

Holonomy from wrapped branes

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Abstract

Compactifications of M-theory on manifolds with reduced holonomy arise as the local eleven-dimensional description of D6-branes wrapped on supersymmetric cycles in manifolds of lower dimension with a different holonomy group. Whenever the isometry group $SU(2)$ is present, eight-dimensional gauged supergravity is a natural arena for such investigations. In this paper we use this approach and review the eleven dimensional description of D6-branes wrapped on coassociative 4-cycles, on deformed 3-cycles inside Calabi–Yau threefolds and on Kähler 4-cycles.

Gravity duals of field theories with low supersymmetry can be constructed wrapping branes on supersymmetric cycles. The resulting field theories are twisted since preserving some amount of supersymmetry after wrapping the brane requires relating the spin connection on the cycle with some external R-symmetry gauge fields. The idea is quite simple: a supersymmetric theory on a curved manifold Σ will break supersymmetry because in general it will not be possible to find a covariantly constant spinor satisfying $(\partial_\mu + \omega_\mu(\Sigma))\epsilon = 0$. However, in the presence of a global R-symmetry an external gauge field can be coupled to the R-current and constant spinors are covariantly constant as well. The coupling to the external field exchanges the spins, resulting into a twisted theory [1], i.e. to preserve supersymmetry branes must wrap a supersymmetric cycle. These twists can be naturally performed within gauged supergravities [2] (see [3] for earlier related work), and may involve in a quite non-trivial way the scalar fields in the theory [4]. The dual supergravity solutions describing branes of diverse dimensions wrapped on various supersymmetric cycles can be naturally constructed in an appropriate gauged supergravity, and eventually lifted to ten or eleven dimensions, an approach that has been applied to wrapped D6-branes [5]-[9],[4], and also has been used to further develop the study of wrapped fivebranes [12] and to obtain solutions for branes of dimension four [13], three [14] and two [15]. Of special interest is the case of D6-branes because they lift to pure geometry in eleven dimensions. In this note, using eight-dimensional gauged supergravity, which is the natural arena to perform twisting for the D6-branes, we will briefly review the eleven-dimensional description of D6-branes wrapped on coassociative 4-cycles [6], deformed special Lagrangian 3-cycles [4] and Kähler 4-cycles [7, 10], thus recovering the lifts considered in [11], and study several interesting limit cases of the solutions. We refer to the original literature for much of the technical details and related generalizations (see also the contribution by J. Gomis to this proceedings).

Maximal gauged supergravity in eight dimensions was constructed through Scherk–Schwarz compactification of eleven-dimensional supergravity on an $SU(2)$ group manifold [16]. The field content in the purely gravitational sector of the theory consists of the metric $g_{\mu\nu}$, a dilaton Φ , five scalars given by a unimodular 3×3 matrix L_α^i in the coset $SL(3, \mathbb{R})/SO(3)$ and an $SU(2)$ gauge potential A_μ , and on the fermion side the pseudo-Majorana spinors ψ_γ and χ_i . We will chose as representation for the Clifford algebra

$$\Gamma^a = \gamma^a \times \mathbb{1}_2, \quad \hat{\Gamma}^i = \gamma_9 \times \tau^i, \quad (1)$$

where as usual $\gamma_9 = i\gamma^0\gamma^1 \dots \gamma^7$, so that $\gamma_9^2 = \mathbb{1}$, while τ^i are Pauli matrices. It will prove useful to introduce $\Gamma_9 \equiv \frac{1}{6i}\epsilon_{ijk}\hat{\Gamma}^{ijk} = -i\hat{\Gamma}_1\hat{\Gamma}_2\hat{\Gamma}_3 = \gamma_9 \times \mathbb{1}_2$.

