# Perturbative Ward identities for Yang-Mills field theory stochastically quantized

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We compute the divergent part of the three-point vertex function of the non-Abelian Yang-Mills gauge field theory within the stochastic quantization approach to the one-loop order. This calculation allows us to find four renormalization constants which, together with the four previously obtained, verify, to the calculated order, some Ward identities.

## I. INTRODUCTION

The stochastic quantization (SQ) approach to gauge theories introduced by Parisi and Wu[1] seems to be an interesting alternative to the usual quantization procedure, which in the non-Abelian Yang-Mills (YM) case leads to the Faddeev-Popov (FP) functional which includes a gauge-fixing term. This is the source of the Gribov ambiguity[2] or, in perturbation theory, leads to the introduction of fermionic ghost fields which are an obstacle to Monte Carlo simulations.[3] The SQ method avoids the gauge-fixing term by introducing an artificial fifth time coordinate $t$ in addition to the usual four Euclidean variables, and assumes that the system evolves according to stochastic differential equations: Langevin equations with an external Gaussian white noise or Fokker-Planck equations. The Euclidean Green's functions of the quantum field theory involved may then be reproduced as the $t\to\infty$ limit of equal-time stochastic correlation functions. Moreover, Parisi and Wu stated that, in such a limit and for gauge theories, only gauge-invariant quantities should give the same result as the usual quantization procedure, while non-gauge-invariant quantities should diverge. In order to handle, in perturbation theory, non-gauge-invariant quantities such as the free propagator, it is useful to introduce a generalized gauge-fixing term[4] (Zwanziger). In this paper, we shall deal with the SQ approach to Euclidean Yang-Mills field theory. We will specifically compute the divergent part of the three-point vertex function to the one-loop order (Fig. 2). This calculation together with previous results[5] will allow us to establish some Ward identities between renormalization constants. We derive the Feynman rules (Fig. 1) from a stochastic generating functional which has deep relations with functionals previously used in the SQ context[6] and also in nonequilibrium critical dynamics.[7] The specific features of our approach are (1) the artificial time $t$ is integrated between $-\infty$ and $+\infty$ (we shall Fourier transform it and obtain simpler Feynman rules),[8] and (2) no separation between transversal and longitudinal parts is needed.[9]

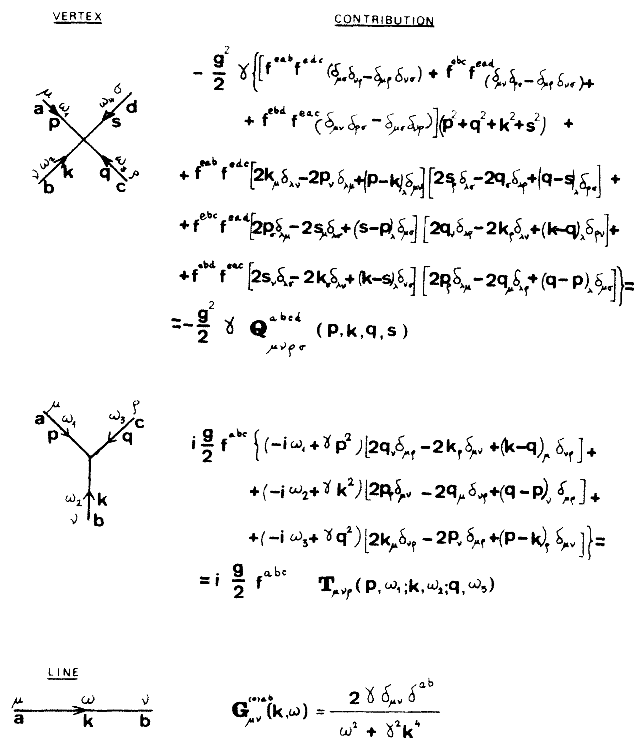


FIG. 1. Symmetrized vertices that contribute to the three-point vertex function to order $g^2$.

