\pi$ resonance, rearrangement.

In this work we study the QCD leading N_c behavior of axial vector mesons. In this case there is no Effective Theory available with higher order terms with a clear chiral counting and N_c behavior. Thus, we are working under the assumption, already shown to work remarkably well [16, 17, 18], that they are predominantly dynamically generated states, and can be generated by a coupled channel unitarization of an Effective chiral Lagrangian describing the interaction between light vectors (V) and the pseudo GB octet (P). This meson-meson state interpretation comes out naturally as long as the cutoff in our model has a very natural size in terms of f_π and meson physics, although some tetraquark arrangements have similar N_c behavior and cannot be excluded.

In brief, the formalism of [17] uses the standard construction of nonlinear Effective Lagrangians [19] to build a chiral Lagrangian with the lowest number of derivatives that follows, which, properly normalized reads [20]:

$$\mathcal{L} = -\frac{1}{4}\{(\nabla_\mu V_\nu - \nabla_\nu V_\mu)(\nabla^\mu V^\nu - \nabla^\mu V^\nu)\}, \tag{1}$$

where $\nabla_\mu V_\nu = \partial_\mu V_\nu + [\Gamma_\mu, V_\nu]$ is the covariant derivative SU(3) matrix with the SU(3) connection defined as $\Gamma_\mu = (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger)/2$, $u = \exp(P/\sqrt{2} f_\pi)$ and

$$P \equiv \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} \end{pmatrix}, \tag{2}$$

$$V_\mu \equiv \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^0 & \phi \end{pmatrix}_\mu .$$

Note that ideal $\phi - \omega$ mixing has been assumed and that the neglected η' effects could be included at higher orders.

Expanding the Lagrangian of Eq. (1) to two vectors and two pseudoscalars one obtains the simple result of

$$\mathcal{L} = -\frac{1}{4f_\pi^2}\langle [V^\mu, \partial^\nu V_\mu][P, \partial_\nu P]\rangle, \tag{3}$$

which has been used in refs. [16, 17].

The VP $\rightarrow$ V′P′ amplitudes [17] are now easily obtained and their dynamics depends on just one parameter, the pion decay constant f_π. The relevant remark here is that QCD fixes $f_\pi \sim O(\sqrt{N_c})$ whereas the V and P masses behave as $O(1)$.

In ref.[17], the unitarization of the tree level T-matrix was carried out within a coupled channel Bethe-Salpeter formalism for the two meson states:

$$T = -[1 + V\hat{G}]^{-1} V\, \vec{\epsilon} \cdot \vec{\epsilon}\,', \tag{4}$$

with $\vec{\epsilon}, \vec{\epsilon}\,'$ the V and V' polarization vectors, and V of Eq. (4) is the s-wave projected scattering amplitude for the vector mesons with pseudoscalar mesons obtained from Eq. (1):

$$V_{ij}(s) = -\frac{\epsilon\cdot\epsilon'}{8f^2} C_{ij}\left[3s - (M^2+m^2+M'^2+m'^2) - \frac{1}{s}(M^2-m^2)(M'^2-m'^2)\right]. \tag{5}$$

where $M(M')$, $m(m')$ correspond to the initial(final) vector mesons and initial(final) pseudoscalar mesons respectively. The indices i and j represent the initial and final VP states respectively and the C_{ij} coefficients are given in ref. [17].

In ref. [17] a separation is made in the longitudinal and transverse parts of the amplitude and the poles are shown to appear in the transverse part, which is the one shown in Eq. (4).

As shown in ref.[17], the matrix $\hat{G}$ is diagonal with the $l-$th element $\hat{G}_l = (1+\frac{1}{3}\frac{q_l^2}{M_l^2})\, G_l$ where the term with the on-shell center of mass momentum of the intermediate states q_l amounts to a few percent, and G_l is the two-meson loop function:

$$G_l(P) = \int \frac{i\, d^4q}{(2\pi)^4}\frac{1}{(P-q)^2 - M_l^2 + i\epsilon}\frac{1}{q^2 - m_l^2 + i\epsilon} \tag{6}$$

where P is the total four-momentum, $P^2 = s$. In order to improve slightly the data description, in [17, 18] the finite widths of vector mesons were included in their propagators. This correction plays a secondary role in our context, since these vectors are firmly established $q\bar{q}$ states, and their widths behave as $1/N_c$.

This Bethe-Salpeter approach is widely used in the literature [15], and is related to other unitarization techniques as the IAM [10, 12, 3] or the N/D [14], which differ from the one described above in that they include higher orders, a left cut, or tadpole and crossed channels. These differences may be relevant at very low energies but at higher energies, where we are generating dynamically the present resonances from the lowest order chiral Lagrangians, they have been shown to be minute.

Since the G_l integral above is divergent, it has to be regularized, which was done in [17] either with a cutoff or in dimensional regularization. In an Effective Theory with a well defined counting, the dependence on the regulator could be absorbed into higher order constants, as is done in ChPT and the IAM. However, when generating resonances dynamically from the lowest order Lagrangian the regularization introduces an additional parameter in the amplitudes: either a cut-off, or a subtraction constant for the dimensional regularization case. In ref. [17] it was shown that the low lying axial-vector mesons can be easily generated with a natural cutoff $\Lambda \sim 1$ GeV.

