Exploiting quasiperiodic order in the design of optical devices

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In this work we present a prospective study on the potential capabilities of optical devices based on Fibonacci dielectric multilayers. We perform a detailed analytical comparison of the linear optical response of periodic versus quasiperiodic multilayers. Based on this study we will suggest the use of *hybrid-order devices*, composed of both periodic and quasiperiodic subunits to design microcavities of practical interest, and we provide some illustrative examples. From our study we conclude that the inclusion of quasiperiodically ordered subunits substantially widens the possibilities of engineering modular optical structures.

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I. INTRODUCTION

The notion of *quasiperiodic order* is progressively pervading different arenas of the physics community, not only by spurring the interest in further clarifying its conceptual relationships with more usual arrangements of matter (like periodic and random ones), but also by offering some promising possibilities for technological applications. In this work we will consider man-made quasiperiodic (QP) structures, leaving aside the broad fields of both quasicrystals[1] and biopolymers,[2,3] where quasiperiodicity spontaneously arises from the physical interactions among their basic building blocks. In turn, we will focus on QP multilayers that, since the first fabrication of Fibonacci semiconductor heterostructures by Merlin and co-workers,[4] have received much attention both theoretically and experimentally.[5]

Broadly speaking, new capabilities from devices based on a QP stacking of different layers can be expected from the presence of *two different kinds of order in the same sample at different length scales*. In fact, at the atomic level we have the usual periodic order determined by the crystalline arrangement of atoms in each layer; but, at longer scales the QP order determined by the sequential deposition of the different layers plays the major role. This long-range aperiodic order is artificially imposed during the system growth process and can be precisely controlled. Then, since different physical phenomena have their own relevant physical scales, we can exploit the physical properties related to the aperiodic order we have introduced in the system by properly matching the characteristic length scales of elementary excitations propagating through it. Such a possibility opens promising avenues in the physics of condensed matter and materials science engineering.[6–8]

Theoretically, the study of QP structures was originally motivated by the prediction that these systems should exhibit peculiar electron and phonon critical states,[9,10] associated with highly fragmented, fractal energy spectra.[11–14] From an experimental point of view, however, severe limitations appear due to electron-phonon, electron-electron or spin-orbit interaction effects. Then, the study of *classical* waves propagating through a QP substrate offers a number of advantages over the study of *quantum* elementary excitations. Accordingly, a number of experimental studies dealing with the propagation of elastic waves,[15,16] third sound,[17] and ultrasonic waves[18] in Fibonacci systems have been reported, confirming that characteristic self-similar features in the transmission spectra are observable when the long-range aperiodic modulation is established at different scale lengths. Similarly, the introduction of the Fibonacci dielectric multilayer (FDM) by Kohmoto and collaborators[19] spurred the interest for both possible optical applications[20,21] and theoretical aspects of light transmission in aperiodic media.[22–26] This interest has motivated several theoretical works aimed to understand the interplay between the optical properties and the underlying aperiodic order of the system through the study of exciton optical absorption[27] and fluorescence decay in aperiodic lattices.[28] At the same time, new insights into the optical capabilities of aperiodically ordered systems have been recently demonstrated by a number of experimental achievements, involving second-[29] and third-harmonic generation,[30] as well as the possible localization of light waves in FDM's.[31,32]

Underlying all these theoretical and experimental efforts a crucial fundamental question remains concerning whether quasiperiodically ordered devices would achieve better performance than usual periodic ones for some specific applications. Thus, in the case of second-order nonlinear optical effects it has been properly illustrated that the second-harmonic spectrum of a FDM is different from that of a periodic one due to its different space-group symmetry. In fact, a QP multilayer can provide more reciprocal vectors to the quasi-phase-matching optical process, and this ultimately results in a more plentiful spectrum structure than that of a periodic multilayer.[29] The importance of the role played by the quasiperiodicity of the substrate is further highlighted when considering third-harmonic generation, where it has been shown that the conversion efficiency in a QP multilayer is increased by a factor of 8 in comparison with the two-step process required for a third-harmonic generator constructed by two periodic superlattices.[30] Quite interestingly, the possibility of designing Fibonacci-based structures able to simultaneously phase match any two nonlinear interactions by introducing a QP modulation of the nonlinear coefficient in ferroelectric devices has been recently discussed.[33]

Motivated by these recent results in this work we will present a prospective study on the potential capabilities of optical devices based on QP arrangements in their structural design. To this end, we shall consider the *linear* response of FDM's and perform a detailed analytical comparison of the optical response of periodic versus QP optical multilayers. In this way, we will gain a deeper understanding on the rich behavior of light propagating through layered QP media under general incidence conditions. The obtained analytical expressions allow us to study the relationship between the resonant wavelengths and the QP structure of the substrate. Based on this relationship we will suggest the use of sandwiched arrays of FDM's to design optical microcavities of practical interest, and we shall provide some illustrative examples. From our study we conclude that the inclusion of quasiperiodically ordered structures substantially widens the possibilities of richer designs. This enrichment can be properly exploited by considering *hybrid devices*, composed of both periodic and QP subunits.

