Bichromatically-controlled entanglement between asymmetric quantum dots in a photonic cavity

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Abstract. Entanglement generation and control of two spatially separated asymmetric quantum dots with broken inversion symmetry and mediated by a photonic cavity is studied using a quantum master equation formalism. The quantum dots are coherently driven by a bichromatic laser consisting of a strong optical field nearly resonant with the optical transition of each quantum dot, and a low frequency field. The optical field dresses each quantum dot, and due to the presence of large permanent dipole moments in the quantum dots they are coupled by the low frequency field. We make use of the generated dressed-state scheme for entanglement control. The master equation which describes the interaction with the cavity modes and the coherent fields is numerically solved. In order to gain some insight on the role of the external parameters on entanglement, an effective Hamiltonian for the atomic subsystem is derived in the dressed state representation by adiabatically eliminating the cavity field operators. It is found that steady-state entanglement can be controlled by means of the amplitude and frequency of the low frequency field.

Keywords: entanglement, permanent dipole moments, concurrence

Submitted to: *J. Phys. B: At. Mol. Opt. Phys.*

1. Introduction

Entanglement is one of the key elements for implementing quantum computation and quantum information [1, 2, 3, 4] via protocols such as teleportation [5] and cryptography [6]. However quantum entanglement is too fragile to play a real role in the real world because a real quantum system is, in general, influenced by its surrounding environment leading to the so-called environment-induced decoherence [7]. This is the most serious problem for all entanglement manipulation in quantum information processing.

As well as avoiding decoherence, entanglement must also be generated and controlled. Up to now, various methods have been proposed to suppress decoherence, such as quantum error correction [8], decoherence-free subspaces [9], quantum feedback control [10], and dynamical decoupling [11]. The study of the controlled entanglement dynamics is of considerable importance to the prospects of maintaining quantum information [12, 13, 14]. Thus, detailed analyses and design of cavity systems are used to overcome decoherence [15, 16], enhance entanglement [17], and prolong entangled time [18] by means of cavity quantum electrodynamics (QED) and circuit QED [19]. In addition, model systems that theoretically exhibit the rebirth of entanglement have been proposed and discussed in several cases [20, 21, 22]. Scheme for controllably entangling the internal states of two atoms trapped in a high-finesse optical cavity by using quantum-reservoir engineering have been put forward [23, 24]. In Refs. [25, 26] it has been shown that white noise may play a positive role in generating the controllable entanglement in some specific conditions, and some methods of controlling entanglement in a two-qubit system by changing a relative phase of the driving pulses [27], or via dissipation-engineering techniques in waveguide-quantum-electrodynamic systems [28, 29] have been proposed. Furthermore, quantum entanglement due to reservoir-induced decoherence using novel entanglement-preserving approach for the photon scattering loss [30] and material loss [31] scenarios, have been addressed. It has recently been shown that, in specific scenarios, it is possible to produce a delayed creation of entanglement, the so-called entanglement sudden birth [32, 33]. These entanglement features have been explored in both Markovian [34] and non-Markovian [35] regimes in several different contexts.

From a practical point of view, qubits can be realized with different systems such as photons [36, 37], trapped ions and atoms [38, 39, 40], spins [41], nitrogen vacancy centers in diamond [42, 43], superconducting qubits [44, 45], hybrid plasmonic nanostructures [46], and quantum dots (QDs) [47, 48]. Also, quantum dot molecules consisting of QDs which are coupled by tunneling and separated by barriers have received attention both theoretically [49] and experimentally [50]; in addition, quantum information properties of a double quantum dot system connected to resonators were studied [51].

