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Light scalars as tetraquarks or two-meson states from large N_c and unitarized Chiral Perturbation Theory.

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By means of unitarized Chiral Perturbation Theory it is possible to obtain a remarkable description of meson-meson scattering amplitudes up to 1.2 GeV, and generate poles associated to scalar and vector resonances. Since Chiral Perturbation Theory is the QCD low energy effective theory, it is possible then to study its large N_c limit where $q\bar{q}$ states are easily identified. The vectors thus generated follow closely a $q\bar{q}$ behavior, whereas the light scalar poles follow the large N_c behavior expected for a dominant tetraquark or two-meson structure.

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1. Introduction

We review here how the spectroscopic nature of the lightest scalar resonances can be obtained from the large N_c behavior[1] of their associated poles[2] in the meson-meson scattering one-loop Chiral Perturbation Theory unitarized[3] amplitudes.

The existence and nature of the lightest scalar resonances is a longstanding controversial issue for hadron spectroscopy. Their relevance is twofold: First, one of the most interesting features of QCD is its non-abelian nature, so that gluons interact among themselves and could produce glueballs, which are isoscalars. Moreover, the lightest ones are expected to be also scalars. Naively, once all quark multiplets are identified in the scalar-isoscalar sector, whatever remains are good candidates for glueballs. Unfortunately, the whole picture is more messy due to mixing, since the resonances we actually see are a superposition of different states. Second, it is also understood that QCD has an spontaneous breaking of the chiral symmetry since its vacuum is not invariant under chiral transformations. The study of the scalar-isoscalar sector is relevant to understand the QCD vacuum, which has precisely those quantum numbers.

Although QCD is firmly established as the fundamental theory of strong interactions and its predictions have been thoroughly tested to great accuracy in the perturbative regime (above 1-2 GeV), it becomes non-perturbative at low energies, and helps little to address the existence and nature of light scalar mesons. Hence, the usual perturbative expansion in terms of quarks and gluons has to be abandoned in favor of somewhat less systematic approaches in terms of mesons. As a matter of fact, most chiral descriptions of meson dynamics do not include quarks and gluons and are hard to relate to QCD, and the spectroscopic nature is thus imposed from the start. In contrast, models with quarks and gluons, even those inspired in QCD, have problems with chiral symmetry, small meson masses, etc...

An exception is the formalism of Chiral Perturbation Theory[4,5] (ChPT), which is the most general effective Lagrangian made out of pions, kaons and etas, that respects the QCD chiral symmetry breaking pattern. These particles are the QCD low energy degrees of freedom since they are Goldstone bosons of the QCD spontaneous chiral symmetry breaking. For meson-meson scattering ChPT is an expansion in even powers of momenta, $O(p^2), O(p^4)$..., over a scale $\Lambda_\chi \sim 4\pi f_0 \simeq 1\,\text{GeV}$. Since the u, d and s quark masses are so small compared with Λ_χ they are introduced as perturbations, giving rise to the π, K and η masses, counted as $O(p^2)$. At each order, ChPT is the sum of *all terms* compatible with the symmetries, multiplied by "chiral" parameters, that absorb loop divergences order by order, yielding finite results. ChPT is thus *the* quantum effective field theory of QCD, and it allows for a systematic *and model independent* analysis of low energy mesonic processes.

Since ChPT is an expansion in momenta and masses, it is limited to low energies. As the energy grows, the ChPT truncated series violate unitarity. Nevertheless, in recent years ChPT has been extended to higher energies by means of unitarization [6,7,8,9,10,11,3]. The results are remarkable[3], extending the one-loop ChPT calculations of two body π, K or η scattering up to 1.2 GeV, but keeping simultaneously the correct low energy expansion and with chiral parameters compatible with standard ChPT. Furthermore it *generates* the poles associated to the $\rho(770)$, $K^*(892)$ and the octet ϕ vector resonances, as well as those of the controversial scalar states, namely: the σ, κ, $a_0(980)$, and the $f_0(980)$.

Among these states, the most controversial are the σ, now called $f_0(600)$ in the Particle Data Group (PDG) Review [12], and the κ. They appear as broad resonant structures in the scalar channels in meson-meson scattering since they do not display a Breit-Wigner shape. Still, many groups[7,8,11,13,15] do find an associated pole in the amplitude, but deep in the complex plane. For a compilation of σ and κ poles see the nice overview in [16]. Let us remark that meson-meson scattering data[17] are hard to obtain, since they are extracted from reactions like meson-N →meson-meson-N, but with assumptions like a factorization of the four meson amplitude, or that only one meson is exchanged and that it is more or less on shell, etc... All these approximations introduce large systematic errors. Very recently, however, other experiments on meson-meson interactions have become available, like the very precise determination of a combination of $\pi\pi$ phase shifts from K_{l4} decays [18], or the results from charm decays [19]. The latter have analyzed the pole structure of their amplitudes and seem to find both the σ and κ poles in reasonable agreement with the works mentioned above but in completely different processes.

The controversy about their spectroscopic nature is even stronger. The interest of unitarized ChPT is that these states are generated from chiral symmetry and unitarity, without any prejudice toward their existence or structure. Amazingly the nine scalars are generated together, suggesting they form an SU(3) multiplet, with a similar composition, although probably mixed with other states when possible.

Once again, ChPT has the advantage that it is the QCD low energy effective theory, and should behave as such in certain limits. For this reason we have studied[1] the large N_c expansion[21], which is the only analytic approximation to QCD in the whole energy region. Remarkably, it provides a clear definition of $\bar{q}q$ states, that become bound states when $N_c \to \infty$, whereas tetraquark or two meson states do not. Starting from the unitarized ChPT meson-meson scattering, and scaling the parameters with N_c according to the QCD rules[22], it has been possible to show that vector mesons follow remarkably well their $q\bar{q}$ expected behavior, whereas all scalar nonet candidates behave as if their main component had a tetraquark or two-meson structure.

In the next two sections we review in more detail the description of meson-meson scattering with unitarized ChPT. In Section 4 we determine the parameters of the poles in those amplitudes, and in Section 5 we will determine how those poles behave in the large N_c limit. Finally we will present some brief conclusions.

