Experimental evidence of chaos in the bound states of ^{208}Pb

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Abstract. Theoretical calculations, especially shell-model calculations, have shown a strongly chaotic behavior of bound states at higher excitation energy, in regions of high level density. However, it had not been possible up to now to observe chaos in the experimental bound energy levels of any single nucleus. In this paper we study the spectral fluctuations of the ^{208}Pb nucleus using the complete experimental spectrum of 151 states up to excitation energies of 6.20 MeV. For natural parity states the results are very close to the predictions of Random Matrix Theory (RMT) for the nearest-neighbor spacing distribution. By contrast, the results for unnatural parity states are far from RMT behavior. We interpret these results as a consequence of the strength of the residual interaction in ^{208}Pb, which, according to experimental data, is much stronger for natural than for unnatural parity states. In addition our results show that chaotic and non-chaotic nuclear states coexist in the same energy region of the spectrum.

1. Introduction

The atomic nucleus is generally considered a paradigmatic case of quantum chaos. Intuitively one can expect that fast moving nucleons interacting with the strong nuclear force and bound in the small nuclear volume should give rise to a chaotic motion. The quest for chaos in nuclei has been quite intensive, both with theoretical calculations using nuclear models and with detailed analyses of experimental data. Statistical spectroscopy studies in nuclei have been also motivated by a desire to understand the implications of chaotic behavior in many-body quantum systems. Theoretical calculations, especially shell-model calculations, have shown a strongly chaotic behavior of bound states at higher excitation energy, in regions of high level density. However, as we discuss below, it has not been possible up to now to observe chaos in the experimental bound energy levels of any single nucleus. For a comprehensive review of chaos in nuclei see for example Gómez *et al.* [1] and Weidenmüller and Mitchell [2].

In this paper we first present in Section 2 a brief outline of the spectral properties that characterize quantum chaos using two different approaches, the Random Matrix Theory (RMT) and the time series approach. Then we look at the theoretical and experimental evidence for the existence of chaos in nuclei in Section 3, and we study in Section 4 the spectral fluctuations of the lowest 151 states in ^{208}Pb, which have been recently identified with spin and parity assignements [3]. It turns out that nuclear motion in ^{208}Pb is strongly chaotic at least for the natural parity states [4]. For unnatural parity states the evidence is less clear, but it seems to be intermediate

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between chaotic and regular motion. Finally we compare our results with those of another recent study of chaos in ^{208}Pb [5].

2. Quantum chaos and spectral fluctuations

Quantum chaos has been the subject of many investigations during the last decades. The pioneering work of Berry, Bohigas *et al.*, and others [6, 7] leads to an important and concise statement: the spectral properties of simple systems known to be ergodic in the classical limit follow very closely those of the Gaussian orthogonal ensemble (GOE) of random matrices. On the contrary, integrable systems lead to level fluctuations that are well described by the Poisson distribution, i.e., levels behave as if they were uncorrelated.

Fluctuations are the departure of the actual level density from a local uniform density. Therefore it is essential to eliminate the smooth part of the exponential increase of the nuclear density, mapping the actual spectrum onto a quasiuniform spectrum with mean spacing $\langle s \rangle = 1$. This step, called unfolding, is delicate and of utmost importance, because some of the unfolding procedures used in the literature can lead to completely wrong results on the behavior of level fluctuations [8].

Short-range correlations between energy states are usually measured by the nearest neighbor spacing (NNS) distribution $P(s)$. The Poisson distribution is given by $P_P(s) = \exp(-s)$. The GOE distribution has no analytical formula, but is well approximated by the Wigner surmise, $P_W(s) = (\pi s/2)\exp(-\pi s^2/4)$. Note that $P_P(0) = 1$ and $P_W(0) = 0$. Quantum chaos is characterized by strong state correlations and level repulsion. An assesment of chaotic or regular behavior of a system is given by comparison of the NNS distribution to Poisson and Wigner. Real complex systems are often not fully chaotic or fully regular. These intermediate situations are often described in terms of a single parameter ω by means of the Brody distribution $P_B(s,\omega)$, which is just an interpolation formula between the Poisson ($\omega = 0$) and Wigner ($\omega = 1$) distributions. Although it has no obvious physical meaning, the Brody parameter ω is a simple and widely used measure of chaoticity of a system. When the number of spacings is not very large, it is preferable to fit the cumulative distribution, $I(s) = \int_0^s P(x)dx$.