Spin(7) holonomy: We will first construct a supergravity solution corresponding to D6-branes wrapped on a coassociative 4-cycle in a seven manifold of G_2 holonomy. Coas-

sociative 4-cycles are supersymmetric cycles preserving 1/16 supersymmetry. Therefore, the solution will be a supergravity dual of a three-dimensional gauge theory with $N = 1$ supersymmetry and when lifted to eleven dimensions will correspond to M-theory on an eight manifold with Spin(7) holonomy group [11, 6]. The symmetry group of the wrapped branes is $SO(1, 2) \times SO(4) \times SO(3)_R$. The twisting is performed by identifying the structure group of the normal bundle, $SO(3)_R$, with $SU(2)_L$ in $SO(4) \simeq SU(2)_L \times SU(2)_R$. This leads to a pure gauge theory in three dimensions with two supercharges. There are no scalars because the bundle of anti self-dual two-forms is trivial; therefore $L_\alpha^i = \delta_\alpha^i$. The deformation on the worldvolume of the D6-brane will be described by the metric ansatz

$$ds_8^2 = \alpha^2 ds_4^2 + e^{2f} dx_{1,2}^2 + d\rho^2, \quad (2)$$

with the 4-cycle taken as a 4-sphere with metric de Sitter's metric on S^4 ,

$$ds_4^2 = \frac{1}{(1 + \xi^2)^2} \left(\frac{\xi^2}{4} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + d\xi^2 \right), \quad (3)$$

where the left-invariant Maurer–Cartan $SU(2)$ 1-forms satisfy $d\sigma_i = \frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k$. A useful representation in terms of the Euler angles θ , ψ and ϕ , is

$$\sigma_1 \pm i\sigma_2 = e^{\pm i\psi} (\sin \theta d\phi + id\theta), \quad \sigma_3 = d\psi + \cos \theta d\phi. \quad (4)$$

We will use for the 8-bein the basis $e^7 = d\rho$, $e^8 = (1 + \xi^2)^{-1} d\xi$, etc. From the structure equations, the spin connections on S^4 can be easily shown to be

$$\omega_{i8} = \frac{1 - \xi^2}{1 + \xi^2} \frac{\sigma_i}{2}, \quad \omega_{ij} = \frac{1}{2} \epsilon_{ijk} \sigma_k. \quad (5)$$

The twisting, which amounts to an identification of the spin connection with the R -symmetry, is performed by turning on an $SU(2)$ gauge field obtained from the self-dual combinations of the spin connection on S^4 , $A^1 = -\frac{1}{2}(-\omega_{81} - \omega_{23})$ (+ cyclic). The gauge field is then that for the charge one $SU(2)$ instanton on S^4 ,

$$A = \frac{1}{2} \frac{1}{1 + \xi^2} i \sigma_i \tau^i. \quad (6)$$

Consistency of the Killing spinor equations requires the four projections

$$\Gamma_{8i} \epsilon = -\frac{1}{2} \epsilon_{ijk} \hat{\Gamma}_{jk} \epsilon, \quad \Gamma^7 \epsilon = -i \Gamma^9 \epsilon. \quad (7)$$

These projections lead to Spin(7) invariant Killing spinors since they imply the conditions

$$\left(\Gamma_{\alpha\beta} + \frac{1}{6} \Psi_{\alpha\beta\gamma\delta} \Gamma_{\gamma\delta} \right) \epsilon = 0, \quad (8)$$

where $\Psi_{\alpha\beta\gamma d}$ is the totally antisymmetric 4-index tensor that is invariant under the Spin(7) subgroup of $SO(8)$. To prove that we use the standard splitting $\alpha = (a, 8)$, with $a = 1, 2, \dots, 7$ and denote by $\psi_{abc} = \Psi_{abc8}$ the octonionic structure constants. In the basis $a = (7, i, \hat{i} = i + 3)$, $i = 1, 2, 3$ we have that

$$\begin{aligned}\psi_{ijk} &= \epsilon_{ijk} , & \psi_{i\hat{j}\hat{k}} &= -\epsilon_{ijk} , & \psi_{7i\hat{j}} &= \delta_{ij} , \\ \psi_{7ij\hat{k}} &= \epsilon_{ijk} , & \psi_{7\hat{i}\hat{j}\hat{k}} &= -\epsilon_{ijk} , & \psi_{ij\hat{m}\hat{n}} &= \delta_{im}\delta_{jn} - \delta_{in}\delta_{jm} ,\end{aligned}\tag{9}$$

and we may easily see that (7) imply the conditions (8) for a Spin(7) invariant Killing spinor. These projections and the gauge field (6) lead the Killing spinor equations to

$$\frac{df}{d\rho} = \frac{1}{3} \frac{d\Phi}{d\rho} = -\frac{1}{2} \frac{e^\Phi}{\alpha^2} + \frac{1}{4} e^{-\Phi} , \quad \frac{1}{\alpha} \frac{d\alpha}{d\rho} = \frac{e^\Phi}{\alpha^2} + \frac{1}{4} e^{-\Phi} .\tag{10}$$

