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Cloudy bag model: Convergent perturbation expansion for the nucleon

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A previously published bound on the probability of finding n pions in the dressed nucleon in Chew-Low theory is improved. The proof is then extended to the recently derived cloudy-bag-model Hamiltonian. Together with a bound on the average number of pions (0.9 ± 1.0), our result strongly suggests a rapid convergence of the perturbation expansion in the cloudy bag model.

I. INTRODUCTION

The concept of chiral symmetry has been of great importance in elementary-particle physics for many years.[1-3] In the context of massless quarks and quantum chromodynamics it is, of course, an exact symmetry, and should survive the proof of confinement in some way. It is not surprising, therefore, that immediately after the presentation of the original MIT bag model[4] Chodos and Thorn[5] attempted to repair its obvious violation of chiral symmetry. Their method relied on introducing massless, elementary σ and $\vec{\pi}$ fields which coupled to the quarks, only at the bag surface, in such a way as to restore exact chiral symmetry.

In the past year or so, interest in this problem has been dramatically revived. For a longer review of these developments we refer to the discussion of Ref. 6, but a few brief comments will be useful here. Brown and collaborators have argued that the $\vec{\pi}$ field is actually a crucial aspect of the confinement process for the nucleon.[7] That is, chiral symmetry should be manifest in the Wigner-Weyl mode inside (no pions), and in the Nambu-Goldstone mode outside (the pion is the Goldstone boson).[3] In their purely classical model this external pion exerts a large pressure on the bag, resulting in a confinement volume of a few tenths of a fermi for the nucleon. They argue further that such a picture (the "little bag") would be more consistent with classical nuclear physics.

On the other hand, Jaffe[8] and others[9] have developed classical models of a bag surrounded by a pion field which merely acts as a small perturbation on the usual MIT ground state. Once again the pion field appears as a Goldstone boson, excluded from the interior of the bag. Recent work by Johnson has also shown the importance of collective $q\bar{q}$ excitations in the volume about the MIT bag.[10] However, the phenomenological replacement of such excitations by a pion field has not yet been clarified.

At the same time as these developments using a classical pion field were taking place, the TRIUMF–University of Washington group has constructed a quantized version of the theory—the "cloudy bag model" (CBM).[11-13] In order to avoid technical problems, the CBM (like that of Chodos and Thorn) allows the pion field to penetrate the bag. By working only to lowest order in the pion field, which is assumed to be small, it was possible to derive a Hamiltonian (see Sec. II) describing an interacting system of (bare) nucleons, deltas, and pions. In Ref. 12, hereafter CBM-1, this Hamiltonian was used to settle the longstanding problem of the nature of the (3, 3) resonance. In CBM-2 (Ref. 13) this model has been used, with considerable success, to calculate pionic corrections to the MIT model of the nucleon (g_A, magnetic moment, and charge radii).

In the CBM work the contribution from the pion field inside the bag was rather small, and could be justified as a crude approximation to the effect of virtual $q\bar{q}$ pairs inside the bag. Indeed it is just this point which was made recently by DeTar.[14,15] His work provides some formal link between the theory of Jaffe[8] with the pion excluded from the bag and the CBM Hamiltonian, which he also used in a calculation of nucleon properties.

There is a great deal of interesting physics in these developments, but for our present purpose we note only that all groups, except Stony Brook, rely on a perturbative treatment of pionic effects. This perturbative treatment has two aspects. First, the exponential coupling at the bag surface

$\bar{q}\exp(i\vec{\tau}\cdot\vec{\phi}\gamma_5/f)q$ is replaced by $\bar{q}(1+i\vec{\tau}\cdot\vec{\phi}\gamma_5/f)q$, and the covariant derivative of the pion field reduces to a normal derivative $(D_\mu\vec{\phi}\to\partial_\mu\vec{\phi})$. Second, the resulting linear Hamiltonian describing the coupling of a pion to a baryon is used only in lower-order perturbation theory to obtain the pionic corrections for nucleon observables. In this paper we shall only address the second aspect of this problem.

In his classical treatment of the problem Jaffe extracted a parameter ϵ, related to the strength of the pion field at the bag surface $[\epsilon=g_A/(8\pi f^2R^2)]$, which should be small if perturbation theory is to work. For the usual MIT parameters his ϵ is quite small, but it certainly is not small for the "little bag". In the calculations using a quantized pion field, that is the CBM Hamiltonian, only the one- and two- pion terms have been retained.