The paper is organized as follows. In Sec. II we describe our unified analytical approach and briefly summarize relevant previous results. In Sec. III we study the quasiperiodicity effects in the light transmission through a FDM, obtaining closed analytical expressions for the transmission coefficient under normal- and oblique-incidence geometries. In this section we also present a detailed comparison between the optical response for periodic and QP multilayers, highlighting the most significant differences between them. On the basis of these results, in Sec. IV we propose some optical devices based on quasiperiodically stacked dielectric layers. Final comments and suggestions for further studies are contained in Sec. V.

II. UNIFIED ANALYTICAL APPROACH

In order to properly compare the optical response of both periodic and QP systems we will rely on the transfer-matrix technique. This approach is particularly well suited to our purposes, since it describes the optical response of the global system in terms of the light behavior in contiguous layers. In fact, when studying the propagation of light in multilayered systems one must consider the propagation across the interface separating two neighboring layers along with the light propagation within each layer. Then, the transmission of light through the entire multilayer can be properly described in terms of a product involving the matrices

$$K_{AA}=\begin{pmatrix}\cos\delta_A & -\sin\delta_A\\ \sin\delta_A & \cos\delta_A\end{pmatrix}$$

$$K_{AB}=\begin{pmatrix}\cos\delta_B & -\sin\delta_B\\ u^{-1}\sin\delta_B & u^{-1}\cos\delta_B\end{pmatrix},$$

$$K_{BA}=\begin{pmatrix}\cos\delta_A & -\sin\delta_A\\ u\sin\delta_A & u\cos\delta_A\end{pmatrix},\quad K_{BB}=\begin{pmatrix}\cos\delta_B & -\sin\delta_B\\ \sin\delta_B & \cos\delta_B\end{pmatrix}, \tag{1}$$

where $\delta_i \equiv n_i d_i k/\cos\theta_i$, n_i are the refractive indices, d_i are the widths of the layers, k is the wave vector in vacuum, θ_i are the incidence angles and we have defined $u\equiv\rho\beta$, where $\rho\equiv n_A/n_B$ measures the refractive-index contrast, and $\beta\equiv\cos\theta_A/\cos\theta_B>0$. Therefore, to obtain the global transfer matrix, we must evaluate a matrix product involving different types of transfer matrices which, in addition, may be ordered either periodically or quasiperiodically. In this way, the long-range order of the system properly determines the overall structure of the matrix product. Thus, the different kinds of long-range order characterizing the periodic and QP arrangements of layers will ultimately emerge in the mathematical structure of the global transfer matrix.

A. Periodic dielectric multilayers

Let us consider a multilayer made of ν bilayers AB that repeat periodically. In this case the global transfer matrix can be straightforwardly expressed in terms of the auxiliary matrices (1) in the form $M(N)=(K_{AB}K_{BA})^{\nu}\equiv Q^{\nu}$, where $N=2\nu$ is the total number of layers. Since Q is unimodular (i.e., its determinant is unity) we can make use of the Cayley-Hamilton theorem for unimodular matrices[34] in order to explicitly evaluate the matrix $M(N)$ in terms of Chebyshev polynomials of the second kind as

$$Q^{\nu}=U_{\nu-1}(z)\,Q-U_{\nu-2}(z)\,I, \tag{2}$$

where I denotes the identity matrix and $U_m(z)\equiv\sin[(m+1)\varphi]/\sin\varphi$, where

$$z\equiv\frac{1}{2}\mathrm{Tr}\,Q=\left[\cos\delta_A\cos\delta_B-\frac{u+u^{-1}}{2}\sin\delta_A\sin\delta_B\right]\equiv\cos\varphi \tag{3}$$

are Chebyshev polynomials of the second kind. Therefore, making use of the relationships $U_{m+1}-2zU_m+U_{m-1}=0$ and $T_m=(U_m-U_{m-2})/2$, the global transfer matrix can be expressed in the closed form

$$M(N)=\begin{pmatrix} T_{\nu}+\dfrac{U_{\nu-1}}{2}(u^{-1}-u)\sin\delta_A\sin\delta_B & -U_{\nu-1}(\sin\delta_A\cos\delta_B+u\cos\delta_A\sin\delta_B)\\ U_{\nu-1}(\sin\delta_A\cos\delta_B+u^{-1}\cos\delta_A\sin\delta_B) & T_{\nu}-\dfrac{U_{\nu-1}}{2}(u^{-1}-u)\sin\delta_A\sin\delta_B\end{pmatrix}, \tag{4}$$

where $T_\nu \equiv \cos(\nu\varphi)$ are Chebyshev polynomials of the first kind. From this expression the dispersion relation for the periodic multilayer can be easily obtained from the condition $\cos[q\nu(d_A+d_B)] = \mathrm{Tr}\, M(N)/2$, which leads to the well-known expression[35]

$$\cos[q(d_A+d_B)] = \cos\delta_A \cos\delta_B - \left(\frac{u^2+1}{2u}\right)^2 \sin\delta_A \sin\delta_B . \tag{5}$$

