Isotropic charged cosmologies in infrared-modified electrodynamics

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It has long been known that the covariant formulation of quantum electrodynamics conflicts with the local description of states in the charged sector. Some of the solutions to this problem amount to modifications of the subsidiary conditions below some arbitrarily low photon frequency. Such infrared-modified theories have been shown to lead to Maxwell equations modified with an additional classical electromagnetic current induced by the quantum charges. The induced current only has support for very small frequencies and cancels the effects of the physical charges on large scales. In this work, we explore the possibility that this deelectrification effect could allow for the existence of isotropic charged cosmologies, thus evading the stringent limits on the electric charge asymmetry of the Universe. We consider a simple model of infrared-modified scalar electrodynamics in the cosmological context and find that the charged sector generates a new contribution to the energy-momentum tensor of which the dominant contribution at late times is a cosmological constantlike term. If the charge asymmetry was generated during inflation, the limits on the asymmetry parameter in this model in order not to produce a too-large cosmological constant are very stringent $\eta_Q < 10^{-131}$–10^{-144} for a number of e-folds $N = 50$–60 in typical models. However, if the charge imbalance is produced after inflation, the limits are relaxed in such a way that $\eta_Q < 10^{-43}(100 \text{ GeV}/T_Q)$, with T_Q the temperature at which the asymmetry was generated. If the charge asymmetry has ever existed and the associated electromagnetic fields vanish in the asymptotic future, the limit can be further reduced to $\eta_Q < 10^{-28}$.

I. INTRODUCTION

One of the long-standing questions in cosmology, dating back to the works of Bondi and Lyttleton in the late 1950s [1], is the possibility that the Universe could have a net electric charge density. It soon became apparent that this kind of charged cosmologies, even respecting large-scale homogeneity, are necessarily anisotropic. Indeed, it is well established in the context of Maxwell electrodynamics that the presence of a nonvanishing charge density ρ generates electric fields such that $\nabla \cdot \mathbf{E} = \rho$. Even if we assume that both the electric field and the charge density are spatially uniform within our Hubble horizon, the corresponding electromagnetic energy-momentum tensor is nonetheless anisotropic. This implies a departure from the Robertson-Walker geometry, which can conflict with the observed isotropy of different backgrounds. Thus, introducing the charge asymmetry parameter as $\eta_Q = n_Q/n_\gamma$, where the charge density is $|e|n_Q$, it has been shown that the isotropy of the cosmic microwave background imposes a limit $\eta_Q \lesssim 10^{-30}$, whereas the isotropy of the observed cosmic ray distribution sets $\eta_Q \lesssim 10^{-39}$ [2]. On the other hand, the electromagnetic interaction with the electrostatic potential generated by the net charge density induces an effective mass shift for any charged particle present in the cosmic plasma. These shifts introduce changes in the nucleosynthesis mechanism, which can be translated into very stringent limits on the charge asymmetry $\eta_Q \lesssim 10^{-43}$ [3]. The mentioned constrains were obtained ignoring the high conductivity of the cosmic plasma. When conductivity effects are taken into account, the limits can be improved, setting $\eta_Q \lesssim 10^{-35}$ for the cosmic microwave background case [4].

On the theory side, several models have been proposed to generate a cosmological charge imbalance. In Refs. [5–7],

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spontaneous breaking of gauge invariance was considered, the symmetry being restored at late times in order to comply with present experimental results. Other mechanisms involve the generation of charge by the presence of a photon mass [8] or prior to the grand unified theory phase transition in braneworld scenarios. Quantum fluctuations of charged fields during inflation have also been considered in Refs. [9–11] to generate charge fluctuations on super-Hubble scales.

Even though the possibility of having a charge asymmetry compatible with current observations seems to be very limited in the context of Maxwell electrodynamics, this is not the case in modified electromagnetic theories. Thus, Barnes [12] realized for the first time that the Proca generalization of electrodynamics, which propagates an additional longitudinal polarization for the photon, admits the possibility that the Universe could possess a net electric charge density uniformly distributed throughout space, while possessing no electric or magnetic fields, thus allowing for homogeneous and isotropic Robertson-Walker solutions and evading most of the aforementioned limits. In particular, for constant photon mass and assuming charge conservation, the electromagnetic energy-momentum tensor in this theory behaves, for homogeneous fields, as that of a perfect fluid with equation of state $p_P = \rho_P$ (see Ref. [13]) so that the energy density in a charge-dominated universe would scale as a^{-6}. However, this scaling suggests that in order to avoid large contribution in the early Universe, the potential contribution of charge in the present Universe should be tiny. Extensions of these ideas in which the photon mass can depend on time were considered in Refs. [14,15], in which a model for cosmic acceleration was proposed.

In this work, we reexamine the cosmology of a charged universe in Maxwell electrodynamics (with two propagating physical modes) from a different perspective. The well-known local and covariant formulation of quantum electrodynamics contains two fundamental ingredients: on one hand, the dynamics, which is provided by Maxwell equations, and, on the other hand, the constraints, which allow one to eliminate the unphysical degrees of freedom and are given in the canonical formalism by the Gupta-Bleuler [16,17] subsidiary conditions,

$$(\nabla_\mu A^\mu)^{(-)}|\Phi\rangle = 0, \tag{1}$$

where $(\nabla_\mu A^\mu)^{(-)}$ is the negative frequency part of the operator $\nabla_\mu A^\mu$ and $|\Phi\rangle$ denotes a physical state. However, this formulation of quantum electrodynamics present certain difficulties in the charged sector which have been known since the 1970s [18,19]. In particular, Maison and Zwanziger [20] proved a general result that states that there is no localized charged state in covariant QED which satisfies the above subsidiary conditions. In other words, either we abandon locality in the description of the charged

states or, if we insist on a local description of charges, we must assume that all charged particles are produced from the decay of neutral states.

A possible way out of this limitation of covariant QED is the modification of the subsidiary condition in the infrared. An explicit implementation of these ideas is presented by Zwanziger in Ref. [21] (see also Ref. [22,23]) and amounts to the introduction of an additional classical conserved current which is generated by the quantum current. In momentum space, this current has only support in the infrared, i.e., below the cutoff frequency, and cancels the effects of the quantum charges on very large scales. This property suggests that the cosmology of charged universes could exhibit important differences in this kind of modified electrodynamics, and in particular, this opens up the possibility of having isotropic charged solutions without including additional polarizations for the photon field.

Even though the Zwanziger model [21] is relatively old, its cosmological implications have not been analyzed so far. The aim of the present work is precisely to evaluate the cosmological viability of that model in the context of charged cosmologies and dark energy models. We will find that the use of the modified formalism generates new terms in the electromagnetic energy-momentum tensor which are not present in standard QED and of which the dominant contribution at late times is a cosmological constant term. By imposing such terms to be compatible with current observations, we will set upper limits on the charge asymmetry of the Universe in this scenario.