2. Meson-meson scattering and Chiral Perturbation Theory

Chiral Perturbation Theory[4,5] (ChPT) is built as the most general derivative expansion of a Lagrangian containing *only* pions, kaons and the eta. These particles are the Goldstone bosons associated to the spontaneous chiral symmetry breaking of massless QCD. In practice, and for meson-meson scattering, ChPT becomes an expansion in even powers of energy or momenta, denoted as $O(p^2), O(p^4)$, etc..., over the scale of the spontaneous breaking, i.e.,$4\pi f_0 \simeq 1.2\,\text{GeV}$. Of course, quarks are not massless, but their masses are small compared with typical hadronic scales, and they are introduced as perturbations in the same power counting, giving rise to the π, K and η masses, counted as $M = O(p^2)$. The main advantage of ChPT is that it provides a Lagrangian that allows for true Quantum Field Theory calculations, and a chiral power counting to organize *systematically* the size of the corrections at low energies. For example, in the isospin limit, the leading order Lagrangian is universal since it only depends on f_0, the pion decay constant in the chiral limit, and the leading order masses M^0_π, M^0_K and M^0_η. However, it is possible to calculate meson loops, whose divergences are renormalized in a finite set of chiral parameters at each order in the expansion. The dependence on the QCD dynamics only comes through higher order parameters. For instance, meson-meson scattering to one loop depends on eight L_i parameters[5], whose values are shown in Table 1. As usual after renormalization, they depend on a regularization scale μ:

$$L_i(\mu_2) = L_i(\mu_1) + \frac{\Gamma_i}{16\pi^2} \log \frac{\mu_1}{\mu_2}, \tag{1}$$

where Γ_i are constants[4]. Of course, in physical observables the μ dependence is canceled through the regularization of the loop integrals. This procedure can be repeated obtaining finite results at each order. As long as we remain at low energies, only a few orders are necessary and the theory is predictive, since once the set of parameters up to that order is fixed from some experiments, it *should* describe, to that order, any other physical process involving mesons.

Table 1. $O(p^4)$ chiral parameters ($\times 10^3$) and their N_c scaling. In the ChPT column, L_1, L_2, L_3 come from[23] and the rest from[4]. The IAM columns correspond to different fits[3]

$O(p^4)$ Parameter	N_c Scaling	ChPT $\mu = 770\,\text{MeV}$	IAM I $\mu = 770\,\text{MeV}$	IAM II $\mu = 770\,\text{MeV}$	IAM III $\mu = 770\,\text{MeV}$
L_1	$O(N_c)$	0.4 ± 0.3	0.56 ± 0.10	0.59 ± 0.08	0.60 ± 0.09
L_2	$O(N_c)$	1.35 ± 0.3	1.21 ± 0.10	1.18 ± 0.10	1.22 ± 0.08
L_3	$O(N_c)$	-3.5 ± 1.1	-2.79 ± 0.14	-2.93 ± 0.10	-3.02 ± 0.06
L_4	$O(1)$	-0.3 ± 0.5	-0.36 ± 0.17	0.2 ± 0.004	0 (fixed)
L_5	$O(N_c)$	1.4 ± 0.5	1.4 ± 0.5	1.8 ± 0.08	1.9 ± 0.03
L_6	$O(1)$	-0.2 ± 0.3	0.07 ± 0.08	0 ± 0.5	-0.07 ± 0.20
L_7	$O(1)$	-0.4 ± 0.2	-0.44 ± 0.15	-0.12 ± 0.16	-0.25 ± 0.18
L_8	$O(N_c)$	0.9 ± 0.3	0.78 ± 0.18	0.78 ± 0.7	0.84 ± 0.23
$2L_1 - L_2$	$O(1)$	-0.55 ± 0.7	0.09 ± 0.10	0.0 ± 0.1	-0.02 ± 0.10

Other salient features of ChPT are its model independence and that it has been possible to obtain[5,22] from QCD the L_i large N_c behavior, given in Table 1. Also, these parameters contain information about heavier[24] meson states that have not been included as degrees of freedom in ChPT.

However, ChPT is limited to low energies, since the number of terms allowed by symmetry increases dramatically at each order, because the ChPT series violate unitarity as the energy increases, and finally, due to resonances that appear rather soon in meson physics. These states are associated to poles in the second Riemann sheet of the amplitudes that cannot be accommodated by the series of ChPT, which are polynomial (there are also logarithmic terms, irrelevant for this discussion).

For the above reasons, in recent years ChPT it has been extended to higher energies by means of unitarization [6,7,8,9,10,11,3], that we discuss next.

3. Unitarized Chiral Perturbation Theory

In order to compare with experiment it is customary to use partial waves t_{IJ} of definite isospin I and angular momentum J. For simplicity we will omit the I, J subindices in what follows, so that the chiral expansion becomes $t \simeq t_2 + t_4 + ...$, with t_2 and t_4 of $O(p^2)$ and $O(p^4)$, respectively. The unitarity relation for the partial waves t_{ij}, where i, j denote the different available states, is very simple: when two states, say "1" and "2", are accessible, it becomes

$$\mathrm{Im}\, T = T\,\Sigma\, T^* \quad \Rightarrow \quad \mathrm{Im}\, T^{-1} = -\Sigma \quad \Rightarrow \quad T = (\mathrm{Re}\, T^{-1} - i\,\Sigma)^{-1} \tag{2}$$

with

$$T = \begin{pmatrix} t_{11} & t_{12} \\ t_{12} & t_{22} \end{pmatrix} \quad , \quad \Sigma = \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} , \tag{3}$$

where $\sigma_i = 2q_i/\sqrt{s}$ and q_i is the C.M. momentum of the state i. The generalization to more than two accessible states is straightforward in this notation. Note that, since $\mathrm{Im}\, T^{-1}$ is fixed by unitarity *we only need to know the real part of the Inverse Amplitude*. Note that Eq.(2) is non-linear and cannot be satisfied exactly with a perturbative expansion like that of ChPT, although it holds perturbatively, i.e,

$$\mathrm{Im}\, T_2 = 0, \qquad \mathrm{Im}\, T_4 = T_2\,\Sigma\, T_2^* + O(p^6). \tag{4}$$

The use of the ChPT expansion $\mathrm{Re}\, T^{-1} \simeq T_2^{-1}(1 - (\mathrm{Re}\, T_4)T_2^{-1} + ...)$ in eq.(2), guarantees that we reobtain the ChPT low energy expansion and that we are taking into account all the information included in the chiral Lagrangians (both about N_c and about heavier resonances). In practice, all the powers of $1/f_0$ in the amplitudes are rewritten in terms of physical constants f_π or f_K or f_η. At leading order this difference is irrelevant, but at one loop, we have three possible choices for each power of f_0 in the amplitudes, all equivalent up to $O(p^6)$. It is however possible to substitute the f_0's by their expressions in terms of f_π or f_K or f_η in such a way that they cancel the $O(p^6)$ and higher order contributions in eq.(4), so that

$$\mathrm{Im}\, T_2 = 0, \qquad \mathrm{Im}\, T_4 = T_2\,\Sigma\, T_2^*. \tag{5}$$

We will call these conditions "exact perturbative unitarity". From eqs.(2),(5), and the $\mathrm{Re}\, T^{-1}$ ChPT expansion, we find

$$T \simeq T_2(T_2 - T_4)^{-1}T_2, \tag{6}$$

which is the coupled channel IAM that has been used to unitarize simultaneously all the one-loop ChPT meson-meson scattering amplitudes[3]. Since all the pieces are analytic it is straightforward to continue analytically to the complex s plane and look for poles associated to resonances. Indeed, the analytic continuation to the complex s plane has also been justified in terms of dispersion relations in the elastic case[6]. Alternative methods have been proposed and applied successfully to the one loop ChPT, for instance for $\pi\pi$ scattering[25]. However, in this brief review we concentrate just on the IAM due to its remarkable success and simplicity, since it only involves algebraic manipulations on the ChPT series.

