

Light scalar and vector mesons in the large N_c limit from unitarized chiral perturbation theory

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Abstract. We review the large N_c behavior of light scalar and vector resonances generated from unitarized meson-meson scattering amplitudes at one loop in Chiral Perturbation Theory. The vectors nicely follow the behavior expected for $\bar{q}q$ states, whereas scalar mesons do not. This suggests that the main component of light scalars is not $\bar{q}q$. We also comment on t-channel vector exchange as well as on the large N_c behavior of the mass splittings between the vectors generated from the inverse amplitude method.

Keywords: mesons, scalars, large N_c , chiral perturbation theory, unitarization

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INTRODUCTION

Chiral Perturbation Theory (ChPT) is the QCD low energy Effective Lagrangian built as the most general derivative expansion respecting SU(3) symmetry and containing only π, K and η mesons [1]. These particles are the QCD low energy degrees of freedom since they are Goldstone bosons of the QCD spontaneous chiral symmetry breaking. For meson-meson scattering, ChPT is an expansion in even powers of momenta, $O(p^2), O(p^4), \dots$, over a scale $\Lambda_\chi \sim 4\pi f_0 \simeq 1 \text{ GeV}$. Since the u, d and s quark masses are so small compared with Λ_χ they are introduced as perturbations, giving rise to the π, K and η masses, counted as $O(p^2)$. At each order, ChPT is the sum of *all terms* compatible with the symmetries, multiplied by “chiral parameters”, that absorb loop divergences order by order, yielding finite results. The leading order is universal, containing only one parameter f_0 , that sets the scale of spontaneous symmetry breaking. Different underlying dynamics correspond to different values of the higher order parameters, called L_i , that, once renormalized, depend on a regularization scale as $L_i(\mu_2) = L_i(\mu_1) + \Gamma_i \log(\mu_1/\mu_2)/16\pi^2$, where Γ_i are constants[1]. In physical observables the μ dependence is canceled with that of the loop integrals.

We will also make use of the large N_c expansion [2], which is the only analytic approximation to QCD in the whole energy region and provides a clear definition of $\bar{q}q$ states, that become bound, and whose masses and widths behave as $O(1)$ and $O(1/N_c)$, respectively. In fact, the π, K, η masses scale as $O(1)$ and f_0 as $O(\sqrt{N_c})$. The L_i parameters that determine meson-meson scattering up to $O(p^4)$ and their N_c scaling [1, 3] is given in Table 1. In order to apply the large N_c expansion, the μ scale, a dependence suppressed by $1/N_c$, has to be chosen[1] between $\mu = 0.5$ and 1 GeV .

In recent years ChPT has been extended to higher energies by means of unitarization [4, 5, 6, 7]. The main point is that the partial waves, t_{IJ} , of definite angular momentum

TABLE 1. $O(p^4)$ chiral parameters ($\times 10^3$) and their N_c scaling. In the ChPT column, L_1 , L_2 , L_3 come from [8] and the rest from [1]. The IAM columns correspond to different fits [4].

$O(p^4)$ Parameter	N_c Scaling	ChPT $\mu = 770\text{MeV}$	IAM I $\mu = 770\text{MeV}$	IAM II $\mu = 770\text{MeV}$	IAM III $\mu = 770\text{MeV}$
L_1	$O(N_c)$	0.4 ± 0.3	0.56 ± 0.10	0.59 ± 0.08	0.60 ± 0.09
L_2	$O(N_c)$	1.35 ± 0.3	1.21 ± 0.10	1.18 ± 0.10	1.22 ± 0.08
L_3	$O(N_c)$	-3.5 ± 1.1	-2.79 ± 0.14	-2.93 ± 0.10	-3.02 ± 0.06
L_4	$O(1)$	-0.3 ± 0.5	-0.36 ± 0.17	0.2 ± 0.004	0 (fixed)
L_5	$O(N_c)$	1.4 ± 0.5	1.4 ± 0.5	1.8 ± 0.08	1.9 ± 0.03
L_6	$O(1)$	-0.2 ± 0.3	0.07 ± 0.08	0 ± 0.5	-0.07 ± 0.20
L_7	$O(1)$	-0.4 ± 0.2	-0.44 ± 0.15	-0.12 ± 0.16	-0.25 ± 0.18
L_8	$O(N_c)$	0.9 ± 0.3	0.78 ± 0.18	0.78 ± 0.7	0.84 ± 0.23
$2L_1 - L_2$	$O(1)$	-0.55 ± 0.7	0.09 ± 0.10	0.0 ± 0.1	-0.02 ± 0.10