Until now there has been no rigorous proof of convergence in any of these calculations. This paper takes the CBM Hamiltonian as given, and provides such a proof. Of course, if the $\Delta N\pi$ coupling were omitted from this model it would be identical in form to the static Chew-Low model[16,17] for the πN system For that model a great deal of formal work has been done to establish convergence properties. For example, Alvarez-Estrada[18-20] has proven rigorously that the perturbation expansion of the physical nucleon state in the Chew-Low model does converge, in the sense that a rigorous least upper bound (LUB) P_n can be placed on the probability of finding n pions in it. P_n does tend to zero as n goes to infinity, but the *convergence is very slow*. For example, his LUB on P_n is not a useful limit, that is, it is not less than one, until $n=5$.

Henley and Thirring were aware of this problem[21]: "For a long time it has been one of the main goals of meson theory to analyze the physical nucleon in terms of the bare nucleon and its surrounding meson cloud. This problem led into a dead-end road.... The reason is that the ... resonant state of the nucleon is not important for the ground state." It is exactly on this point that the CBM has something new to say. As stressed in CBM-1, the quark model has an elementary Δ which carries most of the strength of the P_{33} scattering. Therefore, we do not need such a large bare coupling constant, or such a high cutoff momentum.[13] Consequently, one is led to hope that the theory may be more convergent.

In this paper we first improve the original bound of Alvarez-Estrada (for the Chew-Low model) by a factor of 4, corresponding to the spin-isospin degeneracy of the nucleon. The proof is also generalized to the CBM Hamiltonian by extending the space of bare baryon states. For the parameters of the CBM (Refs. 12 and 13)—or indeed any reasonable parameters near those of the MIT bag model—this leads to a remarkable proof of convergence of the perturbation expansion of the dressed nucleon state. Indeed we find that the probability of finding three pions about the (bag) core of the nucleon is strictly less than 12%. In view of the weakness of the bound, the real probability is almost certainly a factor (2–3) smaller. Even more impressive is the bound and standard deviation on the mean number of pions in the physical nucleon. For the CBM (Ref. 13) these numbers are 0.9 and 1.03, in comparison with the Chew-Low values of 2.16 and 2.22, respectively.

This rapid convergence of perturbation theory for strong interactions is a novel feature of the CBM. It comes about because of the large size of the pion source. As we point out in the final section, this rapid convergence has important consequences not only for the calculation of nucleon properties[13,15] and the N-N force,[22] but also for such exotic questions as the proposed tests of grand unified theories in the search for proton decay.

II. THE CLOUDY-BAG-MODEL HAMILTONIAN

The Hamiltonian of the cloudy bag model (CBM) of Ref. 12 takes the form

$$H=H_0+H_I, \tag{2.1a}$$

$$H_0=\sum_\alpha m_\alpha N_\alpha^\dagger N_\alpha+\sum_k \omega_k a_k^\dagger a_k, \tag{2.1b}$$

and

$$H_I=\sum_{\alpha,\beta,k}(v_k^{\alpha\beta}N_\alpha^\dagger N_\beta a_k+\text{H.c.}) . \tag{2.1c}$$

Here N_α $(N_\alpha^\dagger)$ are annihilation (creation) operators for the static baryon bag states $|\alpha\rangle$ of bare mass m_α. In our application the states $|\alpha\rangle$ include the single-particle states $|n,s,t\rangle$ of the nucleon with spin $s_n=\frac{1}{2}$ and isospin $t_n=\frac{1}{2}$, and the single-particle states $|\Delta st\rangle$ of the Δ with spin $s_\Delta=\frac{3}{2}$ and isospin $t_\Delta=\frac{3}{2}$. The labels s and t are the spin and isospin projections, respectively. The sum over the index k represents the integration over the momentum $\vec{k}$ and the sum over isospin projections j of the pion,

$$\sum_k\equiv\sum_{j=1,2,3}\int\frac{d^3k}{(2\pi)^3} .$$

Since there is no renormalization of the pion in the theory, the rest mass μ and the bare mass of the pion are identical, and the pion energies in Eq. (2.1b) are given by

$$\omega_k=(\vec{k}^2+\mu^2)^{1/2} .$$

The interaction Hamiltonian (2.1c) allows transitions between a nucleon and a Δ with the emission or absorption of a pion. An important feature of the cloudy bag model is that the interaction matrix elements $v^{\alpha\beta}$ are highly constrained by the underlying quark structure of the baryons, and are determined by a single coupling strength f and a single form factor $u(kR)$. Explicitly,