Although the N_c behavior of the cutoff is not known from QCD, it is however clear that it cannot grow faster than the cutoff of the Effective Theory itself, which is of the order of the scale of symmetry breaking $\Lambda \simeq 4\pi f_\pi$. Otherwise we would have the absurd situation that we can extend the validity of the loop integral beyond the applicability of the theory. Therefore, a natural integral cutoff, as it is the case here, could scale as $\sqrt{N_c}$, but no faster.

Of course, we will consider the possibility that the cutoff may scale slower than $\sqrt{N_c}$, since it would be $O(1)$ if it was given by the mass of heavier $q\bar{q}$ mesons, which cannot be generated from low-energy two-meson dynamics, and therefore have been integrated out. Actually, the unitarization can generate $q\bar{q}$ resonances, like the vector nonet, but requires cutoffs at the TeV scale, utterly unnatural, or the explicit values of further parameters that encode information beyond two-body dynamics [10, 12, 3]. Of course, if Λ starts growing like $\sqrt{N_c}$, *for sufficiently large* N_c, it will reach the first $q\bar{q}$ meson mass and will behave as $O(1)$. Hence, we expect a natural behavior in between $O(\sqrt{N_c})$ and $O(1)$. This will be nicely confirmed later on, with the use of the Weinberg sum rule.

However, a priori, we do not know when the $O(1)$ behavior sets in, because the lightest heavy state might not have a $q\bar{q}$ nature and its mass might not be $O(1)$, also, the width of $q\bar{q}$ states decreases very fast as $1/N_c$ and, as N_c grows, their effect is only felt if the energy is very close to their mass.

Let us now remark that, actually, we are interested in the N_c behavior not far from $N_c = 3$, since we study the nature of the physical states, which, most likely, have a small admixture of different kinds of "bare" or "preexistent" states with different N_c behavior. Since the proportions in this admixture could change with N_c, we are not actually interested in very large N_c since the meson composition at such large N_c could be completely different from the one observed at $N_c = 3$. For instance, since $q\bar{q}$ states survive as $N_c \to \infty$, even the tiniest $q\bar{q}$ admixture may show up for a sufficiently large N_c. It is then crucial not to take $N_c \to \infty$. In other words, we are *only* studying the *leading* N_c behavior of a given pole that should be visible not far from $N_c = 3$. Hence, we keep $N_c < 20$, i.e., a variation by less than an order of magnitude.

Thus, by scaling $f_{\pi N_c} \to f_\pi \sqrt{N_c/3}$ and Λ in the amplitudes of ref.[17] we obtain the large N_c dependence of the poles associated to the different axial-vector mesons. Since N_c is not too large, we cannot guarantee that the $O(1)$ scaling regime is reached and therefore we study two cases: $\Lambda \to \Lambda\sqrt{N_c/3}$, which is the largest allowed growth, and Λ constant. The non-trivial fact that our results do not depend on that choice allows us to make a firm statement about the axial-vectors nature.

Thus, in Fig. 1 we show the N_c evolution of the masses and widths of the axial-vector resonances, normalized to their values at $N_c = 3$. We study the $a_1(1260)$, $b_1(1235)$, $h_1(1170)$, $h_1(1380)$, $f_1(1285)$ and the two $K_1(1270)$ resonances. In ref. [17] these states were found from the interaction of several coupled channels in each case and it was found that they coupled most importantly to the $\rho\pi$, $K^*\bar{K}$, $\rho\pi$, $K^*\bar{K}$, $K^*\bar{K}$ and $K^*\pi$, ρK, respectively, with different quantum numbers.

Both the masses and widths are taken from the associated pole position, using $\sqrt{s_{pole}} \simeq M - i\Gamma/2$, which is a very good approximation since all these resonances have $\Gamma << M$. For reference, we have plotted as a dash-dotted line the *leading* behavior for $q\bar{q}$ states, namely, $M_{N_c}/M_3 = 1$ and $\Gamma_{N_c}/\Gamma_3 = 1/N_c$. From the mass behavior not much can be concluded, since, although there is an increase in its value, it is of the order of $30\% \simeq 1/3$ for $N_c \approx 20$, not incompatible with a possible $q\bar{q}$ nature, either with Λ behaving as $\sqrt{N_c}$ or as a constant. This is nevertheless the order of magnitude of the mass in the case of the non $q\bar{q}$ states σ or κ scalar mesons found in [1], although in the case of the σ there was a large dispersion of the values.

However, we find that the widths *do grow* with N_c in sharp contrast with the QCD $1/N_c$ behavior for the widths of $q\bar{q}$ states. Let us remark that this happens irrespective of whether we assume that the cutoff behaves as $O(1)$ or $\sqrt{N_c}$. Moreover, the slowest growth of the widths is given for the maximum allowed growth for the natural cutoff $\Lambda \simeq \sqrt{N_c}$, which means that for the other allowed behaviors of the cutoff, that grow slower, the N_c dependence of the widths separates even more from the $q\bar{q}$ behavior. Our result is thus stable with respect to the different cutoff N_c dependences.