All of these theoretical studies have been analysed in the framework of the usual two-level model (atoms or symmetric QDs) in which inversion symmetry is assumed. In such systems all diagonal matrix elements of the dipole coupling between cavity and atom vanish because of the inversion symmetry of the Coulomb potential. This

means that there is a pure coupling to the radiation field. However in an atomic system with broken inversion symmetry, the permanent dipole matrix elements μ_{11} and μ_{22} are non-zero, and therefore will exhibit an additional longitudinal coupling. For instance, the existence of a non-zero permanent dipole moment (PDM) has been experimentally observed in exciton states of several kinds of QDs [52, 53, 54, 55]. Large PDMs in excited quantum rings have been reported [56] and were also revealed in both the exciton and biexciton states [57]. Direct observation of large PDMs of excitonic complexes in InGaN/GaN in QDs of the order $3e$ nm have been obtained [58]. It has been shown that the presence of PDMs considerably influences the optical response of such materials [59, 60, 61, 62, 63, 64, 65]. The multiphoton resonant excitation of a quantum system subjected with PDMs modify saturation of absorption on intersubband transitions [66] and, more interestingly, allows correlated photons pairs generation at variable frequencies [67].

From an experimental point of view, it is highly desirable to control entanglement, via a macroscopic, easily-adjustable external parameter (e.g., bias) or manipulating entanglement by classical driving fields. In this context, systems with broken inversion symmetry are of special interest. Formally, in such systems not only transversal (proportional to Pauli pseudospin operators σ_x and σ_y) but also longitudinal (proportional to σ_z operator characterizing population inversion) coupling with electromagnetic field becomes possible [68]. The latter opens a radiative channel between adjacent dressed levels at the Rabi frequency [69]. By combining few coherent driving sources, one can achieve a further degree of control of the quantum dynamics. In particular, the resonance fluorescence of a two-level atom in a strong bichromatic field was analyzed in [60] and the control of the spontaneous emission by periodically shifting the atomic transition frequency from the atom-cavity resonance has been theoretically demonstrated [70]. Furthermore, the application to a two-level atom of a sequence of pulses [71], or rather intense low-frequency coherent fields [72, 73] lead to spontaneous emission control as well. On the other hand, incorporation of the asymmetry into the quantum system can radically change its emission pattern and lead to the opening of optical transitions which were forbidden in the symmetric case [74, 75, 76, 77].

The aim of this paper is to develop a theoretical approach on the interaction of a hybrid system formed by two quantum emitters with broken inversion symmetry located inside a photonic cavity with a bichromatic field. Such field is composed of an optical field which dresses the atomic levels and a low frequency field (LF field). The mechanism of the dressing through the optical field splits each level into two giving rise to two new qubits with energy difference given by the effective Rabi frequency Ω_R (see below). If a second LF field with the angular frequency close to the Rabi frequency Ω_R drives the atom, an additional Rabi oscillation on dressed states of the new qubits is induced which exhibits undamped oscillations with a resonance at the Rabi frequency. Thus, an advantage of our time-dependent driving scheme is the substantial decoherence suppression by use of ac-fields [68, 78]. The new qubits have a smaller energy splitting than those corresponding to the bare states. The transition amplitudes between these

Figure 1. Schematic of the hybrid system composed of two asymmetric QDs embedded in a photonic cavity. The system is driven by a bichromatic excitation consisting of an intense optical field with amplitude E_L and frequency ω_L and a low frequency field with amplitude E_s and frequency ω_s close to the cavity resonance frequency ω_c . The intense optical field dresses the atomic transition of each QD resulting in a doublet with energy splitting of the order of Rabi frequency Ω_R , whereas the LF field redistribute the populations among the dressed states.

qubits depend strongly on the amplitude of the LF field, so entanglement can be controlled via this external LF field, and through the Rabi frequency of the optical field as well. A further advantage of materials with broken inversion symmetry considered in this paper is that the interaction between trapped polar molecules [79] provides novel mechanisms for entanglement generation and control that are not possible with atoms. In fact coherent dipole-dipole exchange interactions between polar molecules in microwave-driven 3D optical lattices can be controlled by external fields [80].