$$v_k = \frac{i\mathfrak{F}u(kR)}{\mu(2\pi)^{3/2}(2\omega_k)^{1/2}} T_j\vec{S}\cdot\vec{k}, \qquad (2.2a)$$

with

$$\mathfrak{F} \equiv \begin{pmatrix} \mathfrak{F}^{nn} & \mathfrak{F}^{n\Delta} \\ \mathfrak{F}^{\Delta n} & \mathfrak{F}^{\Delta\Delta} \end{pmatrix} = \tfrac{6}{5} f \begin{pmatrix} 5 & +4\sqrt{2} \\ 4\sqrt{2} & 10 \end{pmatrix} \qquad (2.2b)$$

and

$$u(kR) = \frac{3j_1(kR)}{kR} . \qquad (2.2c)$$

The parameter R is the bag radius and j_1 is a spherical Bessel function of order one.

The Hermitian transition spin operator $\vec{S}$ of Eq. (2.2a), which acts in the spin subspace of the baryon states, is defined by

$$\langle s_\alpha s|\vec{S}\cdot\vec{k}|s_\beta s'\rangle = \sum_m (-1)^{1/2-s}\begin{pmatrix} s_\alpha & s_\beta & 1 \\ -s & s' & m \end{pmatrix} k_m . \qquad (2.3)$$

Here, k_m is the spherical component of $\vec{k}$, and the $3j$ symbol couples the spins s_α and s_β (nucleons or Δ's) to the angular momentum of the pion. The isospin transition operator T_j, coupling the isospins of α, β, and the pion, is defined similarly.[23]

The standard Hamiltonian of the Chew theory may be recovered from Eqs. (2.1) by taking f as the unrenormalized pseudovector coupling constant[16,17] and restricting the sum over α to the nucleon states only or, alternatively, by setting

$$\mathfrak{F}^{n\Delta} = \mathfrak{F}^{\Delta n} = \mathfrak{F}^{\Delta\Delta} = 0 . \qquad (2.4)$$

The transition operators for n-n transitions are proportional to the usual Pauli spin and isospin operators

$$\sqrt{6}\,\vec{s} = \vec{\sigma}, \quad \sqrt{6}\,\vec{T} = \vec{\tau}, \qquad (2.5)$$

so in this case we obtain the standard interaction of the Chew-Low theory,

$$v_k^{nn} = \frac{if}{\mu}\,\frac{u(k)}{(2\pi)^{3/2}(2\omega_k)^{1/2}}\,\tau_j\vec{\sigma}\cdot\vec{k} . \qquad (2.6)$$

III. BOUNDS

The physical nucleon of mass $\tilde{m}$ with isospin projection t and spin projection s is described in the model by a state $|\tilde{n}st\rangle \equiv |\tilde{n}\rangle$ which is a solution of [12,16,17]

$$H|\tilde{n}\rangle = \tilde{m}|\tilde{n}\rangle . \qquad (3.1)$$

Since the interaction (2.1c) conserves baryon number, the expansion of this state in terms of the eigenstates of the bare Hamiltonian (2.1b) may be restricted to states containing a single baryon $|\alpha\rangle$ and arbitrary numbers of the field quanta:

$$|\tilde{n}st\rangle = Z_2^{1/2}|nst\rangle + \sum_{r=1}^{\infty}\sum_{\alpha}\sum_{k_1\ldots k_r} c_r(\alpha;k_1\ldots k_r;\tilde{n}st) \times \frac{1}{(r!)^{1/2}} a_{k_1}^\dagger a_{k_2}^\dagger \cdots a_{k_r}^\dagger|\alpha\rangle , \qquad (3.2)$$

with

$$\delta_{s's}\delta_{t't} Z_2^{1/2} = \langle nst|\tilde{n}s't'\rangle \qquad (3.3)$$

and

$$c_r = \frac{1}{(r!)^{1/2}}\langle\alpha|a_{k_r}a_{k_{r-1}}\cdots a_1|\tilde{n}st\rangle . \qquad (3.4)$$

The bare Δ (with no pions) does not appear on the right-hand-side of (3.2), since it has a different total spin and isospin from the nucleon,

$$\langle\Delta st|\tilde{n}st\rangle = 0 . \qquad (3.5)$$

The matrix element c_r is the probability amplitude for finding r pions with momenta $\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_r$ and isospin projections $j_1, j_2, \ldots, j_r$ surrounding the bag state α (either a nucleon or a Δ with spin s' and isospin t', depending on the index α) in the physical nucleon with spin s and isospin t.