## II. GENERATING FUNCTIONAL

Consider the Langevin equation for Yang-Mills SQ with the Zwanziger gauge-fixing term $v^b$:

$$\frac{\partial A_\mu^a(x,t)}{\partial t}=-\gamma\left[\frac{\delta S}{\delta A_\mu^a(x,t)}+D_\mu^{ab}v^b\right]+\eta_\mu^a(x,t)\ , \quad (2.1)$$

where $S=\int dx^4\frac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a$ is the standard Euclidean Yang-

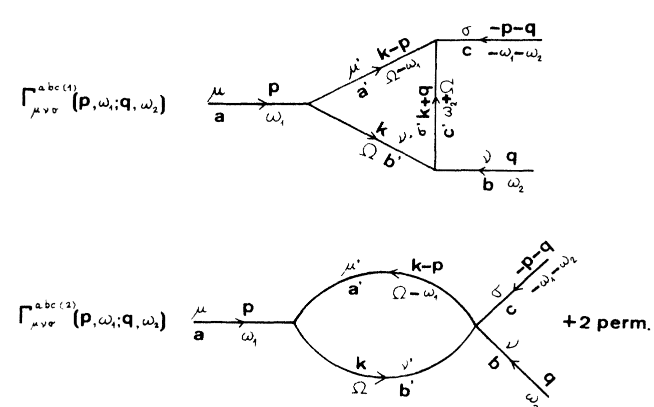


FIG. 2. Feynman diagrams for the three-point vertex function to order $g^2$.

Mills action with

$$F^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+gf^{abc}A^b_\mu A^c_\nu$$

and the covariant derivative $D^{ab}_\mu=\partial_\mu\delta^{ab}-gf^{abc}A^c_\mu$ acts on the gauge-fixing term that we have chosen for our calculation to be $v^b=(1/\alpha)\partial_\mu A^b_\mu$, with $\alpha=1$ (for practical calculations). $\eta^a_\mu(x,t)$ is a Gaussian white noise, that is, with zero average and correlation

$$\langle\eta^a_\mu(x,t)\eta^b_\nu(x',t')\rangle=2\gamma\delta^{ab}\delta_{\mu\nu}\delta^{(4)}(x-x')\delta(t-t') \ .$$

The partition function for (2.1) reads

$$Z_v=\int[DA][D\eta]\times\exp\left(-\frac{1}{4\gamma}\int d^4x\,dt(\eta^a_\mu)^2\right)\delta(A^a_\mu-A^{a,\eta}_\mu) \ , \quad (2.2)$$

where $A^{a,\eta}_\mu$ represents fields which are solutions of (2.1). Introducing a source $J^a_\mu$ for the gauge field and performing the Gaussian integration in $\eta^a_\mu$, we arrive at the generating functional

$$Z_v[J]=\int[DA]\exp\left[J^a_\mu A^a_\mu-\frac{1}{4\gamma}\int d^4x\,dt\left(\frac{\partial A^a_\mu}{\partial t}+\gamma\frac{\delta S}{\delta A^a_\mu}-\gamma D^{ab}_\mu v^b\right)^2\right]\det\left[\frac{\delta}{\delta A^a_\mu(x',t')}\left(\frac{\partial A^a_\mu(x,t)}{\partial t}+\gamma\frac{\delta S}{\delta A^a_\mu}-\gamma D^{ab}_\mu v^b\right)\right] \ . \quad (2.3)$$

The Jacobian in (2.3) may be calculated[7] and, not surprisingly, gives rise to terms which include $\delta^{(4)}(0)$. However, as we have shown before,[6] these cancel in perturbation theory with other quartically (d=4) divergent integrals leaving us with the usual quadratic divergence degree. We regularize the integrals by means of dimensional regularization[10] in the Euclidean coordinates while it is not necessary in the fifth time coordinate.

