



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Physics Letters B 560 (2003) 140–148

PHYSICS LETTERS B

www.elsevier.com/locate/npe

Dynamical generation of a gauge symmetry in the double-exchange model

J.M. Carmona^a, A. Cruz^{a,e}, L.A. Fernández^{b,e}, S. Jiménez^{a,e}, V. Martín-Mayor^{b,e},
A. Muñoz-Sudupe^b, J. Pech^{a,e}, J.J. Ruiz-Lorenzo^{c,e}, A. Tarancón^{a,e}, P. Téllez^d

^a Departamento de Física Teórica, Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza, Spain

^b Departamento de Física Teórica I, Facultad de CC. Físicas, Universidad Complutense de Madrid, 28040 Madrid, Spain

^c Departamento de Física, Facultad de Ciencias, Universidad de Extremadura, 06071 Badajoz, Spain

^d Servicio de Instrumentación Científica, Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza, Spain

^e Instituto de Biocomputación y Física de Sistemas Complejos (INBIFI), Facultad de Ciencias, Universidad de Zaragoza, 50009 Zaragoza, Spain

Received 6 January 2003; received in revised form 4 March 2003; accepted 12 March 2003

Editor: L. Alvarez-Gaumé

Abstract

It is shown that a bosonic formulation of the double-exchange model, one of the classical models for magnetism, generates dynamically a gauge-invariant phase in a finite region of the phase diagram. We use analytical methods, Monte Carlo simulations and finite-size scaling analysis. We study the transition line between that region and the paramagnetic phase. The numerical results show that this transition line belongs to the universality class of the antiferromagnetic RP^2 model. The fact that one can define a universality class for the antiferromagnetic RP^2 model, different from the one of the $O(N)$ models, is puzzling and somehow contradicts naive expectations about universality.

© 2003 Published by Elsevier Science B.V. Open access under [CC BY license](https://creativecommons.org/licenses/by/4.0/).

PACS: 11.10.Kk; 75.10.Hk; 75.40.Mg

Keywords: Universality; Lattice; Restoration; Gauge-symmetry

1. Introduction

The problem of the dynamical restoration of a gauge symmetry (see, e.g., [1,2] and references therein) has received considerable attention in the recent 10 years, because of the problem of introducing a chiral gauge theory in the lattice. Although the Ginsparg–

Wilson [3] method has somehow superseded this approach, the question remains an interesting one. Naively, the problem could seem a trivial one [2]: the non-gauge-invariant terms in the action generate a high-temperature-like gauge-invariant expansion with a finite radius of convergence. The subtle question is whether the radius of convergence of this expansion will remain finite in the continuum limit or not. In this Letter, we want to address a related, but different question, namely, the generation of a local invariance in the

E-mail address: laf@lattice.fis.ucm.es (L.A. Fernández).

low-temperature (broken-symmetry) phase of a system without any explicitly gauge-invariant term in the action. We have found this intriguing phenomenon in a numerical study of a simplified version of the Double-Exchange Model (DEM) [4,5], one of the most general models for magnetism in condensed matter physics, still under active investigation [6]. The local invariance does not follow from a high-temperature-like expansion, but from the infinite degeneracy of the ground-state (see Section 2), which occurs at a unique value of the control parameter at zero temperature, then extending to a finite region of the phase diagram at finite temperature. This phenomenon reminds one of the so-called quantum-critical point phenomenology [7].

We have studied the model using Monte Carlo simulations and finite-size scaling techniques [8–10]. We have found that the critical exponents are fully compatible with the ones [11] of the antiferromagnetic (AFM) RP^2 model in three dimensions¹ [11,13,14], which has an explicit local Z_2 invariance. This might not be surprising, given the strong similarities in the ground-state of both models (see Section 2). However, as we shall argue below, the fact that one can explicitly show (by exhibiting a different model belonging to it) that there is a universality class associated to the AFM- RP^2 model is surprising. Indeed, the most ambitious formulation (as stated, for instance, in Ref. [15]) of the universality hypothesis states that the critical properties of a system undergoing a second order phase transition are fully determined by the space dimensionality and its symmetry groups at high-temperature (\mathcal{G}) and low-temperature (\mathcal{H}). Moreover, systems with a locally isomorphic coset space \mathcal{G}/\mathcal{H} are expected to have the same critical behavior. There is an enormous number of applications of the universality hypothesis in high-energy and condensed-matter physics (see, for instance, [16]). Just to mention, an example, where only the *local structure* of \mathcal{G}/\mathcal{H} matters, let us recall the quite successful application [17] of the $O(3)$ non-linear σ model ($\mathcal{G} = O(3)$, $\mathcal{H} = O(2)$), to the two-dimensional antiferromagnetic quantum Heisenberg model ($\mathcal{G} = \text{SU}(2)$, $\mathcal{H} = \text{U}(1)$), arising in the high-temperature superconductivity context.