The paper is organized as follows. In Sec. II, we review the infrared problems of covariant QED and their implications in the definition of charged states. In Sec. III, we present the Zwanziger subsidiary conditions and obtain the consistency condition for the classical current. In Sec. IV, we derive the equations of motion and energy-momentum tensors for the different components. Section V is devoted to the energy density and pressure of the scalar field. In Sec. VI, we calculate the induced electromagnetic energy density when different boundary conditions are imposed on the classical b field and obtain the limits on the charge asymmetry. Finally, in Sec. VII, we present the main conclusions of the work.

II. INFRARED PROBLEM OF PERTURBATIVE QED

Let us start by reviewing the long-standing infrared problem of QED and its connection with the definition of charged particles.

In the standard description of scattering processes in perturbative field theory, the assumption is made that in the initial and final asymptotic regions the interactions can be switched off so that (in Minkowski space-time) the fields can be expanded in plane-wave solutions with the corresponding creation and annihilation free operators. In the interaction picture, these free field solutions can then be

used to construct the corresponding interacting solution using the Green function for the interaction term. Even though this method can be straightforwardly applied in some toy models, in the case of unbroken gauge theories such as QCD or QED, it exhibits important difficulties. Thus, in the QCD case, low-energy confinement prevents the definition of asymptotically free quark states. In the case of QED, the difficulty is less obvious since electrons are not confined; however, Kulish and Fadeev [18] showed that the masslessness of the photon implies that the electromagnetic interaction does not decay sufficiently fast at long distances so as to neglect it in the asymptotic regions. This residual interaction implies that asymptotic charged fields can no longer be described as plane waves but they appear "dressed" by an electromagnetic field. This in practice prevents the definition of local charged states in covariant QED.

Let us then review in detail how the problem arises in the standard manifestly covariant formulation of QED in the Lorentz gauge. Here, we will closely follow the analysis in Ref. [20].

The equations of motion in Minkowski space-time read [24]

$$\partial_\mu F^{\mu\nu} - \partial^\nu(\partial_\mu A^\mu) = J^\nu, \tag{2}$$

where J^ν is the conserved current. In order to recover the classical Maxwell equation, the Lorentz condition $\partial_\mu A^\mu = 0$ should be imposed. As is well known [24], this cannot be done at the operator level but only in the weak sense through the Gupta-Bleuler subsidiary conditions. These conditions define the physical Fock space of the theory and are given by

$$(\partial_\mu A^\mu)^{(-)}|\Phi\rangle = 0, \tag{3}$$

where $(\partial_\mu A^\mu)^{(-)}$ is the negative frequency part of the operator $\partial_\mu A^\mu$ and $|\Phi\rangle$ denotes a physical state.

Rewriting Eq. (2) as

$$\Box A_\mu = J_\mu \tag{4}$$

and decomposing the external current as $J_\mu(x) = J_{\mu+}(x) + J_{\mu-}(x)$ with $J_{\mu\pm} = \theta(\pm x^0)J_\mu(x)$, the general interacting solution $A_\mu(x)$ can be written in terms of the free solutions $A_\mu^f(x)$ satisfying $\Box A_\mu^f = 0$ as

$$A_\mu(x) = A_\mu^f(x) + \int \Delta^{\text{ret}}(x-y)J_{\mu+}(y)d^4y$$
$$+ \int \Delta^{\text{adv}}(x-y)J_{\mu-}(y)d^4y, \tag{5}$$

where the retarded and advance propagators are

$$\Delta^{\text{ret}}(x) = \frac{1}{(2\pi)^2}\delta(x^2)\delta(x^0)$$

$$\Delta^{\text{adv}}(x) = \frac{1}{(2\pi)^2}\delta(x^2)\delta(-x^0) \tag{6}$$

and the free field can be expanded in plane-wave solutions as

$$A_\mu^f(x) = \frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega}(a_\mu(\vec{k})e^{-ikx} + a_\mu^\dagger(\vec{k})e^{ikx}), \tag{7}$$

where $\omega = |\vec{k}|$ and a_μ, $a_\mu^\dagger$ denote the *free* annihilation and creation operators satisfying

$$[a_\mu(\vec{k}), a_\nu^\dagger(\vec{k}')] = -\eta_{\mu\nu}2\omega\delta^3(\vec{k}-\vec{k}'). \tag{8}$$

From (5), we can write

$$\partial_\mu A^\mu(x) = \partial_\mu A_f^\mu(x) + \int \Delta(x-y)J^0(y)\delta(y^0)d^4y \tag{9}$$

with

$$\Delta(x-y) = \Delta^{\text{ret}}(x-y) - \Delta^{\text{adv}}(x-y)$$
$$= \frac{i}{(2\pi)^3}\int \frac{d^3k}{2\omega}(e^{-ikx} + e^{ikx}). \tag{10}$$

Thus, substituting back (9) in (3), we get the Gupta-Bleuler subsidiary conditions in Fourier space,

$$(\omega(a_0(\vec{k}) - a_\parallel(\vec{k})) - \rho(\vec{k}))|\Phi\rangle = 0, \tag{11}$$

where $a_0(\vec{k})$ and $a_\parallel(\vec{k}) = \frac{k^i}{\omega}a_i(\vec{k})$ denote the temporal and longitudinal annihilation operators of the free fields, respectively, and $\rho(\vec{k})$ is the charge density operator in Fourier space, i.e.,

$$\rho(\vec{k}) = \frac{1}{(2\pi)^{3/2}}\int e^{-i\vec{k}\vec{x}}J^0(0,\vec{x})d^3x. \tag{12}$$

If $\rho(\vec{k})$ is a smooth function, then

$$\rho(0) = \frac{1}{(2\pi)^{3/2}}\int J^0(0,\vec{x})d^3x = \frac{Q}{(2\pi)^{3/2}} \tag{13}$$

with Q the total charge. It is now possible to obtain solutions of the subsidiary condition (11) for the physical states $|\Phi\rangle$ in the form

$$|\Phi\rangle = \exp\left(-\frac{1}{2}\int(a_0^\dagger(\vec{k}) + a_\parallel^\dagger(\vec{k}))\frac{\rho(\vec{k})}{\omega}\frac{d^3k}{2\omega}\right)|\Psi\rangle \tag{14}$$

with

$$|\Psi\rangle = \mathcal{F}[a_0^\dagger(\vec{k}) - a_\parallel^\dagger(\vec{k}), a_\perp^\dagger(\vec{k})]|0\rangle, \qquad (15)$$

where $a_\perp^\dagger(\vec{k})$ denotes the two transverse creation operators and $\mathcal{F}$ is an analytical function. Thus, the norm squared of a physical state is given by

$$\langle\Phi|\Phi\rangle = \exp\left(\int \frac{|\rho(\vec{k})|^2}{\omega^2} \frac{d^3k}{2\omega}\right). \qquad (16)$$

Notice that, since $\rho(\vec{k})$ is a smooth function with $\rho(0) = Q/(2\pi)^{3/2}$, the above integral is infrared divergent for $Q \neq 0$. This means that the Gupta-Blueler condition has no Fock space solution with finite norm in the charged sector [20].