Long-range correlations are usually studied with the Dyson-Mehta statistic $\Delta_3(L)$. For Poisson $\langle \Delta_3(L) \rangle = \dfrac{L}{15}$ and for GOE $\langle \Delta_3(L) \rangle = \dfrac{1}{\pi^2}\log(L) + b + \mathcal{O}(L^{-1}), \quad L \gg 1$.

There is another approach to quantum chaos based on the analogy between a discrete time series and a quantum energy spectrum, if time t is replaced by the energy E of the quantum states [9]. The spacing between two consecutive levels is $s_i = \varepsilon_{i+1} - \varepsilon_i$. We define the statistic δ_n as a signal,

$$\delta_n = \sum_{i=1}^{n} (s_i - \langle s \rangle) = \varepsilon_{n+1} - \varepsilon_1 - n \tag{1}$$

and the discrete power spectrum is $P(k) = \left|\widehat{\delta_k}\right|^2$, where $\widehat{\delta_k}$ is the Fourier transform of δ_n,

$$\widehat{\delta_k} = \frac{1}{\sqrt{N}} \sum_n \delta_n \exp\left(\frac{-2\pi i k n}{N}\right) \tag{2}$$

and N is the size of the series. It turns out that the energy spectra of chaotic quantum systems are characterized by 1/f noise, and on the contrary, the energy spectra of regular quantum systems are characterized by $1/\mathrm{f}^2$ noise (Brown noise) [9].

3. Chaos in nuclei

3.1. Spectral fluctuations in shell-model spectra

Theoretical calculations provide long sequences of J^π or $J^\pi T$ levels suitable for statistical analysis of fluctuations (no missing levels, no uncertain spin and parity assignments). Calculations

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performed with the spherical shell model, the cranking model, the interacting boson model and other models have shown examples of highly regular energy spectra in deformed nuclei and examples of highly chaotic spectra in spherical nuclei, although there are exceptions in some nuclei.

In spherical nuclei, where the shell model is most appropriate, many calculations have shown that for large configuration spaces, with $J^{\pi}T$ level sequences up to several thousand, the usual fluctuation measures $P(s)$ and $\Delta_3(L)$ agree very well with GOE predictions. To mention just some examples, Fig. 1 illustrates the good agreement of $P(s)$ and $\Delta_3(L)$ with GOE for the $J^{\pi} = 2^{+}$ states of ^{28}Si calculated with the shell model [10]. Fig. 2 illustrates the 1/f noise behavior of shell-model states in ^{34}Na ($\alpha = 1.11$) and ^{24}Mg ($\alpha = 1.06$). The straight line is the best fit of the power spectrum assuming $P(k) = 1/k^{\alpha}$.

In the $2p1f$ shell the configuration space and the level density are much larger than in sd-shell nuclei. Shell-model calculations with a realistic interaction have been performed to investigate the degree of chaos in different isotopes as a function of excitation energy by Molina *et al.* [11]. For example, in ^{46}Sc a total of 25,498 spacings are included in the calculations, ensuring excellent statistics. Fig 3 shows the behavior of $\Delta_3(L)$ for the $J^{\pi} = 0^{+}, T = T_z$ states of ^{46}Ca, ^{46}Sc and ^{46}Ti. Clearly, there is an isospin dependence in the chaoticity of these isobars. For ^{46}Ti the agreement with GOE (dotted line in Fig. 3) is excellent. For ^{46}Sc there is GOE behavior up to $L \sim 40$, and ^{46}Ca clearly deviates from GOE towards Poisson (dashed line) for $L \geq 10$. Furthermore, at low energies the fluctuations in Ca isotopes are more regular than chaotic, for instance $\omega = 0.25$ for the levels up to 5 MeV above yrast in ^{52}Ca. Similar results were obtained for Pb isotopes, with only valence neutrons outside the ^{208}Pb core [12].