The paper is organized as follows: Section 2 establishes the model, i.e., the Hamiltonian of the atomic-cavity system taking into account the terms introduced by the PDMs of the QDs. In Section 3, we present the results obtained when solving the complete master equation, and we make use of concurrence to analyze the entanglement of the two QDs in different initial scenarios. In order to discuss the numerical results and shine light on the numerical results, an effective Hamiltonian in the weak qubit-cavity regime is obtained in Section 4 by adiabatically eliminating the cavity field. Finally, Section 5 summarizes the main conclusions.

2. Theoretical model

We consider a hybrid system composed of two QDs coupled to a photonic cavity. Such a device has already been experimentally demonstrated using a photonic crystal cavity [81, 82, 83], a micropillar cavity [84], or Fabry-Perot cavity [85]. Each QD is modeled as a two-level system with ground state $|g\rangle_j$ having all electrons in the valence band, and upper state $|e\rangle_j$, ($j = 1, 2$) corresponding to the excitation of single electron-hole pair. The transition frequency is $\omega_{0j} = \omega_{ej} - \omega_{gj}$, ($j = 1, 2$), and the transition electric dipole moment is $\vec{\mu}_{ge}$, as depicted in Fig. 1. Due to the breaking of inversion symmetry the levels $|g\rangle_j$ and $|e\rangle_j$ can exhibit unequal PDMs ($\vec{\mu}_{gg} \neq \vec{\mu}_{ee}$). The single-mode cavity field is represented by bosonic annihilation and creation operators a and $a^\dagger$.

The optical transition $|g\rangle_j \leftrightarrow |e\rangle_j$ is driven by a polarized laser field of frequency

ω_L given by

$$\hat{E}_L(t) = \frac{1}{2} E_L \left(e^{-i(\omega_L t - \vec{k}_L \cdot \vec{r})} + c.c. \right) \hat{u}, \quad (1)$$

where E_L is the slowly varying amplitude, $\vec{k}_L$ is the propagation wavevector, and $\hat{u}$ the unit polarization vector of the optical laser field. We assume that $\hat{u} || \vec{\mu}_{ge} || \vec{\mu}_{gg} || \vec{\mu}_{ee}$, i.e., they are all parallel. The atomic system is also driven by a monochromatic low frequency field (LF field) of angular frequency $\omega_s \ll \omega_L$ which reads

$$\hat{E}_s(t) = \frac{1}{2} E_s \left(e^{-i(\omega_s t - \vec{k}_s \cdot \vec{r})} + c.c. \right) \hat{u}, \quad (2)$$

$\vec{k}_s$ being the propagation wave vector of the LF field. Within the quantum-field approach, we resort to replace the classical dipole $\mathbf{d}$ with the corresponding operator

$$\mathbf{d} = \frac{\vec{\mu}_{ee} + \vec{\mu}_{gg}}{2} \hat{I} + \sum_{j=1}^2 \left(\frac{\vec{\mu}_{ee} - \vec{\mu}_{gg}}{2} \hat{\sigma}_z^j + \vec{\mu}_{eg} \sigma_+^j + \vec{\mu}_{eg} \sigma_-^j \right), \quad (3)$$

where $\hat{I}$ is the identity operator. With this in mind, the Hamiltonian of the hybrid system can be expressed as

$$H = H_0^M + H_0^C + H_{A-C,1} + H_{A-C,2} + H_{drive}, \quad (4)$$

where

$$\begin{aligned} H_0^M &= \frac{\hbar}{2} \sum_{j=1}^2 \omega_{0j} \sigma_z^j, \\ H_0^C &= \hbar \omega_c a^\dagger a, \\ H_{A-C,1} &= \hbar \sum_{j=1}^2 g_j \sigma_z^j (a^\dagger + a), \\ H_{A-C,2} &= \hbar \sum_{j=1}^2 \hat{g}_j (\sigma_+^j + \sigma_-^j) (a^\dagger + a). \end{aligned} \quad (5)$$