One of the possible solutions to this problem is the modification of the subsidiary condition in the infrared. In particular, it can be seen [20] that a modified condition given by

$$(\omega(a_0(\vec{k}) - a_\parallel(\vec{k})) - \rho(\vec{k}) + Qf_c(\omega))|\Phi\rangle = 0, \qquad (17)$$

where Q is the charge operator and $f_c(\omega)$ is a cutoff function such that $f_c(0) = (2\pi)^{-3/2}$ and $f_c(\omega) = 0$ for $\omega > \omega_0$, defines a nonempty Fock space of physical states. Indeed, the new term can be seen as a classical current which screens the effect of the quantum charges on large scales, thus allowing for finite norm states in (16) and in this way avoids the infrared problem. Notice that this expression implies that the subsidiary conditions are only modified below an arbitrarily low frequency ω_0, so standard QED is recovered on small scales. In next section, we will describe in detail the implementation of this modified electrodynamics in the Zwanziger model [21].

III. ZWANZIGER SUBSIDIARY CONDITIONS

Let us consider a simple renormalizable scalar electrodynamics theory minimally coupled to gravity. The Lagrangian density of this model can be written as [21]

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\nabla_\mu A^\mu)^2 + (D_\mu\varphi)^*(D^\mu\varphi) - V(|\varphi|), \qquad (18)$$

where $D_\mu = \partial_\mu + iqA_\mu$ and q is the $U(1)$ charge of the scalar field. The fundamental fields appearing here are A_μ, φ, and $g_{\mu\nu}$, and the action is

$$S[A_\mu, \varphi, g_{\mu\nu}] = \int \sqrt{\mathfrak{g}}\mathcal{L}d^4x, \qquad (19)$$

with $\mathfrak{g} \equiv |\det(g_{\mu\nu})|$.

The corresponding equations of motion are

$$\nabla_\alpha F^{\alpha\mu} + \lambda\nabla^\mu(\nabla_\alpha A^\alpha) = J^\mu \qquad (20)$$

and

$$D_\mu(\sqrt{\mathfrak{g}}D^\mu\varphi) + \frac{1}{2}\sqrt{\mathfrak{g}}\frac{V'(|\varphi|)}{|\varphi|} = 0, \qquad (21)$$

where

$$J^\mu = iq[\varphi^*(D^\mu\varphi) - \varphi(D^\mu\varphi)^*] \qquad (22)$$

is the $U(1)$ conserved current, i.e., such that $\nabla_\mu J^\mu = 0$. We will see later how current conservation can be derived from (21), as one would expect. Notice that, although the action in (20) is invariant under the restricted gauge transformations, $A_\mu \to A_\mu + \partial_\mu\Lambda$ with transformation parameter satisfying $\Box\Lambda = 0$, the scalar sector is fully gauge invariant, so current conservation is preserved.

Taking the divergence of (20) and taking into account the conservation of the current, we get

$$\Box(\nabla_\alpha A^\alpha) = 0. \qquad (23)$$

Following Ref. [21], we define the physical states $|\Phi\rangle$ as those given by the following modification of the Gupta-Bleuler condition,

$$(\nabla_\alpha A^\alpha)^{(-)}(x)|\Phi\rangle = b^{(-)}(x)|\Phi\rangle, \qquad (24)$$

where $b(x)$ is a real c-number solution of the wave equation

$$\Box b(x) = 0, \qquad (25)$$

which can be separated into its positive and negative frequency parts as $b(x) = b^{(+)}(x) + b^{(-)}(x)$. In Fourier space $b(x)$ would generate the cutoff function $f_c(\omega)$ in (17).

Then, it can be shown that if $b(x)$ satisfies

$$\int \sqrt{g_\Sigma}\partial_\mu b(x)d\Sigma^\mu = \frac{q}{\lambda} \qquad (26)$$

with Σ a constant-time hypersurface, g_Σ the metric on Σ, $d\Sigma^\mu$ the future-oriented volume element on Σ, and q the charge of the state $|\Phi\rangle$ then $\langle\Phi|\Phi\rangle \geq 0$; i.e., states satisfying (24) have non-negative norm, and a such subspace is invariant under the action of observables $\mathcal{O}$, i.e., $\langle\mathcal{O}\Phi|\mathcal{O}\Phi\rangle \geq 0$. Notice that, unlike in Ref. [21], we are working with an arbitrary λ.

These conditions imply that for the expectation value we get

$$\langle\Phi|(\nabla_\alpha A^\alpha)(x)|\Phi\rangle = b(x), \qquad (27)$$

so the classical Maxwell equations are modified with the introduction of an additional classical current and can be written as [21]

$$\nabla_\alpha F^{\alpha\mu} = J^\mu - \lambda \nabla^\mu b. \tag{28}$$

Notice that, even though the subsidiary conditions are modified, the gauge invariance of the scalar sector preserves the Ward identities of the theory, so we do not expect any uncompensated production of temporal and longitudinal photons in the theory. As a matter of fact, as shown [21], all the cross section formulas of standard QED are recovered in the modified formalism.

IV. CHARGED COSMOLOGIES

We will apply the previous formalism in cosmology for the description of a homogeneous and isotropic universe with a uniform charge density. With that purpose, we consider a spatially flat Robertson-Walker metric,

$$ds^2 = dt^2 - a^2 d\vec{x}^2, \tag{29}$$

where $a \equiv a(t)$ is the scale factor and hence $\sqrt{\mathfrak{g}} = a^3$.

We would like to find nontrivial solutions in which the matter fields (scalar and electromagnetic fields) are also homogeneous and isotropic. With that purpose, we will look for solutions of the classical equation of motion where the vector fields cannot point in any spatial direction, so that $A_i = 0$. Then, the only remaining component of the field is A_0. By homogeneity, spatial derivatives of any field must vanish, which gives us $\partial_i A_\mu = 0$ and $\partial_i \varphi = 0$. With these conditions, current conservation reads

$$a^3(t) J_0(t) = \kappa, \tag{30}$$

with κ being the constant comoving charge density of the scalar field.

For the spatial hypersurface of constant t, we have $\sqrt{g_\Sigma} = a^3(t)$ and $d\Sigma^\mu = d^3x(1, 0, 0, 0)$, so the consistency condition (26) reads

$$\int_V a^3 \partial_0 b(x) d^3x = \frac{1}{\lambda} \int_V a^3 J_0 d^3x \tag{31}$$

with V a given comoving volume. In the cosmological context, it is natural to impose that b is a homogeneous field, i.e., $b = b(t)$, which means that in Fourier space $b(k)$ only has contribution from the zero mode; i.e., the corresponding cutoff frequency would be essentially $k_0 = 0$. In other words, the modification of the subsidiary condition would only affect the zero mode electromagnetic fields. For the rest of the states, the standard Gupta-Bleuler condition is recovered. Notice that, since the tip of the light cone $\omega_0 = 0$ is Lorentz invariant, we do not expect any modification in the subsidiary condition in a boosted frame.