In terms of a new radial variable defined as $dr = e^{-\Phi/3} d\rho$ the solution to the differential equations (10) can be lifted to eleven dimensions using the elfbein in [16], and leads to [6]

$$ds_{11}^2 = ds_{1,2}^2 + \frac{dr^2}{\left(1 - \frac{l^{10/3}}{r^{10/3}}\right)} + \frac{9}{100} r^2 \left(1 - \frac{l^{10/3}}{r^{10/3}}\right) (\Sigma_i - A^i)^2 + \frac{9}{20} r^2 ds_4^2,\tag{11}$$

where the Σ_i 's are the left-invariant Maurer–Cartan 1-forms corresponding to the internal, from an eight-dimensional point of view, $SU(2)$, for which an explicit parametrization similar to (4) may be used (with the Euler angles denoted by θ', ψ' and ϕ'). This is the metric of a Spin(7) holonomy manifold [17], with the topology of an \mathbb{R}^4 bundle over S^4 . We also note that M-theory realizations of the strong coupling description of D6-branes wrapped on coassociative cycles involving more general Spin(7) manifolds can be identically obtained using the non-homogeneous quaternionic spaces of [18].

G_2 holonomy: We will now describe how the supergravity configuration corresponding to D6-branes wrapped on a deformed 3-sphere in a Calabi-Yau threefold can be lifted to M-theory on a seven manifold of G_2 holonomy with an $SU(2) \times SU(2)$ isometry group [4]. The wrapped D6-branes on the deformed 3-cycle are described by a metric ansatz

$$ds_8^2 = \alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + \alpha_3^2 \sigma_3^2 + e^{2f} ds_{1,3}^2 + d\rho^2 .\tag{12}$$

Deformation of the 3-cycle requires the existence of scalars on the coset manifold, $L_\alpha^i = \text{diag}(e^{\lambda_1}, e^{\lambda_2}, e^{\lambda_3})$, with the constraint $\lambda_1 + \lambda_2 + \lambda_3 = 0$. Within this ansatz, the only consistent way to obtain non-trivial solutions to the supersymmetry variations is to impose on the spinor ϵ the projections

$$\Gamma_{ij}\epsilon = -\hat{\Gamma}_{ij}\epsilon , \quad \Gamma^7\epsilon = -i\Gamma^9\epsilon .\tag{13}$$

Among the possible pairs $\{ij\} = \{12, 23, 31\}$ only two are independent, so that (13) represents three conditions and reduces the number of supersymmetries to $32/2^3 = 4$, leading to $N = 1$ in four dimensions.¹ The symmetry group of the wrapped branes is now $SO(1, 3) \times SO(3) \times SO(3)_R$. Supporting covariantly constant spinors on the worldvolume of the brane requires again some twisting or mixing of the spin and gauge connections. However, in the presence of scalars the twisting is not a direct identification of the two connections. Instead, the generalized twisting in terms of a gauge field $A^\alpha = A_i^\alpha \sigma_i$ is [4]

$$A_1^1 = \frac{\alpha_1}{2} \left[-\frac{\omega_1^{23}}{\alpha_1} \cosh \lambda_{23} + e^{\lambda_{21}} \sinh \lambda_{31} \frac{\omega_2^{31}}{\alpha_2} - e^{\lambda_{31}} \sinh \lambda_{12} \frac{\omega_3^{12}}{\alpha_3} \right], \quad (14)$$

and A_2^2 and A_3^3 obtained from cyclicity in the 1, 2, 3 indices. We have used the notation

$$\omega_i^{jk} = \epsilon_{ijk} \frac{\alpha_j^2 + \alpha_k^2 - \alpha_i^2}{2\alpha_j\alpha_k}, \quad (15)$$

