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Abstract

The paper investigates the effects of central bank interventions in financial markets, composed of asymmetrically-informed rational investors and noise traders. If the central bank suspects a bubble, it should lift the real risk-free rate to deflate the bubble in “leaning against the wind”. A rise in the real risk-free rate reduces the risk of rational informed investors, and increases the risk of rational uninformed investors. If the central bank intervenes through the nominal risk-free rate and the Fisher arbitrage condition holds, an increase in the nominal rate is transferred to inflation, thereby dampening the policy effect. Conversely, this implies that the central bank can also deflate the bubble by inducing a reduction in inflationary expectations. The effect on the informed investor risk remains ambiguous, while the risk of the uninformed investor grows, but only if they suffer from money illusion.

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1. Introduction

How the central bank might respond to a suspected bubble in financial markets is a key aspect of public policy. General intuition suggests that the lower is the risk-free rate of return, the higher are asset prices as shareholders' discount rate for future cash flows would be low. Based on this intuition, the “leaning against the wind” policy says that if the central bank wishes to deflate a bubble in financial markets, it should lift interest rates. This leads to issues such as how the policy might have implications on investor risk, as well as the critical role of inflation.

By the classical Fisher (1905) arbitrage definition, the nominal risk-free yield equals the real yield multiplied by inflationary expectations. This idea has recently received attention in the Neo-Fisherian form because of the presence of simultaneous low interest rates and inflation in Japan, Europe and the USA. Among others, Williamson (2016) suggests this as a reasonable explanation for the dilemma. The advice is that central banks should use monetary policy to raise the risk-free rate in order to raise inflation to the target level.

Cochrane (2017) proposes that, with flexible prices, the Neo-Fisherian proposition is in accord with the New Keynesian framework. However, Garcia-Schmidt and Woodford (2019) claim that the Neo-Fisherian result is due to the assumption of rational expectations. Moreover, if it is replaced by a temporary equilibrium with reflective expectations (see Evans and McGouch, 2017), the result changes to the “leaning against the wind” implication. They argue that the system does not converge to the Neo-Fisherian equilibrium in any reasonable time.

In the financial market literature, this is related to higher-order expectations (see Allen, Morris and Shin, 2006), where the average expectations of investors depart from first-order rational expectations, thereby implying the existence of trading within a short horizon. On the other hand, the behavioural finance literature (see Shiller, 1982, and De Bondt and Thaler 1985 among others) has tackled the issue from the point of view of bounded rationality of investors (see Hirshleifer, 2015 for a recent review of the literature).

In this paper, it is assumed that rational investors can be separated in informed and uninformed investors. We assume that informed investors have private noisy information about the true value of

a risky asset, whereas uninformed investors infer the true value from the market price of the financial asset.

In addition, there are irrational noise traders in the market. The basic two-period model closely follows the presentation in Ilomäki and Laurila (2018), who show that an increase in the noise trader effect leads to a reduction in the risk of uninformed investors, and an increase in the risk of uninformed investors.

Starting from seminal studies of Markowitz (1952), Tobin (1958), Sharpe (1964) and Lintner (1965), rational investors are assumed to be risk averse so that they allocate their investments between risky and risk-free assets according to their risk tolerance. In the classical Walrasian equilibrium model, excess demand is zero. Therefore, in a financial market where the initial holdings of risky assets are positive without extra emissions, trading continues until the excess supply of risky assets is also set to zero.

This outcome can be interpreted as a short-term trading period. According to Froot et al. (1992) and Shleifer and Vishny (1997), among others, short-term trading is important for a risk averse investor, who seeks gains from closing their position earlier rather than later. This can create a temporary equilibrium with reflective expectations, in the spirit of Garcia-Schmidt and Woodford (2019).

The paper applies the Walrasian framework to the financial market, and show that the “leaning against the wind” policy works. It is also shown that, if the central bank elevates the real risk-free rate, it makes the risk of informed investors decrease and the risk of uninformed investors increase. Furthermore, elevation of the nominal rate is less effective because of the inflationary effect, implying that the bubble can also be deflated by manipulating inflationary expectations downwards. However, if the central bank controls the nominal risk-free rate or inflationary expectations, money illusion (see Shafir et al., 1997) may hamper the policy effect.