If we further assume that the consistency condition is valid for an arbitrary cosmological volume V, then we finally obtain

$$\lambda \partial_0 b(t) = J_0, \tag{32}$$

which implies that the right-hand side of (28) vanishes; i.e., even though we have a net charge density, the presence of the new current cancels its effects on cosmological scales. This means a vanishing Faraday tensor on large scales, so it is possible to get exact homogeneous and isotropic Robertson-Walker solutions. This *deelectrification* of the electric current, which is decoupled from the electromagnetic fields, is different from the degravitation mechanism [25] of the cosmological constant from gravity. Although both cases resort to infrared modifications of the theory, in the gravitational case, it is the dynamics rather than the subsidiary conditions that is modified in order to absorb the vacuum energy contribution.

On the other hand, notice that by introducing (30) into the Maxwell equations (20) and taking into account the isotropy and homogeneity of A_μ we can write a φ-independent equation of motion for the classical electromagnetic field,

$$\lambda \partial_0 (\nabla_\mu A^\mu) = \kappa a^{-3}, \tag{33}$$

which is compatible with (32) and can be rewritten as

$$\lambda(\ddot{A}_0 + 3H\dot{A}_0 + 3\dot{H}A_0) = \kappa a^{-3}, \tag{34}$$

with $H \equiv \dot{a}/a$ being the Hubble parameter.

The scalar field can be written in terms of a modulus and a phase: $\varphi = fe^{i\theta}$. Introducing this expression in (21) gives, after splitting the resulting equation in its real and imaginary parts,

$$\ddot{f} + 3H\dot{f} - (\dot{\theta} + qA_0)^2 f + \frac{1}{2}V'(f) = 0 \tag{35}$$

and

$$\ddot{\theta}f + 2\dot{f}\dot{\theta} + 3H\dot{\theta}f + 2qA_0\dot{f} + q(\dot{A}_0 + 3HA_0)f = 0. \tag{36}$$

In order to simplify this expression, we write the zero component of the current density in terms of f and θ as

$$\kappa a^{-3} = J_0 = -2q(\dot{\theta} + qA_0)f^2, \tag{37}$$

in such a way that (35) becomes

$$\ddot{f} + 3H\dot{f} - \frac{\kappa^2}{4q^2 a^6 f^3} + \frac{1}{2}V'(f) = 0. \tag{38}$$

Now, we can write (36) in a different way as

$$\frac{1}{fa^3}\frac{d}{dt}[a^3 f^2(\dot\theta + qA_0)] = 0. \tag{39}$$

Thus, comparison with (37) shows us that this equation of motion is completely equivalent to current conservation, as was mentioned before.

The stress-energy tensor is obtained by the variation of the action with respect to the metric as

$$T^{\mu\nu} = -\frac{2}{\sqrt{\mathfrak{g}}}\frac{\delta S}{\delta g_{\mu\nu}}. \tag{40}$$

We get the complete stress-energy tensor as a sum of the contributions from the scalar and electromagnetic fields, $T^{\mu\nu} = T_\varphi^{\mu\nu} + T_A^{\mu\nu}$. For our model, we get its symmetrized components as

$$T_\varphi^{\mu\nu} = 2(D^{(\mu}\varphi)^*(D^{\nu)}\varphi) - g^{\mu\nu}((D_\alpha\varphi)^*(D^\alpha\varphi) - V(|\varphi|)), \tag{41}$$

and

$$T_A^{\mu\nu} = -F^{\mu\alpha}F^\nu{}_\alpha + 2\lambda A^{(\mu}\nabla^{\nu)}(\nabla_\alpha A^\alpha)$$
$$- g^{\mu\nu}\left[-\frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} + \frac{\lambda}{2}(\nabla_\alpha A^\alpha)^2 + \lambda A^\alpha\nabla_\alpha(\nabla_\beta A^\beta)\right]. \tag{42}$$

Although gauge covariant derivatives appear in (41), we see that in our case, using (37), $T_\varphi^{\mu\nu}$ can be written in terms of the modulus of the scalar field f only. As a matter of fact, the only nonvanishing components of (41) and (42) are the energy densities and pressures, obtained as $\rho_{(\alpha)} = T_{(\alpha)}{}^0{}_0$ and $p_{(\alpha)} = -T_{(\alpha)}{}^i{}_i$, where (α) stands for φ or A. Thus, we get

$$\rho_\varphi = \dot f^2 + \frac{\kappa^2}{4q^2 a^6 f^2} + V(f), \tag{43a}$$

$$p_\varphi = \dot f^2 + \frac{\kappa^2}{4q^2 a^6 f^2} - V(f), \tag{43b}$$

and

$$\rho_A = \kappa A_0 a^{-3} - \frac{\lambda}{2}(\nabla_\mu A^\mu)^2, \tag{44a}$$

$$p_A = \kappa A_0 a^{-3} + \frac{\lambda}{2}(\nabla_\mu A^\mu)^2, \tag{44b}$$

where we have made use of (34) to get the standard Coulomb interaction $\kappa A_0 a^{-3}$ term. Notice the difference with respect to the standard electromagnetic energy density in QED since, in addition to the Coulomb term, in the

Zwanziger model a new extra contribution which is proportional to b^2 according to (27) is present.

V. ENERGY AND PRESSURE OF THE SCALAR FIELD

In order to obtain the scaling behavior of the energy density of the scalar field, we will particularize the scalar field potential to the simplest case corresponding to a mass term,

$$V(f) = \frac{1}{2}m^2 f^2. \tag{45}$$

We will solve (38) numerically, so we rewrite it here in a dimensionless form. To do so, we define a dimensionless time as $\tau \equiv mt$ and define τ valued fields and parameters as the barred ones:

$$\bar f(\tau) \equiv \frac{q}{m}f(t),$$
$$\bar a(\tau) \equiv a(t),$$
$$\bar H(\tau) \equiv \frac{1}{\bar a(\tau)}\frac{d}{d\tau}\bar a(\tau). \tag{46}$$

In the following, we shall omit the τ dependence, which is obvious in barred quantities.

Now, the equation of motion (38) with the mass potential can be written as

$$\bar f'' + 3\bar H\bar f' - \frac{\bar\kappa^2}{\bar a^6\bar f^3} + \bar f = 0, \tag{47}$$

where the prime means τ derivative, and we have defined the dimensionless constant

$$\bar\kappa \equiv \frac{\kappa q}{2m^3}. \tag{48}$$

In order to simplify the numerical evaluation of (47), we define

$$\bar g \equiv \bar a^{3/2}\bar f \tag{49}$$

so that the equation of motion becomes

$$\bar g'' - \frac{\bar\kappa^2}{\bar g^3} + \left[1 - \frac{9\bar H^2}{4} - \frac{3\bar H'}{2}\right]\bar g = 0, \tag{50}$$

where the damping term $\bar g'$ does not appear. We will solve for different cosmological eras assuming $\bar a \propto \tau^n$ so that we get the final equation

$$\bar g'' - \frac{\bar\kappa^2}{\bar g^3} + \left(1 - \frac{3n(3n-2)}{4\tau^2}\right)\bar g = 0. \tag{51}$$
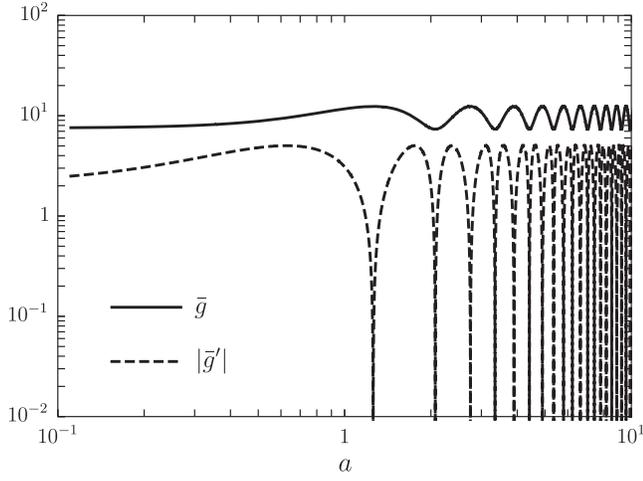
FIG. 1.   Evolution of the modified scalar field in terms of the scale factor.