The first term (H_0^M) describes two two-level atoms with transition frequency ω_{0j} , and $\sigma_\pm^j$, σ_z^j are the Pauli matrices acting in the space spanned by the states $|g\rangle_j$ and $|e\rangle_j$. The second term (H_0^C) corresponds to the energy of the cavity mode, where a ($a^\dagger$) is the annihilation(creation) operator of the cavity mode field with angular frequency ω_c . The third term ($H_{A-C,1}$) represents the interaction between each atom and the cavity mode: note that, in contrast to the standard Jaynes-Cummings model, in systems with broken inversion symmetry, longitudinal coupling proportional to the σ_z^j operator characterizing population inversion, becomes possible. The fourth term ($H_{A-C,2}$) accounts for the usual transversal coupling between the emitters and the cavity. We assume that the cavity mode relies in the THz regime, and $|\mu_{ee} - \mu_{gg}| \gg |\mu_{eg}|$, thus we anticipate that only longitudinal coupling is effective. The couplings g_j and $\hat{g}_j$ ($j = 1, 2$) of the atoms with

the cavity mode in Eq. (5) are given by

$$g_j = (\vec{\mu}_{ee} - \vec{\mu}_{gg}) \cdot \vec{e}_c \sqrt{\left(\frac{\hbar \omega_c}{2\epsilon_0 V_c} \right)}, \quad (6)$$

$$\hat{g}_j = \vec{\mu}_{eg} \cdot \vec{e}_c \sqrt{\left(\frac{\hbar \omega_c}{2\epsilon_0 V_c} \right)},$$

where ϵ_0 is the dielectric constant of the medium, V_c is the cavity mode volume, and $\vec{e}_c$ is the polarization vector of the cavity mode. Finally, the term H_{drive} in Eq. (4) describes the interaction of the atoms with the external electromagnetic fields and can be constructed as follows. The interaction of a classical dipole $\mathbf{d}$ with an external electric field $\mathbf{E}$ is given by the expression $H_{drive} = -\mathbf{d} \cdot \mathbf{E}$. Thus, the interaction Hamiltonian with the external fields can be written as

$$H_{drive} = H_{d,1} + H_{d,2} + H_{d,3} + H_{d,4}, \quad (7)$$

where the different terms in Eq. (7) read

$$H_{d,1} = \hbar \sum_{j=1}^2 (\Omega_j e^{-i\omega_L t} + \text{H.c.}) (\sigma_+^j + \sigma_-^j), \quad (8)$$

$$H_{d,2} = \hbar \sum_{j=1}^2 (\hat{\Omega}_j e^{-i\omega_L t} + \text{H.c.}) \sigma_z^j, \quad (9)$$

$$H_{d,3} = \hbar \sum_{j=1}^2 \left(\frac{G_{sj}}{2} e^{-i\omega_s t} + \text{H.c.} \right) \sigma_z^j, \quad (10)$$

$$H_{d,4} = \hbar \sum_{j=1}^2 \left(\frac{\hat{G}_{sj}}{2} e^{-i\omega_s t} + \text{H.c.} \right) (\sigma_+^j + \sigma_-^j), \quad (11)$$

where H.c. stands for Hermitian conjugate. We have made use of the following shorthand notation: Ω in Eq. (8) stands for the usual Rabi frequency of the optical field, i.e., $\Omega_j = \frac{\mu_{eg} E_L}{2\hbar} e^{i\vec{k}_L \cdot \vec{r}_j}$; G_s in Eq. (10) stands for the Rabi frequency of the LF field, i.e., $G_{sj} = \frac{\mu_0 E_s}{2\hbar} e^{i\vec{k}_s \cdot \vec{r}_j}$ with $\mu_0 \equiv \mu_{ee} - \mu_{gg}$. In addition, we have defined magnitude $\hat{\Omega}_j$ appearing in Eq. (9) as $\hat{\Omega}_j \equiv \frac{\mu_0 E_L}{2\hbar} e^{i\vec{k}_L \cdot \vec{r}_j}$. Finally, we have introduced magnitude $\hat{G}_{sj}$ in Eq. (11) as $\hat{G}_{sj} \equiv \frac{\mu_{eg} E_s}{2\hbar} e^{i\vec{k}_s \cdot \vec{r}_j}$. We also have assumed $\mu_{eg} = \mu_{ge}$ and real. Magnitude G_{sj} possesses an important role in the dynamics of the present system, as we will see later.