The remainder of the paper proceeds as follows. Section 2 presents the basic model specification. Section 3 derives the financial market equilibrium. The analysis of the effects of central bank intervention on asset prices and rational investor risk is presented in Section 4. Some concluding comments are given in Section 5.

2. Model Specification

The basic model follows the basic presentation in Ilomäki and Laurila (2018), with references to Grossman and Stiglitz (1980), Admati (1985), and Mendel and Shleifer (2012). There is a set $[0,1]$ of rational constant absolute risk averse (CARA) investors (with CARA coefficient equal to one) in a Walrasian financial market. The rational investor allocates their investments between risk-free and risky assets. They live for two periods, trading in the first period, and consuming in the second period, maximizing their utility from consumption, which is given as:

$$u(c) = -e^{-c}.$$

All trading occurs in period 1, while the payoffs that enable consumption arise in period 2.

The risk-free asset pays $(1+r)$ units of consumption in period 2, where r is the real risk-free rate of return, and the risky asset pays $\tilde{D} \sim N(\tilde{D}, \sigma_D^2)$ in terms of consumption in period 2. The market price of the risky asset is P per share, expressed in terms of consumption. The rational investor has asymmetric information. The share of informed investors is μ , $0 < \mu < 1$, and they observe a noisy signal, $\tilde{s} = \tilde{D} + \varepsilon$, with $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

The $(1-\mu)$ uninformed investors observe P , and form rational expectations on \tilde{D} based on P . Mendel and Shleifer (2012) also propose that $(1-\mu) > \mu$. In the market, there is also a measure 1 of correlated noise traders, who exchange risk-free assets for $\tilde{N} \sim N(0, \sigma_N^2)$ units of the risky asset purely according to sentiment. The initial allocation of the risky asset is:

$$\mu a_I + (1 - \mu) a_U + a_N = A, \tag{1}$$

where a_I , a_U and a_N denote the possessions of the informed investor, uninformed investor, and noise trader, respectively.

All traders hold an identical amount a_O of the risk-free asset. The informed investors form their expectation of \tilde{D} based on their private signal, \tilde{S} :

$$E[\tilde{D} | \tilde{s}] = \tilde{D} + \beta[\tilde{s} - \tilde{D}]. \quad (2)$$

In equation (2):

$$\beta = \frac{\sigma_D^2}{\sigma_s^2}, \quad (3)$$

where $\sigma_s^2 = \sigma_D^2 + \sigma_\varepsilon^2$. The variance of the informed investors' prediction error is given as:

$$\sigma_I^2 = \frac{\sigma_D^2 \sigma_\varepsilon^2}{\sigma_s^2}. \quad (4)$$

Equation (4) defines the informed investors' risk premium, and thus risk, as the coefficient of absolute risk aversion is 1, according to the CARA assumption. By equation (2), the dividend yield \tilde{D} influences the market price P through the informed investor private signals. Therefore, P depends on \tilde{S} as well as on \tilde{N} , the demand of the noise trader.

In a linear framework, write:

$$P = z + b\tilde{s} + c\tilde{N}, \quad (5)$$

where z , b , and c are unknown parameters. The uninformed investor bases their expectations on their observation on P :

$$E[\tilde{D} | P] = \tilde{D} + \gamma[b(\tilde{s} - \tilde{D}) + c\tilde{N}], \quad (6)$$

where

$$\gamma = \frac{b\sigma_D^2}{b^2\sigma_s^2 + c^2\sigma_N^2}. \quad (7)$$

The unobserved error in the informed investors' signal, \tilde{S} , and of the noise traders' effect, add their components to the expectation. The variance of the uninformed investors' prediction error (risk premium) is given as:

$$\sigma_U^2 = \frac{(b^2\sigma_\varepsilon^2 + c^2\sigma_N^2)\sigma_D^2}{b^2\sigma_s^2 + c^2\sigma_N^2}. \quad (8)$$

Under the CARA assumption, equation (8) defines the uninformed investor risk.