Thus, for the radiation-dominated era with $n = 1/2$, we get

$$\bar{g}'' - \frac{\bar{\kappa}^2}{\bar{g}^3} + \left(1 + \frac{3}{16\tau^2}\right)\bar{g} = 0, \qquad (52\mathrm{a})$$

whereas for matter domination with $n = 2/3$, it reads

$$\bar{g}'' - \frac{\bar{\kappa}^2}{\bar{g}^3} + \bar{g} = 0. \qquad (52\mathrm{b})$$

Solving these equations numerically, we get, for a wide range of initial conditions, an oscillatory behavior for the field around a constant value at late times when $m \gg H$, as shown in Fig. 1, both for $\bar{g}$ and $\bar{g}'$. This translates into an oscillatory behavior for the field $\bar{f}$ with an amplitude that decays with the scale factor as $a^{-3/2}$.

If we introduce this behavior in the scalar field energy density (43a), with the mass potential (45), we will find that it decays at late time as

$$\rho_f \propto \frac{1}{a^3}, \qquad (53)$$

so the scalar field has a matterlike behavior as expected [26]. This means that, provided that initial conditions ensure that it is subdominant with respect to the total matter density, it will be a subdominant component at all times.

VI. ENERGY AND PRESSURE OF THE ELECTROMAGNETIC FIELD

A. Setting the background

In order to study analytically the behavior of both the energy density and pressure in (44), we should solve the equation of motion for $A_0(a)$, given by (34). Changing time derivatives to a-derivatives and integrating twice and once, respectively, from some initial value a_i, we can get, after some manipulations,

$$A_0(a) = \left(\frac{a_i}{a}\right)^3 A_0|_i + \frac{\nabla_\mu A^\mu|_i}{H_0 a^3}(\mathcal{F} - \mathcal{F}_i)$$
$$+ \frac{\kappa}{\lambda H_0^2 a^3}[\mathcal{G} - \mathcal{G}_i - \mathcal{I}_i(\mathcal{F} - \mathcal{F}_i)] \qquad (54\mathrm{a})$$

and

$$\nabla_\mu A^\mu = \nabla_\mu A^\mu|_i + \frac{\kappa}{\lambda H_0}(\mathcal{I} - \mathcal{I}_i), \qquad (54\mathrm{b})$$

where we have defined the primitive functions

$$\mathcal{F} \equiv \int \frac{a'^2}{E(a')}\,da' \qquad (55\mathrm{a})$$

$$\mathcal{G} \equiv \int \frac{a'^2}{E(a')}\,da' \int \frac{da''}{a''^4 E(a'')} \qquad (55\mathrm{b})$$

$$\mathcal{I} \equiv \int \frac{da'}{a'^4 E(a')} \qquad (55\mathrm{c})$$

and $H(a) \equiv H_0 E(a)$. These primitives are evaluated in a except if they have some subscript that indicates a particular constant scale factor.

In order to obtain explicit expressions for the integrals, we will consider the standard cosmological behavior with an initial inflationary phase followed by a reheating phase connecting with the radiation era of standard Λ cold dark matter (ΛCDM) cosmology. We will assume for simplicity that the energy density scales as matter during the reheating phase [27]. This background scheme is depicted in Fig. 2.

We consider a quasi–de Sitter inflationary phase with almost constant Hubble parameter H_I, which can be estimated from the Friedmann equation as

$$H_I = \sqrt{\frac{V_I}{3M_p^2}}, \qquad (56)$$

where $M_p^2 = 1/(8\pi G)$ and $V_I^{1/4}$ is the scale of inflation. On the other hand, we have $a_b = e^{-\mathcal{N}} a_e$, where $\mathcal{N}$ is the total number of inflation e-folds. In addition, since we are assuming the reheating era to be matter dominated, where $\rho \propto a^{-3}$, it is possible to write

FIG. 2.   Principal contributions to the energy density for different eras of the Universe.

$$a_e = \left(\frac{\rho(a_{RH})}{\rho(a_e)}\right)^{\frac{1}{3}} a_{RH}. \qquad (57)$$

Taking $\rho(a_e) = V_I$ and that $\rho(a_{RH}) = \frac{\pi^2}{30} g_* T_{RH}^4$, one can approximate

$$a_e \simeq \left(\frac{T_{RH}^4}{V_I}\right)^{\frac{1}{3}} a_{RH}, \qquad (58)$$

where we have ignored numerical factors of order 1. Finally, we need a_{RH}. Assuming adiabatic expansion after reheating, we have

$$a_{RH} = \frac{T_{eq}}{T_{RH}} a_{eq}, \qquad (59)$$

where $T_{eq} \simeq 0.83$ eV and $a_{eq} = 2.8 \times 10^{-4}$, the standard values for the temperature and scale factor at matter-radiation equality [27]. Thus, we see that the details of the inflationary and reheating phases are encapsulated in the three parameters T_{RH}, V_I, and $\mathcal{N}$.

Thus, up to order-1 numerical factors, the final function to integrate is

$$E(a) = \begin{cases} \frac{1}{H_0}\sqrt{\frac{V_I}{M_p^2}}, & a_b < a < a_e \\ \frac{T_{RH}^2}{H_0 M_p}\left(\frac{T_{eq}}{T_{RH}}\right)^{\frac{3}{2}}\left(\frac{a_{eq}}{a}\right)^{\frac{3}{2}}, & a_e < a < a_{RH} \\ \sqrt{\Omega_\Lambda + \frac{\Omega_M}{a^3} + \frac{\Omega_R}{a^4}}, & a_{RH} < a, \end{cases} \qquad (60)$$

with

$$a_b = e^{-\mathcal{N}}\left(\frac{T_{RH}^4}{V_I}\right)^{\frac{1}{3}} \frac{T_{eq}}{T_{RH}} a_{eq}, \qquad (61a)$$

$$a_e = \left(\frac{T_{RH}^4}{V_I}\right)^{\frac{1}{3}} \frac{T_{eq}}{T_{RH}} a_{eq}, \qquad (61b)$$

$$a_{RH} = \frac{T_{eq}}{T_{RH}} a_{eq}. \qquad (61c)$$

B. Evolution of the energy density

Now, we can study how the energy density of the electromagnetic part evolves in the background described by (60) and (61). This would depend on (a) the initial time where the charge density appears and (b) the boundary conditions that we set for both A_0 and $\nabla_\mu A^\mu$.

