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On nuclear effects in proton decay

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A previous work about inclusive proton decay in a nucleus is critically analyzed. Explicit new calculations are carried out in the deuteron, which turn out to be consistent with previous evaluations by other authors. By relating the inclusive decay rate to the proton Green's function and self-energy in a nucleus, it is shown that it is proportional to A for large enough nuclei. That is, the lifetime per nucleon is an intensive quantity (depending on the nuclear density). Finally, some relationships between inclusive proton decay and one-proton transfer reactions in nuclei are suggested.

In a recent paper, Labastida and Ynduráin proposed a simple and ingenious approach to calculate the total (inclusive) proton decay in a general nucleus of mass number A.[1] Their methods predict a lifetime for a proton bounded in ^{16}O ranging from 1.2×10^{30} to 7×10^{28} yr (the uncertainties came from several sources, the main one being the assumed grand unification mass M_X). As pointed out in Ref. 1, these figures are several times larger than the lifetime of a free proton computed with the same grand unified theory (GUT) parameters. They attribute this peculiar fact to the presence in a nucleus of the three-quark-fusion diagram [Fig. 1(a)], which obviously cannot appear in the decay of a free proton. On the other hand, some extensive calculations for nucleon decay in deuterium and ^{4}He carried out by Dover *et al.*,[2] indicate that nuclear effects, at least in those nuclei, do not imply essential differences between bound and free nucleon decays. Actually, they find that the pure nuclear contribution to the decay width is, at most, $\frac{1}{2}$ of the free width, and this happens for deuteron decay under the assumption (which does not seem very reliable) of no repulsive core between nucleons (see Ref. 2 and our comments below).

Under the preceding circumstances, it seems worthwhile to attempt clarifying the situation and this will be the main purpose of the present paper.

First, for the sake of completeness, we shall summarize the main arguments in Ref. 1. They start from the following effective Lagrangian for proton decay in the framework of SU(5) GUT (see Ref. 3 and references therein for an updated account of GUT's, free nucleon decay, etc.):

$$\mathscr{L}_{\rm eff}(x)=\frac{g_{\rm GUT}{}^2}{2M_X{}^2}\bar{e}^+(x)p(x)\ , \tag{1}$$

where $p(x)$ is a composite operator whose projection over the proton field, $\psi(x)$, is given by

$$p_\alpha(x)=\sum_{\beta=1}^{4}F_{\alpha\beta}\psi_\beta(x)\ ,$$

$$F=\frac{\psi(0,0)}{2^{1/2}}(-5i\gamma_5\gamma^2-7i\gamma^2)\ , \tag{2}$$

$\psi(0,0)$ being the wave function for three quarks in the proton when they are the same point and γ_5,γ^2 are standard Dirac matrices. By reducing the positron and using completeness, they get the following useful formula for the total width of a nucleus A with an outgoing positron with four-momentum k $(k^0,\bar{k})$ (the case of an unobservable neutrino in the final state may be treated in a similar way):

$$\Gamma_A=\frac{g_{\rm GUT}{}^4}{8m_AM_X{}^4}\sum_{\alpha,\beta=1}^{4}\int\frac{d^3\bar{k}}{2k^0}\theta(m_p-|\bar{k}|)\hat{F}_{\alpha\beta}A_s(f)\int d^4x\,\exp ikx\,\langle A\,|\,[\bar{\psi}_\alpha(0),\psi_\beta(x)]\,|\,A\,\rangle\ , \tag{3}$$

where the matrix $\hat{F}=\gamma^0F^\dagger\gamma^0\not{k}F=-F\not{k}F$ and $A_s(f)$ is the enhancement factor.[1]

One of the main points in Ref. 1 is the relationship of Eq. (3) with the imaginary part of forward antiproton-nucleus scattering, which reads [Eq. (6) of Ref. 1]

$$\mathrm{Im}T_{\sigma\sigma}(p,A)=\pi[\bar{v}(p,\sigma)(\not{p}-m_p)]_\alpha \int d^4x \exp(-ipx)\langle A\,|\,[\bar{\psi}_\beta(0),\psi_\alpha(x)]\,|\,A\rangle[(\not{p}-m_p)v(p,\sigma)]_\beta\,. \quad (4)$$