From (2.3) we generate the perturbation theory by standard functional techniques;[11] in this way we have already calculated the infinite part of the two-point function to the one-loop order.[5] In order to absorb it, three renormalization constants have to be introduced: $Z_A$ associated with the field $A^a_\mu$, $Z_\gamma$ with the diffusion constant $\gamma$, and $Z_\alpha$ with the gauge parameter $\alpha$ (for $\alpha=1$). They have the following expressions ($A_0=Z_AA$, $\gamma_0=Z_\gamma\gamma$, $\alpha_0=Z_\alpha 1$):

$$Z_A=1+\tfrac{4}{3}g^2C_2(G)\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)} \ , \quad Z_\gamma=1+\tfrac{13}{12}g^2C_2(G)\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)} \ , \quad Z_\alpha=1-\tfrac{43}{24}g^2C_2(G)\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)}$$
$$(d=4+2\epsilon) \ , \quad (2.4)$$

## III. THREE-POINT VERTEX FUNCTIONS

There are four integrals which contribute to the three-point vertex function $\Gamma^{abc}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$ to the one-loop order that we will call $\Gamma^{abc(i)}_{\mu\nu\sigma}$ ($i=1,\ldots,4$) (Fig. 2), so that

$$\Gamma^{abc}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=\sum_{i=1}^{4}\Gamma^{abc(i)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2) \ .$$

In fact, we will have only to calculate the first two integrals since the other two may be obtained directly from permutations of the legs of $\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$, as is shown in the relations

$$\Gamma^{abc(3)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=\Gamma^{bac(2)}_{\nu\mu\sigma}(q,\omega_2;p,\omega_1), \quad \Gamma^{abc(4)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=\Gamma^{cba(2)}_{\sigma\nu\mu}(-p-q,-\omega_1-\omega_2;q,\omega_2) \ . \quad (3.1)$$

Using the Feynman rules and the notations of our previous works[5,6] (Fig. 1) we have, for the first diagram in Fig. 2,

$$\Gamma^{abc(1)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=i(g\gamma)^3\tfrac{1}{2}Nf^{abc}\int\frac{d\Omega\,d^dk}{(2\pi)^{d+1}}T_{\mu'\mu\nu'}(k-p,\Omega-\omega_1;p,\omega_1;-k,-\Omega)T_{\nu'\nu\sigma'}(k,\Omega;q,\omega_2;-k-q,-\omega_2-\Omega)$$
$$\times T_{\sigma'\sigma\mu'}(k+q,\omega_2+\Omega;-p-q,-\omega_1-\omega_2;p-k,\omega_1-\Omega)$$
$$\times(\Omega^2+\gamma^2k^4)^{-1}[(\Omega-\omega_1)^2+\gamma^2(k-p)^4]^{-1}[(\Omega+\omega_2)^2+\gamma^2(k+q)^4]^{-1} \ , \quad (3.2)$$

where $N$ comes from the SU($N$) gauge group and

$$\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=-i(g\gamma)^3\tfrac{1}{2}f^{aa'b'}\int\frac{d\Omega\,d^dk}{(2\pi)^{d+1}}T_{\mu\nu'\mu'}(p,\omega_1;-k-\Omega;k-p,\Omega-\omega_1)$$
$$\times Q^{a'b'bc}_{\mu'\nu'\nu\sigma}(p-k,k,q,-p-q)(\Omega^2+\gamma^2k^4)^{-1}[(\Omega-\omega_1)^2+\gamma^2(k-p)^4]^{-1} \ . \quad (3.3)$$

The naive divergence degree of $\Gamma^{abc(1)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$ given in (3.2) is at most cubic. In particular, there are 18 divergent integrals: 6 of them contain the cubic divergences, and the remaining 12 are at most logarithmic. However, it is only strictly essential to compute explicitly two of the cubic and four of the logarithmic integrals due to the evident symmetry of $\Gamma^{abc(1)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$, which is

$$\Gamma^{abc(1)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=\Gamma^{bac(1)}_{\nu\mu\sigma}(q,\omega_2;p,\omega_1)=\Gamma^{cba(1)}_{\sigma\nu\mu}(-p-q,-\omega_1-\omega_2;q,\omega_2) \ . \quad (3.4)$$