The IAM was applied first for partial waves in the elastic region, where a single channel is enough to describe the data. This approach was able to generate[6]. the ρ and σ poles in $\pi\pi$ scattering and that of K^* in $\pi K \to \pi K$. Later on, it has been noticed that the κ pole is also obtained within the single channel formalism. Concerning coupled channels, since only the the $\pi\pi$, πK and $\pi\eta$ amplitudes[4,26] were known at that time, at first[8] only the leading order and the dominant s-channel loops were considered, neglecting crossed and tadpole loop diagrams. Despite these approximations, a remarkable description of meson-meson scattering was achieved up to 1.2 GeV, generating the poles associated to the ρ, K^*, f_0, a_0, σ and κ. The price to pay was, first, that only the leading order of the expansion was recovered at low energies. Second, apart from the fact that loop divergences were regularized with a cutoff, thus introducing an spurious parameter, they were not completely renormalized, due to the missing diagrams. Therefore, it was not possible to compare the L_i, which encode the underlying QCD dynamics, with those already present in the literature. In addition, the study of the large N_c limit is cumbersome due to the incomplete renormalization and since we do not know how the cutoff should scale.

As already explained the whole approach is relevant to study the existence and properties of light scalars and it is then very important to check that these poles and their features are not just artifacts of the approximations, estimate the uncertainties in their parameters, and check their compatibility with other data. These considerations triggered the interest in calculating and unitarizing the remaining meson-meson amplitudes within one-loop ChPT. Hence, the $K\bar{K} \to K\bar{K}$ calculation was completed[9], thus allowing for the unitarization of the $\pi\pi$, $K\bar{K}$ coupled system. There was a good agreement of the IAM description with the existing L_i, reproducing again the resonances in that system. More recently, we have completed[3] the one-loop meson-meson scattering calculation, including three new amplitudes: $K\eta \to K\eta$, $\eta\eta \to \eta\eta$ and $K\pi \to K\eta$, but recalculating the other five amplitudes, unifying the notation, ensuring " exact perturbative unitarity", Eq.(5), and also correcting some errors in the literature. Next, we have unitarized these amplitudes with the coupled channel IAM, which allows for a direct comparison with the standard low-energy chiral parameters, in very good agreement with previous determinations. In that work we presented the full calculation of all the one-loop amplitudes in dimensional regularization, and a simultaneous description of the low energy and the resonance regions.

The first check was to use the standard ChPT parameters, given in Table 1 to see if the resonant features were still there, at least qualitatively, and they are. Thus, they are not just an artifact of the approximations and of the values chosen for the parameters. As already commented, this comparison can only be performed now since we have all the amplitudes renormalized in the standard $\overline{MS}-1$ scheme.

After checking that, we made an IAM fit. Systematic uncertainties are the largest contribution to the resulting error bands as well as in the fit parameters in Table 1. These error bands are calculated from a MonteCarlo Gaussian sampling[3] (1000 points) of the L_i sets within their uncertainties, assuming they are uncorrelated. Note that in Table 1 we list three sets of parameters for the IAM fit, fairly compatible among them and with those of standard ChPT. They correspond to different choices when reexpressing the f_0 parameter of the Lagrangian in terms of physical decay constants. The IAM I fit was obtained[3] using just f_π, which is simpler but unnatural when dealing with kaons or etas. There, it could be observed that the $f_0(980)$ region was not very well described yielding a too small width for the resonance.