J and isospin I , in the elastic regime satisfy the unitarity condition:

$$\text{Im}t_{IJ} = \sigma |t_{IJ}|^2, \quad \text{where } \sigma = \frac{2q}{\sqrt{s}} \Rightarrow \text{Im} \frac{1}{t_{IJ}} = -\sigma \Rightarrow t_{IJ} = \frac{1}{\text{Re}t_{IJ}^{-1} - i\sigma}, \quad (1)$$

where q is the meson CM momentum. In order to have a unitary amplitude we only need $\text{Re}t^{-1}$, that can be obtained from ChPT: this is the Inverse Amplitude Method (IAM) [5, 4]. In this way, the IAM generates the ρ , K^* , σ and κ resonances not initially present in ChPT, ensures unitarity in the elastic region and respects the ChPT expansion.

When inelastic two-meson processes occur, the IAM can be generalized [4, 6] to $T \simeq (\text{Re}T^{-1} - i\Sigma)^{-1}$, within a coupled channel formalism, where T is a matrix containing all partial waves between all physically accessible two-body states, whereas Σ is a diagonal matrix with their corresponding phase spaces. Using one-loop ChPT calculations, the coupled channel IAM provides a remarkable description [4] of two-body π , K or η scattering up to 1.2 GeV. In addition, it generates the ρ , K^* , σ , κ , $a_0(980)$, $f_0(980)$ and the octet ϕ . Such states are not included in the ChPT Lagrangian, but each one has an associated pole in the second Riemann sheet of its corresponding partial wave. These poles appear already with the L_i set used for standard ChPT, which is compatible with the L_i sets in Table 1, obtained from fits to data. For narrow, Breit-Wigner like, resonances, their mass and width is roughly given by $\sqrt{s_{pole}} \sim M_R - i\Gamma_R/2$. Furthermore, the IAM respects the $O(p^4)$ correct low energy expansion, with chiral parameters compatible with standard ChPT. Different IAM fits[4] are mostly due to different ChPT truncation schemes, equivalent up to $O(p^4)$, and to the estimation of systematic errors in the data.

Since ChPT amplitudes are renormalized, and therefore scale independent, there are no cutoffs or subtraction constants where a spurious N_c dependence could hide. All the QCD N_c dependence appears correctly through the L_i , f_0 and the π , K , η masses.

Recently[7], by rescaling the ChPT parameters, we have studied how the resonances generated from unitarization behave in the large N_c expansion, around ‘‘real life’’, $N_c = 3$. Thus, in Fig.1 we see that the modulus of partial waves associated to the $\rho(770)$ and $K^*(892)$ vector mesons presents a peak, obtained from a fit to data, that becomes narrower as N_c increases, whereas the mass remains almost the same. This is exactly the behavior expected for a $\bar{q}q$ state, namely, $M \sim O(1)$, $\Gamma \sim O(1/N_c)$.