The probability of finding r pions of any momenta and isospin surrounding the bag state α is then

$$\eta_r^\alpha = \sum_{k_1\ldots k_r}|c_r(\alpha;k_1\ldots k_r;\tilde{n}st)|^2 . \qquad (3.6)$$

The normalization condition from Eqs. (3.2), (3.4), and (3.6) is

$$\langle\tilde{n}|\tilde{n}\rangle = Z_2 + \sum_{r=1}^{\infty}\sum_{\alpha}\eta_r^\alpha = 1 . \qquad (3.7)$$

The probability of finding r pions in and around the cloudy bag is then

$$p_r = \sum_\alpha \eta_r^\alpha \qquad (3.8)$$

In order to construct bounds on P_r, it is useful to define[18-20] a state $|\phi_r\rangle$ by removing r pions of prescribed momenta and isospin from the physical nucleon,

$$|\phi_r\rangle = \frac{1}{(r!)^{1/2}} a_{k_1}\cdots a_{k_r}|\tilde{n}st\rangle . \qquad (3.9)$$

Then, from Eq. (3.4), $c_r = \langle\alpha|\phi_r\rangle$ and

$$p_r = \sum_\alpha \eta_r^\alpha = \sum_\alpha \sum_{k_1\ldots k_r}\langle\phi_r|\alpha\rangle\langle\alpha|\phi_r\rangle . \qquad (3.10)$$

Interchanging the sums over pion states and bag states, and using the completeness of the state $|\alpha\rangle$ in the single-baryon subspace, we have

$$p_r = \sum_{k_1 \ldots k_r} \langle \phi_r | 0\rangle\langle 0|\phi_r\rangle \leq \sum_{k_1 \ldots k_r} \langle \phi_r|\phi_r\rangle = \sum_{k_1 \ldots k_r} \|\phi_r\|^2 \tag{3.11}$$

where $|0\rangle\langle 0|$ is the projector for the pion vacuum times the unit operator in the baryon subspace.

Our aim is to find simple, explicit expressions for $\|\phi_r\|$ in order to place upper bounds on the probabilities P_r using Eq. (3.11). First, consider ϕ_1. The identity

$$|\phi_1\rangle = a_{k_1}|\tilde{n}\rangle = \frac{1}{\tilde{m}_n - \omega_{k_1} - H}[a_{k_1}, H_I]|\tilde{n}\rangle \tag{3.12}$$

is easily established using Eq. (3.1) and the relation $[H_0, a_k] = -\omega_k a_k$. For brevity let us introduce the notation $k_l \equiv l$, and denote the commutator in Eq. (3.12) by $C_1 = [a_{k_1}, H_I]$. For the particular interaction (2.1c), we have

$$C_1 = \sum_{\alpha\beta} N_\beta^\dagger v_k^{\alpha\beta\dagger} N_\alpha . \tag{3.13}$$

By applying a_2 to Eq. (3.12) and using the identity

$$a_k \frac{1}{z-H} = \frac{1}{z-\omega_k - H} a_k + \frac{1}{z-\omega_k-H} C_k \frac{1}{z-H}, \tag{3.14}$$

we find that

$$\begin{aligned} a_2 a_1|\tilde{n}\rangle &= a_2 \frac{1}{\tilde{m}_n - \omega_1 - H} C_1|\tilde{n}\rangle \\ &= \frac{1}{\tilde{m}_n - \omega_1 - \omega_2 - H}\left(C_1 a_2 + C_2 \frac{1}{\tilde{m}_n - \omega_1 - H} C_1\right)|\tilde{n}\rangle \\ &= \frac{1}{\tilde{m}_n - \omega_1 - \omega_2 - H}\left(C_1 \frac{1}{\tilde{m}_n - \omega_2 - H} C_2 + C_2 \frac{1}{\tilde{m}_n - \omega_1 - H} C_1\right)|\tilde{n}\rangle = \sqrt{2}\,|\phi_2\rangle . \end{aligned} \tag{3.15}$$

Repeated application of the identity (3.15) yields the following result: Let $\gamma_1\gamma_2 \ldots \gamma_r$ be an arbitrary permutation of 1, 2, ... r, then

$$|\phi_r\rangle = \frac{1}{(r!)^{1/2}} \frac{1}{\tilde{m}_n - \sum_{l=1}^r \omega_l - H} \sum_{\gamma_1 \ldots \gamma_r} C_{\gamma_1} \frac{1}{\tilde{m}_n - \sum_{l=1, l\neq\gamma_1}^r \omega_l - H} \times C_{\gamma_2} \frac{1}{\tilde{m}_n - \sum_{l=1, l\neq \gamma_1,\gamma_2}^r \omega_l - H} C_{\gamma_3} \cdots \frac{1}{m_n - \omega_{\gamma_r} - H} C_{\gamma_r} |\tilde{n}\rangle . \tag{3.16}$$