If we introduce the fields into the energy density, grouping terms adequately, we get

$$\rho_A = \left[\kappa a_i^3 A_0|_i - \frac{\kappa}{H_0}\left(\nabla_\mu A^\mu|_i - \frac{\kappa \mathcal{I}_i}{\lambda H_0}\right)\mathcal{F}_i - \frac{\kappa^2 \mathcal{G}_i}{\lambda H_0^2}\right]\frac{1}{a^6}$$
$$+ \frac{\kappa}{H_0}\left(\nabla_\mu A^\mu|_i - \frac{\kappa \mathcal{I}_i}{\lambda H_0}\right)\left(\frac{\mathcal{F}}{a^6} - \mathcal{I}\right) - \frac{\kappa^2}{\lambda H_0^2}\left(\frac{\mathcal{I}^2}{2} - \frac{\mathcal{G}}{a^6}\right)$$
$$- \frac{\lambda}{2}\left(\nabla_\mu A^\mu|_i - \frac{\kappa \mathcal{I}_i}{\lambda H_0}\right)^2. \qquad (62)$$

Let us study the evolution of the various energy density terms in different situations.

1. Instantaneous charge density generation

Let us first consider the case in which charge and electromagnetic fields vanish initially and at some $a = a_i$ a net charge density is generated, so we take $A_0|_i = 0$ and $\nabla_\mu A^\mu|_i = 0$ in (62) and obtain

$$\rho_A = \frac{\kappa^2}{\lambda H_0^2}\left(\frac{\mathcal{G} - \mathcal{G}_i - \mathcal{I}_i(\mathcal{F} - \mathcal{F}_i)}{a^6} - \frac{1}{2}(\mathcal{I} - \mathcal{I}_i)^2\right), \qquad (63a)$$

$$p_A = \frac{\kappa^2}{\lambda H_0^2}\left(\frac{\mathcal{G} - \mathcal{G}_i - \mathcal{I}_i(\mathcal{F} - \mathcal{F}_i)}{a^6} + \frac{1}{2}(\mathcal{I} - \mathcal{I}_i)^2\right). \qquad (63b)$$

The value of a_i will be determined by the charge generation mechanism. Thus, as mentioned before, a charge density could be generated, for example, during inflation due to some fluctuation of the charged scalar field [9–11] or at a phase transition [5–7] well inside the radiation era.

Thus, for charge generation occurring during the inflation era ($a_i \ll a_e$), we can see that the terms appearing in (63) behave as shown in Fig. 3. In this plot, $\mathcal{N}_i \equiv \ln\frac{a_e}{a_i} = 2$, i.e., only two inflation e-folds are needed to reach the asymptotic behavior. We can see that at the end of inflation the terms containing $\mathcal{I}$ are 2 orders of magnitude bigger
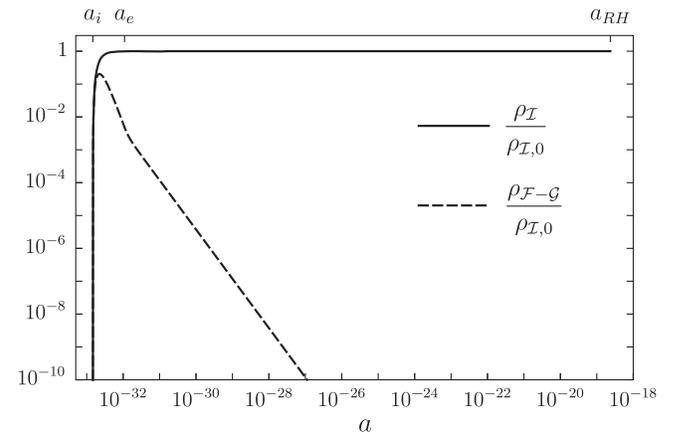

FIG. 3. Behavior of the terms in (62) normalized to the dominant value today, for a charge asymmetry generated during inflation era. Here, $\rho_I \propto (\mathcal{I} - \mathcal{I}_i)^2$, and $\rho_{\mathcal{F}-\mathcal{G}} \propto [\mathcal{G} - \mathcal{G}_i - \mathcal{I}_i(\mathcal{F} - \mathcal{F}_i)]a^{-6}$.

than those with $\mathcal{F}$ and $\mathcal{G}$. After inflation, the difference remains increasing, and nowadays only the $\mathcal{I}$ terms are relevant. Moreover, we see that $\mathcal{I} - \mathcal{I}_i$ remains constant after that, i.e., $\mathcal{I} \simeq \mathcal{I}_e$ for $a > a_e$, so that the dominant contribution at late times is a cosmological constant component.

Since in inflation one has the analytical expression

$$\mathcal{I} = -\frac{M_p H_0}{3\sqrt{V_I}} \frac{1}{a^3}, \tag{64}$$

we can see that at the end of inflation

$$\mathcal{I}_e - \mathcal{I}_i = \frac{M_p H_0}{3\sqrt{V_I}} \frac{1}{a_i^3} (1 - e^{-3\mathcal{N}_i}) \simeq -\mathcal{I}_i, \tag{65}$$

for big enough $\mathcal{N}_i$. Then, the energy density today is

$$\rho_{A,0} \simeq -\frac{\kappa^2}{2\lambda H_0^2} \mathcal{I}_i^2, \tag{66}$$

with

$$\mathcal{I}_i = -\frac{M_p H_0 \sqrt{V_I}}{3 T_{RH} T_{eq}^3} \frac{e^{3\mathcal{N}_i}}{a_{eq}^3}, \tag{67}$$

where we have used (61b) and $a_i = e^{-\mathcal{N}_i} a_e$. Thus, for $\lambda < 0$, it is possible to have a positive cosmological constant today.

We now want to study the range of values of the charge density κ which can provide a value for ρ_A today consistent with dark energy density measurements. In order to do it, we use that

$$\rho_\Lambda = \Omega_\Lambda \rho_c = 3 H_0^2 M_p^2 \Omega_\Lambda. \tag{68}$$

Comparing this expression with (66) and (67), we get

$$-\frac{\kappa^2}{\lambda} \lesssim \frac{54 a_{eq}^6 \Omega_\Lambda}{e^{6\mathcal{N}_i}} \frac{H_0^2 T_{RH}^2 T_{eq}^6}{V_I}. \tag{69}$$

Thus, we get for the constant comoving charge density

$$|\kappa| \lesssim \frac{\sqrt{54 \Omega_\Lambda |\lambda|}}{z_{eq}^3 e^{3\mathcal{N}_i}} \frac{H_0 T_{RH} T_{eq}^3}{\sqrt{V_I}} \tag{70}$$

so that

$$|\kappa| \lesssim 10^{-78} e^{-3\mathcal{N}_i} |\lambda|^{1/2} \left(\frac{T_{RH}}{10^6 \text{ GeV}}\right) \left(\frac{10^{16} \text{ GeV}}{V_I^{1/4}}\right)^2 \text{ eV}^3, \tag{71}$$

which allows us to obtain, assuming individual particles of charge $e = \sqrt{4\pi\alpha}$, the limit on the charge asymmetry as

$$\eta_Q \lesssim [10^{-144}, 10^{-68}] |\lambda|^{1/2} \left(\frac{T_{RH}}{10^6 \text{ GeV}}\right) \left(\frac{10^{16} \text{ GeV}}{V_I^{1/4}}\right)^2 \tag{72}$$

for $\mathcal{N}_i \in [2, 60]$.