We interpret the observed isospin dependence of chaoticity as a result of the strength of the residual interaction. The shell-model residual nn interaction is much weaker than the residual pn interaction. Ca isotopes have only neutrons in the pf shell, but if just one neutron is replaced by a proton, the pn interactions destroy the mean-field order. Therefore Sc or Ti isotopes, having both protons and neutrons in the valence space of the pf shell, exhibit strong chaotic characteristics even in the ground-state region.

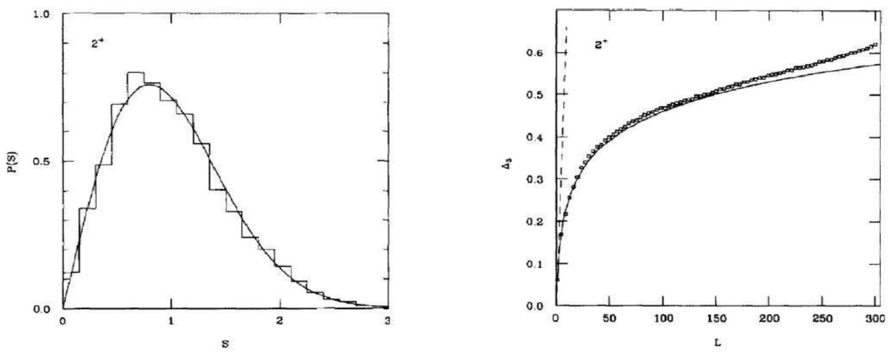

Figure 1. Shell-model $P(s)$ and $\langle\Delta_3(L)\rangle$ values for the $J^{\pi} = 2^{+}$, $T = 0$ states of ^{28}Si. Taken from [10].

3.2. Spectral fluctuations in experimental nuclear bound states

Haq, Pandey and Bohigas [13] analyzed the spectral fluctuations of a very large number of experimentally identified neutron and proton $J^{\pi} = 1/2^{+}$ resonances just above the one-nucleon emission threshold and showed that they agree very well with the spectral fluctuations of GOE.

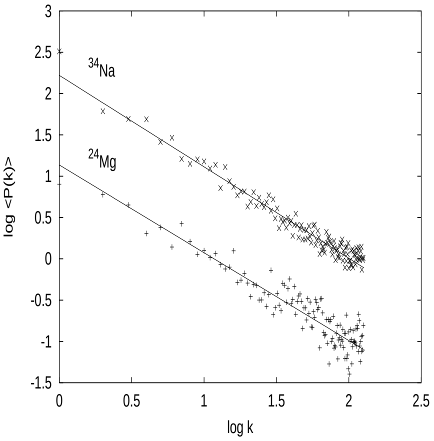

Figure 2. Power spectrum of δ_n statistic for nuclear shell-model spectra exhibiting 1/f noise. Adapted from [9].

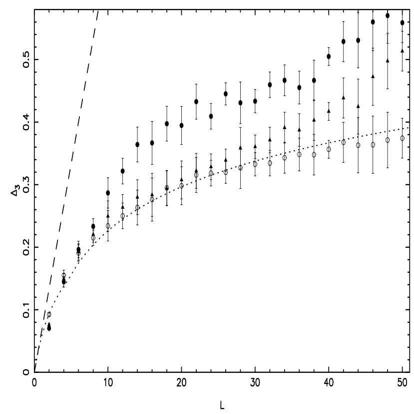

Figure 3. Comparison of $\langle\Delta_3(L)\rangle$ for $J^\pi = 0^+$ states in ^{46}Ca (full circles), ^{46}Sc (triangles) and ^{46}Ti (open circles). Adapted from [11].

