

In memory of Paco Ynduráin: A precise determination of $\pi\pi$ scattering from experiment and dispersion relations

J.R.Peláez^a, R. García Martín^a, R. Kamiński^b and F. J. Ynduráin^c

^a Dept. de Física Teórica II, Univ. Complutense de Madrid. 28040 Madrid. Spain

^bDepartment of Theoretical Physics Henryk Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences, 31-342, Kraków, Poland.

^c Dept. de Física Teórica, C-XI Univ. Autónoma de Madrid, Canto Blanco, E-28049, Madrid, Spain.

This talk is dedicated to the memory of Paco Ynduráin, the original speaker in the conference. After a short account of his scientific career, we briefly review our ongoing collaboration to determine precisely the $\pi\pi$ scattering amplitude including the most recent data by means of Forward Dispersion Relations and Roy Equations. A remarkable improvement in precision over the intermediate energy region is obtained by using once-subtracted Roy Equations in addition to the standard twice-subtracted ones.

1. Paco Ynduráin, in memoriam

F. J. Ynduráin, “Paco” for his friends, was very dear for this QCD96-QCD08 Conference series, since, apart from an active participant, he was member of the Advisory Committee in all editions. For the present QCD08, he was to review our work, but, very unfortunately, he passed away just before the beginning of the Conference. We thank prof. S. Narison for offering us the chance to give the same talk together with a brief account of Prof. Ynduráin outstanding scientific career.

In the University of Zaragoza, Spain, Paco obtained his Master degree in Mathematics in 1962 and defended in 1964 his PhD thesis in Physics, “Definitions of Hamiltonians and Renormalization”, supervised by A. Galindo. In Zaragoza he also held an assistant professorship (1964/66), before moving to New York University as a Fulbright Fellow (1966/67) and Associate Researcher (1967/68). He then became Research Fellow at CERN (1968/70), an institution to which he was specially attached, being senior scientific Associate in 1976 and 1985, and elected Member of the Scientific Policy Committee from 1988 to 1994.

Finally, after a short period in the Complutense University of Madrid in 1970, he became full professor in the Autónoma University of Madrid,

where he had a decisive influence in shaping one of the best Theoretical Physics Departments and Particle Physics Groups in Spain. Actually, he was Director of the Department twice (1974/77-1981/84), dean of the Science Faculty in 1975 and research vice-rector (1978/81).

Paco’s intense activity was rewarded with many friends and collaborations all over the world, becoming visiting researcher at CERN, Michigan, Marseille, NIKHEF, Brookhaven, Orsay, Saclay, Viena, La Plata, Bogotá, Caracas... He was also founding member of the European Physical Society, invited member of the American Association for the Advancement of Science, the New York Academy of Science and both the Mathematical and Physics Spanish Royal Societies. In addition, he became elected member of the European Board of High Energy Physics of the European Physical Society (1983/89), and elected president of the Physics and Chemistry section of the Spanish Royal Academy in 2002. But his influence extended beyond Physics as scientific advisor of IBM (1983/85), of the Institute for Scientific Research of Kuwait in (1980/82), and as elected member, by the Spanish Senate, of the Spanish University Council, as well as elected member of the World Innovation Foundation in London (2001) and the *Academia Europaea*.

Paco was awarded the gold medal of the Spanish Royal Academy and was an honorific collaborator of ICTP in Trieste, La Plata University, and Cavaliere Ufficiale nell'Ordine al Merito della Repubblica Italiana, a country very dear for him.



1.1. Paco, Particle Physics and QCD

Paco authored or coauthored more than 160 publications, including articles and talks. Unfortunately in this space we can only highlight a few. In his New York period he was interested in analyticity methods for scattering theory, that he gathered in a Review of Modern Physics in 1972 [1]. Around that time, QCD, the subject of this conference series, was established as the theory of strong interactions. This is where Paco obtained his most celebrated results, like the calculations of structure functions in the 70's and 80's [2], the hadronic contributions to the muon $g-2$ in the 80's and 00's [3], quarkonium calculations in the 90's [4], as well as light quark and chiral symmetry QCD calculations in the early 80's [5].