Taking norms throughout Eq. (3.16), we have our key result:

$$\begin{aligned} \frac{\|\phi_r\|}{\| |\tilde{n}\rangle \|} &\leq \frac{1}{(r!)^{1/2}} \frac{1}{\sum_{l=1}^r \omega_l} \sum_{\gamma_1 \ldots \gamma_r} \frac{1}{\sum_{l=1, l\neq\gamma_1}^r \omega_l \sum_{l=1, l\neq\gamma_1,\gamma_2}^r \omega_l} \cdots \frac{1}{\omega_{\gamma_r}} \|C_{\gamma_1}\| \, \|C_{\gamma_2}\| \cdots \|C_{\gamma_r}\| \\ &= \frac{1}{(r!)^{1/2}} \prod_{l=1}^r \frac{\|C_l\|}{\omega_l} \end{aligned} \tag{3.17}$$

In deriving (3.17), we have assumed that the spectrum of the total Hamiltonian H begins at $\tilde{m}_n$, the physical mass of the nucleon, so that for any $\omega > 0$ the inequality

$$\left\| \frac{1}{\tilde{m}_n - \omega - H} \right\| \leq \frac{1}{\omega} \tag{3.18}$$

holds.

From the inequalities (3.11) and (3.17), we obtain the central result of this section: The probability of finding r pions in the pion cloud is bounded by

$$P_r \leq \frac{\Lambda^r}{r!}, \tag{3.19}$$

where

$$\Lambda = \sum_k \frac{\|C_k\|^2}{\omega_k^2} . \tag{3.20}$$

Consequently an upper bound for the mean number of pions present in the pion cloud is

$$\langle r\rangle = \sum_r r P_r \leq \Lambda e^{\Lambda} . \tag{3.21}$$

A much tighter bound on the mean number of pions present is given by considering the expectation value of the number operator directly,

$$\langle r\rangle = \frac{1}{\| |\tilde{n}\rangle \|^2} \left\langle \tilde{n} \middle| \sum_k a_k^\dagger a_k \middle| \tilde{n} \right\rangle$$

$$= \sum_k \frac{\|\phi_1\|^2}{\| |\tilde{n}\rangle \|^2} \leq \Lambda . \tag{3.22}$$

It is shown in Appendix A that the uncertainty in the number of pions in the cloud $(\Delta r)^2 = \langle r^2\rangle - \langle r\rangle^2$ is bounded by

$$\Delta r \leq \left(\Lambda^2 + \tfrac{1}{4}\right)^{1/2} . \tag{3.23}$$

With the specific interaction of the CBM, Eq. (2.2), we find that

$$\Lambda = \tfrac{57}{25} f^2 I(R) , \tag{3.24}$$

where

$$I(R) = \frac{3}{\mu^2 (2\pi)^2} \int_0^\infty \frac{k^4 u(kR)^2}{\omega_k^{\,3}} \, dk . \tag{3.25}$$

Some details of the evaluation of Λ are presented in Appendix B.

IV. NUMERICAL RESULTS AND DISCUSSION

First let us consider the case where the Δ is excluded from the single-particle space, i.e., the system is described by the Chew Hamiltonian (2.6). With the coupling matrix (2.4) Eq. (B4) of Appendix B gives

$$\Lambda = f^2 I(R) . \tag{4.1}$$

[A bound of the same form was derived originally by Alvarez-Estrada.[18-20] However, Λ of (4.1) is improved by a factor of 4—through the use of completeness in Eq. (3.11)—corresponding to the spin and isospin degeneracy of the nucleon.]

We also note that Λ of Eq. (4.1) occurs in the expression for the probability Z_2 of Eq. (3.3) when it is evaluated to second order in perturbation theory. In the Chew-Low theory, unlike the cloudy bag model, the functional form of the factor $u(kR)$ is not well determined, and often a simple step function with a cutoff $k = k_{\max}$ is adopted. As described by Henley and Thirring, analysis of experimental data leads (with some ambiguity) to values of about $f^2/4\pi = 0.22$ (compared with $f^2/4\pi = 0.08$ for the renormalized coupling constant) and $k_{\max} \sim 5\mu$. For these values $\Lambda = 2.16$, which, according to Eq. (3.22), is also a bound on the mean number of pions in the nucleon. The corresponding bounds on the probabilities P_r are limited to small values only for $r > 6$, and the uncertainty in the number of pions present in the cloud is from Eq. (3.23) bounded by 2.22. Also listed in Table I are numerical results for the Lorentzian form factor $u(k) \sim \xi^2/(\xi^2 + k^2)$ used by Fubini and Thirring.[24]