3. Financial Market Equilibrium

In order to examine the market demand for the risky asset, consider the expected utility of the informed investor:

$$u(c) = -e^{-x_I E[\tilde{D}|\tilde{s}] - a_0(1+r) + [x_I - a_I]P(1+r) + \frac{x_I^2 \sigma_I^2}{2}}, \quad (9)$$

where x_I is the informed investor demand of the risky asset. Taking the first-order maximum condition and solving leads to:

$$x_I = \frac{E[\tilde{D}|\tilde{s}] - P(1+r)}{\sigma_I^2}, \quad (10)$$

where the numerator depicts the gain for trading, and the denominator is the risk premium. The expected utility of an uninformed investor is given as:

$$u(c) = -e^{-x_I E[\tilde{D}|P] - a_0(1+r) + [x_U - a_U]P(1+r) + \frac{x_U^2 \sigma_U^2}{2}}, \quad (11)$$

which yields:

$$x_U = \frac{E[\tilde{D}|P] - P(1+r)}{\sigma_U^2} \quad (12)$$

for the uninformed investor's demand of the risky asset. Market supply of the risky asset is given in equation (1), and the market clearing condition for the risky asset is given as:

$$\mu x_I + (1-\mu)x_U + \tilde{N} = A. \quad (13)$$

Using equations (10) and (12), and recalling equations (2), (5) and (6), leads to:

$$\mu \left[\frac{\tilde{D} + \beta(\tilde{s} - \tilde{D}) - (z + b\tilde{s} + c\tilde{N})(1+r)}{\sigma_I^2} \right] + (1-\mu) \left[\frac{\tilde{D} + \gamma[b(\tilde{s} - \tilde{D}) + c\tilde{N}] - (z + b\tilde{s} + c\tilde{N})(1+r)}{\sigma_U^2} \right] + \tilde{N} = A. \quad (14)$$

The left-hand side of equation (14) includes a constant, and terms involving \tilde{S} and \tilde{N} , whereas the right-hand side expresses the constant supply of the risky asset. In equilibrium, the constant term on the left-hand side must equal the right-hand side, and the terms including the coefficients of \tilde{S} and \tilde{N} must be zero. The unknown parameters z , b and c in equation (5) can then be solved as:

$$z = -\frac{\sigma_I^2 \sigma_U^2}{[\mu \sigma_U^2 + (1-\mu)\sigma_I^2](1+r)} A, \quad (15)$$

$$b = \frac{\mu \beta \sigma_U^2}{[\mu \sigma_U^2 + (1-\mu)\sigma_I^2](1+r) - \gamma(1-\mu)\sigma_I^2}, \quad (16)$$

$$c = \frac{\sigma_I^2 \sigma_U^2}{[\mu \sigma_U^2 + (1-\mu)\sigma_I^2](1+r) - \gamma(1-\mu)\sigma_I^2}. \quad (17)$$

Finally, substitute equations (15), (16) and (17) into equation (5), which leads to:

$$P = -\frac{\sigma_I^2 \sigma_U^2}{[\mu \sigma_U^2 + (1-\mu) \sigma_I^2](1+r)} A + \frac{\mu \beta \sigma_U^2}{[\mu \sigma_U^2 + (1-\mu) \sigma_I^2](1+r) - (1-\mu) \gamma \sigma_I^2} \tilde{s} + \frac{\sigma_I^2 \sigma_U^2}{[\mu \sigma_U^2 + (1-\mu) \sigma_I^2](1+r) - (1-\mu) \gamma \sigma_I^2} \tilde{N}. \quad (18)$$

Equation (18) presents the composition of the market price of the risky asset in the linear model framework.

4. Central Bank Intervention

Assume now that the central bank can observe the development of bubbles in financial markets, and is able to make appropriate interventions.