B. Fibonacci dielectric multilayers

Let us consider a FDM consisting of two kinds of layers, labeled *A* and *B*, which are arranged according to the Fibonacci sequence, obeying the concatenation rule $S_{j+1} = S_{j-1}S_j$ for $j \geqslant 1$, with $S_0 = B$ and $S_1 = A$.[19] The number of layers is given by $N = F_j$, where F_j is a Fibonacci number obtained from the recursive law $F_{j+1} = F_j + F_{j-1}$, with $F_1 = 1$ and $F_0 = 1$. In order to evaluate the global transfer matrix, involving three different types of transfer matrices quasiperiodically arranged, we shall take advantage of the transfer matrix renormalization technique recently introduced in Ref. 8.[8] The key point of our approach consists of *renormalizing the set of transfer matrices* $K_{i+1,i}$ according to the blocking scheme $Q_B \equiv K_{AA}$ and $Q_A \equiv K_{AB}K_{BA}$. Note that the renormalized transfer-matrix sequence is also arranged according to the Fibonacci sequence and, consequently, the topological order present in the original FDM is preserved by the renormalization process. Now, we realize that the Q_i matrices commute under certain circumstances. In fact, after some algebra we get

$$[Q_A, Q_B] = J(\delta_A, \delta_B) \begin{pmatrix} -\cos\delta_A & \sin\delta_A \\ \sin\delta_A & \cos\delta_A \end{pmatrix}, \tag{6}$$

where

$$J(\delta_A, \delta_B) \equiv \frac{u^2-1}{u} \sin\delta_A \sin\delta_B . \tag{7}$$

The commutator (6) vanishes in three different cases ($u > 0$): (i) the choice $u=1$, which reduces to the trivial periodic case $n_A = n_B$, and the choices (ii) $\delta_A = n\pi$ and (iii) $\delta_B = n\pi$, with $n = 1,2,\ldots$. Therefore, in order to satisfy the commutation condition (6), it is *not* necessary to impose restrictive conditions onto both kinds of layers *simultaneously*. For those wavelengths verifying the condition $[Q_A, Q_B] = 0$, we can express the global transfer matrix of the system as $M(N) \equiv Q_A^p Q_B^q$, where $p = F_{j-2}$ and $q = F_{j-3}$. Note that for a FDM of length *N*, *p* indicates the number of *B* layers present in the system. Since the matrices Q_A and Q_B are unimodular for *any* choice of the system parameters and for *any* value of the light wavelength, we can make use of the Cayley-Hamilton theorem for unimodular matrices[34] in order to explicitly evaluate the matrix $M(N)$ in terms of Chebyshev polynomials of the second kind.[8] In this way, the use of Chebyshev polynomials allows for a *unified mathematical description* of both periodic and QP multilayers. From the knowledge of $M(N)$ the optical response of the multilayers can be described in terms of the transmission coefficient, which can be obtained from the standard expression $T = 4/[\|M(N)\|+2]$, where $\|M(N)\|$ denotes the sum of the squares of the four elements of the global transfer matrix.

III. QUASIPERIODICITY EFFECTS IN THE LIGHT TRANSMISSION

From the different situations we can consider by properly combining the commutation conditions given by Eq. (6), only that corresponding to the case $\delta_A = n\pi$ efficiently exploits the quasiperiodicity of the FDM. In fact, in the case $\delta_A \equiv \delta_B = n\pi$, the half-wavelength condition is satisfied at every layer, so that $Q_A = I$ and $Q_B = (-1)^n I$, and the transparency condition $T=1$ is trivially obtained.[19,20,22–24,31,32] Analogously, in the case $\delta_B = n\pi$, we get $T=1$ for any β as well. In this case the two-component FDM will behave like an equivalent homogeneous *periodic* medium, characterized by an effective thickness $d' \equiv (p+q)d_A$ and a refraction index n_A. Consequently, the FDM will exhibit an effective *optical phase shrinkage* for those wavelengths satisfying the resonance condition.[8,36] Physically these results can be interpreted as follows. When the *B* layers satisfy the effective half-wavelength condition, the transmission properties of the FDM will depend entirely on the interaction of light with the layers of material *A*. Now, since the optical behavior of the double layers *AA* is completely equivalent to that of single *A* layers and, according to the construction rule of the Fibonacci sequence, *B* layers always appear flanked by *A* layers, those wavelengths satisfying the resonance condition $n\lambda = 2n_B d_B/\cos\theta_B$, will effectively *see* a *periodic* distribution of *A* layers separated by fully transparent slabs of constant width d_B. This physical scenario changes substantially if we impose the condition $\delta_A = n\pi$. In this case, the layers of material *A* become fully transparent to the incoming light and, consequently, the transmission properties of the FDM will depend on the interaction of light with the layers of material *B*. The key point now is to realize that these layers are spaced by *two different* distances, d_A and $d_{AA} = 2d_A$, arranged according to the Fibonacci sequence. Hence, those wavelengths satisfying the resonance condition $n\lambda = 2n_A d_A/\cos\theta_A$, will effectively *see* a *quasiperiodic* distribution of *B* layers, instead of a periodic one.