For the Chew Hamiltonian our bounds give no reason to expect that perturbation theory is valid for the values of the coupling constant and form factors required by experiment. Defining a dimensionless parameter λ by $\lambda = k_{\max}/\mu$ we find $I(R) \approx \lambda^2/2$ for reasonable form factors, and if we take from Eq. (4.1) $\lambda f < 1$ as the criterion for the validity of perturbation theory, the unrenormalized coupling constant is restricted to values

$$f < \mu R , \tag{4.2}$$

where $R = k_{\max}^{-1}$. In the Chew-Low theory the "radius" of the nucleon is small, $R \approx 0.28$ fm, and the

TABLE I. Upper bounds for the probabilities P_r of finding r pions surrounding the nucleon in the Chew-Low and cloudy bag models. The column headed $\langle r\rangle$ gives the mean value of the number of pions present, calculated using Eq. (3.22), while the column headed Δr lists the uncertainties in pion numbers, Eq. (3.23). The values a for the Chew-Low model were calculated with a step-function form factor, and for case b a Lorentzian form factor was used. In the cloudy bag model, the results labeled c correspond to the values of the coupling constant f and bag radius R determined in Ref. 13. Those labeled d are constrained, as in Ref. 13 (Théberge *et al.*) to yield in perturbation theory the renormalized value $f_r^2/4\pi = 0.08$.

Theory		$f^2/4\pi$	R (fm)	$I(R)$	P_1	P_2	P_3	P_4	P_5	P_6	$\langle r\rangle$	Δr
Chew-Low	a	0.22	0.28	9.80	2.16	2.33	1.67	0.90	0.39	0.14	2.16	2.22
	b	0.22	0.30	9.03	1.99	1.97	1.31	0.65	0.26	0.09	1.99	2.05
CBM	c	0.078	0.82	5.04	0.90	0.40	0.12	0.03	0.005		0.90	1.03
	d	0.113	0.6	10.44	2.69	3.62	2.73	1.55	0.71	0.27	2.69	2.74
	d	0.109	0.7	7.16	1.78	1.58	0.94	0.56	0.20	0.06	1.78	1.85
	d	0.103	0.8	5.26	1.24	0.76	0.31	0.10	0.024	0.005	1.24	1.34
	d	0.100	0.9	3.98	0.91	0.41	0.12	0.03	0.005	0.001	0.91	1.04
	d	0.096	1.0	3.08	0.68	0.23	0.05	0.009	0.001		0.68	0.85
	d	0.093	1.1	2.43	0.52	0.14	0.023	0.003			0.52	0.72

unrenormalized coupling constant f far too large to satisfy the criterion (4.2).

Turning now to the cloudy bag model, we note that there is an additional factor $\frac{57}{25}$ in Eq. (3.23) due to the presence of the Δ, which tends to increase the bound. However, in the CBM, smaller values of the unrenormalized coupling constant f are needed to fit the observed quantities. In the calculations of Ref. 13, it was found that, with the form factor (2.2c) fixed by the bag model, there was very little freedom in fitting the P_{33} phase shift through the Δ resonance. A bag radius of 0.82 fm and a coupling constant $f^2/4\pi = 0.078$ were determined. Evaluating Λ for these values and the form factor (2.2c) we find that the mean number of pions present in the nucleon is bounded by $\Lambda = 0.9$, and P_r tends to zero quite rapidly, the root-mean-square fluctuation in the pion number being bounded by 1.03. The renormalized coupling constant calculated with these values of f and R, using perturbation theory, is $f_r^2/4\pi = 0.071$, somewhat less than the accepted value of $f_r^2/4\pi = 0.080$.

In Table I under the entries labeled d, we have also listed bounds for the values of f and R taken in the perturbative calculations of Théberge *et al.*[13] of the static properties of the nucleon in the CBM. Here, f and R are constrained to produce the value $f_r^2/4\pi = 0.080$ for the renormalized coupling constant.

It is seen from Table I that for reasonable values of the bag radius $R \approx 0.9$ fm, the use of perturbation theory, or other approximate methods which truncate the number of pions in the pion cloud, is much more acceptable in the case of the CBM than in the Chew-Low theory. The criterion (4.2) is much closer to being satisfied, and the bound on the mean number of pions in the cloud is about unity.