for the spin connection along the 3-sphere expanded as $\omega^{jk} = \omega_i^{jk} \sigma_i$, and $\lambda_{ij} = \lambda_i - \lambda_j$. If we define some new variables, $a_i = e^{-\Phi/3} \alpha_i$, $b_i = e^{2\Phi/3} e^{\lambda_i}$, $e^{2\Phi} = b_1 b_2 b_3$, $dr = e^{-\Phi/3} d\rho$, the Killing spinor equations become

$$\frac{da_1}{dr} = -\frac{b_2}{a_3} F_{31}^2 - \frac{b_3}{a_2} F_{12}^3, \quad \frac{db_1}{dr} = \frac{b_1^2}{a_2 a_3} F_{23}^1 - \frac{1}{2b_2 b_3} (b_1^2 - b_2^2 - b_3^2) \quad (16)$$

and cyclic in the 1, 2, 3 indices, where the field strength components $F_{jk}^i = A_i^j + g A_j^j A_k^k$ in the $\sigma^j \wedge \sigma^k$ basis in (16) are computed using (14) in the new variables,

$$A_1^1 = -\frac{1}{2} \frac{d_2^2 + d_3^2 - d_1^2}{2d_2 d_3} \equiv -\frac{1}{2} \Omega_1^{23}, \quad (17)$$

where $d_i \equiv \frac{a_i}{b_i}$ and cyclic in 1, 2, 3. We see that the generalized twist condition (14) takes the form of the ordinary twist, but for an auxiliary deformed 3-sphere metric obtained by replacing the a_i 's by the d_i 's defined above. The lift to eleven dimensions of our eight-dimensional background is of the form $ds_{11}^2 = ds_{1,3}^2 + ds_7^2$, where

$$ds_7^2 = dr^2 + \sum_{i=1}^3 a_i^2 \sigma_i^2 + \sum_{i=1}^3 b_i^2 (\Sigma_i + c_i \sigma_i)^2, \quad (18)$$

with $c_i = 2A_i^i$. This metric, when the various functions are subject to the conditions (16) and (17), describes G_2 holonomy manifolds with an $SU(2) \times SU(2)$ isometry. This

¹In the basis (9) the projections (13) imply the condition for a G_2 invariant Killing spinor,

$$\left(\Gamma_{ab} + \frac{1}{4} \psi_{abcd} \Gamma_{cd} \right) \epsilon = 0.$$

can be proved explicitly [4] noting that the system of equations (16) can also be derived from self-duality of the spin connection for the seven manifold (18), since self-duality of the spin connection in seven dimensions implies closedness and co-closedness of the associative three-form and, therefore, G_2 holonomy. An extra $SU(2)$ isometry develops when $a_1 = a_2 = a_3$ and $b_1 = b_2 = b_3$. In that case there is no need for scalar fields, and the branes wrap a round 3-sphere. Then, the system (16) simplifies enormously and can be solved explicitly [5], leading naturally to the metric of [17].

Let us now consider several interesting limits of the metric (18). First we will study the case where the radius of the “spacetime” 3-sphere becomes very large so that it can be approximated by \mathbb{R}^3 and the D6-branes are effectively unwrapped. This limit can be taken systematically as follows: consider the rescaling $\sigma_i \rightarrow \epsilon dx_i$, $b_i \rightarrow \epsilon b_i$ and $r \rightarrow \epsilon r$ in the limit $\epsilon \rightarrow 0$. Then, since the functions $c_i = 2A_i^i$ do not scale, the metric (18) takes the form $ds_7^2 = dx_i^2 + ds_4^2$, where we have absorbed a factor of ϵ^2 into a redefinition of the overall Planck scale and where the four-dimensional non-trivial part of the metric is

$$ds_4^2 = dr^2 + \sum_{i=1}^3 b_i^2 \Sigma_i^2 . \quad (19)$$

The system of equations (16) reduce to the statement that the coefficients a_i are constants and therefore they can be absorbed into a rescaling of the new coordinates x_i , as we have already done above, and the simpler system

$$\frac{db_1}{dr} = \frac{1}{2b_2b_3}(b_2^2 + b_3^2 - b_1^2) , \quad \text{and cyclic permutations} , \quad (20)$$

which is nothing but the Lagrange system. Four-dimensional metrics (19) governed by that system correspond to a class of hyperkähler metrics with $SU(2)$ isometry with famous example, if an extra $U(1)$ symmetry develops (for instance when $b_1 = b_2$), the Eguchi–Hanson metric which is the first non-trivial ALE four-manifold. This is in agreement with the fact that the near horizon limit of (unwrapped) D6-branes of type IIA when uplifted to M-theory contains, besides the D6-brane worldvolume, the Eguchi–Hanson metric.