Based on the universality hypothesis, a very general and simple scenario was proposed by Azaria, Delamotte, Delduc and Jolicœur [18]. They considered a generic classical spin model in three dimensions, with $\mathcal{G} = O(3)$ undergoing a continuous transition to a low temperature phase where the symmetry is fully broken (this requires antiferromagnetic interactions). Using the Non-Linear σ Model (NL σ M) of the locally isomorphic manifold $O(3) \times O(2)/O(2)$, they were able to compute the critical exponents in $2 + \epsilon$ dimensions to the first order in ϵ , concluding that the corresponding universality class is the one of the three-dimensional $O(4)$ NL σ M.² This result implies that there are only two possible universality classes for a classical spin model in three dimensions, with $\mathcal{G} = O(3)$, undergoing a second-order phase transition: (1) If $\mathcal{H} = O(2)$ the universality class is that of the $O(3)$ NL σ M and (2) if \mathcal{H} is discrete the universality class must be that of $O(4)$ NL σ M. However, in our case, $\mathcal{G} = O(3)$, and from our numerical study \mathcal{H} seems to be $O(2)$ [19], although we cannot discard a breakdown of the $O(2)$ residual symmetry (to $O(1) = \text{Z}_2$) near the critical point. According to the NL σ M scenario [18], in the former case, the universality class should be the one of the $O(3)$ NL σ M which is ruled out by our results, while in the latter case one expects $O(4)$ NL σ M-like behavior, hardly compatible with our measurements. The possibility of a tricritical behavior can be also safely discarded.

Of course, the NL σ M scenario is not fire-proof. We can think of at least two reasons that could invalidate it. First, it is known that the universality hypothesis fails in the presence of marginally relevant operators, like in the two-dimensional eight-vertex model, for which critical exponents vary continuously with some model parameter [20], or like in the two-dimensional XY model. Second, it has been argued [15,18] that the global structure of the \mathcal{G}/\mathcal{H} manifold could be relevant for the higher-order terms in the $2 + \epsilon$ expansion. Indeed, let us recall that the NL σ M scenario has been challenged by the so-called chiral models [21]. The proposed critical exponents in the Heisenberg case ($\mathcal{G} = O(3)$) are [22] $\nu =$

¹ We recall that the ferromagnetic RP^2 model has a first order phase transition [12].

² However $4 - \epsilon$ calculations predict a first order phase transition, and so, a tricritical point that separates the $O(4)$ behavior from the first order one is also expected.

0.55(3), $\gamma = 1.06(5)$, to be compared with the ones of the $O(4)$ -NL σ M [23]: $\nu = 0.749(2)$, $\gamma = 1.471(4)$. Yet, the situation is still hotly debated: some RG calculations both analytical [24] and numerical [25] maintain that the chiral universality class with $\mathcal{G} = O(3)$ does not exist, all the transitions being first order. On the other hand, a chiral fixed point and the associated universality class has been found in a six-loops calculation in the fixed dimension scheme [22]. The situation is further complicated by the fact that the generic apparent critical exponents for weak first order transitions [26], $\nu = 0.5$, $\gamma = 1$, are almost indistinguishable from the critical exponents of the proposed chiral fixed-point. Clearly more work is needed in order to settle the issue. On the other hand, we have no doubts that the here studied phase transition is continuous, thus our results really pose a problem for the NL σ M scenario.

2. The model

Although some powerful techniques have been developed [27] for the double-exchange model [4] (involving dynamical fermions), lattice sizes beyond $L = 16$ are extremely hard to study with the present generations of computers. Thus, one may turn to the simplified version proposed by Anderson [4], where a purely bosonic Hamiltonian is considered. This simplified model has been recently studied [5] by extensive Monte Carlo simulation. Yet, previous studies missed several phases in the phase diagram (see Fig. 1 and below).

Specifically, we consider a three-dimensional cubic lattice of side L , with periodic boundary conditions. The dynamical variables, $\vec{\phi}_i$, live on the sites of the lattice and are three-component vectors of unit modulus. The Hamiltonian contains the Anderson version of the double-exchange model plus an antiferromagnetic first-neighbor Heisenberg interaction [28]:

$$-H = \sum_{\langle i,j \rangle} J \vec{\phi}_i \cdot \vec{\phi}_j + \sqrt{1 + \vec{\phi}_i \cdot \vec{\phi}_j}, \quad J < 0, \quad (1)$$

where $\langle i, j \rangle$ means first-neighbor sites on the lattice. The partition function reads

$$Z = \int d[\vec{\phi}] e^{-H/T}, \quad (2)$$

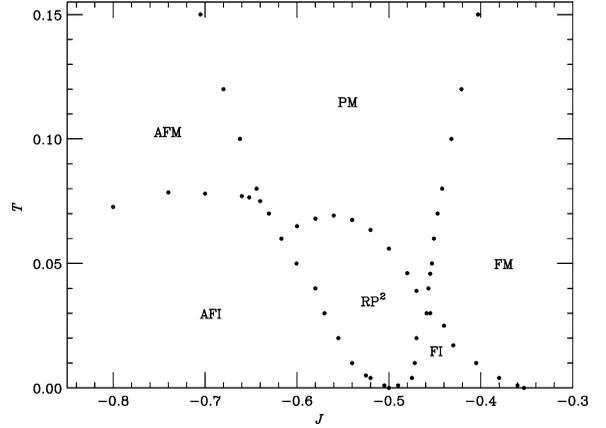


Fig. 1. Phase diagram of the model (1), as obtained from Monte Carlo simulation. The labels correspond to paramagnetic (PM), ferromagnetic (FM), ferrimagnetic (FI), antiferromagnetic (AFI), antiferromagnetic (AFM), and RP^2 -symmetric (RP^2) phases. See, Ref. [19] for details.