In this case, the global transfer matrix for the FDM can be expressed in the closed form

$$M(N) = (-1)^{nF_{j-1}} \begin{pmatrix} T_p & -u\sin\delta_B U_{p-1} \\ u^{-1}\sin\delta_B U_{p-1} & T_p \end{pmatrix}, \tag{8}$$

where $T_p \equiv \cos(p\delta_B)$ and $U_{p-1} \equiv \sin(p\delta_B)/\sin\delta_B$ are Chebyshev polynomials of the first and second kinds, respectively. Then, the explicit evaluation of the transmission coefficient for the FDM under general incidence conditions leads to (we shall assume henceforth that the incoming light enters the FDM through an *A* layer)

$$T_{QP} = \frac{1}{1 + a(\rho,x)\sin^2[p\, b_n(\rho,x,\eta)]}, \tag{9}$$

where we have made use of Snell's law $\rho \sin\theta = \sin\theta_B$ to introduce the auxiliary variable $x \equiv \sin^2\theta$, describing the light incidence geometry, and we have also introduced the auxiliary functions

$$a(\rho,x) \equiv \left[\frac{\rho^2-1}{2\rho}\right]^2 (1-x)^{-1}(1-\rho^2 x)^{-1}, \tag{10}$$

$$b_n(\rho,x,\eta) \equiv n\pi y \sqrt{\frac{1-x}{1-\rho^2 x}}, \tag{11}$$

where $y \equiv \eta/\rho = n_B d_B / n_A d_A$ measures the phase ratio between both dielectric layers, and $\eta \equiv d_B/d_A$ measures the filling factor.

On the other hand, the transmission coefficient for the periodic multilayer can be obtained from Eq. (4) as

$$T_P = \frac{1}{1+\left[\frac{u^2-1}{2u}\right]^2 U_{\nu-1}^2(z)\sin^2\delta_B}. \tag{12}$$

This expression holds for any arbitrary wavelength impinging onto the system at any arbitrary incidence angle. As discussed above, in order to compare the relative performance of QP and periodic dielectric multilayers, it is convenient to focus on the case $\delta_A = n\pi$, determining the working wavelength $n\lambda = 2n_A d_A/\cos\theta$. In so doing, we finally get

$$T_P = \frac{1}{1+a(\rho,x)\sin^2[\nu\, b_n(\rho,x,\eta)]}. \tag{13}$$

At first sight the overall appearance of Eqs. (9) and (13) looks rather similar. However, we will see that they enclose quite different physical behaviors in many instances.

A. Normal-incidence conditions

Under normal-incidence geometry ($x=0$) Eqs. (9) and (13) simplify to

$$T_{QP} = \frac{1}{1+\left[\frac{\rho^2-1}{2\rho}\right]^2 \sin^2[np\pi y]} \tag{14}$$

and

$$T_P = \frac{1}{1+\left[\frac{\rho^2-1}{2\rho}\right]^2 \sin^2[n\nu\pi y]}. \tag{15}$$

We see that the only difference between Eqs. (14) and (15) is the replacement of the number of bilayers, ν, by the number of B layers, p, in the FDM. Thus, when $p=\nu$ the transmission of light through both kinds of structures is exactly the same. This indicates that for any arbitrary FDM we can find a periodic one exhibiting the same optical response. However, since the set of integer numbers is larger than the set of Fibonacci ones, the converse case does not hold. In what follows we shall study the optical performance of multilayers having the same *total* number of layers, N, but different kinds of topological order in the sequence in which these layers appear. Let $N_P = 2\nu$ be the number of layers present in the periodic multilayer and $N_{QP} = 2p+q$ be the number of layers present in the FDM. By equating $N_P \equiv N_{QP}$ we get $\nu = p + q/2$. This relationship determines the length of the multilayers we are interested in. By induction we readily find that the general solution can be expressed as $\nu = F_{3l} + F_{3l-1}/2$, with $l = 1,2,\ldots$, which implies $p = F_{3l}$. For the sake of clarity, we shall consider the simpler solution ($l=1$) in more detail, since it can be properly considered as a representative instance of the general behavior.

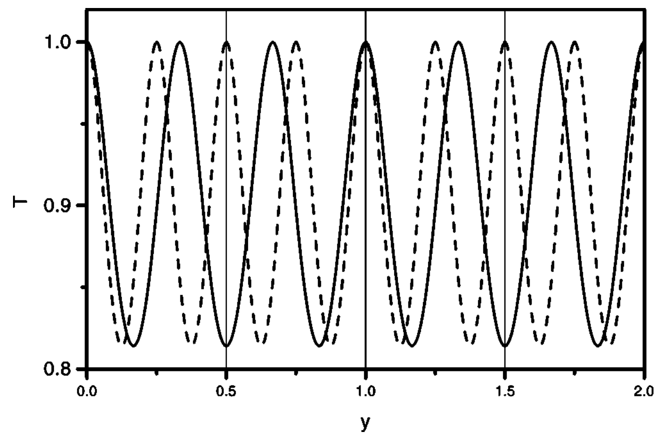

FIG. 1. Dependence of the transmission coefficient on the phase ratio y for a periodic multilayer with $\nu=4$ (dashed line) and a Fibonacci dielectric multilayer with $p=3$ (solid line) containing $N=8$ layers under normal-incidence geometry and $n=1$. The vertical lines indicate the phase ratios satisfying the condition $y=1/2$ and $y=3/2$, respectively.