Thus, in this sense, it is clear that nuclei are very chaotic in the energy region just above the one-nucleon emission threshold. But for bound states, the situation is not so clear, because a good analysis of fluctuations in experimental energy spectra requires the knowledge of sufficiently long, pure and complete sequences, i.e. with the same $J^\pi T$ values and without missing levels or $J^\pi T$ misassignments. But this ideal situation is rarely found in nuclei. For very light nuclei the number of bound levels is not sufficient for statistical purposes. For medium and heavy nuclei the identified levels are limited to the ground state region, because at higher energy the level density becomes very high and the experimental identification of the energy and J^π values becomes generally impossible.

In order to improve statistics, level spacings from different nuclei can be combined into a single set to analyze the behavior of the NNS distribution $P(s)$. An extensive analysis of low-lying energy levels was performed by Shriner *et al.* [14] using experimental data along the whole nuclear chart. A total of 988 spacings from 60 different nuclei were included in the analysis, giving $\omega = 0.43 \pm 0.05$, which is closer to Poisson than to GOE. Separating the data in six different mass regions a clear trend from GOE to Poisson is observed as A increases. For $A \leq 50$, the fit gives $\omega = 0.72 \pm 0.16$, and for $A > 230$ it gives $\omega = 0.24 \pm 0.11$. Generally spherical nuclei are closer to GOE and deformed nuclei are closer to Poisson. The latter is not necessarily a manifestation of regular behavior for the low-lying states of these nuclei, because a deviation towards Poisson may be also due to the omission of some symmetry. In the present case of deformed nuclei it may be due to omission of the K quantum number.

4. Chaos in the experimental bound states of ^{208}Pb

Recently the complete experimental spectrum of the lowest 151 states in ^{208}Pb, up to 6.20 MeV excitation energy, has been identified with spin and parity assignments. This represents the largest ensemble known up to now which can be used for a statistical investigation of the chaotic behavior among bound states in an atomic nucleus. We consider that these accurate data on ^{208}Pb enable a meaningful statistical analysis of level fluctuations, with pure, complete, and reasonably long sequences. There are 14 J^π sequences with a minimum of 5 and up to 19 known consecutive states.

Separate unfolding has been performed for all J^π sequences using the constant temperature

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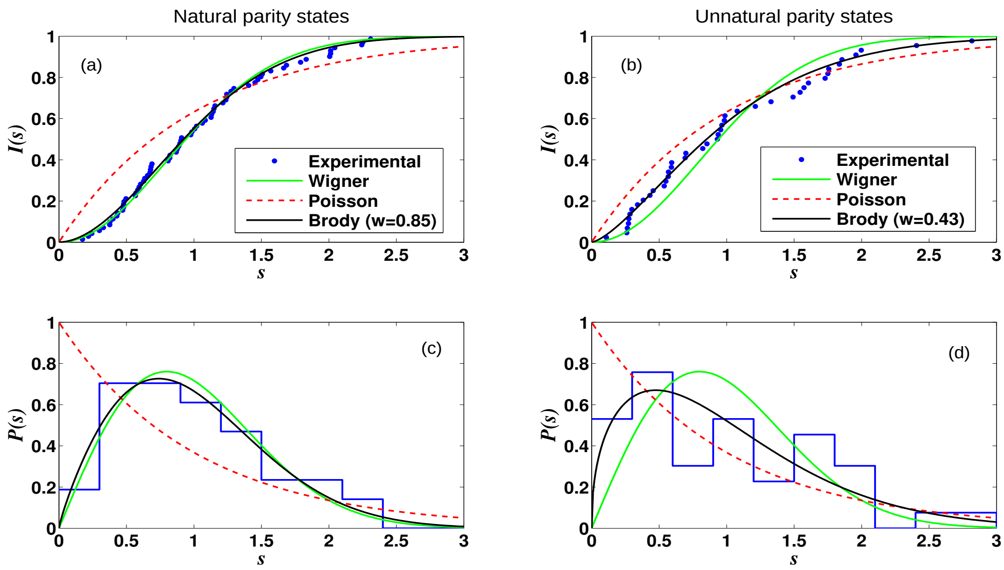