the integration measure being the standard rotationally-invariant measure on the sphere. In the following, expectation values will be indicated as $\langle \dots \rangle$.

The zero temperature limit is dominated by the spin configurations that minimize the energy. Exploratory Monte Carlo simulations showed that these configurations have a bipartite structure. Indeed, let us call a lattice site even or odd, according to the parity of the sum of its coordinates ($\vec{r}_i = (x_i, y_i, z_i)$, $x_i + y_i + z_i$ even or odd). Then the spins on the (say) even lattice are all parallel, while the spins on the odd sublattice are randomly placed on a cone of angle Ω ($\vec{\phi}_i \cdot \vec{\phi}_j = \cos \Omega$) around the direction of the even lattice. The energy when T goes to zero is simply

$$H_0(\Omega) = -3L^3 (J \cos \Omega + \sqrt{1 + \cos \Omega}). \quad (3)$$

Now, $\Omega(J)$ is obtained by minimizing $H_0(\Omega)$. For $J > -\sqrt{2}/4$, $\Omega = 0$, meaning a ferromagnetic vacuum. For $-\sqrt{2}/4 > J > -0.5$, one has $0 < \Omega < \pi/2$ (ferrimagnetic vacuum), while for all $J < -0.5$, it is $\pi/2 < \Omega < \pi$ (antiferromagnetic vacuum). The full antiferromagnetic configuration ($\Omega = \pi$) is never stable at zero temperature. The $J = -0.5$ point is very peculiar: much like for the AFM- RP^2 model [11,13, 14], spins in the even sublattice are randomly aligned or antialigned with the (say) Z -axis, while the spins in the odd sublattice are randomly placed in the X, Y plane. Since spins in the two sublattices are perpendicular to each other, one can arbitrarily re-

verse every spin. A local Z_2 symmetry is dynamically generated and, as we will see, it extends to finite temperatures. An operational definition of dynamical generation of a gauge symmetry is the following. One must calculate the correlation-length for non-gauge-invariant operators (ξ_{NGI}) and compare it to the correlation-length corresponding to gauge-invariant quantities (ξ_{GI}). In the continuum-limit ($\xi_{\text{GI}} \rightarrow \infty$), one should have $\xi_{\text{NGI}}/\xi_{\text{GI}} \rightarrow 0$. We have checked that the correlation-length associated to the spin–spin correlation function (non- Z_2 gauge-invariant) is smaller than 0.3 for all temperatures at $J = -0.5$. The alert reader will notice that the symmetry group at the point $(T, J) = (0, -0.5)$ is rather a local $O(2)$, besides the local Z_2 previously discussed. However, the associated correlation-length at finite temperature grows enormously when approaching the critical temperature (tenths of lattice-spacings already at $T = 0.9T_c$), and probably diverges. More details on this will be given in Ref. [19].

A further analytical evidence for this fact can be obtained by performing a Taylor expansion of the action at $J = -1/2$ assuming that $\vec{\phi}_i \cdot \vec{\phi}_j$, which just vanishes at $J = -1/2$ and zero temperature, is small:

$$-H = 1 - \frac{1}{8} \sum_{\langle i,j \rangle} (\vec{\phi}_i \cdot \vec{\phi}_j)^2 + \frac{1}{16} \sum_{\langle i,j \rangle} (\vec{\phi}_i \cdot \vec{\phi}_j)^3 - \frac{5}{128} \sum_{\langle i,j \rangle} (\vec{\phi}_i \cdot \vec{\phi}_j)^4 + \dots \quad (4)$$

We can assume that this expansion has a finite radius of convergence and so we can extend this series to the non-zero temperature region. Notice that the first term in the expansion is just the AFM-RP² Hamiltonian modified by terms that are no longer gauge invariant (those with odd powers in the scalar product). We can argue that those terms, which break explicitly the gauge invariance, are irrelevant operators, in the Renormalization Group sense, at the PM to AFM-RP² critical point and so, our model at finite temperature should belong to the same universality class as the AFM-RP² model. Obviously, were the transition of the first order, the argument would not apply.