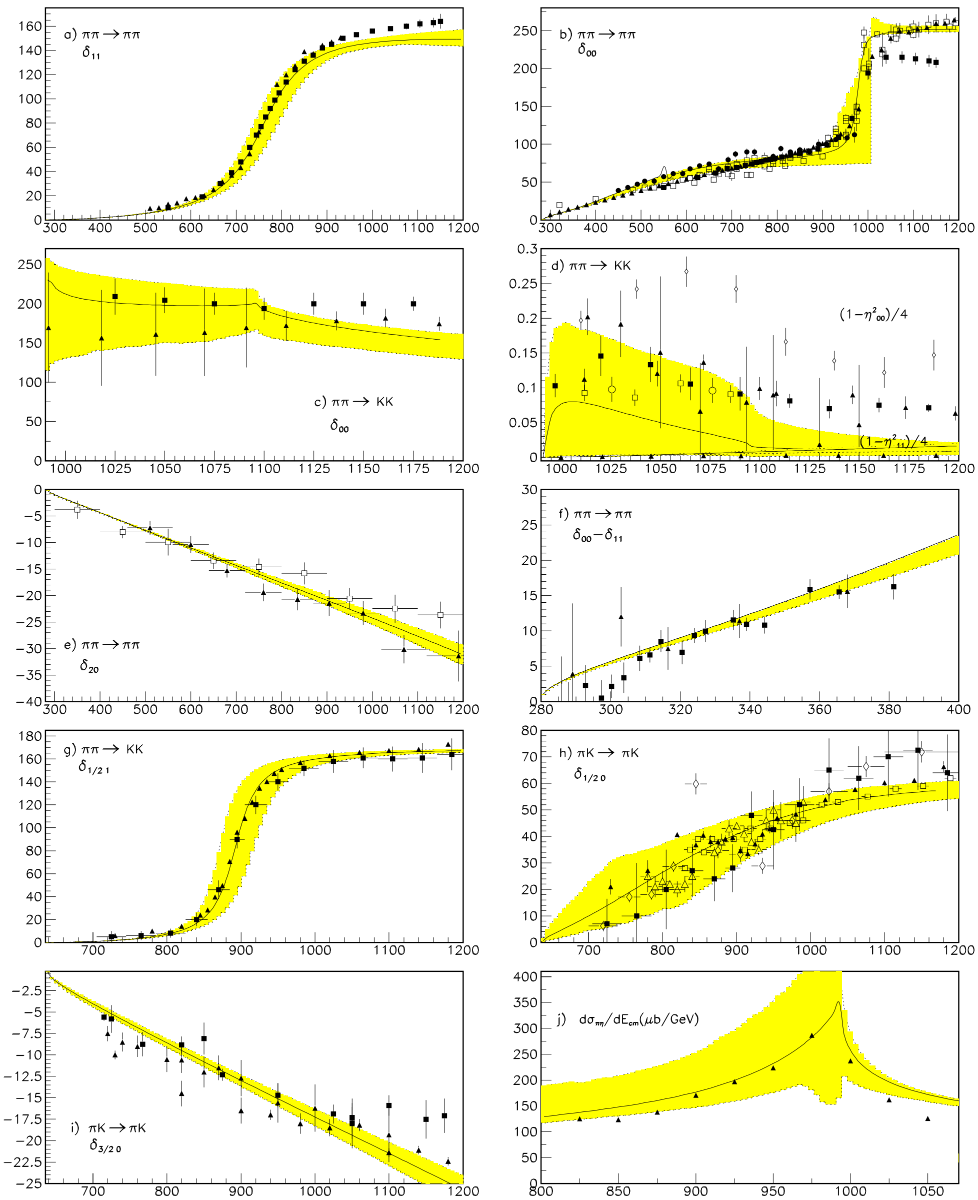

Fig. 1. IAM fit to meson-meson scattering data, set II in Table 1. The uncertainty bands are dominated by systematic errors in the data[17].

For that reason, Fig.1 shows the results of a second fit[2] (IAM II) using amplitudes written in terms of f_K and f_η when dealing with processes involving kaons or etas. Let us remark that the data in the $f_0(980)$ region is well within the uncertainties. In particular, in the $O(p^2)$ one factor of $1/f_\pi$ has been replaced[2] by $1/f_K$

for each two kaons present between the initial or final state, or by $1/f_\eta$ for each two etas appearing between the initial and final states. In the special case $K\eta \to K\pi$ $1/f_\pi^2$ has been changed by $1/(f_K f_\eta)$. The difference between the two ways of writing the leading order amplitudes is $O(p^4)$, and is therefore included in the next to leading order contribution using the relations[5,3] between the decay constants and f_0. The $1/f_0$ factors in each loop function at $O(p^4)$ (generically, the $J(s)$ given in the appendix of [3]) are changed to satisfy "exact perturbative unitarity", eqs.(5). According to the ChPT counting, the amplitudes are the same up to $O(p^4)$, but numerically they are slightly different. A similar choice has been suggested[27] independently within non-unitarized standard SU(3) ChPT to avoid the uncertainties arising from fluctuation of vacuum $s\bar{s}$ pairs. In particular, it is suggested to obtain elastic amplitudes A_{PQ} from the safe combinations $F_\pi^4 A_{\pi\pi}$ or $F_\pi^2 F_K^2 A_{\pi K}$, etc... For external fields this amounts to our choice, and the normalization of internal loop function is then dictated by exact perturbative coupled channel unitarity. From Table 1, we see that the only sizable change is in the parameters related to the decay constants, i.e., L_4 and L_5. For illustration we give in Table 1 a third fit, IAM III, obtained as IAM II but fixing $L_4 = 0$, its large N_c limit, as it is done in recent K_{l4} $O(p^4)$ determinations. As seen in Table 1, these chiral parameters are compatible with those from standard ChPT, which means that we have a simultaneous description of the low energy and resonance regime. Finally, since the expressions are fully renormalized, all the QCD N_c dependence appears correctly through the L_i and cannot hide in any spurious parameter.

At this point one might wonder about higher order effects. There is a simple way to extend the IAM to higher orders[6], first applied to two loop $\pi\pi$ scattering and the pion form factor[28], with remarkable results. This study has been extended[29] with a careful analysis of the uncertainties. The amplitude depends on the $O(p^4)$ and $O(p^6)$ parameters through six combinations, called b_i. Despite the poor knowledge about these two-loop parameters a good description of the data is found[29], including the σ and ρ regions, with parameters compatible within errors with those in the literature. The error analysis[29] is also of relevance because it was shown that the IAM crossing violations, *taking into account the present experimental uncertainties*, are " not very large in percentage terms".

4. Poles associated to resonances

The most interesting feature of chiral unitary approaches is that the poles thus generated are not included in the original ChPT Lagrangian and hence appear without theoretical prejudices toward their existence, classification in multiplets, or nature. Remarkably, the scalar resonances $\sigma, \kappa, a_0(980)$ and $f_0(980)$ appear together in those chiral unitarized amplitudes, and it seems natural to interpret them as a nonet (see also[30]). Nevertheless, we should distinguish two different resonance generation mechanisms: it had already been noted[11], that to generate the scalars just the leading order and a cutoff was enough, whereas the vector mesons require the chiral parameters, particularly[7,8] L_1, L_2 and L_3. Of course, the chiral parameters are always present, but changes within errors in their values do not affect the existence of the light scalar poles, but just the details of their description. Since the vectors are fairly well established $q\bar{q}$ states, this difference suggests that scalars like the σ, κ, etc, may have a different nature. With the amplitudes described in the previous sections we expect to reach a more conclusive statement.

Thus, in Table 2 we show the pole position for the resonances[2], including uncertainties, for the different IAM parameter sets given in Table 1. For comparison we also provide those obtained in the "approximated" IAM[8], whereas we list in Table 3 the poles listed presently in the PDG[12]. These results deserve some comments:

- The vectors $\rho(770)$ and $K^*(892)$, are very stable within chiral unitary approaches. Their positions are almost the same irrespective of whether one uses the single channel[6], the approximated coupled channel[8] or the complete IAM[3].
- The σ and κ pole positions are robust within these approaches. No matter what version of the IAM is used. Note the small uncertainties in some of their parameters, in very good agreement with recent experiments[19].
- The $f_0(980)$ has a sizable decay to two different channels and therefore it can only be studied in a coupled channel formalism. In practice, both the approximated and complete IAM generate a pole associated to this state at approximately the same mass. However, as already remarked, the unitarized ChPT amplitudes using just[3] f_π, yield a too narrow width. The good news is that it can be well accommodated[2] using f_π, f_K and f_η.
- The $a_0(980)$ also requires a coupled channel formalism, and the data on this region is well described either by the approximated or the complete IAM. However, the presence of a pole is strongly dependent on whether we write the ChPT amplitudes only in terms of f_π, or using the three f_π, f_K and f_η. In the first case, it has been pointed out[31], that the use of the "approximated" IAM with just f_π, favored a "cusp" interpretation of the $a_0(980)$ enhancement in $\pi\eta$ production. With the complete IAM we do not find a pole near the $a_0(980)$ enhancement and indeed the $\pi\eta$ phase-shift does not cross $\pi/2$ and it neither has a fast phase movement. In contrast, when expressing f_0 in terms of f_π, f_K and f_η as described in the previous section, we do find a pole and its associated fast phase movement through $\pi/2$, either with the approximated or complete IAM. Thus, this pole is rather unstable as can be noticed from its large uncertainties in Table 2.
- We are more favorable toward the pole interpretation for the $a_0(980)$ because, first, it is also able to describe better the $f_0(980)$ width. Second, we have already remarked that even within standard SU(3) ChPT, the scattering amplitude calculations are safer against $s\bar{s}$ vacuum fluctuations if these fluctuation terms, related to L_4 and L_6 are absorbed[27] in terms of physical decay constants f_π, f_K and f_η when expanding the anplitudes.

Let us remark that the $f_0(980)$ and $a_0(980)$ resonances are very close to the $K\bar{K}$

Table 2. Pole positions (with errors) in meson-meson scattering. When close to the real axis the mass and width of the associated resonance is $\sqrt{s_{pole}} \simeq M - i\Gamma/2$.

$\sqrt{s_{pole}}$(MeV)	ρ	K^*	σ	f_0	a_0	κ
IAM Approx (no errors)	759-i 71	892-i 21	442-i 227	994-i 14	1055-i 21	770-i 250
IAM I	760-i 82	886-i 21	443-i 217	988-i 4	cusp?	750-i 226
(errors)	± 52± i 25	± 50± i 8	± 17± i 12	± 19± i 3		±18±i 11
IAM II	754-i 74	889-i 24	440-i 212	973-i 11	1117-i 12	753-i 235
(errors)	± 18± i 10	± 13± i 4	± 8± i 15	${}^{+39}_{-127}\ {}^{+i\,189}_{-i\,11}$	${}^{+24}_{-320}\ {}^{+i\,43}_{-i\,12}$	± 52± i 33
IAM III	748-i68	889-i23	440-i216	972-i8	1091-i52	754-i230
(errors)	± 31± i 29	± 22± i 8	± 7± i 18	${}^{+21}_{-56}$± i 7	${}^{+19}_{-45}\ {}^{+i\,21}_{-i\,40}$	± 22± i 27

Table 3. Mass and widths or pole positions of the light resonances quoted in the PDG[12] Recall that for narrow resonances $\sqrt{s_{pole}} \simeq M - i\Gamma/2$

PDG2004	$\rho(770)$	$K^*(892)^\pm$	σ or $f_0(600)$	$f_0(980)$	$a_0(980)$	κ
Mass (MeV)	775.8 ± 0.5	891.66 ± 0.26	(400-1200)-i (300-500)	980 ± 10	984.7 ± 1.2	not
Width (MeV)	150.3 ± 1.6	50.8 ± 0.9	(we list the pole)	40-100	50-100	listed

threshold, which can induce a considerable distortion in the resonance shape, whose relation to the pole position could be far from that expected for narrow resonances. In addition these states have a large mass and it is likely that their nature should be understood from a mixture with heavier states.

5. Resonance Behavior in the large N_c limit

First of all, let us recall that QCD not only predicts $\bar{q}q$ states to become bound states in the $N_c \to \infty$ limit, but it tells us exactly *what is the large N_c dependence of their mass and width*: their mass should remain constant, whereas their width vanishes as $1/N_c$. For instance, the π, K, η masses behave as $O(1)$ and f_0, their decay constant in the chiral limit, as $O(\sqrt{N_c})$.

The N_c scaling of the L_i parameters[4,22] is listed in Table 1. Let us then scale $f_0 \to f_0\sqrt{N_c/3}$ and $L_i(\mu) \to L_i(\mu)(N_c/3)$ for $i = 2, 3, 5, 8$, keeping the masses, $2L_1 - L_2, L_4, L_6$ and L_7 constant. We use IAM set II in Table 1 for the parameters at $N_c = 3$. In Fig. 2, we show how the poles, represented by a dot, move in the lower half of the complex plane as N_c changes. On top, we see the $\rho(770)$ and $K^*(892)$ vector mesons, whose poles move toward the real axis. That is, the vectors widths vanish at large N_c, thus becoming bound states. In contrast, in the bottom, we see that both the σ and κ poles move *away* from the real axis, which means that they dissolve in the continuum.

Not only we can see that vectors become bound states and scalars do not, but we can determine what is the precise N_c dependence. Before doing that, we should nevertheless recall that it is not known at what scale μ to apply the large N_c to the L_i. Certainly, the logarithmic part in eq.(1) is subleading, but it has been pointed out[32] that it can be rather large for $N_c = 3$, which is our starting point for the N_c evolution. In addition, the scale dependence is suppressed by $1/N_c$ for

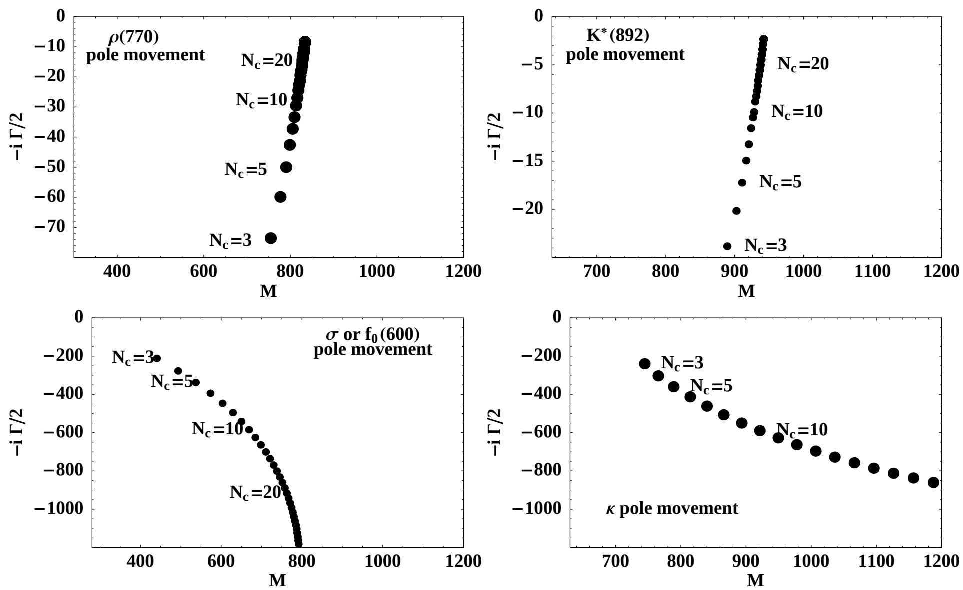

Fig. 2. Large N_c behavior of poles in the lower half of the second Riemann sheet of the unitarized one-loop meson-meson scattering amplitude from ChPT. For each value of N_c the pole is represented by a dot, in different meson-meson scattering channel. Note that the σ and κ behavior is opposite to that of well know vector states as the ρ and K^*.