Now the following facts are to be noticed. First, as already pointed out in Ref. 1, k in Eq. (3) fulfills $k^2\simeq 0$, whereas for actual antiprotons $p^2=m_p^2$. Second, and more important, the signs of four-momenta in the exponentials of Eqs. (3) and (4) are opposite. Then, a comparison between both expressions is not possible unless both four-momenta vanish (i.e., $k,p\to 0$ and *not only* $p^2\to 0$). This makes the relationship between proton decay in a nucleus and forward antiproton-nucleus scattering unreliable, because the sign of the exponent is crucial in determining which intermediate states do appear. (Actually, it is this sign which makes intermediate states in pA and $\bar{p}A$ so different, then implying very different forward amplitudes for both reactions.) Consequently, the extrapolation from $p=0$ to the physical threshold cannot be safely done because of the great incertitude about the error involved. In fact, it is easy to see that by identifying the nuclear matrix element in Eq. (3) with the imaginary part of the forward $\bar{p}A$ amplitude, one is overestimating the integral. The reason is that many meson states are included which cannot be produced in nucleon decay [they indeed contribute to $\mathrm{Im}(\bar{p}A\to\bar{p}A)$]. Thus, a decay larger than the actual one is predicted. A similar kind of argument can be applied to demonstrate that inclusive nucleon decay in a nucleus is not related to forward nucleon-nucleus scattering either. Again, the relevant intermediate states are very different. In a "many-body language," we would say that the former reaction explores hole excitations whereas the latter is connected to particle excitation in the nucleus. Summarizing, we hope to have clearly stated that relationships between Eq. (3) and either pA or $\bar{p}A$ forward scattering are not useful because the extrapolations involved could drastically alter the results.

Explicit estimates show that although the term

$$\int d^4x \exp ikx\langle A\,|\,\psi_\beta(x)\bar{\psi}_\alpha(0)\,|\,A\rangle$$

does not vanish exactly, its contribution is always restricted to a very narrow integration interval near $|\bar{k}|=m_p$ (even for large A) and hence it can be regarded as negligible. Consequently, its inclusion in the right-hand side of Eq. (3) does not give rise to quantitative modifications. It is interesting to notice that the nuclear matrix elements of the commutator in Eq. (3) can be related to a Green's function in the limit of a large nucleus ($A\gg 1$) to which a perturbation theory could, in principle, be applied. Also, it will be shown that the said matrix element is the one which appears in the so-called one-proton-transfer reactions in nuclei. All of this will be further discussed below. First, we shall deal with the opposite case of the "simplest complex" nucleus, i.e., deuteron decay.

Let us consider inclusive nucleon decay in the deuteron. Of course, this nucleus is of no practical interest, but it is reasonably expected that the study of nuclear effects in the deuteron will shed some light on nuclear effects in other nuclei, especially in ^{16}O which is indeed the interesting nucleus from an experimental point of view. In deuteron decay, the possible final states $|F\rangle$ are $|ne^+\rangle$, $|ne^+M^0\rangle$, $|pe^+M^-\rangle$, $|n(p)MM\rangle$, ..., etc., where M is a meson with the appropriate charge and we have restricted our treatment to final states with a positron (the case of final states with a neutrino can be dealt with in a similar way, but they are scarcely interesting since the neutrino is unobservable). It is commonly believed (though some controversy exists) that final states with more than a meson are not very relevant, so we will neglect them here. Now the decay $d\to ne^+$ is due to proper nuclear effects, whereas some care is needed when dealing with $d\to ne^+M^0$, $d\to pe^+M^-$. This is so because the main contribution to these reactions comes from "quasifree" nucleon decay, i.e., $p(n)\to e^+M^0(M^-)$ where the nucleon which does not decay participates merely as a spectator. Thus, nuclear effects on "quasifree" decay are due exclusively to the binding energy and therefore are negligible. The proper nuclear effects on the said reactions are of two types: (i) rescattering of the emitted (real) meson by the spectator nucleon, and (ii) rescattering of a virtual meson in the deuteron producing a real meson in the final state. Effect (i) has been studied by Sparrow[4] and since it has no influence on the decay rate will not be considered here any longer. Effect (ii) has been carefully treated in Ref. 2, where the conclusion is reached that its influence on the decay rate is very small.