In the Z_2 gauge-invariant phase, the vectorial magnetizations defined as

$$\vec{M}_u = \frac{1}{L^3} \sum_i \vec{\phi}_i,$$

$$\vec{M}_s = \frac{1}{L^3} \sum_i (-1)^{x_i+y_i+z_i} \vec{\phi}_i, \quad (5)$$

are zero. Thus, we define proper order parameters, invariant under that gauge symmetry, in terms of the spin field, $\vec{\phi}_i$ and the related spin-2 tensor field (which is Z_2 invariant):

$$\tau_i^{\alpha\beta} = \phi_i^\alpha \phi_i^\beta - \frac{1}{3} \delta^{\alpha\beta}, \quad \alpha, \beta = 1, 2, 3. \quad (6)$$

Then we define:

$$\tau_i^u = \tau_i, \quad \tau_i^s = (-1)^{x_i+y_i+z_i} \tau_i, \\ M_{u,s} = \frac{1}{V} \sum_i \tau_i^{u,s}. \quad (7)$$

The different phases we find (see [19] for details) are: paramagnetic, ferromagnetic ($\langle \vec{M}_u \rangle \neq 0, \langle \vec{M}_s \rangle = 0$), ferrimagnetic ($\|\langle \vec{M}_u \rangle\| > \|\langle \vec{M}_s \rangle\| > 0$), antiferrimagnetic ($\|\langle \vec{M}_s \rangle\| > \|\langle \vec{M}_u \rangle\| > 0$), antiferromagnetic ($\langle \vec{M}_s \rangle \neq 0, \langle \vec{M}_u \rangle = 0$), and RP² ($\langle M_u \rangle, \langle M_s \rangle \neq 0$ but with vanishing vectorial magnetizations). The phase diagram can be seen in Fig. 1. Notice the strong similarities of the point $(T = 0, J = -0.5)$ with a quantum critical point [7]. A detailed analysis of this phase diagram will appear soon [19].

Let us end this section by defining the observables actually used in the simulation. They are obtained in terms of the Fourier transform of the tensor fields:

$$\widehat{\tau}_p^{u,s} = \sum_{\vec{r} \in L^3} e^{-i\vec{p} \cdot \vec{r}} \tau_{\vec{r}}^{u,s}, \quad (8)$$

and their propagators

$$G_{u,s}(\vec{p}) = \frac{1}{V} \langle \text{tr} \widehat{\tau}_p^{u,s} \widehat{\tau}_p^{u,s \dagger} \rangle, \quad (9)$$

where

$$\vec{p} = \frac{2\pi}{L} \vec{n}, \quad n_i = 0, \dots, L-1. \quad (10)$$

Notice that $G_u(0) = V \text{tr} \langle (M_u)^2 \rangle$, and $G_u(\pi, \pi, \pi) = G_s(0, 0, 0) = V \text{tr} \langle (M_s)^2 \rangle$. Then we have the usual (χ_u) and staggered (χ_s) susceptibilities,

$$\chi_{u,s} = G_{u,s}(0, 0, 0). \quad (11)$$

Having those two order parameters, we must expect the following behavior for the propagators in the thermodynamic limit, in the scaling region and for

$T > T_c$:

$$G^{u,s}(\vec{p}) \simeq \frac{Z_{u,s} \xi_{u,s}^{-\eta_{u,s}}}{\vec{p}^2 + \xi_{u,s}^{-2}},$$

$$\xi_{u,s} = A_{u,s} t^{-\nu} (1 + B_{u,s} t^{-\nu\omega} + \dots), \quad (12)$$

where ξ_u and ξ_s are correlation-lengths, $t = (T - T_c)/T_c$ is the reduced temperature and $Z_{u,s}$, $A_{u,s}$ and $B_{u,s}$ are constants. On the other hand, the anomalous dimensions η_u and η_s need not be equal: we can relate them to the dimensions of the composite operators following the standard way: $d - 2 + \eta_u = 2 \dim(\tau^u)$ and $d - 2 + \eta_s = 2 \dim(\tau^s)$, and in general $\dim(\tau^s) \neq \dim(\tau^u)$.

To study the propagators (12) on a finite lattice, we need to use the minimal momentum propagators

$$F_{u,s} = \frac{1}{3} (G_{u,s}(\pi/L, 0, 0) + (\text{permutations})). \quad (13)$$

With those quantities in hand, one can define a finite-lattice correlation-length [29] for the staggered, and non-staggered sectors:

$$\xi_{u,s} = \left(\frac{\chi_{u,s}/F_{u,s} - 1}{4 \sin^2(\pi/L)} \right)^{1/2}. \quad (14)$$

Other quantities of interest are the dimensionless cumulants

$$\kappa_{u,s} = \frac{\langle (\text{tr}(M_{u,s})^2)^2 \rangle}{(\text{tr}(M_{u,s})^2)^2}. \quad (15)$$

Besides the above quantities, we also measure the energy (1), which is used in a reweighting method [30], and temperature derivatives of operators through their crossed correlation with the energy.

3. Critical behavior

For an operator O that diverges as $|t|^{-x_O}$, its mean value at temperature T in a size L lattice can be written, in the critical region, assuming the finite-size scaling ansatz as [8]

$$O(L, T) = L^{x_O/\nu} (F_O(\xi(L, T)/L) + O(L^{-\omega})), \quad (16)$$

where F_O is a smooth scaling function and ω is the universal leading correction-to-scaling exponent.