$L_i = L_2, L_3, L_5, L_8$, but not for $2L_1 - L_2, L_4, L_6$ and L_7. The choice μ, or, in other words, the choice of "initial values", is a systematic uncertainty typical of the large N_c approach. Customarily, the uncertainty in the μ where the N_c scaling applies is estimated[4] varying μ between 0.5 and 1 GeV. We will check that this estimate is correct with the vector mesons, firmly established as $\bar{q}q$ states. In addition, and in order to show that the choice of IAM sets in Table 1 is irrelevant for our analysis, we change now to set III. The behavior is exactly that already found in Fig.2 and in previous works[1,2].

Let us then look back at vector mesons. In Fig.3 we show, for increasing N_c, the modulus of the $(I, J) = (1, 1)$ and $(1/2, 1)$ amplitudes with the Breit-Wigner shape of the ρ and $K^*(892)$, respectively. There is always a peak at an almost constant position, becoming narrower as N_c increases. We also show the evolution of the ρ and K^* pole positions, related to their mass and width as $\sqrt{s_{pole}} \simeq M - i\Gamma/2$. We have normalized both M and Γ to their value at $N_c = 3$ in order to compare with the expected $\bar{q}q$ behavior: M_{N_c}/M_3 constant and $\Gamma_{N_c}/\Gamma_3 \sim 1/N_c$ The agreement is remarkable within the gray band that covers the uncertainty $\mu = 0.5 - 1\,\mathrm{GeV}$ where to apply the large N_c scaling. We have checked that outside that band, the behavior starts deviating from that of $\bar{q}q$ states, which confirms that the expected scale range where the large N_c scaling applies is correct.

In Fig.4, in contrast, all over the σ and κ regions the $(0, 0)$ and $(1/2, 0)$ amplitudes decrease as $N_c \to \infty$. Their associated poles show a totally different behavior,

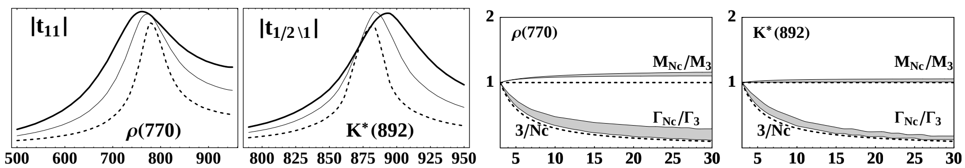

Fig. 3. Left: Modulus of $\pi\pi$ and πK elastic amplitudes versus $\sqrt{s}$ for $(I,J)=(1,1),(1/2,1)$: $N_c=3$ (thick line), $N_c=5$ (thin line) and $N_c=10$ (dotted line), scaled at $\mu=770\,\mathrm{MeV}$. Right: $\rho(770)$ and $K^*(892)$ pole positions: $\sqrt{s_{pole}}\equiv M-i\Gamma/2$ versus N_c. The gray areas cover the uncertainty $N_c=0.5-1\,\mathrm{GeV}$. The dotted lines show the expected $\bar{q}q$ large N_c scaling.

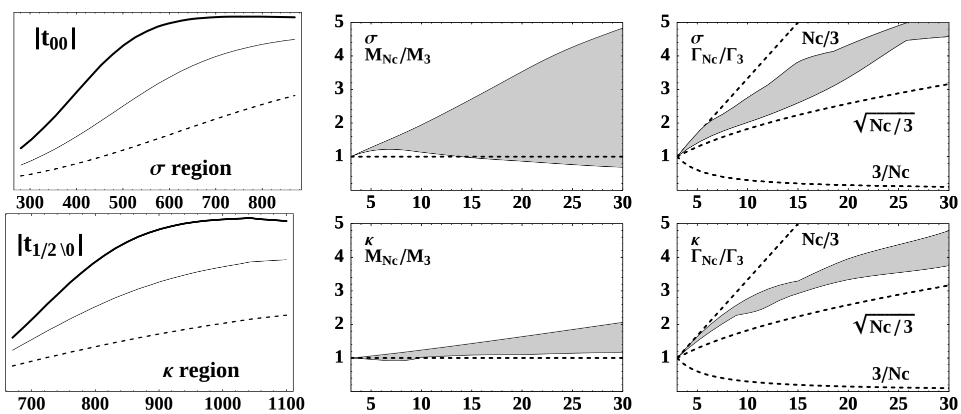

Fig. 4. Top) Left: Modulus of the $(I,J)=(0,0)$ amplitude, versus $\sqrt{s}$ for $N_c=3$ (thick line), $N_c=5$ (thin line) and $N_c=10$ (dotted line), scaled at $\mu=770\,\mathrm{MeV}$. Center: σ mass N_c behavior. Right: σ width N_c behavior. Bottom: The same but for the $(1/2,0)$ amplitude and the κ.

since *their width grows with* N_c, in conflict with a $\bar{q}q$ interpretation. (We keep the M, Γ notation, but now as definitions). This is also suggested using the ChPT leading order unitarized amplitudes with a regularization scale[11,33].

In order to determine their spectroscopic nature, we abandon momentarily the poles and look at the amplitudes in the real axis and the representative Feynman diagrams given in in Fig. 5. Imaginary parts are generated from s-channel intermediate states, as in Fig.5(a) or 5(c), when they are physically accessible. If there was a $\bar{q}q$ meson, with mass $M\sim O(1)$ and $\Gamma\sim 1/N_c$, we would expect at precisely $\sqrt{s}\simeq M$ that $\mathrm{Im}\,t\sim O(1)$ and a peak in the modulus, as it is indeed the case of the ρ and K^*. However, we have checked that *in the whole* σ and κ regions, $\mathrm{Im}\,t\sim O(1/N_c^2)$ and $\mathrm{Re}\,t\sim O(1/N_c)$, which means that from $\bar{q}q$ states, the σ and κ can only get real contributions from ρ or K^* t-channel exchange, respectively, as in Fig.5(b). The leading s-channel contribution for the κ and σ comes from Fig.5(c).