The result of a lengthy and extremely careful calculation is

$$(\Gamma^{abc(1)}_{\mu\nu\sigma} p,\omega_1;q,\omega_2)_{\rm div}$$
$$= ig^3 N f^{abc}\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)}\frac{1}{32}\Big\{ i\omega_1[\delta_{\nu\sigma}(32p_\mu+11q_\mu)+\delta_{\mu\sigma}(-53q_\nu)+\delta_{\mu\nu}(-21q_\sigma-32p_\sigma)]$$
$$+i\omega_2[\delta_{\mu\sigma}(-32q_\nu-11p_\nu)+\delta_{\nu\sigma}53p_\mu+\delta_{\mu\nu}(21p_\sigma+32q_\sigma)]$$
$$+\frac{\gamma p^2}{6}[\delta_{\nu\sigma}364(p+2q)_\mu+\delta_{\mu\sigma}(-533p_\nu-658q_\nu)+\delta_{\mu\nu}(-125p_\sigma-658q_\sigma)]$$
$$+\frac{\gamma q^2}{6}[\delta_{\mu\sigma}(-364)(q+2p)_\nu+\delta_{\mu\nu}(125q_\sigma+658p_\sigma)+\delta_{\nu\sigma}(533q_\mu+658p_\mu)]$$
$$+\frac{\gamma(p+q)^2}{6}[\delta_{\mu\nu}364(p-q)_\sigma+\delta_{\mu\sigma}(-533p_\nu+125q_\nu)+\delta_{\nu\sigma}(-125p_\mu+533q_\mu)]$$
$$+26\gamma[2(p_\mu p_\nu-q_\mu q_\nu)(p+q)_\sigma+p_\mu q_\nu(p-q)_\sigma]\Big\}\ . \tag{3.5}$$

On the other hand, $\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$ given in (3.3) can be written as

$$\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)=-i(g\gamma)^3\tfrac{1}{2}Nf^{abc}\int\frac{d\Omega\, d^dk}{(2\pi)^{d+1}}T_{\mu\nu'\mu'}(p,\omega_1;-k,-\Omega;k-p,\Omega-\omega_1)$$
$$\times\Delta_{\mu'\nu'\nu\sigma}(k,p,q)(\Omega^2+\gamma^2k^4)^{-1}[(\Omega-\omega_1)^2+\gamma^2(k-p)^4]^{-1} \tag{3.6}$$

with

$$\Delta_{\mu'\nu'\nu\sigma}(k,p,q)=[2k_{\mu'}\delta_{\lambda\nu'}+2(k-p)_{\nu'}\delta_{\lambda\mu'}+(-2k+p)_\lambda\delta_{\mu'\nu'}][-2(p+q)_\nu\delta_{\lambda\sigma}-2q_\sigma\delta_{\lambda\nu}+(2q+p)_\lambda\delta_{\nu\sigma}]$$
$$+\tfrac{1}{2}[-2(k-p)_\sigma\delta_{\lambda\mu'}+2(p+q)_{\mu'}\delta_{\lambda\sigma}+(k-2p-q)_\lambda\delta_{\mu'\sigma}][2q_{\nu'}\delta_{\lambda\nu}-2k_\nu\delta_{\lambda\nu'}+(k-q)_\lambda\delta_{\nu\nu'}]$$
$$-\tfrac{1}{2}[-2(p+q)_{\nu'}\delta_{\lambda\sigma}-2k_\sigma\delta_{\lambda\nu'}+(p+q+k)_\lambda\delta_{\sigma\nu'}][2(p-k)_\nu\delta_{\lambda\mu'}-2q_{\mu'}\delta_{\lambda\nu}+(q+k-p)_\lambda\delta_{\mu'\nu}]$$
$$+\tfrac{3}{2}(\delta_{\mu'\sigma}\delta_{\nu\nu'}-\delta_{\mu'\nu}\delta_{\sigma\nu'})[k^2+q^2+(p+q)^2+(k-q)^2]\ . \tag{3.7}$$

Performing the $\Omega$ integration we can write $\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$ as the sum of three divergent integrals:

$$\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)$$
$$=-ig^3\gamma^2\tfrac{1}{4}Nf^{abc}\Big\{\int\frac{d^dk}{(2\pi)^d}(i\omega_1+\gamma p^2)\frac{k^2+(k-p)^2}{k^2(k-p)^2\{\omega_1^2+\gamma^2[k^2+(k-p)^2]^2\}}$$
$$\times[(-2k+p)_\mu\delta_{\mu'\nu'}+2(k-p)_{\nu'}\delta_{\mu\mu'}+2k_{\mu'}\delta_{\mu\nu'}]\Delta_{\mu'\nu'\nu\sigma}(k,p,q)$$
$$+\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2\{i\omega_1+\gamma[k^2+(k-p)^2]^2\}}[(p+k)_{\mu'}\delta_{\mu\nu'}-2k_\mu\delta_{\mu'\nu'}-2p_{\nu'}\delta_{\mu\mu'}]\Delta_{\mu'\nu'\nu\sigma}(k,p,q)$$
$$+\int\frac{d^dk}{(2\pi)^d}\frac{1}{(k-p)^2\{i\omega_1+\gamma[k^2+(k-p)^2]^2\}}[(k-2p)_{\nu'}\delta_{\mu\mu'}+2p_{\mu'}\delta_{\mu\nu'}-2(k-p)_\mu\delta_{\mu'\nu'}]\Delta_{\mu'\nu'\nu\sigma}(k,p,q)\Big\}. \tag{3.8}$$

The first integral on the right-hand side (RHS) of (3.8) is at most linearly divergent, and so we will need to take into account only the terms in $\Delta_{\mu'\nu'\nu\sigma}(k,p,q)$ of orders $k$ and $k^2$, respectively. The remaining two integrals may be seen to be identical, as follows: when the change $k\leftrightarrow p-k$, $\mu'\leftrightarrow\nu'$ is performed, the second integral transforms into the third because $\Delta_{\mu'\nu'\nu\sigma}(k,p,q)\rightarrow-\Delta_{\mu'\nu'\nu\sigma}(k,p,q)$ and

$$[(p+k)_{\mu'}\delta_{\mu\nu'}-2k_\mu\delta_{\mu'\nu'}-2p_{\nu'}\delta_{\mu\mu'}]\rightarrow-[(k-2p)_{\nu'}\delta_{\mu\mu'}+2p_{\mu'}\delta_{\mu\nu'}-2(k-p)_\mu\delta_{\nu'\nu'}]\ .$$

The second integral is at most cubically divergent and at least logarithmically, so that it has to be calculated completely.

A very lengthy and detailed calculation yields finally

$$[\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1,q,\omega_2)]_{\rm div}$$

$$= -ig^3Nf^{abc}\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)}\frac{1}{32}\Bigg( i\omega_1[\delta_{\nu\sigma}24(2q+p)_\mu+\delta_{\mu\sigma}(-37p_\nu-51q_\nu)+\delta_{\mu\nu}(-51q_\sigma-14p_\sigma)]$$
$$+\frac{\gamma p^2}{6}[\delta_{\nu\sigma}512(2q+p)_\mu+\delta_{\mu\sigma}(-1087p_\nu-802q_\nu)+\delta_{\mu\nu}(-802q_\sigma+285p_\sigma)]$$
$$+\frac{\gamma q^2}{6}[\delta_{\mu\sigma}(-268p_\nu-78q_\nu)+\delta_{\nu\sigma}332p_\mu+\delta_{\mu\nu}(712p_\sigma+78q_\sigma)]$$
$$+\frac{\gamma(p+q)^2}{3}[\delta_{\mu\nu}(190p_\sigma-78q_\sigma)+\delta_{\mu\nu}(-278p_\nu+78q_\nu)+\delta_{\nu\sigma}p_\mu(-332)]$$
$$+\frac{\gamma}{3}[250p_\mu p_\nu(p+q)_\sigma+78p_\nu q_\mu(p+q)_\sigma-78p_\sigma q_\mu q_\nu+172p_\mu q_\sigma q_\nu]\Bigg) \;. \tag{3.9}$$

Using (3.1) we can express the sum of the last three diagrams as

$$\sum_{i=2}^{4}[\Gamma^{abc(2)}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)]_{\rm div}$$