In order to eliminate the unknown F_O function, we use the method of quotients [9–11,31]. One studies the

behavior of the operator of interest in two lattice sizes, L and rL (typically $r = 2$):

$$Q_O = \frac{O(rL, t)}{O(L, t)}. \quad (17)$$

Then one chooses a value of the reduced temperature t , such that the correlation-length in units of the lattice size is the same in both lattices [11]. One readily obtains

$$Q_O|_{Q_\xi=r} = r^{x_O/\nu} + O(L^{-\omega}). \quad (18)$$

Notice that the matching condition $Q_\xi = r$ can be easily tuned with a reweighting method. The usual procedure consists on fixing $r = 2$, and obtaining the above quotients for several L values in order to perform an infinite volume extrapolation.

In order to obtain the critical exponents, we use as operators $\chi_{u,s}$ ($x_\chi = \gamma = 2 - \eta$), $\partial_T \xi_s$ ($x_{\partial_T \xi_s} = \nu + 1$). Notice that several quantities can play the role of the correlation-length in Eq. (18): ξ_u , ξ_s , $L\kappa_u$ and $L\kappa_s$. This simply changes the amplitude of the scaling corrections, which will turn out to be quite useful.

Another interesting quantity is the shift of the apparent critical point (i.e., $r\xi(L, T_c^{L,r}) = \xi(rL, T_c^{L,r})$), with respect to the real critical point:

$$T_c^{L,r} - T_c \propto \frac{1 - r^{-\omega}}{r^{1/\nu} - 1} L^{-\omega-1/\nu}. \quad (19)$$

4. The simulation

We have studied the model (1) in lattices $L = 6, 8, 12, 16, 24, 32, 48$ and 64 , with a Monte Carlo simulation at $J = -0.5$. The algorithm has been a standard Metropolis with 2 hits per spin. The trial new spin is chosen randomly in the sphere. The probability of finally changing the spin at least once is about 50%.

We have carried out 20 million full-lattice sweeps (measuring every 2 sweeps) at each lattice size at $T = 0.056$. For the $L = 64$ lattice we have also performed 20 million sweeps at $T = 0.0558$. The largest autocorrelation time measured is about 1400 sweeps (corresponding to χ_s). To ensure the thermalization we have discarded a minimum of 150 times the autocorrelation time.

The computation was made on the RTN3 cluster of 28 1.9 GHz PentiumIV processors at the University of

Table 1

Results of the infinite volume extrapolation with Eq. (19) to obtain T_c and ω . Q is the quality-of-fit parameter. Our final values are the bold values

L_{\min}	T_c	ΔT_c	ω	$\Delta\omega$	χ^2/D	D	Q
6	0.0559075	0.0000034	0.959	0.021	3.11	23	0.00
8	0.0558946	0.0000039	0.862	0.025	1.33	19	0.15
12	0.0558951	0.0000055	0.817	0.050	0.76	15	0.72
16	0.0558984	0.0000078	0.815	0.277	0.72	11	0.72

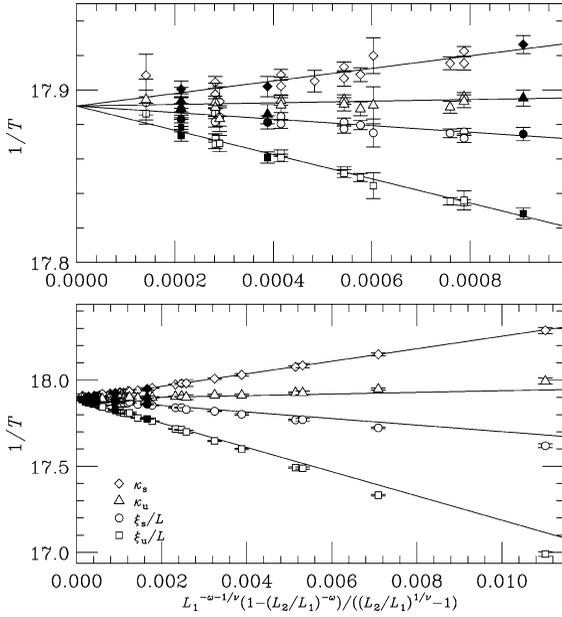


Fig. 2. Shift of the apparent critical temperature Eq. (19) with the lattice size, using the four possible kinds of dimensionless operators: ξ_u/L , ξ_s/L , κ_u and κ_s . The top panel is a magnification of the leftmost region. Only filled data points, corresponding to $r = 2$, were used in the fit.