Then, one can immediately interpret the κ as a $\bar{q}\bar{q}qq$ (or two meson) state, which is predicted[34] to unbound and become the meson-meson continuum when $N_c\to\infty$. However, in the large N_c limit, $\bar{q}\bar{q}qq$ and glueball exchange count the same, and our N_c argument *alone* cannot decide between both structures for the σ. Nevertheless,

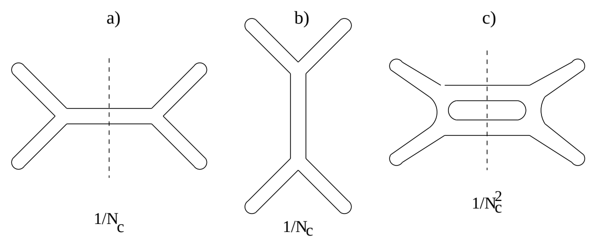

Fig. 5. Representative diagrams contributing to meson-meson scattering and their N_c scaling.

given the fact that glueballs are expected to have masses above 1 GeV, and that the κ, which has strangeness and thus cannot mix with a glueball, is a natural $SU(3)$ partner of the σ, a dominant $\bar{q}\bar{q}qq$ or two-meson component for the σ seems the most natural interpretation, although most likely with some glueball mixing.

Let us finally turn to the other members of the hypothesized scalar nonet: the $f_0(980)$ and the $a_0(980)$. These resonances are more complicated due to the distortions caused by the nearby $\bar{K}K$ threshold, and poles are harder to follow. Still, by looking in Fig.6 at the modulus of the amplitude $(0,0)$ in the vicinity of the $f_0(980)$, we see that the characteristic sharp dip of the $f_0(980)$ vanishes when $N_c \to \infty$, at variance with a $\bar{q}q$ state. For $N_c > 5$ it follows again the $1/N_c^2$ scaling compatible with $\bar{q}\bar{q}qq$ states or glueballs. The $a_0(980)$ behavior, shown in Fig.7, is more complicated. When we apply the large N_c scaling at $\mu = 0.55 - 1$ GeV, its peak disappears, suggesting that this is not a $\bar{q}q$ state, and $\mathrm{Im}\, t_{10}$ follows roughly the $1/N_c^2$ behavior in the whole region. However, as shown in Fig.5, the peak does not vanish at large N_c if we take $\mu = 0.5\,$GeV. Thus we cannot rule out a possible $\bar{q}q$ nature, or a sizable mixing with $\bar{q}q$, although it shows up in an extreme corner of our uncertainty band.

Before concluding we want to remark that similar results have been obtained[35] for all resonances and just for central values of the L_i plus a cutoff, using the approximated IAM[8]. Furthermore, either the tetraquark structure of these light scalar states, or the fact that the lightest $\bar{q}q$ states are above ~1 GeV, has received further support from other large N_c studies[33,36] as well as other kind of analysis[37].

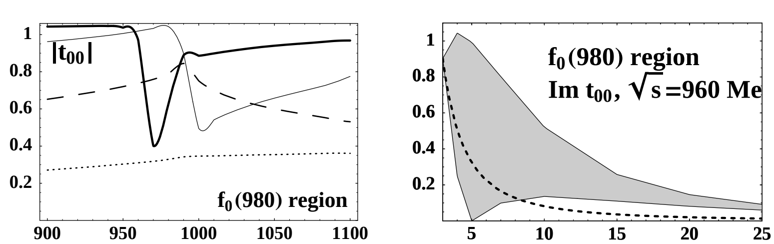

Fig. 6. Left: Modulus of a $(I,J) = (0,0)$ amplitude, versus $\sqrt{s}$, for $N_c = 3$ (thick), $N_c = 5$ (thin), $N_c = 10$ (dashed) and $N_c = 25$ (dotted), scaled at $\mu = 770\,$MeV. Right: $\mathrm{Im}\, t_{00}$ versus N_c.

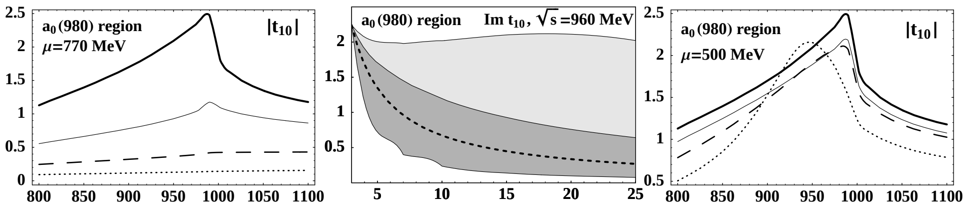

Fig. 7. Left: Modulus of a $(I, J) = (1, 0)$ scattering amplitude, versus $\sqrt{s}$, for $N_c = 3$ (thick), $N_c = 5$ (thin), $N_c = 10$ (dashed) and $N_c = 25$ (dotted), scaled at $\mu = 770\,\text{MeV}$. Right: scaled at $\mu = 500\,\text{MeV}$. Center: $\text{Im}\, t_{00}$ versus N_c. The dark gray area covers the uncertainty $\mu = 0.55 - 1\,\text{GeV}$, the light gray area from $\mu = 0.5$ to $0.55\,\text{GeV}$.

6. Conclusions

We have reviewed a recent set of works[1,2,3] in which we show how the unitarized one-loop Chiral Perturbation Theory (ChPT) amplitudes generate poles associated to the lightest vector and scalar resonances and we study their large N_c behavior. This amplitudes respect the chiral expansion and are fully renormalized. Indeed they provide a remarkable description of two body scattering of pions, kaons and etas up to 1.2 GeV using just the $O(p^4)$ ChPT parameters, with values compatible with previously existing determinations in the literature.

We have then studied[1] the evolution of the poles, mass and width associated to each one of these resonances, through the QCD large N_c scaling inherited by the ChPT parameters. We have found that the ρ and K^* vector mesons follow remarkably well their expected $\bar{q}q$ behavior, both qualitatively and quantitatively. In contrast, the σ, κ, $f_0(980)$ and $a_0(980)$ large N_c behavior is in conflict with a $\bar{q}q$ nature (not so conclusively for the $a_0(980)$), and strongly suggests a $\bar{q}\bar{q}qq$ or two meson main component, maybe with some mixing with glueballs, when possible.