In Fig. 1 we show the dependence of the transmission coefficient on the phase ratio y for a periodic multilayer with $\nu=4$ (dashed line) and a FDM with $p=3$ (solid line), both of them containing $N=8$ layers, under normal-incidence conditions. For the sake of illustration we have taken $n_A = 1.45$ and $n_B = 2.30$ as suitable representative values (see Table I). From this figure we observe that the curves $T(y)$ are symmetrical with respect to the axis $y=1$ ($\eta=\rho$), and exhibit a series of maxima ($T_{max}=1$) and minima ($T_{min} = \{1+[(\rho^2-1)/2\rho]^2\}^{-1} = 0.8140\ldots$). Physically the ori-

TABLE I. Experimental values for several parameters used in optical devices as reported in literature.

Material	n	d (nm)	λ (nm)
SiO_2	1.46	108	400–700
TiO_2	2.35	67	400–700
Na_3AlF_6	1.34	90	633
ZnSe	2.5–2.8	90	633
		d (μm)	λ (μm)
$LiTaO_3$	2.0	3.6	0.8
$LiNbO_3$	2.14	11−25	1.3
$KTiOPO_4$	1.74	1.3−1.4	0.5-1

gin for such an oscillatory pattern can be understood as follows. Under the A layer half-wavelength resonance condition, which fixes the value for the incoming light wavelength, the constructive or destructive nature of the interferences, arising from the interaction of light with the distribution of B layers, will be strongly dependent on the precise relationship between λ and the structural parameters d_A and d_B. Thus, when $y=1$ ($n_A d_A = n_B d_B$) both A and B layers simultaneously satisfy the resonance condition, and the long-range order of the layers' sequence becomes irrelevant. However, for $y \neq 1$ the optical response of periodic and QP multilayers progressively differs as the phase ratio is progressively increased (or decreased), in the way displayed in Fig. 1.

A particularly interesting situation emerges if we choose the phase ratio value in such a way that the wavelengths satisfying the half-wavelength condition at the A layers of the periodic multilayer verify the quarter-wavelength condition at the B layers of the FDM. In this case, the QP distribution of B layers efficiently backscatters the incoming light, resulting in a significant reduction of the transmission-coefficient value. Conversely, the periodic multilayer exhibits full transmission for the same wavelength. From Eqs. (14) and (15) we obtain that this condition is satisfied when $y = k/2n|p-\nu|$, with $k=1,3,5 \ldots$. The cases corresponding to $k=1$ and $k=3$ are indicated by vertical lines in Fig. 1. This behavior opens the possibility of constructing a mixed device composed of both periodically and quasiperiodically arranged multilayers so that the refractive-index constrast and the layers' thicknesses determining the phase-ratio value, can act as *control design parameters* able to determine the optical response of their different constitutive substructures from that corresponding to a selective filter ($T=1$) to that proper of a reflective coating (T_{min}). It is worth highlighting, at this point, that such a mixed device can be viewed as a *hybrid device* made of two different kinds of subunits, each one exhibiting a *different kind of topological ordering*, which, respectively, give rise to a reversal in the value of the corresponding transmission coefficients. The key point here is that such a *complementary behavior* can be obtained by just changing the kind of topological order in the stacking sequence of layers composing each subunit, so that both the chemical nature of the different layers and the wavelength of the incoming light remains unchanged. This is quite a remarkable result, since the pertinent codes to alternate between periodic and quasiperiodic orderings in the sequence of the different layers can be easily implemented in current state-of-the-art deposition processes.

B. Oblique-incidence geometry

In the more general case we must consider the transmission coefficients as given by Eqs. (9) and (13). In Fig. 2 we show the dependence of the transmission coefficient on the angle of incidence θ for the periodic multilayer (dashed line) and the FDM (solid line) previously considered in Fig. 1 for the case $\eta=\rho$. Both transmission curves exhibit a series of maxima, whose number increases with the length of the system. From Eq. (9) we see that the number of full transmis-