The participants in this QCD08 conference might be more familiar with his QCD work, but he also made other well known contributions to the Standard Model phenomenology, like the calculation of radiative corrections to WW scattering [6]. He also made some well known phenomenology beyond the Standard Model [7] but always limited within his very cautious and critical attitude towards Standard Model extensions.

In recent years, motivated by the very recent and precise experimental results, he was looking back to some of his initial interests, about analytic properties of pion-pion scattering amplitudes [8,9], the subject of this talk.

1.2. Books

Paco dedicated an enormous effort and enthusiasm to communicate Physics and Science. Of special relevance for the participants in this QCD Conference is his famous book – a classic – “QCD: An introduction to the Theory of Quarks and Gluons” (Springer-Verlag 1983). For many of us in the young – well, maybe not so young – generations of Particle Physicists, Paco's book has been THE BOOK where to learn QCD. He kept improving it throughout four different editions (from the second it is entitled “The Theory of Quark and Gluon Interactions”). More recently he also wrote a book on “Relativistic Quantum Mechanics: and introduction to Field Theory” (Springer-Verlag 1996), and another one on “Mecánica Cuántica” (In Spanish, 2nd Ed. Ariel S.A. 2003).

Less known to the international community – they are written in Spanish– are his books on Popular Science and “Speculation”, as he called them, whose translated titles are: “Unified Theories and the constituents of matter”, “Who walks out there”, “Electrons neutrinos and quarks” and “The Challenges of Science”.

1.3. Why the name “Paco” and farewell

Anyone called “Francisco” in Spain can also be called “Paco”. The reason is that St. Francesco de Assisi – San Francisco in Spanish–, was the founder of Franciscan monastic community and so he was the “Father of the Community” or “*Pater Comunalis*” in latin. Abbreviated: *Pa.Co*.

But in the case of F.J. Ynduráin, “Paco” was much more than just a nickname. In dire times for Spanish Scientific research, he was able to produce first level results and was able to gather a research group on Particle and Theoretical Physics that later on became the Theoretical Physics Department of the Autónoma University in Madrid, extending his influence and scientific attitude much beyond. With his career and example he

really became one of the “*PAter COMunalis*” of the Spanish Particle Physics Community.

Paco, we will miss you as a friend, but also your enthusiastic and contagious vitality not only about Physics but life as a whole. *Sit tibi terra levis* — may the earth rest lightly on you.

Of course, knowing Paco, I guess that by now he would be highly embarrassed and yelling from the back... Enough is enough! Let’s have fun with Physics!!... – Let’s not make him wait any longer.

2. Dispersive approach to $\pi\pi$ scattering

A precise determination of the $\pi\pi$ scattering amplitude at low energies is relevant for the study of Chiral Perturbation Theory (ChPT), quark masses, the chiral condensate [10] and, at intermediate energies, for the properties of the controversial sigma meson. However, the existing experimental information from $\pi\pi$ scattering has many conflicting data sets at intermediate energies and for many years very little data in the low energy region close to threshold. The interest in the topic has been renewed with recent [11] and precise experiments on kaon decays, relatively easy to relate to $\pi\pi$ scattering.

From the theoretical side, this process is very special due to the strong constraints from isospin, crossing and chiral symmetries, but mostly from analyticity. The latter allows for a very rigorous dispersive integral formalism that relates the $\pi\pi$ amplitude at any energy with an integral over the whole energy range, increasing the precision and providing information on the amplitude even at energies where data are poor, or in the complex plane. Remarkably, it is model independent since it makes the data parametrization irrelevant once it is included in the integral. The dispersive approach is thus well suited to study the threshold region or poles in the complex plane associated to resonances. Our recent works [8,9] make use of two complementary dispersive approaches, in brief:

- *Forward Dispersion Relations (FDRs)*: Calculated at $t = 0$ so that the amplitude unknown large- t behavior is not needed. There are two symmetric and one asymmetric isospin combinations, to cover the isospin basis. The symmetric

ones, $\pi^0\pi^+$ and $\pi^0\pi^0$, have two subtractions

$$\text{Re } F(s, 0) - F(4M_\pi^2, 0) = \frac{s(s-4M_\pi^2)}{\pi} \text{P.P.} \int_{4M_\pi^2}^{\infty} \frac{(2s'-4M_\pi^2) \text{Im } F(s', 0) \text{d } s'}{s'(s'-s)(s'-4M_\pi^2)(s'+s-4M_\pi^2)} \quad (1)$$

where F stands for the $F_{0+}(s, t)$ or $F_{00}(s, t)$ amplitudes. They are very precise since all the integrand contributions are positive. The antisymmetric isospin combination $I_t = 1$ reads

$$F^{(I_t=1)}(s, 0) = \frac{2s-4M_\pi^2}{\pi} \text{P.P.} \int_{4M_\pi^2}^{\infty} \text{d } s' \frac{\text{Im } F^{(I_t=1)}(s', 0)}{(s'-s)(s'+s-4M_\pi^2)}. \quad (2)$$

All FDRs are calculated up to $\sqrt{s} \simeq 1420$ MeV

- *Twice subtracted Roy Equations (RE)*: they are an infinite set of coupled equations [13], equivalent to nonforward dispersion relations plus some $t - s$ crossing symmetry. They are well suited to study poles of resonances since they are written in terms of partial waves $f_l^{(I)}$ of definite isospin I and angular momentum l . The complicated left cut contribution is rewritten as a series of integrals over the physical region:

$$\text{Re } f_l^{(I)}(s) = C_l^{(I)} a_0^{(0)} + C_l^{\prime(I)} a_0^{(2)} + \sum_{l', I'} \text{P.P.} \int_{4M_\pi^2}^{\infty} \text{d } s' K_{l, l'; I, I'}(s', s) \text{Im } f_{l'}^{(I')}(s'), \quad (3)$$

where the $C_l^{(I)}$, $C_l^{\prime(I)}$ constants and $K_{l, l'; I, I'}$ kernels are known. The calculation is truncated at $l < 2$ and at some cutoff energy s_0 . The $l \geq 2$ waves and the high energy are input. Although RE are valid for $\sqrt{s} \leq 8M_\pi \simeq 1120$ MeV, for now we have implemented them only up to $\sqrt{s} \simeq 2M_K$.

The aim of our group when using RE combined with FDR has been to improve the precision of scattering data analysis, to test ChPT, and, still in progress, to obtain precise determinations of the σ resonance. The Bern group [14] has also carried out a series of RE analysis, with and without ChPT constraints, using as input phenomenological parametrizations for the $l \geq 2$ waves and above 800 MeV for the other waves, as well as some Regge input. When using ChPT constraints, they find, $a_0^{(0)} = 0.220 \pm 0.005 M_\pi^{-1}$ and $a_0^{(2)} = -0.0444 \pm 0.0010 M_\pi^{-1}$, an extremely

precise claim, together with predictions for other scattering lengths and the S and P wave phase shifts up to 800 MeV. Although of the original input, particularly the Regge theory and the D waves, was questionable [15], it certainly seems to have a very small influence in the threshold region of the scalar waves [16]. In addition, the Krakow-Paris [17] and Paris [18] groups have performed other RE analysis. The former resolved a long-standing ambiguity, discarding the so-called "up" solution, including in their analysis a study using polarized target data. The latter checked the calculation in [14] and claimed a small discrepancy in the Olsson sum rule.

3. Sketch of our analysis

The approach we have followed over a series of works [8] can be sketched as follows:

(1) We obtain simple "Unconstrained Fits to Data" (UFD) to each $\pi\pi$ scattering wave so that it can be improved independently if needed. High energy fits use Regge theory. As our precision improves, we refine our fits with more flexible parametrizations or with new and more precise data, as, for example, in [9] where we included the newest and very precise K_{l_4} data [11].

(2) We then check how well these UFD satisfy FDRs and RE and some sum rules. To our surprise some of the most widely used data parametrizations fail to satisfy FDRs or the sum rules. Thus we choose the data parametrizations in better agreement with FDRs.

(3) Our best results are obtained by imposing dispersion relations (both FDRs and RE) and some crossing sum rules (SR) on the data fits, now called "Constrained Fits to Data" (CFD), which are thus consistent with analyticity, unitarity, crossing, etc...