Note that the Rabi frequencies of the driving fields depend on the positions of the atoms and can be different for the atoms located at different points. For example, if the dipole moments of the atoms are parallel, the Rabi frequencies G_{si} and G_{sj} of two arbitrary atoms separated by a distance r_{ij} are related by

$$G_{sj} = G_{si} e^{i\vec{k}_s \cdot \vec{r}_{ij}} \quad (12)$$

where $\vec{r}_{ij}$ is the vector in the direction of the interatomic axis and $|\vec{r}_{ij}| = r_{ij}$ is the distance between the atoms. Thus, the Rabi frequencies differ by the phase factor $e^{i\vec{k}_s \cdot \vec{r}_{ij}}$ arising from different position coordinates of the atoms. For proper spatial configuration of the two atoms, we may obtain $G_{si} = -G_{sj}$. This can be achieved by choosing the propagation vector $\vec{k}_s$ of the laser along the interatomic axis, and the atomic separations such that $k_s r_{21} = (2n + 1)\pi, n = 0, 1, 2, \dots$, which corresponds to a situation where the atoms are separated by a distance $r_{21} = (2n + 1)\lambda_s/2, n = 0, 1, 2, \dots$.

For the specific system of coupled QD-cavity, there are two major sources of loss: dissipation of the cavity field to the environment and an effective dipole decay rate of the QD excitons. Thus, taking into account these losses, the master equation for the density operator of the system is given by [86]

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H, \rho] + L_{QD}\rho + L_c\rho, \quad (13)$$

where L_{QD} and L_c represent the irreversible decay of the QDs and the cavity, respectively, and can be written as

$$\begin{aligned} L_{QD}\rho &= -\sum_{j=1}^2 \frac{\gamma_j}{2} \left[\sigma_+^{(j)} \sigma_-^{(j)} \rho - 2\sigma_-^{(j)} \rho \sigma_+^{(j)} + \rho \sigma_+^{(j)} \sigma_-^{(j)} \right], \\ L_c\rho &= -\frac{\kappa}{2} [a^+ a \rho - 2a \rho a^+ + \rho a^+ a], \end{aligned} \quad (14)$$

where κ takes into account the cavity decay rate, and γ_j is the effective dipole decay rate of the QD excited state which takes into account the interaction with the free space and cavity modes. In the following we assume $\gamma_i = \gamma_j \equiv \gamma$.

In order to remove the fast oscillating terms in Eqs. (8)-(11), we perform the unitary transformation to the singly rotating frame given by

$$U_1 = e^{-i \sum_{j=1}^2 \sigma_z^j \omega_L / 2t}. \quad (15)$$

In this frame, the new density matrix $\rho^{(1)}$ is given by $\rho^{(1)} = U_1^\dagger(t) \rho^{(s)}(t) U_1(t)$, and the transformed Hamiltonian (4) is given by $H^{(1)}(t) = U_1^\dagger(t) H(t) U_1(t) - i\hbar U_1^\dagger(t) \frac{\partial U_1(t)}{\partial t}$ yielding