We will now decouple just one of the coordinates in the cycle. First we consider the case where $a_1 = a_2 = a$ and $b_1 = b_2$ so that an extra $U(1)$ symmetry develops. The resulting simplified system of equations in (16) is still quite complicated and there is no explicit solution to it, up to date, leading to regular metrics. Next, let the change of variables $\psi' = \varphi + x/\epsilon$, $\psi = x/\epsilon$, followed by setting $a_3 = \epsilon$, and by taking the limit $\epsilon \rightarrow 0$. We are then left with a five-dimensional field theory with $N = 1$ supersymmetry, and the metric splits as $ds_{11}^2 = ds_{1,4}^2 + ds_6^2$, where the non-compact variable x parametrizes the fifth dimension. The system (16) becomes

$$\frac{da}{dr} = \frac{1}{2} \frac{b_3}{a} , \quad \frac{db_3}{dr} = 1 - \frac{1}{2} \frac{b_3^2}{a^2} - \frac{1}{2} \frac{b_3^2}{b_1^2} , \quad \frac{db_1}{dr} = \frac{1}{2} \frac{b_3}{b_1} \quad (21)$$

and also $c_1 = c_2 = 0$ and $c_3 = -1$. Hence, we recover the case of D6-branes wrapped on a sLag₂ cycle (special Lagrangian 2-sphere) studied in [5], with ds_6^2 the metric for the resolved conifold [19].

SU(4) holonomy: D6-branes wrapped on a Kähler 4-cycle inside a Calabi–Yau three-fold lift to M-theory on a Calabi–Yau four-fold [11, 7, 10]. The symmetry group of the branes is $SO(1, 2) \times SO(4) \times U(1)_R$. The R-symmetry is broken to the $U(1)_R$ corresponding to the two normal directions to the D6-branes that are inside the three-fold. The twisting amounts to the identification of this $U(1)_R$ with a $U(1)$ subgroup in one of the $SU(2)$ factors in $SO(4)$. The remaining scalar after the twisting, the vector and two fermions preserved by the diagonal group of $U(1) \times U(1)_R$ determine the field content of $N = 2$ three-dimensional Yang–Mills. Following [10] we take the 4-cycle to be a product of two 2-spheres of different radii, $S^2 \times \bar{S}^2$. The worldvolume of the D6-branes will be described by a metric

$$ds_8^2 = e^{2f} ds_{1,2}^2 + d\rho^2 + \alpha^2 d\Omega_2^2 + \beta^2 d\bar{\Omega}_2^2 . \quad (22)$$

Consistency of the Killing spinor equations requires turning on one of the scalars in the coset, $L_\alpha^i = \text{diag}(e^\lambda, e^\lambda, e^{-2\lambda})$, and one of the components of the gauge field,

$$A^3 = -\frac{1}{2}(\sigma_3 + \bar{\sigma}_3) , \quad (23)$$

thus realizing the breaking of the $SU(2)_R$ R-symmetry to $U(1)_R$, and the projections

$$\Gamma_1 \Gamma_2 \epsilon = \bar{\Gamma}_1 \bar{\Gamma}_2 \epsilon = -\hat{\Gamma}_1 \hat{\Gamma}_2 \epsilon, \quad \Gamma_7 \epsilon = -i \Gamma_9 \epsilon . \quad (24)$$