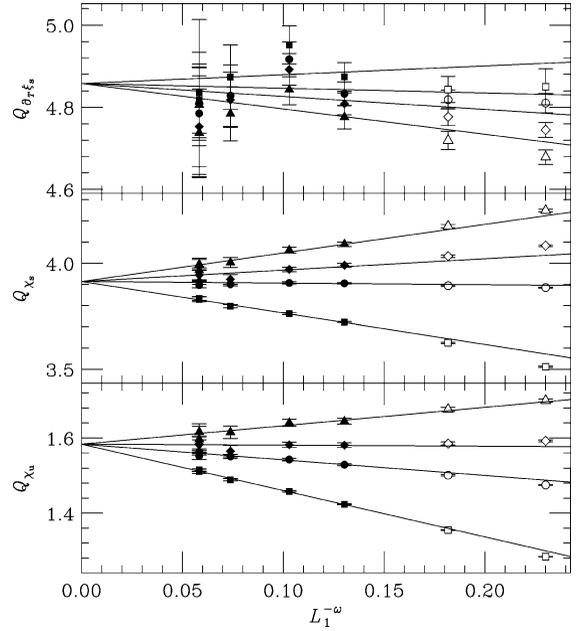


Fig. 3. Quotients defined in Eq. (18), for $\partial_T \xi_s$ (top), χ_s (medium) and χ_u , as a function of $L_1^{-\omega}$, for $r = 2$. We have measured at the crossing of four dimensionless operators, ξ_u/L , ξ_s/L , κ_u and κ_s . Symbols are as in Fig. 2.

Zaragoza and the total simulation time was equivalent to 11 months of a single processor.

5. Critical exponents

The first step, as usual, is to estimate ω from Eq. (19). For this, one needs a rough estimate of ν . Since our data for the quotient of $\partial_T \xi_s$ using ξ_s as correlation-length show very small scaling corrections (see Fig. 3), one can temporarily choose $\nu = 0.78$ and proceed with the determination of ω . Having four possible dimensionless quantities, ξ_u/L , ξ_s/L , κ_u and κ_s , we can perform a joint fit to Eq. (19) constrained to

yield the same T_c . This largely improves the accuracy of the final estimate. The full covariance matrix is used in the fit, and errors are determined by the increase of χ^2 by one-unit.³ Our results are summarized in Fig. 2 and Table 1. Although scaling corrections are clearly visible, good fits are obtained from $L_{\min} = 12$. Thus, we conclude that

$$\omega = 0.82(5), \quad T_c = 0.055895(5). \quad (20)$$

³ We have taken into account the high correlation in fitted parameters: for each one, the error corresponds to the maximum possible deviation from its central value compatible with a one-unit increase of χ^2 .

Table 2

Infinite volume extrapolation for the critical exponents, using Eq. (18) and $\omega = 0.82$. The fits were performed using $L \geq L_{\min}$. The goodness-of-fit (Q) is also indicated. The chosen extrapolation and error are emphasized

L_{\min}	ν	$\Delta\nu$	χ^2/D	D	Q
6	0.7722	0.0041	2.55	23	0.00
8	0.7724	0.0055	1.66	19	0.03
12	0.7811	0.0107	1.03	15	0.42
16	0.7898	0.0179	0.83	11	0.61
<hr/>					
	η_s	$\Delta\eta_s$			
6	0.0238	0.0012	18.4	23	0.00
8	0.0278	0.0016	1.97	19	0.01
12	0.0315	0.0027	0.67	15	0.81
16	0.0359	0.0049	0.56	11	0.87
<hr/>					
	η_u	$\Delta\eta_u$			
6	1.3259	0.0013	8.52	23	0.00
8	1.3334	0.0018	1.56	19	0.06
12	1.3368	0.0031	0.77	15	0.72
16	1.3435	0.0058	0.62	11	0.82

We are now ready for the infinite volume extrapolation of the critical exponents. As usual, one needs to worry about higher-order scaling corrections. Here we shall follow a conservative criterion: we shall perform the fit to Eq. (18) only for $L \geq L_{\min}$, and observe what happens varying L_{\min} . Once we found a L_{\min} for which the fit is acceptable and the infinite volume extrapolation for $L \geq L_{\min}$ and $L > L_{\min}$ are compatible within errors, we take the extrapolated value from the $L \geq L_{\min}$ fit, and the error from the $L > L_{\min}$ fit. Our results can be found in Fig. 3 and in Table 2. As well as for the critical temperature, we used all four dimensionless quantities in a single fit constrained to yield a common infinite volume extrapolation. Our final estimates are

$$\begin{aligned} \nu &= 0.781(18)(1), & \eta_s &= 0.032(5)(2), \\ \eta_u &= 1.337(6)(7), \end{aligned} \quad (21)$$

were the second error is due to the uncertainty in ω .