Of course, one could still wonder if our results require a fine tuning of the cutoff value used to describe the data at $N_c = 3$. Hence, in Fig. 2 we show the N_c dependence of the $h_1(1170)$ width using different cutoffs of natural size. Our previous arguments are again confirmed to be firm, since in all instances the width *grows* with N_c, which is once again against a dominant $q\bar{q}$ behavior, and the larger the cutoff, the faster the growth.

In all cases, we find for N_c not larger than 20 a steady increase of the width of the axial vector mesons. This is in striking difference from the $1/N_c$ scaling of the width of the $q\bar{q}$ states, followed very clearly by the ρ [1]. Nevertheless, in Fig. 1 we can see a very different behavior of the width for different states. In some cases it starts growing steadily from $N_c = 3$ while in other cases the clear growth appears at larger values of N_c. The reason can easily be traced to the different coupled channels entering the formation of each state together with the quite different coupling of the state to these coupled channels [17]. As N_c grows, the thresholds of the important channels are opened and the steady growth of the width appears. Just for the sake of comparison with the scalar mesons of [1] we effectively parameterize the curves for the $a_1(1260)$ and $h_1(1170)$ as $a + b(N_c/3)^\alpha$ with the values: $a = -1.91$, $b = 2.79$ and $\alpha = 0.70$ for $a_1(1260)$ and $\Lambda \sim 1$; $a = -0.51$, $b = 1.51$ and $\alpha = 0.59$ for $a_1(1260)$ and $\Lambda \sim \sqrt{N_c}$ (up to $N_c = 14$); $a = -4.78$, $b = 5.72$ and $\alpha = 0.83$ for $h_1(1170)$ and $\Lambda \sim 1$; $a = -2.41$, $b = 3.57$ and $\alpha = 0.69$ for $h_1(1170)$ and $\Lambda \sim \sqrt{N_c}$. We see an increase of the width slightly slower than $O(N_c)$ which was also the case for both the σ and κ in [1] (where $0.5 < \alpha < 1$). However the coefficient in front of $(N_c/3)^\alpha$ is larger than unity in our case while in [1] was around one, which indicates a faster growth of the width as a function of N_c in the present case.

On the other hand, one could also wonder about the stability of our results on subleading $1/N_c$ corrections to f_π. Actually, such corrections come partly from the $O(p^4)$ terms that renormalize the tree level decay constant f_0 within standard ChPT [9] (see also the second reference in [12] for a simplified expression). They have also been estimated within a quark model in [22]. For our purpose it is enough to observe that all these can be recast under the general form:

$$f_{\pi\,N_c}^2 = \frac{N_c}{3}\frac{f_\pi^2}{1+\epsilon/3}\,(1+\epsilon/N_c) \tag{7}$$

which ensures the correct physical value at $N_c = 3$, the correct leading behavior, but also a subleading contribution of the expected size, namely, 30%, when $\epsilon = 1$. For $\epsilon = 0$ we recover our previous calculation. Actually, this uncertainty roughly covers the subleading terms estimated in [9] and [22]. Thus, in Fig. 3 we show as an example, with the region between the two dashed lines, the effect of this uncertainty on the $h_1(1170)$ resonance N_c dependence. Similar results are found for the other generated resonances. Let us remark that the qualitative behavior is very conclusive, since, once again, both the mass and width increase with N_c, in sharp contrast with the behavior for a $q\bar{q}$ state. Note that, since the uncertainties due to the $1/N_c$ suppressed contributions to f_π are similar both for the $\Lambda \sim \sqrt{N_c}$ and $\Lambda \sim O(1)$ cases, we are not showing the latter, where the difference with a $q\bar{q}$ behavior is even more significant.

Let us comment now on a possible improvement of the Lagrangian considered in the present work. The results shown are evaluated with the kernel V, Eq. (5), of the Bethe-Salpeter equation. However, an improvement of this kernel comes from the addition of the term provided by the $SU(3)$ breaking Lagrangian [25, 26].

$$\mathcal{L} = \lambda_m \langle V_\mu V^\mu \chi_+ \rangle \tag{8}$$

where $\chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u$, with $\chi = diag(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$ and $u = \exp(iP/\sqrt{2}f)$.

The parameter λ_m is readily evaluated form the K^*-ρ mass splitting

$$\lambda_m = \frac{M_{K^*}^2 - M_\rho^2}{4(m_K^2 - m_\pi^2)} \tag{9}$$

which is of order $\mathcal{O}(N_c^0)$ in the $1/N_c$ expansion.