Our bounds are simple, but quite crude, and probably overestimate P_r significantly. The probability of finding one pion in the physical nucleon takes the value 35% in the perturbation calculations of Ref. 13. This value may be compared with our bound of 0.9.

V. CONCLUSION

Within the framework of a static source theory, we have established an improved, rigorous bound on the probability of finding the physical nucleon to contain n pions. For the recently developed Hamiltonian of the cloudy bag model, this bound goes rapidly to zero as n goes to three or more pions. In this model the mean number of pions about the nucleon is less than about 0.9, and the standard deviation is less than 1.0. This represents a remarkable improvement in convergence over earlier models such as the Chew-Low model—essentially because of the inclusion of the bare Δ isobar in the CBM.

It is certainly true that the calculation of pionic corrections to nucleon properties such as magnetic moments and charge radii is more complicated than simple probabilities. This is because of the interference between amplitudes with different numbers of pions. Thus, even though the probability of finding three pions is very small, it is conceivable that the three-pion terms could alter the calculations of Refs. 13 and 15 at a noticeable level. Nevertheless, the convergence properties of the CBM seem to be so good that we do not expect any major change in their conclusions.

Not only do our results give great support to the perturbative approach to single-baryon properties, but one may hope for new insight in several other areas. For example, one might now expect to make progress in the understanding of the long- and intermediate-range N-N force using similar techniques.[22] We might also mention the proposed tests of the various grand unified theories.[25] In particular, there are many experiments under way which look for proton decay modes, such as $p \rightarrow e^+\pi^0$. With few exceptions (e.g., Ref. 26), the assumption is usually made that the nucleon consists of just three quarks, two of which annihilate to an antiquark and a lepton. If the dressed nucleon actually had a cloud of pions like that in the Chew-Low model, the theoretical predictions based on the three-quark picture would be quite unreliable, because of the dominance of multipion decay modes. However, within the CBM our bounds strongly suggest that decays to a lepton and one or two pions will dominate. Detailed calculations on this problem would be very useful.

Our purpose in this paper has been to put bounds on the pion content of the dressed nucleon, within the framework of the linearized equations (2.1). This is a worthwhile exercise in itself, in view of the interest in such Hamiltonians in low- and medium-energy nuclear physics. However, we did remark in the Introduction that Eqs. (2.1) are an approximation to a highly nonlinear, exactly chiral-symmetric theory.[8,13] Unlike the truncated version discussed here that theory is not renormalizable, and discussions of it (e.g., the nonlinear σ model) usually rely on the tree approximation. It is worth observing though, that the reason for this problem is the treatment of the pion as an elementary, pointlike object. Our underlying motivation for introducing the pion is that one expects in the limit of exact SU(2) × SU(2) symmetry, that the pion should appear as a massless Goldstone boson associated with the dynamical breaking of the symmetry of the vacuum. Once the pion has some internal structure the pion sector of the theory will have a natural cutoff too, and one might expect a fairly rapid truncation of the higher-order terms (in f^{-1}), required formally for exact chiral symmetry. Thus, although our results may at first appear to be of somewhat limited interest because they rely on a linearized version of the equations, they may be rather close to reality.

In conclusion we must remark that the convergence of this expansion in number of pions is essential to the internal consistency of the CBM. At present, the internal structure of the pion is ignored in our model, and therefore we should only expect to describe the long-range piece of the pion field about the nucleon—that is, the one- and two-pion pieces. By the time we get to three or more pions we are probing phenomena within one- or two-tenths of a fermi of the bag surface—where the bag model itself, and particularly the static cavity approximation, is probably unrealistic.

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APPENDIX A

The expectation value of the square of the number operator for the pion field is related to the vector $|\phi_2\rangle$ of Eq. (3.9) in the following way:

$$\left\langle \tilde{n}\left|\left(\sum_k a_k^\dagger a_k\right)^2\right|\tilde{n}\right\rangle = \left\langle \tilde{n}\left|\sum_{kk'} a_k^\dagger a_{k'}^\dagger a_k a_{k'}\right|\tilde{n}\right\rangle + \left\langle \tilde{n}\left|\sum_k a_k^\dagger a_k\right|\tilde{n}\right\rangle = 2\sum_{kk'}\langle\phi_2|\phi_2\rangle + \langle\tilde{n}|\tilde{n}\rangle\langle r\rangle\,, \tag{A1}$$

where we have used the commutator $[a_k^\dagger, a_{k'}] = \delta_{kk'}$ and $\langle r\rangle$ is the mean number of pions present.