One can compare these results with other models, once it is decided what is going to play the role of our η_u in the $O(N)$ models. Our candidate is the tensorial

representation [9]^{4 5}

$$\begin{aligned} \text{RP}^2 [11]: \quad \nu &= 0.783(11), & \eta_s &= 0.038(3), \\ & \eta_u &= 1.339(10), \\ O(3) [9,23]: \quad \nu &= 0.710(2), & \eta_\phi &= 0.0380(10), \\ & \eta_{\vec{\phi}\otimes\vec{\phi}} &= 1.427(3), \\ O(4) [9,23]: \quad \nu &= 0.749(2), & \eta_\phi &= 0.0365(10), \\ & \eta_{\vec{\phi}\otimes\vec{\phi}} &= 1.374(5), \\ \text{Tricritical (MF):} \quad \nu &= 1/2, & \eta_\phi &= 0, \\ & \eta_{\vec{\phi}\otimes\vec{\phi}} &= 1. \end{aligned} \quad (22)$$

⁴ In $O(N)$ models it is possible to compute $\eta_{\vec{\phi}\otimes\vec{\phi}}$ using field theoretical methods. We should compute the dimensions of the operators ϕ_i^2 (scalar) and $\phi_i\phi_j$ ($i \neq j$), for instance, introducing them in correlation functions. These dimensions have been computed using field theory in fixed dimension in Ref. [32] (six-loops result), in terms of $\gamma_{\vec{\phi}\otimes\vec{\phi}}^+$. Using $\eta_{\vec{\phi}\otimes\vec{\phi}} = 2 - \gamma_{\vec{\phi}\otimes\vec{\phi}}^+/\nu$ we obtain $\eta_{\vec{\phi}\otimes\vec{\phi}} = 1.42(4)$ for $N = 3$ and $\eta_{\vec{\phi}\otimes\vec{\phi}} = 1.39(8)$ for $N = 4$. We have used the values of ν reported in Ref. [31] computed in fixed dimension.

⁵ In Mean Field (MF) we can relate the tensorial anomalous dimension $\eta_{\vec{\phi}\otimes\vec{\phi}}$ and the upper critical dimension of the theory d_u by means of $d_u - 2 + \eta_{\vec{\phi}\otimes\vec{\phi}} = 2(d_u - 2)$. For instance, if $d_u = 4$ we obtain $\eta_{\vec{\phi}\otimes\vec{\phi}} = 2$. For tricritical mean field exponents, $d_u = 3$ and $\eta_{\vec{\phi}\otimes\vec{\phi}} = 1$. The other exponents are the usual ones: $\nu = 1/2$ and $\eta_\phi = 0$.

6. Conclusions

We have studied numerically a bosonic version of the DEM, Eq. (1), by Monte Carlo simulations, obtaining its full phase-diagram, missed in previous studies [5]. We have studied its critical behavior with finite-size scaling techniques. As Eqs. (21) and (22) show, our results for the critical exponents are fully compatible with the results for the AFM-RP² model, barely compatible with the $O(4)$ model, and fully incompatible with the results for the $O(3)$ model. Our results in the low temperature phase [19] seem to indicate that the scheme of symmetry-breaking is $O(3)/O(2)$, which contradicts universality. Most puzzling is the excellent agreement between the present results and the estimates for the AFM-RP² model. This seems to indicate that the AFM-RP² model really represents a new universality class in three dimensions, in plain contradiction with the universality-hypothesis (at least in its more general form). This seems to imply that the local isomorphism of \mathcal{G}/\mathcal{H} is not enough to guarantee a common universality class. Of course, it could happen that we were seeing only *effective* exponents and that in the $L \rightarrow \infty$ limit a more standard picture arises. Yet, we do not find any obvious reason for two very different models to have such a similar *effective* exponents.

An important remark is that in this model one of the local symmetries of the fixed point which controls the phase transition was completely hidden in our Hamiltonian. This is another example of an interesting fact: the symmetries of the fixed points are those that control the universality classes, not just the symmetries of the initial Hamiltonian [18].

Another intriguing effect is that the augmented local Z_2 symmetry of the point ($T = 0$, $J = -0.5$) extends to the finite temperature plane, which is recalling a quantum critical point [7]. In this respect, it is interesting to recall that the full double-exchange model (with dynamical fermions) has been proposed to have a quantum critical point induced by disorder [33]. It could be quite interesting to see whether the PM phase that extends to zero temperature is really PM or, like in this case, an ordering arises in the tensorial sector. Anyhow, in this work we have found that the universality class is the one of an explicitly gauge-invariant model. To our knowledge this is a new effect in (clas-

sical) statistical mechanics, and deserves to be called dynamical generation of a gauge symmetry.

Acknowledgements

We thank very particularly José Luis Alonso for pointing this problem to us, for his encouragement and for many discussions. It is also a pleasure to thank Francisco Guinea for discussions. The simulations were performed in the PentiumIV cluster RTN3 at the Universidad de Zaragoza. We thank the Spanish MCyT for financial support through research contracts FPA2001-1813, FPA2000-0956, BFM2001-0718 and PB98-0842. V.M.-M. is a Ramón y Cajal research fellow (MCyT).