Expanding χ_+ up to two pseudoscalar meson fields, one obtains an interaction term for $VP \to VP$ to be added to that of Eq. (5). If one looks at the $\rho\pi$ channel the new term is proportional to m_π^2 and is negligible compared to Eq. (5). But this is not the case in channels with strangeness, where it is proportional to m_K^2. For instance, for $\rho^+ K^- \to \rho^+ K^-$ the new term can be 30% of the dominant one. Since the corrections are not negligible, a reanalysis of the results of ref. [17] with the new term would be of interest, maybe improving on the semiquantitative agreement with data found in ref. [17]. However, for the purpose of the present work, such exercise does not change the conclusions. Indeed, the new potential is proportional to $\lambda_m m^2/f^2$ and, hence, scales like the one of Eq. (5). But, as we have seen, as N_c grows, so do the masses of the axial-vectors generated, and hence the variable $s = M_A^2$, as a consequence of which the strength of the λ_m term becomes progressively smaller relative to the one of Eq. (5), used in the present work. As an example, for $N_c = 19$ and the $\rho^+ K^- \to \rho^+ K^-$ channel, the ratio of terms is smaller than 10%.

At this point one might wonder about the Weinberg sum rule [24] that using chiral symmetry imposes a strong relation between the vector and axial spectral functions and is expected to hold independently of the number of colors. We will show next that this relation is naturally satisfied within the uncertainties of our approach, and could even provide a more refined estimate of the cut-off N_c dependence[1]. Given the precision of our whole approach, the usual estimation of the sum rule considering just one pole contribution should be enough. In such case, the sum rule reads

$$F_V^2 - F_A^2 \simeq f_\pi^2, \tag{10}$$

where F_V is the coupling of the vector resonance to the vector current and F_A is the coupling of the axial-vector resonance to the axial current. We will show that our approach can effectively generate an axial coupling constant F_A and that the cutoff N_c behavior needed to satisfy exactly the one-pole estimation of the sum rule lies right in between the two extreme behaviors we have been considering so far.

The definition and normalization of the F_A and F_V couplings is similar as in the familiar case when explicit A_μ fields are considered with the following interaction terms

$$\mathcal{L} = -\frac{F_V}{\sqrt{2}M_V} < \partial_\mu V_\nu f_+^{\mu\nu} >, \tag{11}$$

$$\mathcal{L} = -\frac{F_A}{\sqrt{2}M_A} < \partial_\mu A_\nu f_-^{\mu\nu} >, \tag{12}$$

which are given in an equivalent form in ref. [23] with antisymmetric tensors for the vector meson fields, with

$$\begin{aligned} f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u, \\ F_L^{\mu\nu} &= \partial^\mu l^\nu - \partial^\nu l^\mu - i[l^\mu, l^\nu], \\ F_R^{\mu\nu} &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu], \\ r^\mu &= v^\mu + a^\mu, \qquad l^\mu = v^\mu - a^\mu. \end{aligned} \tag{13}$$

Expanding $f_\pm^{\mu\nu}$ up to one pseudoscalar meson field, the former Lagrangians give us the coupling of the explicit axial-vector resonance, A_μ, to an axial-vector current, a_μ, through F_A and the coupling of VP to the same current via F_V.

[1]We are indebted to the referee for this suggestion.

Of course, in our scheme the axial-vector resonances are dynamically generated, and do not appear explicitly in the Lagrangian, but the resonance is still linked to the axial-vector current via a VP loop, as depicted in Fig. 4(b). For the particular case of the $a_1(1260)$ resonance, generated basically through $\rho\pi$ loops, which is largely the dominant channel, from Fig. 4(b) we read

$$F_A = \frac{F_V}{\sqrt{2}f}\,\frac{M_A}{M_V}\,\frac{s-m_\pi^2+m_\rho^2}{s}\,G_{\rho\pi}(s)\,g_{\rho\pi}, \tag{14}$$

where $G_{\rho\pi}(s)$ is the loop function of $\rho\pi$ (see Eq. (6)) appearing in Fig. 4(b) and $g_{\rho\pi}$ the coupling constant of the dynamically generated $a_1(1260)$ to $\rho\pi$ in isospin $I=1$, which is obtained within our approach from the residues of the $\rho\pi \to \rho\pi$ amplitude at the resonance pole. In the numerical evaluation of Eq. (14), we take $M_V = M_\rho$ and $M_A = \mathrm{Re}(\sqrt{s_{pole}})$. Eq. (14) provides complex values for the F_A coupling. Thus, in order to compare with the F_A defined in Eq. (12), which has different phase, we take for F_A in the following its absolute value.

Let us recall that we have shown that, although for our calculations we have used a central value $\Lambda(N_c=3) = 1000$ MeV, we still obtain a fair description of data and the non-$q\bar{q}$ behavior within a much wider range of the cutoff. If we now impose the one-pole approximation of the Weinberg sum rule, to get exactly $f_\pi = 92$ MeV in Eq. (10), using $s = s_{pole}$ in Eq. (14) we find $\Lambda(N_c=3) = 785$ MeV. This simple one-pole estimate shows that within our approach it is fairly simple to accommodate the Weinberg sum rule for physical $N_c = 3$. Note that, given the precision of our approach, we can neglect other possible resonance and continuum contributions[2].