These leave in total four independent components for the spinor. If we redefine variables as $dr = e^{-\Phi/3} d\rho$, $a = e^{-2\lambda+2\Phi/3}$, $a_1 = \alpha e^{-\Phi/3}$, $a_2 = \beta e^{-\Phi/3}$, $a_3 = e^{\lambda+2\Phi/3}$, the Killing spinor equations become

$$\frac{da}{dr} = 1 - \frac{a^2}{2} \left(\frac{1}{a_1^2} + \frac{1}{a_2^2} + \frac{1}{a_3^2} \right) , \quad a_1 \frac{da_1}{dr} = \frac{a}{2} , \quad \text{and cyclic in } 1, 2, 3 . \quad (25)$$

The general solution to the system (25) is

$$a_1^2 = R^2 + l_1^2, \quad a_2^2 = R^2 + l_2^2, \quad a_3^2 = R^2, \quad a^2 = R^2 U^2(R) , \quad (26)$$

where

$$U^2(R) = \frac{3R^4 + 4(l_1^2 + l_2^2)R^2 + 6l_1^2 l_2^2 + 3C/R^4}{6(R^2 + l_1^2)(R^2 + l_2^2)} \quad (27)$$

and the two variables r and R relate via the differentials $dr = 2dR/U(R)$. Here we have denoted three of the constants of integration by l_1, l_2 and C and we have absorbed the fourth one by an appropriate shift in the variable R . When lifted to eleven dimensions the solution factorizes into the three-dimensional flat space-time and a Calabi–Yau four-fold

$$ds_{11}^2 = ds_{1,2}^2 + \frac{4dR^2}{U^2(R)} + a_1^2 d\Omega_2^2 + a_2^2 d\bar{\Omega}_2^2 + a_3^2 d\hat{\Omega}_2^2 + a^2 (\hat{\sigma}_3 - \sigma_3 - \bar{\sigma}_3)^2 . \quad (28)$$

Topologically the Calabi–Yau four-fold is a \mathbb{C}^2 bundle over $S^2 \times S^2$ and was also constructed with a different method in [20] (for $C = 0$). This result includes those obtained in [7, 8], where both spheres in the four-cycle were taken to have the same radius, so that $l_1 = l_2$. Having unequal radii makes possible to consider the limit where the radius of one of the three spheres tends to infinity. Indeed, if we take the limit $l_2 \rightarrow \infty$, we have that $a_2^2 d\bar{\Omega}_2^2 = dx_1^2 + dx_2^2$, i.e. $\bar{S}^2 \rightarrow \mathbb{R}^2$, and the metric (28) becomes $ds_{1,4}^2 + ds_6^2$, where ds_6^2 is the metric of the resolved conifold [19] in its standard form, up to a factor of 6 and after letting $l_1 = \sqrt{6}a$ and $C = 0$. Note also that, in this limit, the systems (25) and (21), after an appropriate renaming of the variables, coincide, as it should be.

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Note added: After completion of this note, we learned from reference [21], where the construction in [4] was extended to include all metrics of G_2 holonomy with $S^3 \times S^3$ principal orbits through an extension of the twisting procedure.

References

- [1] M. Bershadsky, C. Vafa and V. Sadov, Nucl. Phys. **B463** (1996) 420, [hep-th/9511222](#).
- [2] J. Maldacena and C. Núñez, Int. J. Mod. Phys. **A16** (2001) 822, [hep-th/0007018](#) and Phys. Rev. Lett. **86** (2001) 588, [hep-th/0008001](#).
- [3] A. H. Chamseddine and M. S. Volkov, Phys. Rev. Lett. **79** (1997) 3343, [hep-th/9707176](#) and Phys. Rev. **D57** (1998) 6242, [hep-th/9711181](#).
- [4] R. Hernández and K. Sfetsos, Phys. Lett. **B536** (2002) 294, [hep-th/0202135](#).
- [5] J.D. Edelstein and C. Núñez, JHEP **0104** (2001) 028, [hep-th/0103167](#).
- [6] R. Hernández, Phys. Lett. **B521** (2001) 371, [hep-th/0106055](#).