Let us now study the effect of the three-quark-fusion process of Fig. 1(a) on deuteron decay via the reaction $d\to ne^+$ shown in Fig. 1(b). In this figure, M is any meson which can be exchanged between nucleons in the deuteron. We shall actual-

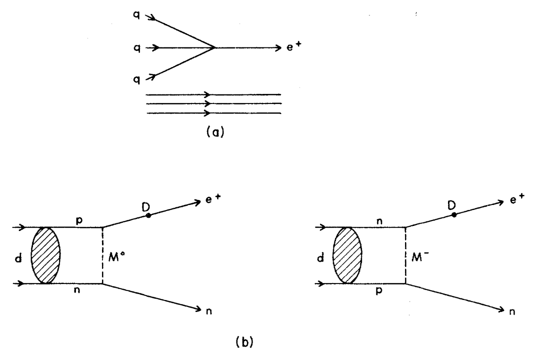

FIG. 1. (a) Diagram of the three-quark-fusion process in a nucleus. (b) The decay $d \rightarrow ne^+$. D shows the "decay point," i.e., where the reaction p(virtual)$\rightarrow e^+$ takes place via the three-quark-fusion process of Fig. 1(a).

ly consider π, η, ρ, and ω mesons. The decay process is governed by the effective Lagrangian of Eq. (1). By means of standard procedures, we compute the diagrams in Fig. 1(b) (coupling constants have been taken from Ref. 5). Then, upon dividing by $\Gamma_d(\text{free}) = \Gamma_p + \Gamma_n$, where $\Gamma_{p(n)}$ refers to the decay width of $p \rightarrow M^0 e^+$ ($n \rightarrow M^- e^+$) (all M's included), we get the quantity of interest to the present paper, i.e.,

$$R_d = \frac{\Gamma(d \rightarrow ne^+)}{\Gamma_d(\text{free})} . \tag{5}$$

As expected, we find that the main contribution to R_d comes from pion exchange, for which R_d can be readily expressed:

$$(R_d)_\pi = \frac{3g^2}{2m_p} |\vec{k}|^2 \frac{|D|^2}{|f(q_f^0, q_f)|^2} , \tag{6}$$

where g ($\simeq 13$) is the πN coupling constant, $|\vec{k}| \simeq \frac{3}{4} m_p$ is the momentum ($\simeq$ energy) of the emitted positron. We have introduced a form factor f in the πNN vertex. q_f^0 and q_f refer, respectively, to the energy and momentum of the pion in free nucleon decay ($N \rightarrow e^+ \pi$). Notice that $q_f^0 \simeq q_f \simeq m_p/2$. Finally D is given by

$$D = \int d^3\vec{k}' \phi_d(\vec{k}') \left[\left[\frac{m_p^2}{16} - m_\pi^2 \right] - |\vec{k} - \vec{k}'|^2 \right]^{-1} \times |f(q^0, q)|^2 , \tag{7}$$

where ϕ_d is the dueteron function in momentum space and q^0 (q) is the energy (momentum) of the exchanged virtual pion ($q^0 = m_p - |\vec{k}|$, $q = |\vec{k} - \vec{k}'|$).

Now let us notice that by setting $f = 1$ in Eq. (7), i.e., we include no form factor, we get the result of Ref. 2, because

$$D(f=1) = \int d^3\vec{k}' \phi_d(\vec{k}') \left[\left[\frac{m_p^2}{16} - m_\pi^2 \right] - |\vec{k} - \vec{k}'|^2 \right]^{-1}$$
$$= \frac{1}{|\vec{k}|} \int d^3 r\, \psi_d(r) \exp(iQ_0 r) \sin(|\vec{k}| r) = \frac{1}{|\vec{k}|} F , \tag{8}$$

where $Q_0 \equiv m_p^2/16 - m_\pi^2$, F being the function used in Ref. 2. Then Eq. (6) reads

$$(R_d)_\pi = \frac{3g^2}{2m_p} |F|^2 \text{ (no form factor)} \tag{6'}$$

which coincides with Eq. (2.5) of Ref. 2.