But now we can impose the one-pole approximation of the Weinberg sum rule for larger N_c, thus estimating the cutoff N_c dependence, which, for the sake of simplicity, we will parameterize as $\Lambda \sim O(N_c^\delta)$. Thus, in Fig. 5 we plot the resulting N_c dependence of F_A from Eq. (14) for different δ values. Note that for $\delta \sim 0.35$ we find the $F_A \simeq F_A(N_c=3)\sqrt{N_c/3}$ behavior. Of course, this value of δ is just an estimate, since we are using the one-pole approximation of the Weinberg sum rule, and we have not allowed for subleading $1/N_c$ uncertainties in F_V, Λ or f_π. However, it is remarkable and reassuring to note that it comes out naturally right in-between the two extreme cases of cutoff behavior that we had already considered, namely, $\delta = 0$ and $\delta = 1/2$. Hence, we have shown that the Weinberg sum rule can

[2]JRP thanks S. Leupold for comments about this continuum contribution.

be easily accommodated within our approach and the predominantly non-$q\bar{q}$ behavior of the axial-vector resonances.

In summary, we have studied the N_c behavior of the axial-vector mesons using the unitarized chiral approach with a phenomenological Lagrangian for vector and pseudoscalar mesons, which has been shown to generate and describe the known phenomenology of axial-vector resonances. This model does not require any fine-tuning since it has just two parameters: the pion decay constant, whose N_c dependence is known from QCD, and a cutoff of a very natural size. We have shown here that assuming a natural N_c scaling of the cutoff, either $O(1)$ if it is due to more massive mesons, or $O(N_c)$ if it is due to the chiral scale, the resulting N_c behavior of the generated resonances is not that of predominantly $q\bar{q}$ states. In particular, their widths grow as N_c increases not too far from the physical $N_c = 3$ value, contrary to the QCD N_c behavior of $q\bar{q}$ states. Of course, a smaller $q\bar{q}$ component may not be excluded, but it is not predominant. This growth is always obtained in our approach and is faster than that found for other non-predominantly $q\bar{q}$ states, like the light scalars. This suggests a rather natural interpretation of the axial vector resonances as non-$q\bar{q}$ states.

We also estimated the Weinberg sum rule that, using chiral symmetry, imposes a strong relation between the vector and axial spectral functions and should hold independently of the number of colors. For the physical $N_c = 3$ case, and using the usual one narrow resonance saturation approximation of the sum rule, we have shown that it can be accommodated within the natural uncertainties of the cutoff. If such approximation of the sum rule is also imposed for larger values of N_c, the cutoff behavior lies naturally in-between the two extreme cases considered in this work. Consequently, without the need of any fine tuning, the sum rule is found to be consistent with our predominantly non-$q\bar{q}$ behavior of axial vectors.

We should also note that the results obtained are not trivial at all if we look at them from the perspective of the findings of [5, 6] where, in the case of the meson-baryon interaction generating two $\Lambda(1405)$ states, in the large N_c limit the pole associated with the singlet becomes a bound state, with zero width while the other state fades away.

The method we have followed is remarkably simple and could be easily extended to other systems where dynamically generated states are obtained within the Chiral Unitary approach with a natural cutoff and with constants whose leading QCD N_c behavior is known. We consider that the technique we have presented could provide a method to identify resonance candidates whose spectroscopic nature may not be predominantly that of a $q\bar{q}$ state.

Acknowledgments: L. S. Geng thanks the Ministerio de Educacion y Ciencia in the Program of estancias de doctores y tecnologos extranjeros for finantial support. This work is partly supported by DGICYT contracts FPA2007-29115-E, FIS2006-03438, FPA2005-02327 and FPA2007-62777, Santander/Complutense contract PR27/05-13955-BSCH, the Fundación Séneca contract 02975/PI/05, the Generalitat Valenciana and the EU Integrated Infrastructure Initiative Hadron Physics Project under contract RII3-CT-2004-506078.

References

[1] J. R. Pelaez, Phys. Rev. Lett. **92**, 102001 (2004).

[2] J. R. Pelaez and G. Rios, Phys. Rev. Lett. **97**, 242002 (2006).

[3] J. R. Pelaez, Mod. Phys. Lett. A **19**, 2879 (2004).

[4] C. Garcia-Recio, J. Nieves and L. L. Salcedo, Phys. Rev. D **74** (2006) 036004.

[5] T. Hyodo, D. Jido and L. Roca, Phys. Rev. D **77** (2008) 056010.

[6] L. Roca, T. Hyodo and D. Jido, Nucl. Phys. A **809** (2008) 65.

[7] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974); E. Witten, Annals Phys. **128**, 363 (1980).

[8] R. L. Jaffe, Proceedings of the Intl. Symposium on Lepton and Photon Interactions at High Energies, University of Bonn (1981) [ISBN: 3-9800625-0-3] and AIP Conf. Proc. **964**, 1 (2007) [Prog. Theor. Phys. Suppl. **168**, 127 (2007)] [arXiv:hep-ph/0701038].

[9] S. Weinberg, Physica **A96**, 327 (1979); J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984); Nucl. Phys. B **250**, 465 (1985).