Let us now consider the case in which the charge generation takes place well inside the radiation era at a temperature $T = T_Q$, corresponding to an initial scale factor,

$$a_i = a_Q = \frac{T_{eq}}{T_Q} a_{eq}. \tag{73}$$

In Fig. 4, we can see that, again, the $\mathcal{I}$ terms are several orders of magnitude bigger than the $\mathcal{F}$ and $\mathcal{G}$ ones. Moreover, after a_{eq}, $\mathcal{I} - \mathcal{I}_i$ has again a constant value until today, so we can approximate the energy density today as the energy density in the radiation-matter equality. We use the analytical solution for $\mathcal{I}$ during a radiation-dominated universe,

$$\mathcal{I} = -\frac{1}{\sqrt{\Omega_R} a}, \tag{74}$$

in order to write

$$\mathcal{I}_{eq} - \mathcal{I}_i = \frac{1}{\sqrt{\Omega_R}} \frac{1}{a_{eq}} \left(\frac{T_Q}{T_{eq}} - 1\right) \approx -\mathcal{I}_i, \tag{75}$$

and then, following the same steps as in (68)–(70), one can estimate

$$|\kappa| = \sqrt{6 \Omega_\Lambda \Omega_R |\lambda|} H_0^2 M_p a_{eq} \frac{T_{eq}}{T_Q}, \tag{76}$$
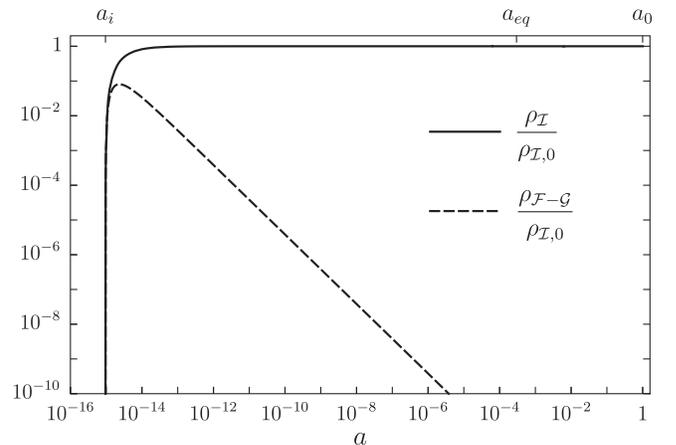
which gives the limit on the charge asymmetry

FIG. 4.   Same as in Fig. 3 but for a charge asymmetry generated at $T_Q = T_{\text{EW}} = 246$ GeV.

$$\eta_Q \lesssim 10^{-43} |\lambda|^{1/2} \left(\frac{100 \text{ GeV}}{T_Q} \right). \qquad (77)$$

Thus, we see that for $|\lambda| = \mathcal{O}(1)$ a tiny charge asymmetry of order $\eta_Q \simeq 10^{-43}$ produced at the electroweak phase transition would generate a cosmological constant compatible with observations. The possibility of generating a cosmological constant in the context of modified electrodynamics was considered in the uncharged case in Refs. [28,29]. Notice that in the uncharged sector with $q = 0$, i.e., $\kappa = 0$, the only possible homogeneous solution of the consistency condition (26) is a constant b field, which contributes to ρ_A in (44a) as a pure cosmological constant.

2. Vanishing fields in the asymptotic future

Another type of solutions corresponds to those in which charge density has ever been present but the induced electromagnetic field vanishes asymptotically in the future as the charge density decreases as $\kappa \propto a^{-3}$, i.e.,

$$0 = \lim_{a \to \infty} \nabla_\mu A^\mu = \nabla_\mu A^\mu |_i - \frac{\kappa \mathcal{I}_i}{\lambda H_0} + \frac{\kappa}{\lambda H_0} \lim_{a \to \infty} \mathcal{I}. \qquad (78)$$

For $a \gg 1$, only the Ω_Λ term will survive, and $\mathcal{I}$ vanishes in the limit. Then, this yields

$$\nabla_\mu A^\mu |_i = \frac{\kappa \mathcal{I}_i}{\lambda H_0}. \qquad (79)$$

Now, taking the same limit in (54a), one can see that it is automatically satisfied for any $A_0|_i$ taking the previous result for $\nabla_\mu A^\mu|_i$. By introducing (79) in the energy density (62), we can see that this is equivalent to removing the constant mode, as one would expect. Moreover, the term $\mathcal{F}a^{-6} - \mathcal{I}$ is removed, too. The energy density is then

$$\rho_A = \left[\kappa a_i^3 A_0|_i - \frac{\kappa^2 \mathcal{G}_i}{\lambda H_0^2} \right] \frac{1}{a^6} - \frac{\kappa^2}{\lambda H_0^2} \left(\frac{\mathcal{I}^2}{2} - \frac{\mathcal{G}}{a^6} \right), \qquad (80)$$

and the pressure

$$p_A = \left[\kappa a_i^3 A_0|_i - \frac{\kappa^2 \mathcal{G}_i}{\lambda H_0^2} \right] \frac{1}{a^6} + \frac{\kappa^2}{\lambda H_0^2} \left(\frac{\mathcal{I}^2}{2} + \frac{\mathcal{G}}{a^6} \right). \qquad (81)$$

As mentioned above, we can safely neglect the term with $A_0|_i$, which makes the analysis easier. Now, let us define the dimensionless functions

$$\bar{\rho}_A \equiv \frac{\mathcal{I}^2}{2} - \frac{\mathcal{G} - \mathcal{G}_i}{a^6}, \qquad (82a)$$

$$\bar{p}_A \equiv -\frac{\mathcal{I}^2}{2} - \frac{\mathcal{G} - \mathcal{G}_i}{a^6}. \qquad (82b)$$

We solve (55b) and (55c) for the different epochs:

(1) Inflation:

$$\bar{\rho}_A = \frac{M_p^2 H_0^2}{V_I} \left(\ln a + \frac{1}{6} + \mathcal{G}_i \right) \frac{1}{a^6}, \qquad (83a)$$

$$\bar{p}_A = \frac{M_p^2 H_0^2}{V_I} \left(\ln a - \frac{1}{6} + \mathcal{G}_i \right) \frac{1}{a^6}. \qquad (83b)$$

(2) Reheating:

$$\bar{\rho}_A = \frac{4 H_0^2 M_p^2}{3 T_{RH} T_{eq}^3} \frac{1}{(a a_{eq})^3} + \frac{\mathcal{G}_i}{a^6}, \qquad (84a)$$

$$\bar{p}_A = \frac{\mathcal{G}_i}{a^6}. \qquad (84b)$$

(3) Radiation:

$$\bar{\rho}_A = \frac{3}{4 \Omega_R a^2} + \frac{\mathcal{G}_i}{a^6}, \qquad (85a)$$

$$\bar{p}_A = -\frac{1}{4 \Omega_R a^2} + \frac{\mathcal{G}_i}{a^6}. \qquad (85b)$$