At this point, however, we would like to stress the convenience of introducing a form factor in the πNN vertex; otherwise, the probability of a pion being absorbed by a nucleon is overestimated. Nevertheless, it is true that if a deuteron wave function with a repulsive core between nucleons is used, the influence of short distances, or equivalently, of large momenta, is reduced. Accordingly, the role of the form factor is then less important than it is when a "soft" function (such as Hulthen's) is employed. For convenience, we recall here the results of Ref. 2 with three different wave functions:

$$(R_d)_\pi = \begin{cases} 0.45, \text{ Hulthen's function, no hard core} , \\ 0.022, \text{ hard core, with } r_c = 0.43 \text{ fm} , \\ 0.052, \text{ hard core, with } r_c = 0.50 \text{ fm} . \end{cases}$$

These figures clearly show the importance of the detailed structure of the deuteron at short distances when no form factor in the πNN vertex is introduced. On the other hand, by introducing the said form factor the contribution of very short distances diminishes considerably. To see this, consider for instance a form factor of monopole type, i.e.,

$$f(q^0,q)\equiv f(t)=\frac{\Lambda^2-m_\pi{}^2}{\Lambda^2-t}(\Lambda\simeq 0.85\ \text{GeV})\ . \tag{9}$$

By using this form factor we get

$$D=\frac{1}{|\vec{k}|}\int_0^\infty dr\,\psi_d(r)\sin(|\vec{k}|r)\left[\exp iQ_0 r-\exp(-\Lambda r)\left[1+\frac{\Lambda r}{2}\right]\right]. \tag{10}$$

Thus, for ReD [Re$D \gg$ ImD, so that $|D|^2\simeq(\text{Re}D)^2$] the factor multiplying ψ_d in the integrand goes to zero as r^2 when $r\to 0$, instead of linearly as it does in Eq. (8). Therefore, the contribution of very short distances is considerably reduced as compared with the case of no form factor. Of course, a small contribution still remains and, furthermore, we meet now the problem of disposing of a reliable form factor. However, it is shown below that three different form factors which describe reasonably well NN and πN low-energy interactions produce similar results for R_d, so that a very precise knowledge of the πNN vertex function is not necessary in practice.

The form factors mentioned above are as follows.

(1) "Monopole". See Eq. (9).

(2) "Veneziano-type"[6]:

$$f(q^0,q)\equiv f(t)=\frac{2}{[2-\alpha(t-m_\pi{}^2)][1-\alpha(t-m_\pi{}^2)]}\ , \tag{11}$$

where $\alpha=0.83\ \text{GeV}^{-2}$ is the universal slope of Regge trajectories, and we have set the parameter β (see Ref. 6) as $\beta=3$.

(3) "Cloudy-bag" model[7] (CBM):

$$f(q^0,q)\equiv f(t)=\frac{3j_1(qR)}{qR}\ , \tag{12}$$

where $R=0.82$ fm is the bag radius and j_1 is a spherical Bessel function.

Upon using these form factors, we obtain the results shown in Table I, where the numbers in parentheses refer to the corresponding contributions of pion exchange. In all three cases, a Hulthen's wave function has been used as in Ref. 2, and the form factor for η, ρ, and ω mesons is the same as that for the pion. Of course, this last assumption has no theoretical base, but because of the small contribution of these mesons, it is harmless in practice.

From Table I, it can be concluded that nuclear effects on nuclear decay in the deuteron are likely to represent less than 15% of the decay rate. Had we used a wave function with a hard core, we would get a smaller value for R_d. For instance, by using a wave function with a core radius $r_c=0.5$ fm (see Ref. 2) and a monopole form factor, we get for the pion contribution, $(R_d)_\pi=0.046$. This is to be compared with 0.10 that we get with Hulthen's function (Table I), and with 0.052, obtained in Ref. 2 with a wave function with the same hard core and no form factor. The latter figures justify our preceding comments about form factors and short-distance behavior of the deuteron wave function. Concerning the decays $d\to ne^+M^0$ and $d\to pe^+M^-$, a similar treatment can be developed. However, as already shown in Ref. 2, nuclear effects on these reactions are very small (less than 3% on the decay width), so that they can be safely neglected.