[10] T. N. Truong, Phys. Rev. Lett. **61**, 2526 (1988); Phys. Rev. Lett. 67, 2260 (1991); A. Dobado *et al.*, Phys. Lett. B **235**, 134 (1990); A. Dobado and J. R. Pelaez, Phys. Rev. D **47**, 4883 (1993); Phys. Rev. D **56**, 3057 (1997).

[11] J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. **80**, 3452 (1998); Phys. Rev. D **59**, 074001 (1999) [Erratum-ibid. D **60**, 09990 (1999)]; Phys. Rev. D **62**, 114017 (2000).

[12] F. Guerrero and J. A. Oller, Nucl. Phys. B **15**, 459 (1999) [Erratum-ibid. B **602**, 641 (2001)]; A. Gómez Nicola and J. R. Peláez, Phys. Rev. D **65**, 054009 (2002).

[13] J. A. Oller and E. Oset, Nucl. Phys. A **620**, 438 (1997) [Erratum-ibid. A **652**, 407 (1999)].

[14] J. A. Oller and E. Oset, Phys. Rev. D **60**, 074023 (1999).

[15] N. Kaiser, P. B. Siegel and W. Weise, Phys. Lett. B **362**, 23 (1995); J. Nieves and E. Ruiz Arriola, Nucl. Phys. A **679**, 57 (2000); C. Garcia-Recio *et al.* Phys. Rev. D **67**, 076009 (2003); E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **585**, 243 (2004); U. G. Meissner and J. A. Oller, Nucl. Phys. A **673**, 311 (2000); T. Hyodo *et al.* Phys. Rev. C **68**, 018201 (2003); B. Borasoy, U. G. Meissner and R. Nissler, Phys. Rev. C **74**, 055201 (2006).

[16] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A **730**, 392 (2004).

[17] L. Roca, E. Oset and J. Singh, Phys. Rev. D **72**, 014002 (2005).

[18] L. S. Geng, E. Oset, L. Roca and J. A. Oller, Phys. Rev. D **75**, 014017 (2007).

[19] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969); C. G. Callan *et al.* Phys. Rev. **177**, 2247 (1969).

[20] M. C. Birse, Z. Phys. A **355**, 231 (1996).

[21] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33**, 1 (2006).

[22] D. Espriu, E. de Rafael and J. Taron, Nucl. Phys. B **345**, 22 (1990) [Erratum-ibid. B **355**, 278 (1991)].

[23] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311.

[24] S. Weinberg, Phys. Rev. Lett. **18** (1967) 507.

[25] J. Prades, Z. Phys. C **63** (1994) 491 [Erratum-ibid. C **11** (1999) 571].

[26] V. Cirigliano, G. Ecker, M. Eidemuller, R. Kaiser, A. Pich and J. Portoles, Nucl. Phys. B **753** (2006) 139.