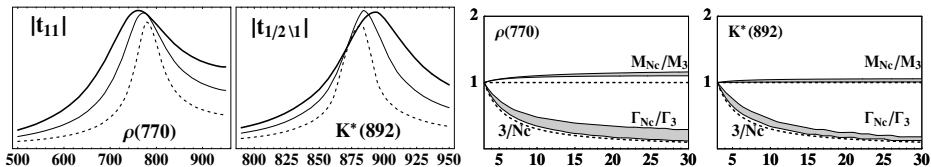


FIGURE 1. Left: Modulus of $\pi\pi$ and πK elastic amplitudes versus \sqrt{s} for $(I, J) = (1, 1), (1/2, 1)$: $N_c = 3$ (thick line), $N_c = 5$ (thin line) and $N_c = 10$ (dotted line), scaled at $\mu = 770$ MeV. Right: $\rho(770)$ and $K^*(892)$ pole positions: $\sqrt{s_{pole}} \equiv M - i\Gamma/2$ versus N_c . The gray areas cover the uncertainty $\mu = 0.5 - 1$ GeV. The dotted lines show the large N_c scaling expected for a $\bar{q}q$ state.

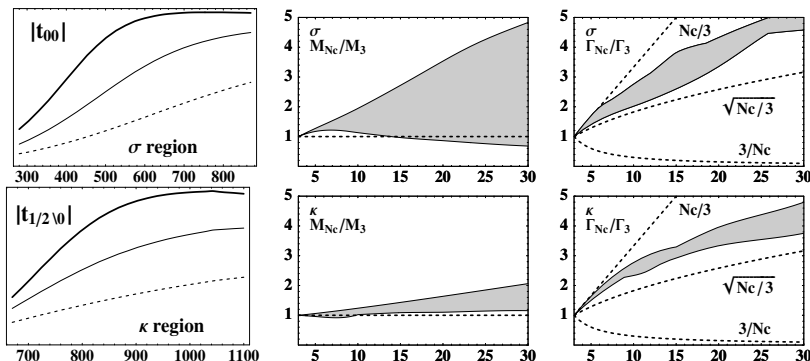


FIGURE 2. (Top) Left: Modulus of the $(I, J) = (0, 0)$ scattering amplitude, versus \sqrt{s} for $N_c = 3$ (thick line), $N_c = 5$ (thin line) and $N_c = 10$ (dotted line), scaled at $\mu = 770$ MeV. Center: N_c evolution of the σ mass. Right: N_c evolution of the σ width. (Bottom): The same but for the $(1/2, 0)$ amplitude and the κ .

In contrast, in Figure 2 we see the corresponding behavior for the σ (or $f_0(600)$) and the κ . The results for the $f_0(980)$ and $a_0(980)$ (the latter, except in a corner of parameter space) are roughly similar, but more subtle [7]. It is evident that these *scalars behave completely different to $\bar{q}q$* : *The modulus of their partial waves in the resonance region vanish and their widths grow as N_c increases from $N_c = 3$, as $O(N_c^{1/2}) < \Gamma < O(N_c)$.* These results have been confirmed later [9]. Thus, we can conclude the following

- The dominant component of the σ and κ in meson-meson scattering does not behave as a $\bar{q}q$.
 - Why “dominant”? Because, most likely, scalars are a mixture of different kind of states. If the $\bar{q}q$ was *dominant*, they would behave as the ρ or the K^* in Figure 1. *But it cannot be excluded that there is some smaller fraction of $\bar{q}q$.*
 - Also, since scalars could be an admixture of states with different nature and wave functions, the small $\bar{q}q$ component could be concentrated in the core and better seen in other reactions, whereas in scattering we are seeing mostly the outer region.
- *Two-meson and some tetraquark states [10] have a consistent “qualitative” behavior*, i.e., both disappear in the continuum of the meson-meson scattering amplitude as N_c increases (also the glueballs for the σ case, but not for the κ). Waiting for more

quantitative results, we have not been able to establish yet the nature of that dominant component, but two-meson states or some kind of tetraquarks are, qualitatively, candidates to form that dominant component.

Next we will address comments concerning the role of vector mesons in the IAM.