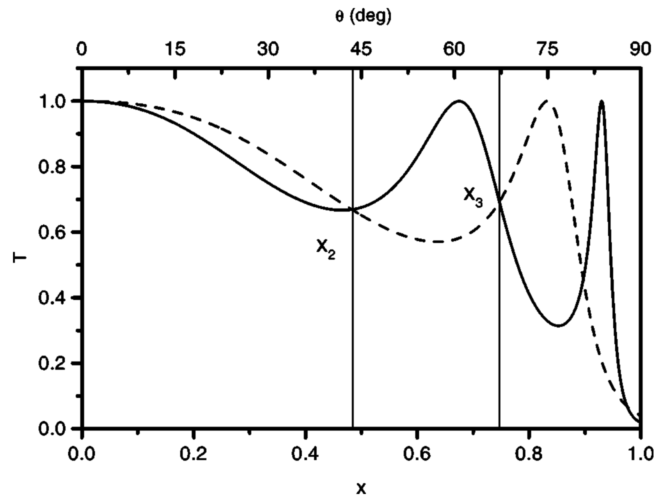

FIG. 2. Dependence of the transmission coefficient on the angle of incidence θ for a periodic multilayer with $\nu=4$ (dashed line) and a Fibonacci dielectric multilayer with $p=3$ (solid line) containing $N=8$ layers and $n=1$ for the case $y=1$.

sion peaks for the FDM depends on the number of B layers, p, and their positions are given by the expression ($\rho<1$)

$$x_{l,QP} = \frac{\gamma^2 p^2 - l^2}{\gamma^2 p^2 - \rho^2 l^2}, \quad 1 \leqslant l \leqslant \lfloor \gamma p \rfloor, \tag{16}$$

where $\gamma \equiv ny$, and $\lfloor \cdots \rfloor$ indicates that we take the integer part of the quantity inside the brackets. Similarly, the number of full transmission peaks for the periodic multilayer depends on the number of bilayers ν, and their positions are obtained from Eq. (13) as

$$x_{l,P} = \frac{\gamma^2 \nu^2 - l^2}{\gamma^2 \nu^2 - \rho^2 l^2}, \quad 1 \leqslant l \leqslant \lfloor \gamma \nu \rfloor. \tag{17}$$

By inspecting Fig. 2 we can distinguish two different regimes. At low incidence angles the optical response of both periodic and QP multilayers is quite similar, although the transmission curve for the FDM systematically departs from that corresponding to the periodic multilayer, exhibiting lower transmission values. The second regime starts at the critical angle value $x_2 \simeq 0.48$ where a crossing point between both transmission curves occurs. Afterwards, the transmission curve for the FDM suddenly grows as the incidence angle is increased, reaching a broad peak at $x \simeq 0.67$ and it rapidly decreases again to reach another crossing point with the periodic multilayer-transmission curve at $x_3 \simeq 0.75$. A similar oscillatory pattern repeats itself as the incidence angle is further increased, determining the subsequent crossing points.

In Fig. 3 we compare the dependence of the transmission coefficient on the incidence angle θ for a periodic multilayer (dashed line) and a FDM (solid line) for the case $y=3/2$. The remaining parameters coincide with those indicated in Fig. 1. The overall qualitative behavior of the optical response is analogous to that observed in Fig. 2, albeit the respective transmission curves take different initial values when $\theta=0$,

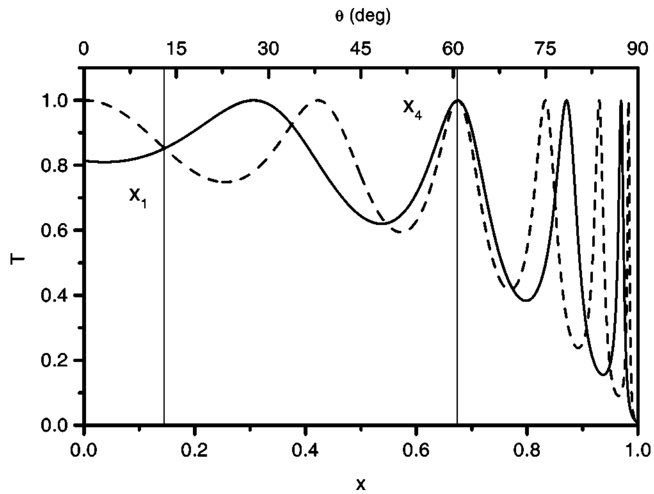

FIG. 3. Dependence of the transmission coefficient on the angle of incidence θ for a periodic multilayer with $\nu=4$ (dashed line) and a Fibonacci dielectric multilayer with $p=3$ (solid line) containing $N=8$ layers and $n=1$, for the case $y=3/2$.

as should be expected when $y\neq 1$ (see Fig. 1). In addition, the high-angle regime characterized by the presence of a series of crossing points, starts at the incidence angle $x_1=0.15$, which is sensibly lower than the value observed for the first crossing point in Fig. 2. Physically, the differences between the optical responses shown in Figs. 2 and 3 are attributable to the fact that, in the later case, the symmetry imposed by the phase-ratio coincidence $y=1$ has been removed. Consequently, we have an additional design parameter playing a role in the propagation of light through both kinds of multilayers.

The most relevant feature of the transmission curves shown in both Figs. 2 and 3 is the existence of broad incidence-angle intervals where the periodic and QP multilayers, respectively, exhibit *complementary optical responses*, in the sense that when one of them exhibits high T values the other one exhibits low T values (and vice versa). This complementary behavior indicates that, by properly choosing the incidence-angle geometry, the same device can act either as a mirror or as a full transmission material for a given working wavelength. In this way, the complementary optical response previously reported in Sec. III A under normal-incidence conditions also holds for the more general case of oblique-incidence geometry.