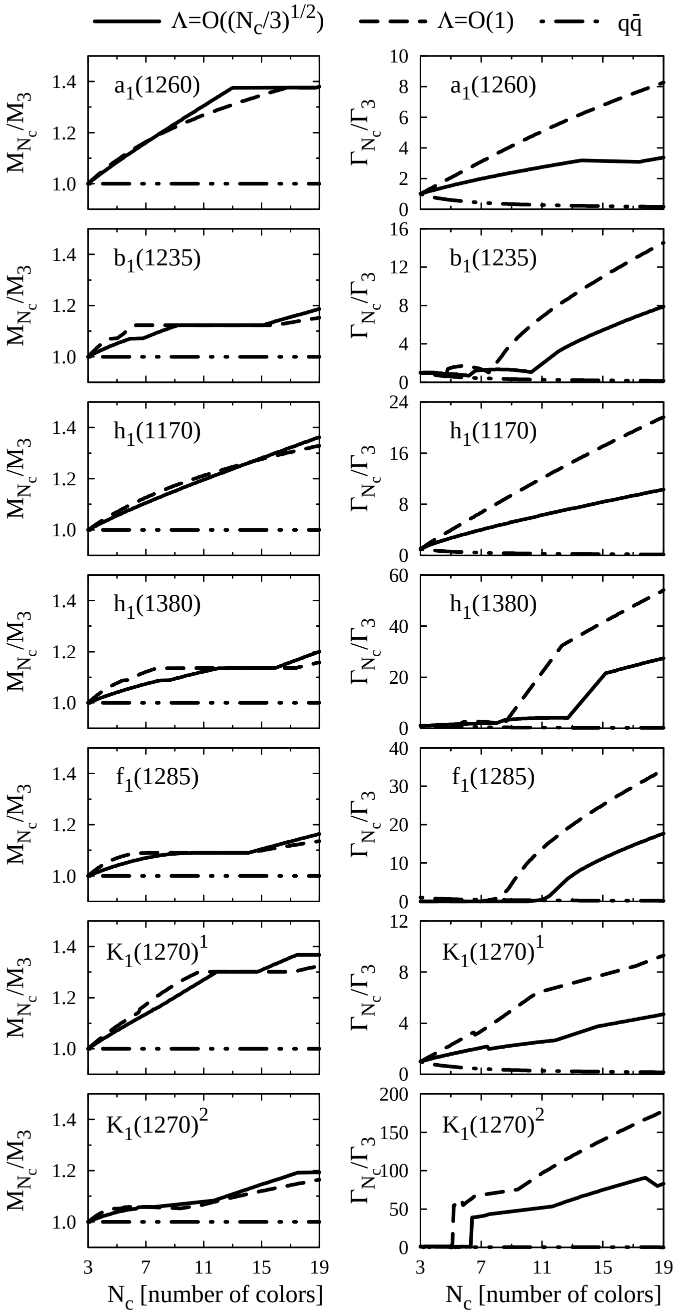

Figure 1: Mass and width N_c behavior of the axial-vector mesons. $K_1(1270)^1$ and $K_1(1270)^2$ denote the low-energy and high-energy states associated to the nominal $K_1(1270)$. The $f_1(1285)$ has a zero width at $N_c = 3$ in our model, but for convenience of comparison, we assign it its width of 24.2 MeV in the PDG [21] at $N_c = 3$.

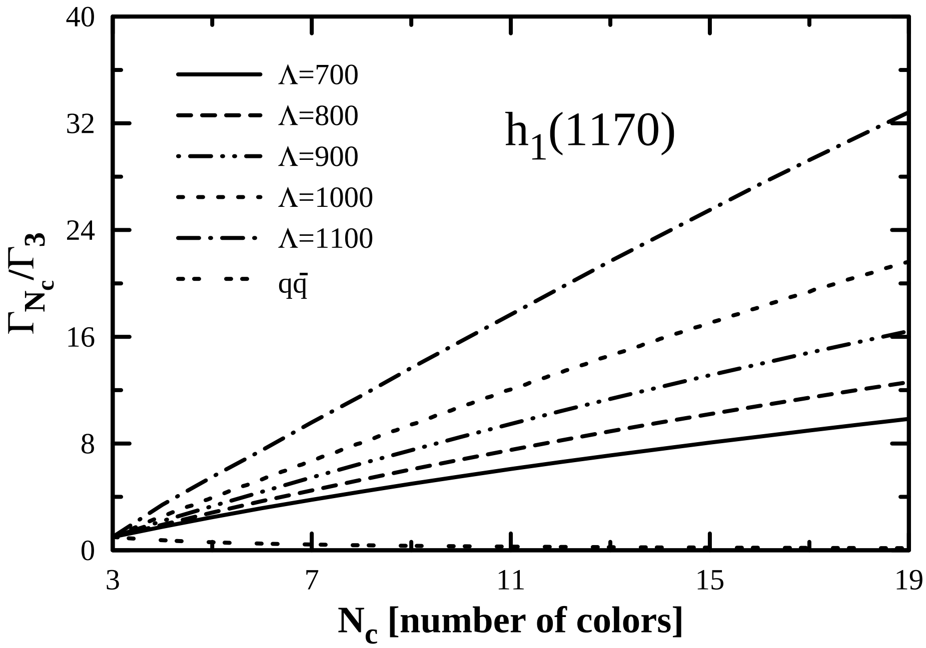

Figure 2: N_c behavior of the $h_1(1170)$ width for different choices of a natural cutoff scaling with N_c as $O(1)$. Note that in all cases the width grows with N_c in contrast with the $1/N_c$ behavior of $q\bar{q}$ states. Similar results are found for the other light axial-vector mesons.