This leads to the following proposition.

Proposition 1: *If the central bank suspects a bubble in financial markets and wants to deflate the bubble in the long run, it should lift the real risk-free rate.*

Proof: Following Milgrom and Stokey (1982), set $A = 0$ in equation (18), such that, in the Walrasian equilibrium, all investors are content with their current holdings. It follows that:

$$P = \frac{\mu \beta \sigma_U^2 \tilde{s} + \sigma_I^2 \sigma_U^2 \tilde{N}}{[\mu \sigma_U^2 + (1-\mu) \sigma_I^2](1+r) - (1-\mu) \gamma \sigma_I^2}. \quad (19)$$

As P is supposedly positive, it follows that the denominator is also positive. Rewriting the denominator as:

$$\mu(1+r)\sigma_U^2 + (1-\mu)(1+r-\gamma)\sigma_I^2,$$

It follows that:

$$(1+r) > \gamma. \quad (20)$$

Concentrating on the slope of the linear model specification, taking the partial derivative of P against r in equation (19) leads to:

$$\frac{\partial P}{\partial r} = -\frac{\sigma_U^2(\mu\beta\tilde{s} + \sigma_I^2\tilde{N})[\mu\sigma_U^2 + (1-\mu)\sigma_I^2]}{[(\mu\sigma_U^2 + (1-\mu)\sigma_I^2)(1+r) - (1-\mu)\gamma\sigma_I^2]^2} < 0. \quad (21)$$

The sign is negative, such that that increasing the risk-free rate lowers the market price of the risky asset, which deflates the bubble. Therefore, the “leaning against the wind” policy works. ***QED***

The previous result leads to the following corollary.

Corollary: *The increase in the real risk-free rate affects the risk of rational informed and uninformed investors.*

Proof: Recall definitions (4) and (8) and the CARA assumption, and use equation (19) to solve for the informed investor risk, σ_I^2 :

$$\sigma_I^2 = \frac{\mu\sigma_U^2[\beta\tilde{s} - (1+r)P]}{(1-\mu)(1+r-\gamma)P - \sigma_U^2\tilde{N}}. \quad (22)$$

Likewise, solve the uninformed investor risk, σ_U^2 , to obtain:

$$\sigma_U^2 = \frac{(1-\mu)(1+r-\gamma)\mu\sigma_I^2P}{\mu[\beta\tilde{s} - (1+r)P] + \sigma_I^2\tilde{N}}. \quad (23)$$

As the values of σ_I^2 and σ_U^2 in equations (22) and (23), respectively, must be positive, the following conditions must hold:

$$(1 - \mu)(1 + r - \gamma)P > \sigma_U^2 \tilde{N}, \quad (23)$$

$$\beta \tilde{s} > (1 + r)P. \quad (24)$$

By condition (20), condition (24) implies that:

$$\beta \tilde{s} > \gamma P. \quad (25)$$

Taking partial derivatives from equations (21) and (22) against r , leads to the following:

$$\frac{\partial \sigma_I^2}{\partial r} = \frac{\sigma_U^2 \tilde{N} - (1 - \mu)(\beta \tilde{s} - \gamma P)}{[(1 - \mu)(1 + r - \gamma)P - \sigma_U^2 \tilde{N}]^2} \mu \sigma_U^2 P < 0, \quad (26)$$

$$\frac{\partial \sigma_U^2}{\partial r} = \frac{\sigma_I^2 \tilde{N} + \mu(\beta \tilde{s} - \gamma P)}{[\mu(1 + r)P - \mu\beta \tilde{s} - \sigma_I^2 \tilde{N}]^2} (1 - \mu) \sigma_I^2 P > 0. \quad (27)$$

The signs in equations (26) and (27), as confirmed in conditions (23)-(25), make it clear that the rise in the real risk-free rate decreases the risk of the informed investor, and increases the risk of the uninformed investor. *QED*

The previous result leads to the following proposition.