(4) Matter:

$$\bar{\rho}_A = \frac{4}{9 \Omega_M a^3} + \frac{\mathcal{G}_i}{a^6}, \qquad (86a)$$

$$\bar{p}_A = \frac{\mathcal{G}_i}{a^6}. \qquad (86b)$$

(5) Dark energy:

$$\bar{\rho}_A = \frac{1}{3 \Omega_\Lambda} \left(\ln a + \frac{1}{6} + \mathcal{G}_i \right) \frac{1}{a^6}, \qquad (87a)$$

$$\bar{p}_A = \frac{1}{3 \Omega_\Lambda} \left(\ln a - \frac{1}{6} + \mathcal{G}_i \right) \frac{1}{a^6}. \qquad (87b)$$

The evolution of the equation of state is shown in Fig. 5. We can see that far from the transition regions, i.e., neglecting $\mathcal{G}_i$, $w_A = -1/3$ in the radiation era; $w_A = 0$ in the matter era; and during the accelerated expansion eras, the behavior goes as

$$w_A(a) = 1 - \frac{2}{1 + 6 \ln a}, \qquad (88)$$

which tends asymptotically to that of a stiff fluid $w_A = 1$.

In Fig. 6, we show the evolution of the dimensionless $\bar{\rho}_A$ compared to $E^2(a)$, which follows the scaling of $\rho_{\Lambda CDM}$. As we can see, the maximum contribution occurs in the matter-dominated era, when the ratio $R = \bar{\rho}/E^2$ reaches a maximum value $R_{max} = 4.8$. Thus, in order for the charge-induced energy density not to spoil the predictions of
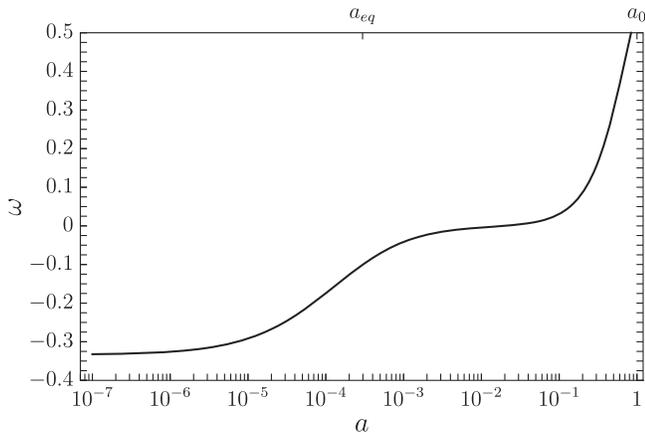
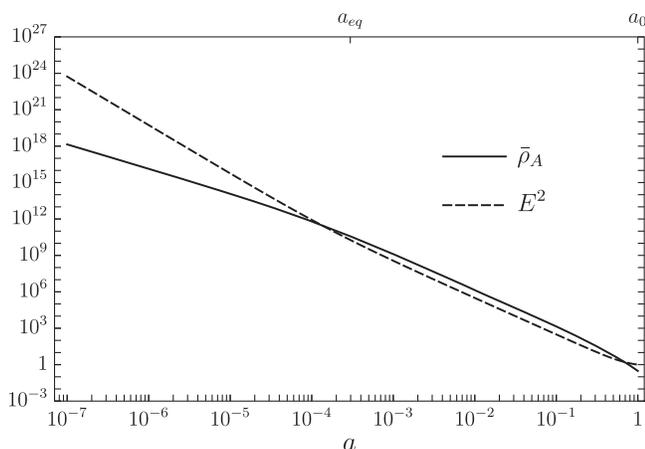FIG. 5.   Evolution of the effective electromagnetic equation of state w_A.

FIG. 6.   Evolution of the normalized electromagnetic energy density $\bar{\rho}_A(a)$ (solid) and normalized ΛCDM energy density $E^2(a)$ (dashed).

standard ΛCDM, we impose $\rho_A \lesssim 10^{-2}\rho_{\Lambda\text{CDM}}$, which implies

$$R_{\max}\frac{8\pi G}{3}\frac{\kappa^2}{|\lambda|H_0^4} \lesssim 10^{-2}, \qquad (89)$$

which, in turn, can be translated into a limit on the charge asymmetry as

$$\eta_Q \lesssim 10^{-28}|\lambda|^{1/2}. \qquad (90)$$

This limit relaxes in several order of magnitude the present bounds on the charge asymmetry in standard Maxwell electrodynamics mentioned before.

VII. CONCLUSIONS

We have explored the possibility of constructing homogeneous and isotropic cosmologies with a nonvanishing charge density in the context of modified Maxwell electrodynamics. Unlike previous works which considered theories that include a small photon mass and thus propagate 3 degrees of freedom, we have limited ourselves to the Zwanziger model of electrodynamics, with two propagating polarizations, but with modified subsidiary conditions. The modification affects only the physical photon Fock space in the infrared.

We show that in the context of this model the induced classical current counterbalances the effects of the physical (quantum) charges so that the Faraday tensor vanishes on cosmological scales, thus allowing for the construction of exact Robertson-Walker geometries. Depending on the boundary conditions imposed on the classical $b(t)$ field, different scenarios are possible. Thus, if $b(t)$ vanishes at some initial time when the charge density is generated, then the dominant contribution to the electromagnetic energy-momentum tensor is a cosmological constantlike term. Imposing the value of the induced constant to be smaller than the observed one sets stringent limits on the comoving charge density, which translates into limits on the charge asymmetry which can range from $\eta_Q \lesssim 10^{-131}$ if charges are produced during inflation (for typical inflationary models) to $\eta_Q \lesssim 10^{-43}|\lambda|^{1/2}(100\,\text{GeV}/T_Q)$ if the charge density is generated in the radiation era at a temperature T_Q. In the case in which the b field vanishes asymptotically in the future when the charge density also vanishes, the cosmological constantlike term is absent, and the dominant contribution appears as an extra matter density in the matter-dominated era. Imposing again compatibility with the observed matter density sets a weaker limit $\eta_Q \lesssim 10^{-28}|\lambda|^{1/2}$, several orders of magnitude below the limits in standard Maxwell electrodynamics.

The deelectrification mechanism discussed in this work only takes place on cosmological scales, and, *a priori*, could not prevent the appearance of effects on smaller scales. However, the small charge densities suggest that such effects could be actually suppressed. Thus, for example, in the highest-density case, corresponding to $\eta_Q \simeq 10^{-28}$, the corresponding density of charged particles today would be $n_Q \simeq 10^{-26}\,\text{cm}^{-3}$. Of course, density perturbations would induce also charge density perturbations, which could be enhanced in high-density objects. The study of the evolution of these charge fluctuations is, however, beyond the scope of the present work.

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