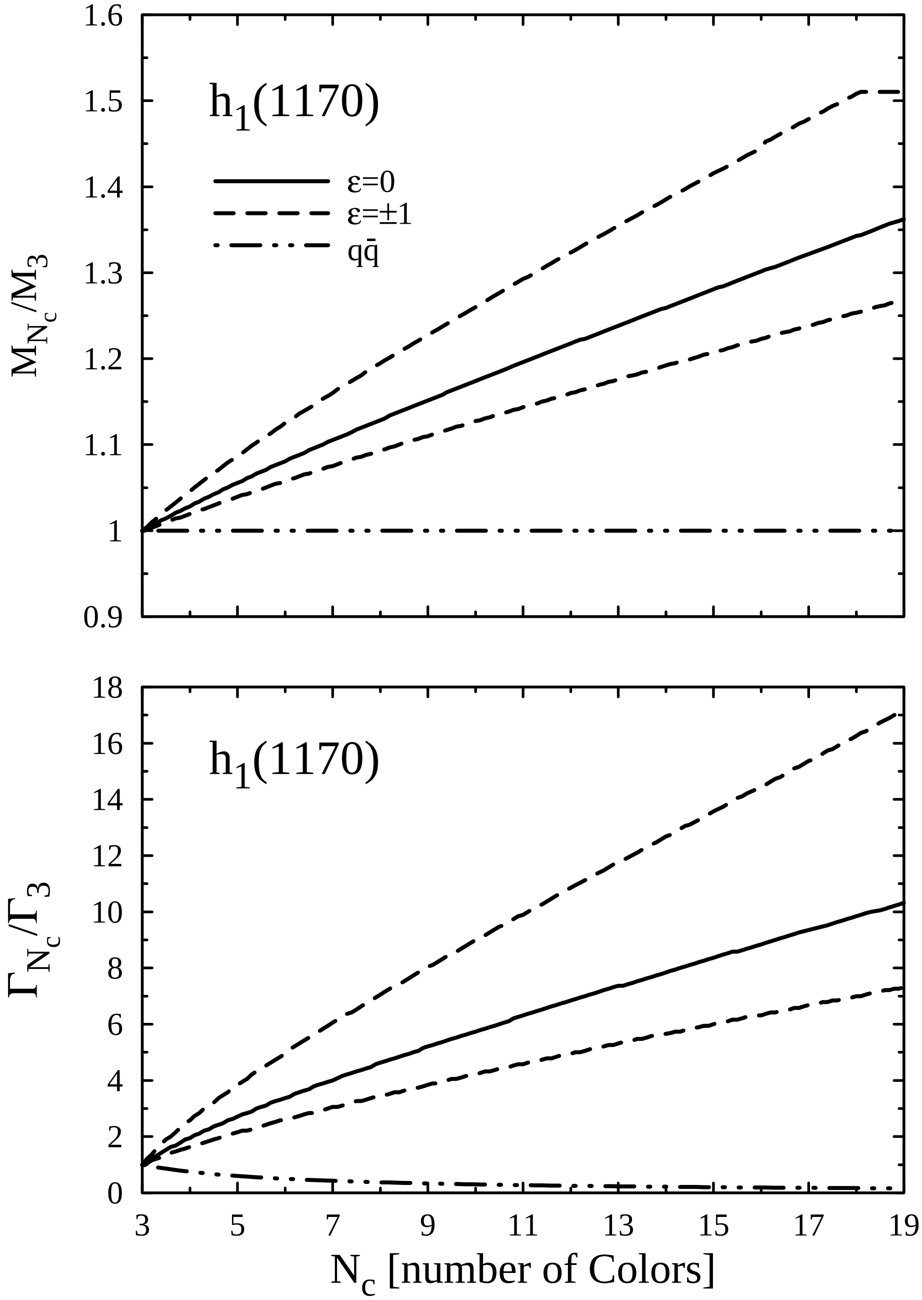

Figure 3: The dashed lines cover the estimated uncertainty in the N_c behavior of the $h_1(1170)$ mass and width due to subleading $1/N_c$ contributions to f_π. The continuous line assumes f_π scaling just as $\sqrt{N_c}$. Even though we show the case with the cutoff scaling as $\sqrt{N_c}$, the behavior is still completely at odds with a dominant $q\bar{q}$ nature (dot-dot-dashed line). Similar results are found for the other resonances.

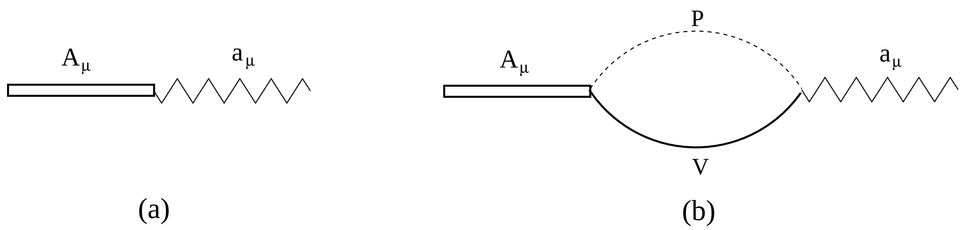

Figure 4: Coupling of an axial-vector resonance to an external axial-vector current via the Lagrangian of Eq. (12), (a), or in the case of the resonance being dynamically generated, (b).

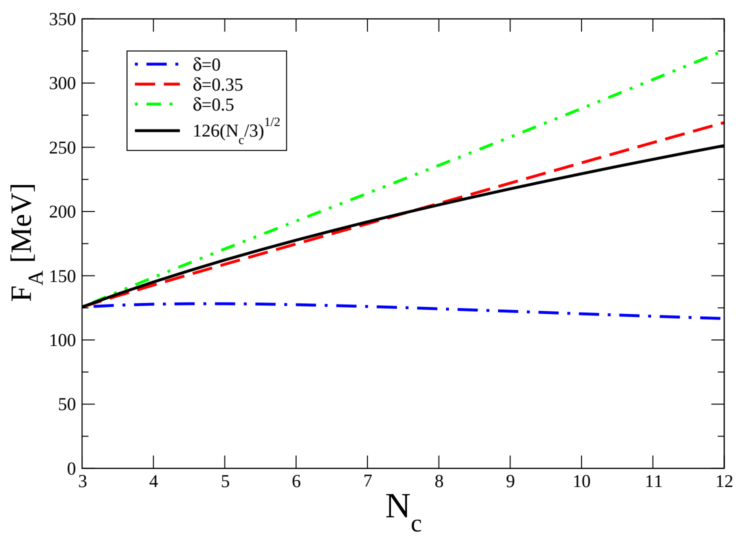

Figure 5: Dependence on the number of colors, N_c, of the axial-vector coupling, F_A, for different N_c dependences of the cutoff.