We now consider the case of a large nucleus ($A\gg 1$). Here, our main results will be (1) to establish the announced relationship of the total width Γ_A to the proton Green's function or self-energy in the large nucleus, regarded as a many-

TABLE I. Values of R_d for three different form factors (FF's) and without any form factor (see text). Numbers in parentheses are the contributions of pion exchange.

	No FF	FF (1)	FF (2)	FF (3)
R_d	0.56	0.11	0.14	0.10
	(0.45)	(0.10)	(0.12)	(0.07)

body system for which perturbation-theoretic techniques are available in principle, and (2) to show that, for large A, the lifetime per nucleon $\tau=A/\Gamma_A$ is independent of A, i.e., it is an intensive quantity instead of behaving as proportional to $A^{1/3}$ as proposed in Ref. 1. (See, however, footnote 9 in Ref. 1.)

We shall start by rewriting Eq. (3) as follows ($k^0 \simeq |\bar{k}|$):

$$\Gamma_A = \frac{g_{\rm GUT}{}^4}{4M_X{}^2} V A_s(f) \int \frac{d^3\bar{k}}{2k^0} \frac{1}{(2\pi)^3} \theta(m_p - |\vec{k}|) \times \left\{ \sum_{\alpha=1}^{4} (\gamma^0\hat{F})_{\alpha\alpha} B_{\alpha\alpha} + \tfrac{1}{2} \sum_{\substack{\alpha,\beta=1\\ \alpha>\beta}} \{ (B_{\alpha\beta}+B_{\beta\alpha})[(\gamma^0\hat{F})_{\beta\alpha}+(\gamma^0\hat{F})^*_{\beta\alpha}] + (B_{\alpha\beta}-B_{\beta\alpha})[(\gamma^0\hat{F})_{\beta\alpha}-(\gamma^0\hat{F})^*_{\beta\alpha}]\} \right\}, \tag{13}$$

where (i) we have used the fact that $\langle A \,|\, A\rangle = (2\pi)^{-3} 2m_A V$, V being the nuclear volume so that $A/V = 3/4\pi r_0{}^3$ ($\sim 2\times 10^{38}$ particles/cm^3) is kept finite, and (ii) we used translational invariance and the fact that $\gamma_0\hat{F}$ is Hermitean [so that $(\gamma^0\hat{F})_{\beta\alpha} - (\gamma^0\hat{F})^*_{\beta\alpha}$ for $\beta>\alpha$ is purely imaginary]. At this point, we introduce the Fourier transform of the proton's Green's function in the large nucleus, treated as a many-body system ($Z=N=A/2$ is assumed throughout this paper), as[8]

$$\tilde{G}(k)_{\alpha\beta} = -i(\langle A\,|\,A\rangle)^{-1} \int d^4x \exp ikx \langle A \,|\, T[\psi_\alpha(x/2), \psi^\dagger_\beta(-x/2)] \,|\, A\rangle , \tag{14}$$

where the symbol T denotes the time-ordered product. Upon comparing Eqs. (13) and (14), direct manipulations yield, for any $\alpha,\beta=1,2,3,4$,

$$B_{\alpha\beta}+B_{\beta\alpha} = 2[\mathrm{Im}\tilde{G}(k)_{\alpha\beta} + \mathrm{Im}\tilde{G}(k)_{\beta\alpha}] , \tag{15}$$

$$B_{\alpha\beta}-B_{\beta\alpha} = 2i[\mathrm{Re}\tilde{G}(k)_{\beta\alpha} - \mathrm{Re}\tilde{G}(k)_{\alpha\beta}] \tag{16}$$