A dispersion relation i is well satisfied at a point s_n if the difference Δ_i between the left and right sides in Eqs.(1),(2) and (3) is small relative to its uncertainty $\delta\Delta_i$. Thus, when the average discrepancy (similar to an averaged $\chi^2/(d.o.f)$)

$$\bar{d}_i^2 \equiv \frac{1}{\text{number of points}} \sum_n \left(\frac{\Delta_i(s_n)}{\delta\Delta_i(s_n)} \right)^2 \leq 1 \quad (4)$$

it implies that the corresponding dispersion relation is satisfied within uncertainties. In practice,

the values of s_n are taken at intervals of 25 MeV. This measure is also used [8] to obtain the Constrained Fit to Data, by minimizing:

$$\chi^2 \equiv W \sum_i \bar{d}_i^2 + \bar{d}_I^2 + \bar{d}_J^2 + \sum_k \left(\frac{p_k - p_k^{\text{exp}}}{\delta p_k} \right)^2, \quad (5)$$

where p_k^{exp} are all the parameters of the different UFD parametrization for each wave or Regge trajectory, thus ensuring the data description, and d_I and d_J are the discrepancies for a couple of crossing sum rules. The weight $W = 9$ was estimated from the typical number of degrees of freedom needed to describe the shape of the amplitude.

The result of this program is, on the one hand, a set of precise Constrained Fits to Data, that satisfy very well all dispersion relations within uncertainties. Remarkably, all the waves are given in terms of very simple parametrizations that can be found in [8] which are very easy to use for phenomenological purposes. On the other hand, we have the outcome of Roy Eqs. themselves that allows us to extend the calculation to the complex plane and look for poles associated to resonances and study their parameters.

In particular the best determination of threshold parameters is obtained by using the CFD set directly or inside appropriate sum rules [8]. For the S0 and S2 waves, we find: $a_0^{(0)} = 0.223 \pm 0.009 M_\pi^{-1}$ and $a_0^{(2)} = -0.0444 \pm 0.0045 M_\pi^{-1}$, in remarkable agreement with the predictions in [14], that extends also to the P wave scattering length. There are however, some disagreements of 2 to 3 standard deviations in the P-wave slope, and also in some D wave parameters. In general, for the controversial S0 wave, the agreement is fairly good only up to roughly 450 MeV, but from that energy up to 800 MeV those predictions deviate slightly from our data analysis. This means that we find from our Roy Eqs. and FDR data analysis a somewhat different pole from that of the Bern group (see R. Kaminski talk in this conference). Let us emphasize that we are talking about a deviation of a few degrees that only affects the sigma mass and width determination by ten or twenty MeV at most, which is a remarkable improvement compared with the situation just a few years ago and the huge and extremely conser-

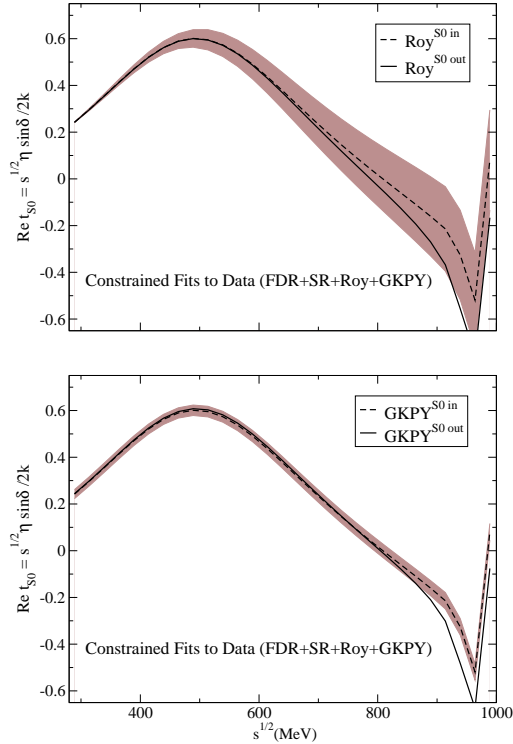


Figure 1. Above 500 MeV, the uncertainty in standard S0 Roy Eqs. (Top) is much larger than with once subtracted Roy Eqs. (Bottom).

vative uncertainties of hundreds of MeV quoted in the PDG at present [19].