Consequently for $(\Delta r)^2 = \langle r^2\rangle - \langle r\rangle^2$, we have the expression

$$(\Delta r)^2 = 2\sum_{kk'}\frac{\|\phi_2\|^2}{\||\tilde{n}\rangle\|^2} + \langle r\rangle - \langle r\rangle^2\,. \tag{A2}$$

Since the maximum value of $\langle r\rangle - \langle r\rangle^2$ is $\frac{1}{4}$, and from Eq. (3.17) and definition (3.20),

$$2\sum_{kk'}\frac{\|\phi_2\|^2}{\||\tilde{n}\rangle\|^2} \leq \Lambda^2\,, \tag{A3}$$

the uncertainty in the number of pions in the cloud is bounded by

$$\Delta r \leq (\Lambda^2 + \tfrac{1}{4})^{1/2}\,. \tag{A4}$$

APPENDIX B: EVALUATION OF Λ

To evaluate Λ, we seek the maximum value of the magnitude of the vector $C_k|\psi\rangle$, C_k given by Eq. (3.13), as the normalized vector $|\psi\rangle$ ranges over the complete single-particle space, i.e., if $|\psi\rangle = \sum_\alpha d_\alpha|\alpha\rangle$ and $\sum_\alpha |d_\alpha|^2 = 1$, the expansion coefficients d_α must be chosen to maximize the quantity

$$\sum_k \frac{\rho_k^2}{\omega_k^2} = \sum_k \frac{1}{\omega_k^2}\sum_\gamma\left|\sum_\alpha d_\alpha\langle\gamma|v_k^{\gamma\alpha\dagger}|\alpha\rangle\right|^2. \tag{B1}$$

After introducing spin and isospin labels explicitly by setting $|\alpha\rangle = |\alpha st\rangle$, $|\beta\rangle = |\beta s''t''\rangle$, and $|\gamma\rangle = |\gamma s't'\rangle$, and substituting the interaction of Eq. (2.2a), expression (B1) becomes

$$\sum_k \frac{\rho_k^2}{\omega_k^2} = \sum_j \int d^3k\,\frac{u(kR)^2}{\mu^2(2\pi)^3 2\omega_k^3}\sum_{\substack{\alpha st\\ \beta s''t''}} d^*_{\alpha st} d_{\beta s''t''}\sum_\gamma \mathfrak{F}^{\alpha\gamma}\mathfrak{F}^{\dagger\gamma\alpha}\sum_{s'}\langle s_\alpha s|\vec{S}\cdot\vec{k}|s_\gamma s'\rangle\langle s_\gamma s'|\vec{S}\cdot\vec{k}|s_\beta s''\rangle \times\sum_{t'}\langle t_\alpha t|T_j|t_\gamma t'\rangle\langle t_\gamma t'|T_j|t_\beta t''\rangle\,. \tag{B2}$$

The evaluation is simplified by integrating over the angles of $\vec{k}$ first, using

$$\int k_i k_j d\Omega_k = 4\pi/3\,\delta_{ij}k^2\,.$$

The spin and isospin sums may then be performed with the help of the identities

$$\sum_i\sum_{s'}\langle s_\alpha s|S_i|s_\gamma s'\rangle\langle s_\gamma s'|S_i|s_\beta s''\rangle = \delta_{s's''}\delta_{\alpha\gamma}(2s_\alpha+1)^{-1}$$

and

$$\sum_j\sum_{t'}\langle t_\alpha t|T_j|t_\gamma t'\rangle\langle t_\gamma t'|T_j|t_\beta t''\rangle = \delta_{t't''}\delta_{\alpha\gamma}(2t_\beta+1)^{-1}\,, \tag{B3}$$

which follow from the definition (2.3) of the spin and isospin transition operators. The result is

$$\sum_k \frac{\rho_k^2}{\omega_k^2} \sum_{\alpha s t} \frac{|d_{\alpha s t}|^2}{2(2\pi)3\mu^2} \frac{1}{(2s_\alpha+1)(2t_\alpha+1)} \times \sum_\gamma \mathfrak{F}^{\alpha\gamma}\mathfrak{F}^{\dagger\gamma\alpha} \frac{4\pi}{3}\int \frac{k^4}{\omega_k^3} u(kR)^2 dk\,. \quad \text{(B4)}$$

With the specific values of the coupling matrix of Eq. (2.2b), we find the maximum value of (4.4) is attained when $d_{\Delta s t}=0$, giving the results, Eqs. (3.24) and (3.25), of the text.

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