which reduce the determination of the B's and hence of Γ_A/A to the construction of $\tilde{G}(k)_{\alpha\beta}$ [or to that of $\mathrm{Im}\tilde{G}(k)_{\alpha\beta}$ if the Lehmann representation[9] is used to obtain $\mathrm{Re}\tilde{G}(k)_{\alpha\beta}$ in terms of the former]. For both relativistic and nonrelativistic nucleons, $\tilde{G}(k)_{\alpha\beta}$ depends on the nucleon density A/V, but neither on A nor V for large A (i.e., it is an intensive quantity), a fact which is fully supported by the perturbation expansion for $\tilde{G}(k)_{\alpha\beta}$ (or rearrangements thereof.) This implies that the right-hand side (RHS) of Eq. (14) and, hence, the lifetime per nucleon $\tau=A/\Gamma_A$ depends on A/V for large A.

In general, once a given nucleon-nucleon potential or, more generally, a given strong-interaction Hamiltonian (including π's, Δ's, etc.) has been assumed, $\tilde{G}(k)_{\alpha\beta}$ can be calculated through the Dyson equation,

$$\tilde{G}(k)_{\alpha\beta} = \tilde{G}^{(0)}(k)_{\alpha\beta} + \sum_{\rho,\sigma=1}^{4} \tilde{G}^{(0)}(k)_{\alpha\rho} \Sigma^*(k)_{\rho\sigma} \tilde{G}(k)_{\sigma\beta} . \tag{17}$$

Here ($\epsilon \to 0^+$),

$$\tilde{G}^{(0)}(k)_{\alpha\beta} = \left[\frac{\theta(|\vec{k}|-k_F)}{k^0-E_k+i\epsilon} + \frac{\theta(k_F-|\vec{k}|)}{k^0-E_k-i\epsilon} - \frac{1}{k^0+E_k-i\epsilon} \right] \left[\frac{\not{k}+m_p}{2E_k}\gamma^0 \right]_{\alpha\beta} , \tag{18}$$

where $k_F=(9\pi/8)^{1/3} r_0{}^{-1}$ is the Fermi momentum and $E_k=(m_p{}^2+\bar{k}^2)^{1/2}$, $\tilde{G}^{(0)}_{\alpha\beta}$ is the relativistic free-nucleon Green's function given by Eq. (14) when ψ_α is a free Dirac nucleon field and $|A\rangle$ has been replaced by a Fermi sea of free nucleons filled up to k_F. It is easy to see that the contribution of $\tilde{G}^{(0)}$ alone to the RHS of Eq. (13) vanishes exactly due to the fact that $k^0 < E_k$. On the other hand, $\Sigma^*(k)_{\rho\sigma}$ is the *proper* nucleon self-energy in the large nucleus which, in turn, is given formally through an infinite set of many-body Feynman diagrams (or rearrangements thereof). Even if detailed estimates of $\Sigma^*_{\rho\sigma}$ lie outside the scope of this note, we shall be able to outline a rather rough estimate of Γ_A as follows. Coming back to Eq. (3), we shall assume that the magnitude of Γ_A can be assessed if the Dirac proton fields $\psi,\bar{\psi}$ are replaced by their nonrelativistic limits which are two-component (Pauli) spinors, so that $\alpha,\beta=1,2$ only. Even if, *a priori*, this approximation is not truly justified, we hope that it will not give rise to too large an error, provided that the positron momentum $|\bar{k}|$ does not exceed, say, 600 MeV appreciably. Moreover, such an approximation is consistent with the previous approach to nucleon decay in the deuteron. Then, one sees that Eqs. (14)–(16) remain formally valid (with $\alpha,\beta=1,2$ only) where

$$\tilde{G}(k)_{\alpha\beta}=\tilde{G}(k)\delta_{\alpha\beta},\quad \Sigma^*(k)_{\alpha\beta}=\Sigma^*(k)\delta_{\alpha\beta}\,,$$
$$\tilde{G}^{(0)}(k)_{\alpha\beta}=\tilde{G}^{(0)}(k)\delta_{\alpha\beta}\,,$$

due to invariance under rotations and reflections. Equations (13), (17), and (18) now simplify, respectively, to