Proposition 2: *Manipulation of the nominal risk-free rate is less effective, as it causes inflation in terms of consumption.*

Proof: Suppose that the Fisherian arbitrage condition holds:

$$(1 + r_t^n) = (1 + r_t^r) E_t(1 + \pi_{t+1}).$$

Define the gross real risk-free yield as:

$$1 + r \equiv \frac{1 + r^n}{E_t(1 + \pi_{t+1})}, \quad (28)$$

where r^n is the nominal rate and $E_t(1 + \pi_{t+1})$ denotes the inflationary expectation of the price of one unit of consumption in period 2. Using (28) in equation (19), and taking partial derivatives against r^n , leads to:

$$\frac{\partial P}{\partial r^n} = \frac{1}{E_t(1 + \pi_{t+1})} \frac{\partial P}{\partial r}. \quad (29)$$

Therefore, under the Fisherian condition, manipulation of the nominal risk-free rate is less effective than manipulation of the real rate as inflationary expectations deflate the effect. ***QED***

The previous result leads to the following proposition.

Proposition 3: *The bubble in financial markets can also be deflated by controlling inflation.*

Proof: Using definition (28) in equation (19), and taking partial derivatives against inflationary expectations, leads to:

$$\frac{\partial P}{\partial E_t(\pi_{t+1})} = \frac{\sigma_U^2 (\mu\beta\tilde{s} + \sigma_I^2 \tilde{N}) [\mu\sigma_U^2 + (1-\mu)\sigma_I^2]}{\left\{ [\mu\sigma_U^2 + (1-\mu)\sigma_I^2] \frac{1+r^n}{E_t(1+\pi_{t+1})} - (1-\mu)\gamma\sigma_I^2 \right\}^2} \frac{1+r^n}{E_t(1+\pi_{t+1})^2} > 0. \quad (30)$$

The last equation states that controlling consumption price inflation, $\partial E_t(\pi_{t+1}) < 0$, also deflates the price in financial markets. ***QED***

The previous result leads to the following proposition.

Corollary: *A change in inflationary expectations may affect the risk of rational investors.*

Proof: Using the Fisherian condition (28) in equation(19), and solving for σ_I^2 and σ_U^2 , leads to the following results:

$$\sigma_I^2 = \frac{\mu\sigma_U^2 \left[\beta\tilde{s} - \frac{1+r^n}{E_t(1+\pi_{t+1})} P \right]}{(1-\mu) \left(\frac{1+r^n}{E_t(1+\pi_{t+1})} - \gamma \right) P - \sigma_U^2 \tilde{N}}, \quad (31)$$

$$\sigma_U^2 = \frac{(1-\mu) \left(\frac{1+r^n}{E_t(1+\pi_{t+1})} - \gamma \right) \mu\sigma_I^2 P}{\mu \left[\beta\tilde{s} - \frac{1+r^n}{E_t(1+\pi_{t+1})} P \right] + \sigma_I^2 \tilde{N}}. \quad (32)$$

According to equations (31) and (32), the following conditions must hold:

$$\frac{1+r^n}{E_t(1+\pi_{t+1})} > \gamma, \quad (20')$$

$$\beta\tilde{s} > \frac{1+r^n}{E_t(1+\pi_{t+1})} P, \quad (24')$$

which imply that $\beta\tilde{s} > \gamma P$ must also hold. Differentiate equations (31) and (32) partially against

$E_t(\pi_{t+1})$, recall (30), and denote $\frac{\partial P}{\partial E_t(\pi_{t+1})} = P'$. After manipulation, the respective effects can

be given as:

$$\frac{\partial \sigma_U^2}{\partial E_t(\pi_{t+1})} = \mu \sigma_U^2 \frac{(P' - \frac{1}{E_t(1+\pi_{t+1})}P) \frac{(1+r^n)\sigma_U^2 \tilde{N}}{E_t(1+\pi_{t+1})} + (1-\mu) \left[\frac{(1+r^n)P(\beta\tilde{s} - \gamma P)}{E_t(1+\pi_{t+1})^2} - (\frac{1+r^n}{E_t(1+\pi_{t+1})} - \gamma)\beta\tilde{s}P' \right]}{\left\{ (1-\mu) \left(\frac{1+r^n}{E_t(1+\pi_{t+1})} - \gamma \right) P - \sigma_U^2 \tilde{N} \right\}^2}, \quad (33)$$