T-CHANNEL VECTOR MESON EXCHANGE

A common concern for people who have modeled meson-meson scattering including explicitly resonance fields, is the t-channel vector meson exchange, since it should contribute sizably to $\pi\pi$ or πK scattering. Since the ChPT Lagrangian does not have an explicit ρ or K^* field, and the usual resummations in the literature involve only s-channel loops, it may seem that we are not taking into account this contribution in the IAM. However, the resonance saturation mechanism [11] explains the L_i values as the contact terms that remain once resonances heavier than pions, kaons and etas, are integrated out from a chiral Lagrangian including all resonances. In other words, resonance propagators are reduced to constants when $M_V^2 \gg s$. From vector resonances some L_i get contributions of the form

$$L_i^V \sim 1/M_V^2, \quad (2)$$

where M_V is the typical mass of the ρ and K^* multiplet.

Hence, a t-channel vector exchange, after integration of the heavy fields, becomes a combination of L_i parameters in the effective theory, and is thus *effectively* included in the one loop amplitudes later used in the IAM [7]. This is another important reason to consider the completely renormalized $O(p^4)$ ChPT amplitudes in the IAM, because we can then use L_i parameters compatible with standard ChPT, which also ensure the correct *low energy* expansion and the *effective* crossed channel resonance exchange.

ρ AND K^* MASS SPLITTINGS FROM THE LARGE N_C IAM

As shown in [7], using L_i parameters compatible with those of standard ChPT, the IAM is able to fit the experimental data on many meson-meson channels below 1.2 GeV, and in particular the different physical masses of the ρ and K^* resonances. However, it has been remarked [12] that the vector mass splittings are

$$M_{K^*}^2 = M_\rho^2 + O(M_{\pi,K}^2) \simeq M_V^2 + O(M_{\pi,K}^2) \quad (3)$$

and that the $O(p^4)$ IAM does not yield this dependence in the large N_C limit. Certainly, it does not, but if we took into account the vector mass splittings, Eq.(3), in the resonance saturation hypothesis, Eq.(2), we would find,

$$L_i^{V+split} \sim \frac{1}{(M_V^2 + O(M_{\pi,K}^2))} \sim \frac{1}{M_V^2} (1 + O(M_{\pi,K}^2)) \sim L_i^V (1 + O(M_{\pi,K}^2)).$$

But in ChPT, $O(M_{\pi,K}^2)$ counts as $O(p^2)$. Since L_i^V is multiplied by an $O(p^4)$ operator, the splitting term contributes, at least, at $O(p^6)$. Obviously, just with $O(p^4)$ ChPT one

does not have to get the $O(p^6)$ splitting terms. In particular, this does *not* mean that “a systematic expansion in powers of quark masses has not been performed” [12], but that it has only been performed up to $O(p^4)$ in ChPT. Therefore, *nothing is to blame on the IAM, but just on the truncation at $O(p^4)$* . Any other unitarization scheme matching ChPT only up to $O(p^4)$ could also fail at $O(p^6)$ in some observable. In particular, even using $O(p^4)$ ChPT *without unitarization* to fit the low energy $\pi\pi$ and πK scattering in the vector channels, the resulting L_i will not correspond to splittings including $O(p^6)$ corrections. Of course, when dealing with a truncated expansion to describe data, the L_i absorb the effect of higher orders, and contain information on the physical splittings.

In summary, the observation that the $M_{\pi,K}^2$ dependence of the splittings is not obtained at $O(p^4)$ is interesting, but the IAM is not to blame for it, but just the $O(p^4)$ truncation. In particular, as we see in Fig.1, it cannot be concluded [12] that the IAM at $O(p^4)$ *does not yield parametrically reasonable large N_c dependence for the vector masses, $O(1)$* , which is the only relevant issue for our discussion on the nature of resonances. For mass splittings, which is another issue, one should consider the $O(p^6)$ at least [13].

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