IV. OPTICAL DEVICES BASED ON QUASIPERIODIC ARRANGEMENTS

According to the analytical results obtained in the previous sections we shall consider two different kinds of devices based on a QP arrangement of layers. The first class will be referred to as *hybrid-order devices* and, as it was indicated at the end of subsection Sec. III A, will be composed of two different kinds of *subunits*, each one exhibiting a *different kind of topological ordering*. This mixed structure is aimed to obtain *complementary optical responses* by properly choosing the incidence-angle geometry. In this way, a given multilayer structure can act either as a mirror or as a full

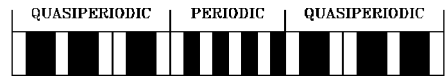

FIG. 4. Scheme of a hybrid-order optical resonating microcavity based on three subunits. The full transmission, periodically ordered unit, is encased by two quasiperiodically ordered ones, acting as optical mirrors.

transmission material for the same incoming wavelength. The second class will be entirely based on QP arrangements of multilayers, where the role played by the A and B layers is, respectively, permuted.

A. Hybrid order devices

In Fig. 4 we show a sketch illustrating a possible design for an optical device based on the mixed architecture just described. It corresponds to a resonant cavity where a high-transmission periodic multilayer is encased by high-reflectivity Bragg reflectors based on quasiperiodic FDM's. Current devices require transmission-coefficient values below $T\simeq 0.2$ for the mirrors. According to Figs. 2 and 3 the wider difference in the magnitude $\Delta\equiv 1-T_{min}$ is attained for oblique-incidence geometries for the phase ratio $y\neq 1$. On the other hand, for a fixed angle of incidence, the value of Δ can be significantly increased by increasing the refractive-index contrast of the materials composing the FDM subunits. For the sake of illustration in Table I we list pertinent data for materials commonly used in optoelectronic devices. In Fig. 5 we compare the dependence of the transmission coefficient with the incidence angle θ for FDM subunits based on different materials. From these plots we realize that efficient mirrors, on reflectances over 80%, can be obtained by employing Na_3AlF_6/ZnSe-based QP multilayers. In fact, periodic multilayers based on these materials have been recently shown to act as promising candidates to attain multidirectional reflectors.[37]

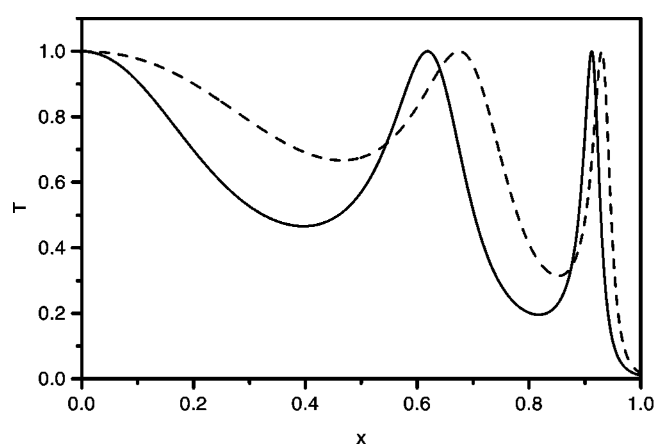

FIG. 5. Comparison of the dependence of the transmission coefficient on the angle of incidence θ for two Fibonacci dielectric multilayers of different compositions: Na_3AlF_6/ZnSn (solid line) and TiO_2/SiO_2 (dashed line), with $p=3$, $N=8$ layers, and $n=1$, for the case $y=1$.

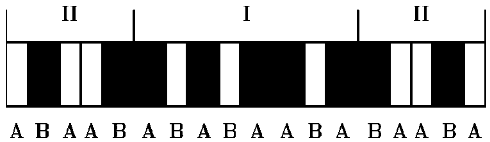

FIG. 6. Scheme of a permuted-order optical resonating microcavity based on three quasiperiodic subunits, where a Fibonacci dielectric multilayer of the first kind (with $N=8$), showing high transmission, is encased by two Fibonacci multilayers of the second kind (with $N=5$), acting as optical mirrors. Dark (white) layers correspond to low (high) refractive-index materials, respectively.

B. Permuted-order quasiperiodic devices

An interesting property of FDM's is that their characteristic long-range QP order is preserved under transformations involving the permutation operation $A \rightarrow B$ and $B \rightarrow A$. This invariance, which follows from the very definition of the Fibonacci sequence, has interesting physical implications, which we will exploit as follows. In Fig. 6 we show a sketch illustrating a possible design for an optical device based on two kinds of QP subunits. In the first kind (labeled I), the A (B) layers are composed of low (high) refractive-index materials. In the second kind (labeled II), the values of the refractive indices assigned to the layers A and B are reversed, so that the total-internal-reflection angle condition is achieved when $x_0 = \rho^{-2} \simeq 0.4$. Consequently, the FDM behaves as a *perfect mirror* for incidence angles verifying $x > x_0$. The key point here is the possibility of *combining both kinds of designs* in order to construct *highly efficient* optical microcavities.