References

- [1] W. Bock, J. Smit, J.C. Vink, Nucl. Phys. B 416 (1994) 645.
- [2] G. Parisi, Nucl. Phys. Proc. Suppl. BC 29 (1992) 247.
- [3] P.H. Ginsparg, K.G. Wilson, Phys. Rev. B 25 (1982) 2649; M. Lüscher, Lectures given at the International School of Subnuclear Physics, Erice, 2000, hep-th/0102028.
- [4] C. Zener, Phys. Rev. 82 (1951) 403; P.W. Anderson, H. Hasegawa, Phys. Rev. 100 (1955) 675; P.-G. de Gennes, Phys. Rev. 118 (1960) 141.
- [5] S.-H. Tsai, D.P. Landau, J. Magn. Magn. Mater. 226–230 (2001) 650; S.-H. Tsai, D.P. Landau, J. Appl. Phys. 87 (2000) 5807.
- [6] See, e.g., E. Dagotto, T. Hotta, A. Moreo, Phys. Rep. 344 (2001) 1.
- [7] S. Sachdev, Quantum Phase Transitions, Cambridge Univ. Press, Cambridge, 1999; S. Sachdev, Science 288 (2000) 475.
- [8] J.L. Cardy (Ed.), Finite-Size Scaling, North-Holland, Amsterdam, 1988.
- [9] H.G. Ballesteros, L.A. Fernández, V. Martín-Mayor, A. Muñoz Sudupe, Phys. Lett. B 387 (1996) 125.
- [10] H.G. Ballesteros, L.A. Fernández, V. Martín-Mayor, A. Muñoz Sudupe, G. Parisi, J.J. Ruiz-Lorenzo, J. Phys. A 32 (1999) 1.
- [11] H.G. Ballesteros, L.A. Fernández, V. Martín-Mayor, A. Muñoz Sudupe, Phys. Lett. B 378 (1996) 207; H.G. Ballesteros, L.A. Fernández, V. Martín-Mayor, A. Muñoz Sudupe, Nucl. Phys. B 483 (1997) 707.
- [12] P.A. Lebowitz, G. Lasher, Phys. Rev. A 6 (1972) 426.
- [13] S. Romano, Int. J. Mod. Phys. B 8 (1994) 3389.
- [14] G. Korhning, R.E. Shrock, Nucl. Phys. B 295 (1988) 36.
- [15] G. Zumbach, Nucl. Phys. B 435 (1995) 753.
- [16] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena, 1st Edition, Oxford Univ. Press, Oxford, 1989.
- [17] P. Hasenfratz, H. Leuwyler, Nucl. Phys. B 343 (1990) 241;

- P. Hasenfratz, F. Niedermayer, Phys. Lett. B 268 (1991) 231;
P. Hasenfratz, F. Niedermayer, Z. Phys. B 92 (1993) 91.
- [18] P. Azaria, B. Delamotte, T. Jolicœur, Phys. Rev. Lett. 64 (1990) 3175;
P. Azaria, B. Delamotte, F. Delduc, T. Jolicœur, Nucl. Phys. B 408 (1993) 485.
- [19] A. Cruz, et al., in preparation.
- [20] R.J. Baxter, Exactly Solved Models in Statistical Mechanics, Academic Press, San Diego, CA, 1982.
- [21] See, for example, H. Kawamura, Can. J. Phys. 79 (2001) 1447.
- [22] A. Pelissetto, P. Rossi, E. Vicari, Phys. Rev. B 65 (2002) 020403, and references therein.
- [23] M. Hasenbusch, J. Phys. A 34 (2001) 8221.
- [24] M. Tissier, B. Delamotte, D. Mouhanna, Phys. Rev. Lett. 84 (2000) 5208.
- [25] M. Itakura, J. Phys. Soc. Jpn. 72 (2003) 74.
- [26] L.A. Fernández, M.P. Lombardo, J.J. Ruiz-Lorenzo, A. Tarancón, Phys. Lett. B 277 (1992) 485.
- [27] J.L. Alonso, L.A. Fernández, F. Guinea, V. Laliena, V. Martín-Mayor, Nucl. Phys. B 569 (2001) 587.
- [28] J.L. Alonso, L.A. Fernández, F. Guinea, V. Laliena, V. Martín-Mayor, Phys. Rev. B 63 (2001) 64416;
J.L. Alonso, J.A. Capitán, L.A. Fernández, F. Guinea, V. Martín-Mayor, Phys. Rev. B 64 (2001) 54408.
- [29] F. Cooper, B. Freedman, D. Preston, Nucl. Phys. B 210 (1989) 210.
- [30] A.M. Ferrenberg, R.H. Swendsen, Phys. Rev. Lett. 61 (1988) 2635.
- [31] A. Pelissetto, E. Vicari, Phys. Rep. 368 (2002) 549.
- [32] P. Calabrese, A. Pelissetto, E. Vicari, Phys. Rev. E 65 (2002) 046115.
- [33] J. Burgy, M. Mayr, V. Martín-Mayor, A. Moreo, E. Dagotto, Phys. Rev. Lett. 87 (2001) 277202;
J.L. Alonso, L.A. Fernández, F. Guinea, V. Laliena, V. Martín-Mayor, Phys. Rev. B 66 (2002) 104430.