$$\frac{\partial \sigma_U^2}{\partial E_t(\pi_{t+1})} = \mu(1-\mu)\sigma_U^2 \frac{\frac{1+r^n}{E_t(1+\pi_{t+1})} (P' - \frac{1}{E_t(1+\pi_{t+1})}P) \left[\mu(\beta\tilde{s} - \gamma P) + \sigma_U^2 \tilde{N} \right]}{\left\{ (1-\mu) \left(\frac{1+r^n}{E_t(1+\pi_{t+1})} - \gamma \right) P - \sigma_U^2 \tilde{N} \right\}^2}. \quad (34)$$

The effect in equation (33) remains ambiguous, as the two terms in the square brackets in the nominator are of opposite signs. Therefore, the effect of a change in inflationary expectations cannot be determined unambiguously for informed investors. However, the sign of equation (34) is more clear, which is reasonable as the uninformed investors monitor the market price of the risky asset. The sign equals zero if:

$$P' = \frac{1}{E_t(1+\pi_{t+1})} P,$$

that is, if the effect of the change in inflationary expectations, P' , fully deflates the asset's market price, P . This holds in the Fisherian framework. However, if there is money illusion that prevails in the consumption and financial markets, the effect of inflationary expectations on the asset price remains weaker, and it follows that:

$$P' < \frac{1}{E_t(1+\pi_{t+1})} P.$$

Then the sign of equation (34) is unambiguously negative, which states that an increase in inflationary expectations decreases the uninformed investor risk, and vice-versa. *QED*

5. Concluding Remarks

The paper analyzed the effects of central bank interventions in Walrasian financial markets. The financial market considered consists of rational but asymmetrically-informed investors and noise traders, between whom the economy's risky and risk-free assets are initially allocated. Investors make their investment decisions in period 1, and consume the real yields of their risky and risk-free investments in period 2. The objective of the central bank is to defend the market against bubbles.

The analysis provided several insightful findings:

- (i) If the central banks observes a bubble in the market, it can deflate it by elevating the real risk-free rate, which confirms the “leaning against the wind” argument.
- (ii) Lifting the real risk-free rate decreases the risk of rational informed investors, who receive a private signal of the dividends, and increases the risk of rational uninformed investors, who infer the dividends from the market price. More generally, this indicates that higher risk-free rates encourage informed investors and discourage uninformed investors to make risky investments.
- (iii) If the central bank tackles the bubble by lifting the nominal risk-free rate, inflationary expectations dampen the effect. Inversely, this implies that bubbles can also be treated by controlling inflation through chaining inflationary expectations.
- (iv) A reduction in inflationary expectations also has effects on perceived risk.

The sign of the effect remains ambiguous for informed investors, but it is clear for uninformed investors. If the Fisherian arbitrage condition holds, inflationary expectations have no effect at all. However, if the uninformed investors have money illusion, a reduction in inflationary expectations unambiguously increases their risk, and vice-versa.

In the macroeconomic literature, the persistent combination of low risk-free rates, low inflation and ballooning stock markets has lately attracted considerable attention. The analysis in the paper has provided some insights into this puzzle. Reducing the risk-free rate encourages the uninformed investor to undertake risky investments and discourages uninformed investors, who base their

decisions more closely on the fair value of the asset. As $(1-\mu) > \mu$ plausibly holds, the central bank should pay special attention to the mass of uninformed investors.

More importantly, in a Fisherian world, the uninformed investor risk does not depend on inflationary expectations, but money illusion would mean that inflation expectations and risk behave inversely. In short, increased inflationary expectations would cause more risky investments. Therefore, in the presence of low risk-free interest rates, control of inflation may be necessary to curtail uninformed investor risk in order to tackle the bubble.

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