$$= -ig^3Nf^{abc}\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)}\frac{1}{32}\Bigg( i\omega_1[\delta_{\nu\sigma}(38p_\mu+11q_\mu)+\delta_{\mu\sigma}(-65q_\nu)+\delta_{\mu\nu}(-27q_\sigma-38p_\sigma)]$$
$$+i\omega_2[\delta_{\mu\sigma}(-38q_\nu-11p_\nu)+\delta_{\nu\sigma}65p_\mu+\delta_{\mu\nu}(27p_\sigma+38q_\sigma)]$$
$$+\frac{\gamma p^2}{6}[\delta_{\nu\sigma}780(p+2q)_\mu+\delta_{\mu\sigma}(-1365p_\nu-1490q_\nu)+\delta_{\mu\nu}(-125p_\sigma-1490q_\sigma)]$$
$$+\frac{\gamma q^2}{6}[\delta_{\mu\sigma}(-780)(q+2p)_\nu+\delta_{\nu\sigma}(1365q_\mu+1490p_\mu)+\delta_{\mu\nu}(125q_\sigma+1490p_\sigma)]$$
$$+\frac{\gamma(p+q)^2}{6}[\delta_{\mu\nu}780(p-q)_\sigma+\delta_{\mu\sigma}(-1365p_\nu+125q_\nu)+\delta_{\nu\sigma}(1365q_\mu-125p_\mu)]$$
$$+\gamma\frac{250}{3}[2(p_\mu p_\nu-q_\mu q_\nu)(p+q)_\sigma+p_\mu q_\nu(p-q)_\sigma]\Bigg) \;. \tag{3.10}$$

Finally, we get for $[\Gamma^{abc}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)]_{\rm div}$, by summing up (3.5) and (3.10),

$$\Gamma^{abc}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2) = -ig^3Nf^{abc}\frac{1}{(4\pi)^2}\frac{1}{(-\epsilon)}\frac{1}{24}\{9i\omega_1[\delta_{\mu\sigma}2q_\nu+\delta_{\mu\nu}(p+q)_\sigma-\delta_{\nu\sigma}p_\mu]-9i\omega_2[\delta_{\nu\sigma}2p_\mu+\delta_{\mu\nu}(p+q)_\sigma-\delta_{\mu\sigma}q_\nu]$$
$$+52\gamma p^2[\delta_{\mu\nu}2q_\sigma+\delta_{\mu\sigma}2(p+q)_\nu-\delta_{\nu\sigma}(p+2q)_\mu]$$
$$-52\gamma q^2[\delta_{\mu\nu}2p_\sigma+\delta_{\nu\sigma}2(p+q)_\mu-\delta_{\mu\sigma}(q+2p)_\nu]$$
$$-52\gamma(p+q)^2[\delta_{\nu\sigma}2q_\mu-\delta_{\mu\sigma}2p_\nu+\delta_{\mu\nu}2(p-q)_\sigma]$$
$$-43\gamma[2(p_\mu p_\nu-q_\mu q_\nu)(p+q)_\sigma+p_\mu q_\nu(p-q)_\sigma]\} \;. \tag{3.11}$$

The first term, proportional to $\omega_1$, on the RHS of (3.11), has already been derived in a previous work.[12] In order to absorb it and the remaining terms, we can add counterterms to the original (renormalized) Lagrangian

$$\mathcal{L}_R=\frac{1}{4\gamma}\left(\frac{\partial A^a_\mu}{\partial t}+\gamma\Lambda^a_\mu\right)^2 \;;$$

they constitute the counterterm Lagrangian $\mathcal{L}_c$:

$$\mathcal{L}_c=\tfrac{1}{2}g\frac{\partial A^a_\mu}{\partial t}\frac{1}{\alpha}f^{abc}A^c_\mu\partial_\nu A^b_\nu(Z_4-1)+\tfrac{1}{2}g\gamma(-\partial_\nu\partial_\nu A^a_\mu+\partial_\mu\partial_\nu A^a_\nu)f^{abc}[\partial_\sigma(A^c_\sigma A^b_\mu)+A^b_\sigma\partial_\mu A^c_\sigma+A^b_\sigma\partial_\sigma A^c_\mu](Z_5-1)$$
$$+\tfrac{1}{2}g\gamma(-\partial_\nu\partial_\nu A^a_\mu+\partial_\mu\partial_\nu A^a_\nu)\frac{1}{\alpha}f^{abc}A^c_\mu\partial_\sigma A^b_\sigma(Z_6-1)-\tfrac{1}{2}g\gamma\frac{1}{\alpha}(\partial_\mu\partial_\nu A^a_\nu)f^{abc}[\partial_\sigma(A^c_\sigma A^b_\mu)+A^b_\sigma\partial_\mu A^c_\sigma+A^b_\sigma\partial_\sigma A^c_\mu](Z_7-1)$$
$$-\tfrac{1}{2}g\gamma\frac{1}{\alpha^2}(\partial_\mu\partial_\nu A^a_\nu)f^{abc}A^c_\mu(\partial_\sigma A^b_\sigma)(Z_8-1) \;. \tag{3.12}$$