$$\frac{\Gamma_A}{A}=\frac{g_{\rm GUT}{}^4}{4M_X{}^4}|\psi(0,0)|^2\frac{V}{A}A_s(f)\frac{74}{(2\pi)^3}\times\int d^3\bar{k}\,\theta(m_p-|\vec{k}|)\,{\rm Im}\tilde{G}(k)\,, \qquad (19)$$

$$\tilde{G}(k)=\{[\tilde{G}^{(0)}(k)]^{-1}-\Sigma^*(k)\}^{-1}\,, \qquad (20)$$

$$\tilde{G}^{(0)}(k)=[k^0-(m_p+\bar{k}^2/2m_p)]^{-1} \qquad (21)$$

(as $k^0=|\bar{k}|<m_p+\bar{k}^2/2m_p$). Again, if the nonrelativistic proton proper self-energy vanishes or is real in the integration range the RHS of Eq. (19) is zero and, hence, it predicts an infinite proton lifetime in the nucleus. Notice that Eqs. (14) and (20) imply that ${\rm Im}\tilde{G}(k)\geq 0$ [and therefore ${\rm Im}\Sigma^*(k)>0$] for any $\bar{k}$ in the integration range of Eq. (19) (i.e., $|\bar{k}|<m_p$); that is, we are considering ${\rm Im}\tilde{G}$ for values of k^0-m_p below the chemical potential, where it change sign and becomes negative (compare with Ref. 8). Let us assume that as $|\bar{k}|$ increases in the integration range in Eq. (19) (so that $k^0-m_p\simeq|\bar{k}|-m_p$ approaches the chemical potential) ${\rm Im}\tilde{G}$ is a decreasing positive function. This possibility together with the increasing phase-space factor $|\bar{k}|^2$ would imply that the positron spectrum would be peaked at some $|\bar{k}|_{\rm max}$ strictly below m_p. *A posteriori*, this would provide additional support for the preceding nonrelativistic approximation.

Upon integrating over angles and using[1]

$$g_{\rm GUT}{}^2/4\pi=0.02,\quad M_X=4.5\times10^{14}\ {\rm GeV}\,,$$
$$A_s(f)=5,\quad |\psi(0,0)|^2=3\times10^{-5}\ {\rm GeV}^6$$

(see Ref. 10).

Equation (19) leads to

$$A/\Gamma_A\equiv(\tau_p)_A\simeq0.6\times10^{29}\times\rho^{-1}\ {\rm yr}\,, \qquad (22)$$

where

$$\rho=\int_0^1 dy\,y^2\frac{2\,{\rm Im}\hat{\Sigma}^*}{(2\,{\rm Im}\hat{\Sigma}^*)^2+[(y-1)^2+(1+2\,{\rm Re}\hat{\Sigma}^*)]^2} \qquad (23)$$

with $\hat{\Sigma}^*\equiv\Sigma^*/m_p$. Notice that $\Sigma^*(k)\equiv\Sigma^*(k^0=|\bar{k}|,k)$ and $|\bar{k}|=m_p y$.

The use of rough estimates for ${\rm Im}\Sigma^*$ and ${\rm Re}\Sigma^*$ typical of either nuclear matter (optical-model approach) or low-energy πN interaction, leads to values for $(\tau_p)_A$ ranging from ~ 1 to 3×10^{30} yr. This is to be compared with the lifetime of a free proton computed with the same GUT parameters[10] (recall that *only* final states with a positron are included):

$$\tau_p({\rm free})\simeq3.5\times10^{30}\ {\rm yr}$$

We want to point out again that the above numerical estimate for proton decay in a large nucleus is only very rough and cannot be taken as conclusive. Anyhow, the preceding figures give some indication that, contrary to the deuteron case, it could well be that nuclear effects in a complex nucleus (such as ^{16}O) are relevant.