3.1. Latest developments

In this conference we report preliminary results on two issues: First, we have recently implemented once-subtracted Roy Eqs, (denoted GKPY for brevity)[20]. This is motivated by the interest in the $450\text{MeV} < \sqrt{s} < 2M_K$ region of the S0 wave, dominated by the σ -resonance. The large low-energy experimental uncertainties in the S2-wave translate into large uncertainties for the dispersive output for the S0 wave in that region when using the standard twice subtracted Roy Eqs. (shaded area in top panel of Fig 1), but not for the once subtracted ones, which have a much smaller uncertainty (Fig 1, Bottom).

Second, we have improved the matching between the S0 wave parametrizations at low and intermediate energies, which occurs at 932 MeV, imposing also continuity in the first derivative.

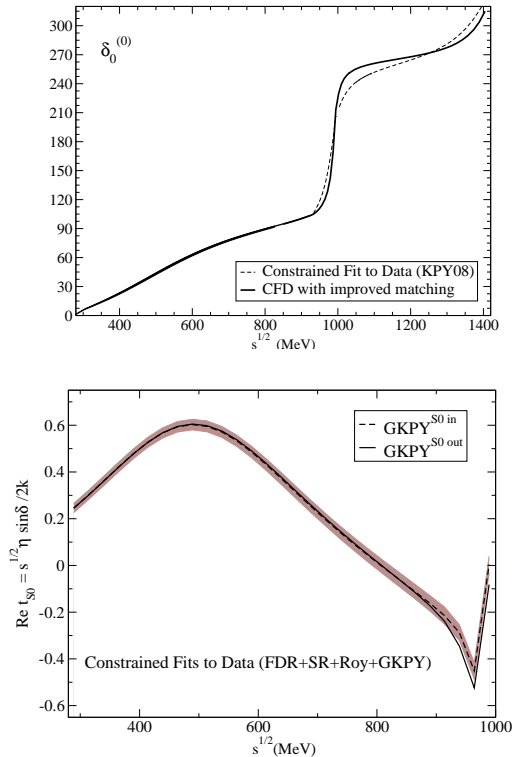


Figure 2. Top: The improved matching only changes slightly the S0 wave above 932 MeV. Bottom: The agreement GKPY Eqs. is slightly improved above 850 MeV (compare with Fig.1 Bottom).

This follows a suggestion [21] to explain the roughly 2σ level discrepancies in the S0 wave between our analysis and that of the Bern group in the 400 – 900MeV region. In Fig.2 we compare the CFD in [8] (dashed line) and the new one with the improved matching (continuous line), which only differ a little above 932 MeV. Note that, for clarity, we do not provide data points, which are nevertheless reasonably well described by both parametrizations when taking into account experimental errors. Hence, the disagreement is *not caused* by a poor matching, but we are studying further suggestions in [21]. Nevertheless this better matching improves the Roy and GKPY Eqs. fulfillment above 850 MeV, as seen by comparing the bottom panels in Figs.1 and 2.

Finally, in order to show how well our preliminary CFD set satisfies the nine dispersion relations, we list in Table 1 the average discrepancies,

FDRs	$s^{1/2} \leq 932 \text{ MeV}$	$s^{1/2} \leq 1420 \text{ MeV}$
$\pi^0\pi^0$	0.13	0.31
$\pi^+\pi^0$	0.83	0.85
$I_{t=1}$	0.13	0.70

Roy Eqs.	$s^{1/2} \leq 992 \text{ MeV}$	
	2 subs.	1 sub. (GKPY)
S0	0.06	0.49
S2	0.13	0.11
P	0.11	0.23

Table 1. Average discrepancies \bar{d}^2 of the Constrained Fit to Data for each forward dispersion relation.

defined in Eq.(4), for the three FDR, up to two different energies, and for the three Roy Eqs. up to $\sim 2M_K$, either with the standard two subtractions or only with one. From that Table it is clear that this CFD set satisfies remarkably well all dispersion relations within uncertainties

In summary the CFD set provides a model independent and very precise description of the $\pi\pi$ scattering data consistent with analyticity and crossing that can be easily used for phenomenological purposes. Work is in progress to obtain an even more accurate description for threshold parameters than the one we had, and (see R. Kaminski talk) by using it inside once-subtracted Roy Eqs., to obtain a precise determination from data of the σ pole parameters.