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References

1. J. R. Pelaez, Phys. Rev. Lett. **92** (2004) 102001, hep-ph/0307018 and hep-ph/0306063.
2. J. R. Pelaez and A. Gomez Nicola, AIP Conf. Proc. **660** (2003) 102, hep-ph/0301049.
3. A. Gómez Nicola and J. R. Peláez, Phys. Rev. D **65** (2002) 054009 and
4. S. Weinberg, Physica **A96** (1979) 327. J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142;
5. J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.
6. T. N. Truong, Phys. Rev. Lett. **61** (1988) 2526. Phys. Rev. Lett. **67**, (1991) 2260; A. Dobado, M.J.Herrero and T.N. Truong, Phys. Lett. **B235** (1990) 134.

7. A. Dobado and J. R. Pelaez, Phys. Rev. D **47** (1993) 4883. Phys. Rev. D **56** (1997) 3057.
8. J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. **80** (1998) 3452; Phys. Rev. D **59** (1999) 074001 and Phys. Rev. D **62** (2000) 114017.
9. F. Guerrero and J. A. Oller, Nucl. Phys. B **537** (1999) 459 [Erratum-ibid. B **602** (2001) 641].
10. J. A. Oller and E. Oset, Nucl. Phys. A **620** (1997) 438;
11. J. A. Oller and E. Oset, Phys. Rev. D **60** (1999) 074023
12. S. Eidelman, *et al.*, Phys. Lett. **B592**,1 (2004).
13. R. Kaminski, L. Lesniak and J. P. Maillet, Phys. Rev. D **50** (1994) 3145. R. Delbourgo and M. D. Scadron, Mod. Phys. Lett. A **10** (1995) 251. S. Ishida *et al.*, Prog. Theor. Phys. **95** (1996) 745. M. Harada, F. Sannino and J. Schechter, Phys. Rev. D **54** (1996) 1991 N. A. Tornqvist and M. Roos, Phys. Rev. Lett. **76** (1996) 1575.
14. A. Bramon and R. Escribano, JHEP **0402**, 048 (2004). Z. Y. Zhou,*et al.* hep-ph/0406271.
15. R.L. Jaffe, Phys. Rev. **D15** 267 (1977); Phys. Rev. **D15**, 281 (1977). E. van Beveren *et al.* Z. Phys. **C30**, 615 (1986). S. Ishida *et al*, Prog. Theor. Phys. 98,621 (1997). D. Black, A. H. Fariborz, F. Sannino, J. Schechter. Phys. Rev. **D58**:054012,1998. E. van Beveren and G. Rupp, Eur. Phys. J. C **22** (2001) 493 H. Q. Zheng,*et al.* Nucl. Phys. A **733**, 235 (2004)
16. E. van Beveren and G. Rupp, hep-ph/0201006.
17. S. D. Protopopescu *et al.*, Phys. Rev. **D7**, (1973) 1279; P. Estabrooks and A.D.Martin, Nucl.Phys.**B79**, (1974) 301. G. Grayer *et al.*, Nucl. Phys. **B75**, (1974) 189. D. Cohen, Phys. Rev. **D22**, (1980) 2595. W. Hoogland *et al.*, Nucl. Phys. **B126** (1977) 109. M. J. Losty *et al.*, Nucl. Phys. **B69** (1974) 185. R. Mercer *et al.*, Nucl. Phys. **B32** (1971) 381. P. Estabrooks *et al.*, Nucl. Phys. **B133** (1978) 490. H. H. Bingham *et al.*, Nucl. Phys. **B41** (1972) 1. S. L. Baker *et al.*, Nucl. Phys. **B99** (1975) 211. D. Aston *et al.* Nucl. Phys. **B296** (1988) 493. D. Linglin *et al.*, Nucl. Phys. **B57** (1973) 64 .
18. S. Pislak *et al.* [BNL-E865 Collaboration], Phys. Rev. Lett. **87** (2001) 221801.
19. E791 Collaboration, Phys. Rev. Lett. **86**,(2001) 770. E. M. Aitala *et al.* [E791 Collaboration], Phys. Rev. Lett. **89** (2002) 121801. I. Bediaga and J. M. de Miranda, hep-ex/0405019.
20. M. Ablikim *et al.* [BES Collaboration], Phys. Lett. B **598**, 149 (2004) D. V. Bugg, Phys. Lett. B **572**, 1 (2003) [Erratum-ibid. B **595**, 556 (2004)], Phys. Rept. **397**, 257 (2004).
21. G. 't Hooft, Nucl. Phys. B **72** (1974) 461. E. Witten, Annals Phys. **128** (1980) 363.
22. A. A. Andrianov, Phys. Lett. B **157**, 425 (1985). A. A. Andrianov and L. Bonora, Nucl. Phys. B **233**, 232 (1984). D. Espriu, E. de Rafael and J. Taron, Nucl. Phys. B **345** (1990) 22 S. Peris and E. de Rafael, Phys. Lett. B **348** (1995) 539
23. J. Bijnens, G. Colangelo and J. Gasser, Nucl. Phys. **B427** (1994) 427.
24. G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321**, 311 (1989).
25. J. Nieves and E. Ruiz Arriola, Nucl. Phys. A **679** (2000) 57
26. V. Bernard, N. Kaiser, U.G. Mei*ss*ner, Phys. Rev. **D43** (1991) 2757; Nucl. Phys. **B357** (1991) 129; Phys. Rev. **D44** (1991) 3698.
27. S. Descotes-Genon, N. H. Fuchs, L. Girlanda and J. Stern, hep-ph/0311120.
28. T. Hannah, Phys. Rev. D **55** (1997) 5613
29. J. Nieves, M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. D **65** (2002) 036002
30. D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. D **59**, 074026 (1999)
31. M. Uehara, hep-ph/0204020.
32. A. Pich, hep-ph/0205030.
33. M. Harada, F. Sannino and J. Schechter, Phys. Rev. D **69**, 034005 (2004)
34. R. L. Jaffe, Proceedings of the Intl. Symposium on Lepton and Photon Interactions at High Energies. Physikalisches Institut, University of Bonn (1981) . ISBN: 3-9800625-0-3
35. M. Uehara, hep-ph/0308241. hep-ph/0401037. hep-ph/0404221.
36. V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, JHEP **0306**, 012 (2003) N. N. Achasov, Nucl. Phys. A **728**, 425 (2003) T. Schafer, Phys. Rev. D **68**, 114017 (2003) L. Maiani, *et al.* hep-ph/0407017.
37. S. Prelovsek, C. Dawson, T. Izubuchi, K. Orginos and A. Soni, hep-lat/0407037. J. Vijande, F. Fernandez and A. Valcarce, hep-ph/0308318, hep-ph/0309319. V. Baru, *et al.* Phys. Lett. B **586**, 53 (2004)