This counterterm gives rise to the following contribution to the three-point vertex function ($\alpha=1$):

$$
\begin{aligned}
&[\Gamma^{abc}_{\mu\nu\sigma}(p,\omega_1;q,\omega_2)]_{ct}\\
&= igf^{abc}\tfrac{1}{2}\big(-i\omega_1(Z_4-1)[\delta_{\mu\nu}(p+q)_\sigma+\delta_{\mu\sigma}2q_\nu-p_\mu\delta_{\nu\sigma}]+i\omega_2(Z_4-1)[\delta_{\mu\nu}(p+q)_\sigma+\delta_{\nu\sigma}2p_\mu-q_\nu\delta_{\mu\sigma}]\\
&\quad+\gamma p^2\{\delta_{\mu\nu}[(Z_7-Z_6)(p+q)_\sigma+2(1-Z_5)q_\sigma]+\delta_{\mu\sigma}[(Z_7-Z_6)q_\nu+2(1-Z_5)(q+p)_\nu]+\delta_{\nu\sigma}(Z_5-1)(2q+p)_\mu\}\\
&\quad+\gamma q^2\{\delta_{\mu\nu}[(Z_6-Z_7)(q+p)_\sigma+2(Z_5-1)p_\sigma]+\delta_{\nu\sigma}[(Z_6-Z_7)p_\mu+2(Z_5-1)(p+q)_\mu]+\delta_{\mu\sigma}(1-Z_5)(2p+q)_\nu\}\\
&\quad+\gamma(p+q)^2\{\delta_{\nu\sigma}[(Z_7-Z_6)p_\mu+2(Z_5-1)q_\mu]+\delta_{\mu\sigma}[(Z_6-Z_7)q_\nu-2(Z_5-1)p_\nu]+\delta_{\mu\nu}(Z_5-1)(p-q)_\sigma\}\\
&\quad+(Z_6-Z_7+Z_5-Z_8)[2(p_\mu p_\nu-q_\mu q_\nu)(p+q)_\sigma+p_\mu q_\nu(p-q)_\sigma]\big)\ . \qquad (3.13)
\end{aligned}
$$

By comparing (3.13) with (3.11), we obtain, in addition to the previously obtained $Z_4$,[12]

$$
\begin{aligned}
Z_4&=1+\frac{3}{8}\frac{g^2N}{(4\pi)^2(-\epsilon)}\ ,\\
Z_5&=1+\frac{13}{3}\frac{g^2N}{(4\pi)^2(-\epsilon)}\ ,\\
Z_6&=Z_7\ ,\\
Z_8&=1+\frac{3}{4}\frac{g^2N}{(4\pi)^2(-\epsilon)}\ .
\end{aligned} \qquad (3.14)
$$

It has to be remarked that we cannot establish the value of $Z_6, Z_7$ from (3.13), since they always appear in the combination $Z_6-Z_7$.

But, if the bare Lagrangian $\mathcal{L}_0=\mathcal{L}_R+\mathcal{L}_c$ is going to have the same structure as the renormalized one, the following identities (which are a direct consequence of the Ward identities) will hold:

$$Z_5=Z_4Z_\gamma Z_\alpha,\quad Z_6=Z_7=Z_4Z_\gamma,\quad Z_8=Z_4Z_\gamma Z_\alpha{}^{-1}\ , \qquad (3.15)$$

where $Z_A, Z_\alpha, Z_\gamma$ are the renormalization constants of the propagator previously given in Eq. (2.4). It can be easily seen that Eqs. (3.15) are satisfied when the expressions (3.14) are used. Because of the remark after Eq. (3.14), it is impossible to check explicitly the second identity in (3.15).

It remains to establish nonperturbatively the Ward or Slavnov-Taylor identities and study their relation with (3.15). This is a subject which is now under study.

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