Acknowledgments

J.R.P. thanks A. Galindo, A. González Arroyo and M.J. Herrero, for kindly providing information about Paco Ynduráin's career and H. Leutwyler for his comments about the matching of the S0 wave.

REFERENCES

1. F. J. Yndurain, Rev.Mod.Phys.**44** (1972) 645.
2. F. J. Yndurain, Phys. Lett. B **74** (1978) 68. A. Gonzalez-Arroyo, C. Lopez and F. J. Yndurain, Nucl. Phys. B **153** (1979) 161. C. Lopez and F. J. Yndurain, Nucl. Phys. B **171**, 231 (1980); Nucl. Phys. B **183**, 157 (1981).
3. J. A. Casas, C. Lopez and F. J. Yndurain, Phys. Rev. D **32** (1985) 736. J. F. De Troconiz and F. J. Yndurain, Phys. Rev. D **65**, 093001 (2002); Phys. Rev. D **71**, 073008 (2005).
4. S. Titard and F. J. Yndurain, Phys. Rev. D **49** (1994) 6007. A. Pineda and F. J. Yndurain, Phys. Rev. D **58** (1998) 094022.
5. C. Becchi, S. Narison, E. de Rafael and F. J. Yndurain, Z. Phys. C **8** (1981) 335.
6. M. J. G. Veltman and F. J. Yndurain, Nucl. Phys. B **325** (1989) 1.
7. C. Jarlskog and F. J. Yndurain, Nucl. Phys. B **149** (1979) 29. M. J. Herrero, L. E. Ibañez, C. Lopez and F. J. Yndurain, Phys. Lett. B **132** (1983) 199; Phys. Lett. B **145** (1984) 430.
8. R. Kaminski, J. R. Pelaez and F. J. Yndurain, Phys. Rev. D **77** (2008) 054015; Phys. Rev. D **74** (2006) 014001
9. R. Garcia-Martin and J. R. Pelaez, F. J. Yndurain, Phys. Rev. D **76** (2007) 074034 J. R. Pelaez and F. J. Yndurain, Phys. Rev. D **71** (2005) 074016; Phys. Rev. D **69** (2004) 114001
10. J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).
11. J. R. Batley *et al.* [NA48/2 Collaboration], Eur. Phys. J. C **54** (2008) 411. S. Pislak *et al.* [BNL-E865 Collaboration], Phys. Rev. Lett. **87** (2001) 221801 [arXiv:hep-ex/0106071].
12. R. Garcia-Martin, R. Kaminski, and J. R. Peláez arXiv:0804.2643 [hep-ph].
13. S. M. Roy, Phys. Lett. B **36**, 353 (1971).
14. G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B **603** (2001) 125 B. Ananthanarayan, G. Colangelo, J. Gasser and H. Leutwyler, Phys. Rept. **353** (2001) 207
15. J. R. Pelaez and F. J. Yndurain, Nucl. Phys. Proc. Suppl. **164** (2007) 93; Phys. Rev. D **68** (2003) 074005
16. I. Caprini, G. Colangelo, J. Gasser and H. Leutwyler, Phys. Rev. D **68** (2003) 074006
17. R. Kaminski, L. Lesniak and B. Loiseau, Phys. Lett. B **551**, 241 (2003)
18. S. Descotes-Genon, N. H. Fuchs, L. Girlanda and J. Stern, Eur. Phys. J. C **24**, 469 (2002)
19. C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667** (2008) 1.
20. R. Kaminski *et al* arXiv:0809.4766 [hep-ph].
21. H. Leutwyler, AIP Conf. Proc. **892** (2007) 58 [arXiv:hep-ph/0612111].