- [7] J. Gomis and T. Mateos, Phys. Lett. **B524** (2002) 170, [hep-th/0108080](#).
- [8] U. Gursoy, C. Núñez and M. Schwelling, JHEP **0206** (2002) 015, [hep-th/0203124](#).
- [9] J. Brugués, J. Gomis, T. Mateos and T. Ramírez, JHEP **0210** (2002) 016, [hep-th/0207091](#); J. D. Edelstein, A. Paredes and A. V. Ramallo, [hep-th/0207127](#).
- [10] R. Hernández and K. Sfetsos, JHEP **0207** (2002) 045, [hep-th/0205099](#).
- [11] J. Gomis, Nucl. Phys. **B606** (2001) 3, [hep-th/0103115](#).
- [12] B. Acharya, J.P. Gauntlett and N. Kim, Phys. Rev. **D63** (2001) 106003, [hep-th/0011190](#); J.P. Gauntlett, N. Kim and D. Waldram, Phys. Rev. **D63** (2001) 126001, [hep-th/0012195](#); M. Schwelling and T.A. Tran, JHEP **0106** (2001) 025, [hep-th/0105019](#); J. Maldacena and H. Nastase, JHEP **0109** (2001) 024, [hep-th/0105049](#); J.P. Gauntlett, N. Kim, D. Martelli and D. Waldram, Phys. Rev. **D64** (2001) 106008, [hep-th/0106117](#); F. Bigazzi, A.L. Cotrone and A. Zaffaroni, Phys. Lett. **B519** (2001) 269, [hep-th/0106160](#); J.P. Gauntlett and N. Kim, Phys. Rev. **D65** (2002) 086003, [hep-th/0109039](#); J. Gomis and J.G. Russo, JHEP **0110** (2001) 028, [hep-th/0109177](#); J. P. Gauntlett, N. W. Kim, D. Martelli and D. Waldram, JHEP **0111** (2001) 018, [hep-th/0110034](#); J. Gomis, Nucl. Phys. **B624** (2002) 181, [hep-th/0111060](#); R. Apreda, F. Bigazzi, A. L. Cotrone, M. Petrini and A. Zaffaroni, Phys. Lett. **B536** (2002) 161, [hep-th/0112236](#); J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, Class. Quant. Grav. **19** (2002) 3927, [hep-th/0202184](#); N. Evans, M. Petrini and A. Zaffaroni, JHEP **0206** (2002) 004, [hep-th/0203203](#); J. P. Gauntlett, D. Martelli, S. Pakis and D. Waldram, [hep-th/0205050](#); P.Di Vecchia, A. Lerda and P. Merlatti, Nucl. Phys. **B646** (2002) 43, [hep-th/0205204](#); T. Mateos, J.M. Pons and P. Talavera, [hep-th/0209150](#).
- [13] C. Núñez, I.Y. Park, M. Schwelling and T.A. Tran, JHEP **0104** (2001) 025, [hep-th/0103080](#); P. Di Vecchia, H. Enger, E. Imeroni and E. Lozano-Tellechea, Nucl. Phys. **B631** (2002) 95, [hep-th 0112126](#).
- [14] H. Nieder and Y. Oz, JHEP **0103** (2001) 008, [hep-th/0011288](#); M. Naka, [hep-th/0206141](#).
- [15] J.P. Gauntlett, N. Kim, S. Pakis and D. Waldram, Phys. Rev. **D65** (2002) 026003, [hep-th/0105250](#).
- [16] A. Salam and E. Sezgin, Nucl. Phys. **B258** (1985) 284.
- [17] R. Bryant and S. Salamon, Duke Math. J. **58** (1989) 829; G.W. Gibbons, D.N. Page and C.N. Pope, Commun. Math. Phys. **127** (1990) 529.
- [18] K. Behrndt, G. Dall'Agata, D. Lüst and S. Mahapatra, JHEP **0208** (2002) 027, [hep-th/0207117](#).
- [19] P. Candelas and X.C. de la Ossa, Nucl. Phys. **B342** (1990) 246; L.A. Pando Zayas and A.A. Tseytlin, JHEP **0011** (2000) 028, [hep-th/0010088](#).
- [20] M. Cvetič, G. W. Gibbons, H. Lu and C. N. Pope, [hep-th/0012011](#).
- [21] J. D. Edelstein, A. Paredes and A. V. Ramallo, JHEP **0301** (2003) 011, [hep-th/0211203](#).