In Fig. 7 we show the dependence of the transmission coefficient on the incidence angle θ for the optical device just described, where the FDM subunits are composed of SiO_2 (TiO_2) whose indices of refraction (at 700 nm) are $n_A = 1.45$ and $n_B = 2.30$, respectively. From this figure we see that an ideal *resonant cavity* to work at specific incidence angles could be constructed by making a structure where a FDM of the first kind I, exhibiting *full transmission* at the incidence angle $x \simeq 0.67 \simeq 55°$, is sandwiched between two FDM's of the second kind II, behaving as perfect mirrors at this angle (see Fig. 6). In this way, the capabilities of the device sketched in Fig. 4 should be properly enhanced.

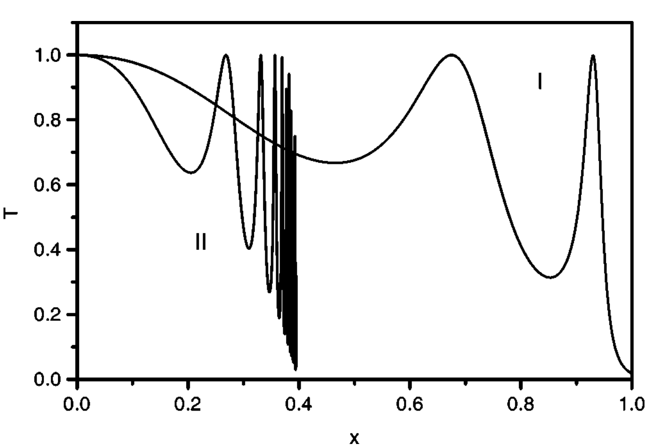

FIG. 7. Dependence of the transmission coefficient on the incidence angle for the permuted-order device sketched in Fig. 6, with $n_A = 1.46$ (SiO_2), $n_B = 2.35$ (TiO_2) (type I FDM) and $n_A = 2.35$ (TiO_2), $n_B = 1.46$ (SiO_2) (type II FDM) for $n=1$ and $y=1$.

V. CONCLUSION

In this work we analyze the role of QP order in the properties of light propagation through multilayered structures, and compare the optical response of QP versus periodic systems in order to ascertain the capabilities associated with the inclusion of QP orderings of matter in the design of optical devices. From a physical viewpoint the convenience of this study stems from the possibility of exploiting the QP order of the system by properly matching the wavelength of the incoming light with some of the different characteristic lengths present in the aperiodic substrate. Thus, by properly choosing the different design parameters, as determined by the refractive-index contrast, ρ, and the multilayer filling factor, η, we can select specific resonance conditions directly associated with the QP order of the substrate. In addition, we can also play with the incidence-angle geometry of the incoming light in order to further exploit the capabilities previously determined by the architecture of the multilayer. In this way, plentiful possibilities for new tailored materials should appear. In the present work we propose just two of them, illustrating in turn, two broad classes of devices based on a proper stacking of different structural *subunits*. The introduction of these subunits allows for the inclusion of an *additional degree of order* in the system, bridging the gap between the atomic-level characteristic of the microstructural domain of each layer and the mesoscale level associated with the long-range order of the entire device as a whole. In turn, since each subunit can sustain a specific kind of order (i.e., periodic or quasiperiodic) we are able to introduce a *modular design* in the multilayer structure by properly selecting different kinds of order for each subunit. Thus, when considering *hybrid-order devices* we take two different kinds of *subunits*, each one exhibiting a *different kind of topological ordering*. This mixed structure exhibits *complementary optical responses*, which can be observed in just the same piece of matter by a judicious choice of the incidence-angle geometry.

Alternatively, we can also think of devices entirely based on QP subunits. In this case, the structural richness of the self-similarity of the different layers is taken into account, for instance, through the QP-order preservation under the permutation of the refractive indices of the different involved layers. In this way, the capabilities previously obtained for hybrid-order devices could be enhanced in some instances. It is also worth noting that by properly selecting the refraction indices of the materials from which the FDM is made, it would be possible to achieve broad multidirectional reflection devices based on QP structures. Such a property has been recently reported for periodic one-dimensional dielectric lattices,[37] so that the possible extension to aperiodically layered structures is quite appealing in order to further explore the new capabilities associated with this novel kind of ordering.[38] I understand that pertinent experimental work along this research line may be undertaken in the near future.[39]

Finally, although we have restricted ourselves to the consideration of QP systems based on the Fibonacci sequence, it is clear that the main tenets presented in this work should be readily extended to other kinds of aperiodic orders as well. In this regard, some recent works illustrating the possibility of designing aperiodic optical devices aimed to obtain enhanced harmonic generation is quite appealing.[40,41] It would also be interesting to extend the approach presented in this work in order to consider *almost periodic* structures instead of quasiperiodic ones. In fact, since it is known that almost periodic structures have much in common with random ones,[42,43] the possibility of observing Anderson-like localization phenomena in almost periodically ordered, layered structures would deserve a closer scrutiny. This will provide another instance of nonperiodic structures with interesting properties.

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