An important check of consistency is provided by taking the $k_F\to0$ limit in Eq. (20), which, phys-

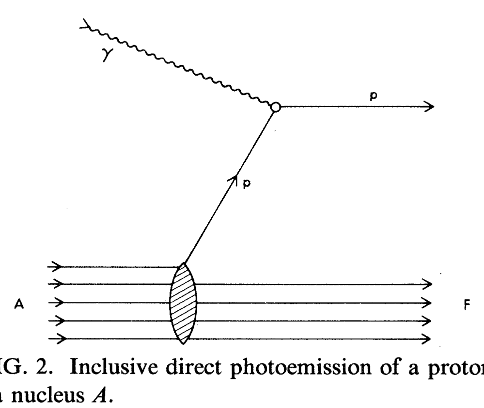

FIG. 2. Inclusive direct photoemission of a proton off a nucleus A.

ically, amounts to considering the inclusive decay of one nucleon in a large nuclear system when the density goes to zero. Then Γ_A/A should tend to the free proton total width, calculated with the same GUT parameters. As a first approximation, we use a typical (meson-theory) πN Hamiltonian, like the CBM one,[7] in lowest-order perturbation theory, which is the usual second-order self-energy diagram.[8] In the $k_F \to 0$ limit, the integral in Eq. (20) gives a ${k_F}^3$ factor which cancels out the ${k_F}^{-3}$ one coming from V/A. Then Γ_A/A is seen to approach the corresponding value for a free proton. Moreover, in the limit $k_F \to 0$ (neglecting ${m_\pi}^2/{m_p}^2$) one gets

$$(V/A)\,\mathrm{Im}\Sigma^*(k) \to \frac{3\pi^2|\vec{k}|}{16{m_p}^2}\frac{g^2}{4\pi}|f(|\vec{k}|)|^2\delta(|\vec{k}|-m_p/2)\,, \tag{24}$$

$$\mathrm{Re}\Sigma^*(k) \simeq -\left[\frac{g^2}{4\pi}\right]\frac{3}{8\pi m_p|\vec{k}|}\int_0^\infty dq\, q^2|f(q)|^2 \ln\left[\frac{2{m_p}^2+2m_p q+(|\vec{k}|+q)^2-2m_p|\vec{k}|}{2{m_p}^2+2m_p q+(|\vec{k}|-q)^2-2m_p|\vec{k}|}\right], \tag{25}$$

where $f(q)$ is the CBM form factor previously used. Then, by introducing Eqs. (25) and (26) into Eqs. (23) and (24) we find $\tau_p \simeq 6.1\times 10^{30}$ yr, which is a reasonable value in view of the simplifications made when computing the proton self-energy. A more detailed study of all these problems is in progress.

Finally, as pointed out before, the nuclear matrix element in Eq. (3) also appears in the so-called one-proton-transfer reactions in nuclei.[11] We have drawn in Fig. (2) one of these reactions: inclusive direct photoemission of a proton off a nucleus A (the final states F may, of course, include mesons). Let us assume that the energy of the incoming particle E_a is large enough (in general $E_a \gtrsim 3m_p$) and that the angle θ between a and the outgoing particle b, as well as the energy of the later E_b are suitably chosen (θ being always close to the forward direction). Then it is easy to show that conditions $p^0>0$ and $p^2 \equiv (p^0)^2 - \vec{p}^2 \simeq 0$ can always be fulfilled [except from a narrow region where $p^0 \simeq 0$, which is not important due to the phase-space factor in Eq. (3)]. A direct calculation shows that

$$\left[\frac{d\sigma}{dt}\right]_{t=0} \sim \sum_{\alpha,\beta=1}^{4} M_{\alpha\beta}\int d^4(x)\exp ipx \times\langle A\,|\,[\bar{\psi}_\alpha(0),\psi_\beta(x)]\,|\,A\rangle\,, \tag{26}$$

where $t=p^2=0$ and $M_{\alpha\beta}$ is a known matrix which includes the nucleon propagator and the corresponding matrices for the upper vertex. Then, the experimental knowledge of the said inclusive cross section would allow us to determine the above nuclear matrix element (actually a distorted-wave Born approximation should be used in general, but we will not enter here into this). Unfortunately, to our knowledge, there seem to exist no data about this kind of high-energy inclusive nuclear reactions at present.

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