Figure 4. Comparison of spectral fluctuation statistics for natural parity states (left panels) and unnatural parity states (right panels) in ^{208}Pb. Panels (a) and (b) show the cumulative nearest neighbor spacing (cumulative NNS) distribution for experimental levels (dots). Panels (c) and (d) show the NNS distribution $P(s)$ for experimental levels (histogram). In all panels the green (light gray) line is the Wigner surmise, the red dashed line is the Poisson distribution, and the black solid line is the best fit Brody distribution. Adapted from [4].

formula [14],

$$\overline{\rho}(E) = \frac{1}{T}\exp[(E - E_0)/T], \tag{3}$$

where T and E_0 are taken as parameters. Gathering the unfolded spacings for all J^π into a single set, there are 115 spacings. The $P(s)$ distribution and the cumulative $I(s)$ distribution for the full set of spacings exhibit shapes much closer to GOE than to Poisson, but still the difference with GOE is substantial. The Brody parameter for the cumulative distribution is $\omega = 0.68$.

The relevant question with these results is how to interpret them regarding chaotic motion in ^{208}Pb. The result $\omega = 0.68$ is similar to the highest value $\omega = 0.72 \pm 0.16$ obtained for experimental states of nuclei with $A < 50$ [14]. Therefore we may wonder whether it represents more or less a practical limit of possible chaos in nuclear bound states.

Heusler *et al.* [3] have shown that the number of identified states at $E_x < 6.20$ MeV nearly agrees with the number of states in this energy interval predicted by a simplified shell-model consisting of 1p-1h mean field configurations. However, 70 unnatural parity states agree with this model within ~ 0.2 MeV, and 20 natural parity states differ by > 0.5 MeV. Hence they conclude that configuration mixing is stronger for natural parity states and therefore the residual interaction is much larger than for unnatural parity states. To check if this effect can be observed in the fluctuation measures we have analyzed separately the spectral fluctuations of those two sets of states and have found that they behave very differently.

Fig. 4 shows the experimental $P(s)$ and $I(s)$ distributions for natural and for unnatural parity states, compared to Wigner and Poisson. Clearly the agreement with GOE is good for natural parity states ($\omega = 0.85$), but for unnatural parity states the results are somewhat closer to Poisson ($\omega = 0.43$). Table 1 summarizes the number of spacings and the value of ω for

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Table 1. Number of spacings, Brody parameter ω and rms deviation from Wigner and Poisson distributions for different combinations of parity in the experimental states of ^{208}Pb at $E_x < 6.20$ MeV.

	Number of spacings		
Parity	all	natural	unnatural
even	45	29	16
odd	70	42	28
all	115	71	44
Brody ω	0.68	0.85	0.43
$(\mathrm{RMSD})_W$	0.040	0.025	0.077
$(\mathrm{RMSD})_P$	0.115	0.129	0.088

the different combinations of parity and natural/unnatural parity. Independently of the Brody parameter, the table gives as well the root mean square deviation (RMSD) of the experimental $I(s)$ from the Wigner and Poisson limits, showing the strong chaoticity of natural parity states.

These results agree with the observation of a stronger residual interaction for natural parity states pointed out in Ref. [3]. In the shell model the mean field gives rise to a regular motion and the residual interaction produces the mixing of basis states in the eigenstates, destroying the regular mean-field motion. This effect has been observed in shell-model calculations introducing a strength parameter to modulate the residual interaction. As the strength parameter increases, the fluctuations measures of energy levels approach GOE behavior and thus the motion becomes chaotic [10].

Finally, let us point out that another analysis of spectral fluctuations has been recently published [5]. They use two chaoticity measures different from ours and do not observe a significant difference between the experimental natural and unnatural parity states. Possibly this difference with our results is due to the different unfolding procedures used. The influence of the unfolding method is important when the number of levels is relatively small, as is the case for the unnatural parity states of ^{208}Pb. On the other hand, the spectral fluctuations have been analyzed by the same authors for two different shell-model calculations in ^{208}Pb. It should be noticed that the shell-model J^π sequences are much larger than the experimental ones and therefore the unfolding is more reliable. The shell-model values of the chaoticity parameters are different for natural and unnatural parity, the natural parity states being more chaotic.

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