$$\begin{aligned} H^{(1)} &= \frac{\hbar}{2} \sum_{j=1}^2 \Delta_{Lj} \sigma_z^{(j)} + \hbar \sum_{j=1}^2 (\Omega_j \sigma_+^j + \text{H.c.}) \\ &+ \hbar \sum_{j=1}^2 \left(\frac{G_{sj}}{2} e^{-i\omega_s t} + \text{H.c.} \right) \sigma_z^j + H_{d,4}^{(1)} \\ &+ \hbar \omega_c a^+ a + \hbar \sum_{j=1}^2 g_j \sigma_z^{(j)} (a^+ + a) \\ &+ \hbar \sum_{j=1}^2 \hat{g}_j (\sigma_+^j + \sigma_-^j) (a^+ + a), \end{aligned} \quad (16)$$

where $\Delta_{Lj} = (\omega_{0j} - \omega_L)$ is the detuning of the optical field with the transition frequency of each atom. In arriving at Eq. (16) we have made use of the RWA keeping terms

involving Ω_j , and neglecting terms involving $\hat{\Omega}_j$ which are rapidly oscillating and average to zero. The term $H_{d,4}^{(1)}$ reads

$$H_{d,4}^{(1)} = \hbar \sum_{j=1}^2 \left(\frac{\hat{G}_{sj}}{2} \sigma_+^j e^{i\delta t} + \text{H.c.} \right), \quad (17)$$

where $\delta = \omega_L - \omega_s$. In what follows we assume that $\omega_L \gg \omega_s$ thus the contribution arising from the coupling of the PDMs with the atomic transition, $H_{d,4}$, as given in Eq. (17) can be safely neglected as it averages to zero. We want to highlight that condition $\omega_L \gg \omega_s$ makes the current scenario far from being the usual pump-probe experiment, thus the coupling between the LF field and the emitters is linked to the broken inversion symmetry through magnitudes G_{sj} . This is not the usual coupling addressed in Ref. [87]. We refer the reader to Section 3 where an estimation of this approximation is addressed in terms of the numerical values used within the simulations.

In addition, we assume that the frequency of the LF field is close to the Rabi frequency of the strong optical field, so the following conditions hold: (i) $\Omega_j \gg G_{sj}$, and (ii) $\Omega_j - \omega_s \equiv \Delta_s \ll \Omega_j, \omega_s$. In view of these two conditions, the strong optical field E_L can be viewed as a dressing field for the QDs. The next step consists in diagonalizing the atomic part of the Hamiltonian plus the interaction of the QDs with the optical laser field

$$H^{(01)} = \hbar \sum_{j=1}^2 \frac{\Delta_{Lj}}{2} \sigma_z^j + \hbar \sum_{j=1}^2 (\Omega_j \sigma_+^j + \text{H.c.}), \quad (18)$$

by means of a canonical transformation $U_2 = e^{-i \sum_{j=1}^2 \theta_j \sigma_y^j}$, where θ_j obeys $\cos(2\theta_j) = \frac{\Delta_{Lj}}{\Omega_{Rj}}$, $\sin(2\theta_j) = \frac{2\Omega_j}{\Omega_{Rj}}$, and $\Omega_{Rj} = \sqrt{\Delta_{Lj}^2 + (2\Omega_j)^2}$. Here $\sigma_y^j = i(\sigma_-^j - \sigma_+^j)$. With these relations in mind, the Hamiltonian $H^{(1)}$ in Eq. (16) becomes

$$\begin{aligned} H^{(2)} &= \hbar \sum_{j=1}^2 \frac{\Omega_{Rj}}{2} R_z^j + \hbar \sum_{j=1}^2 \left(\frac{G_{sj}}{2} e^{-i\omega_s t} + \text{H.c.} \right) \\ &\times [R_z^j \cos(2\theta_j) - (R_-^j + R_+^j) \sin(2\theta_j)] + \hbar \omega_c a^\dagger a \\ &+ \hbar \sum_{j=1}^2 g_j [R_z^j \cos(2\theta_j) - (R_-^j + R_+^j) \sin(2\theta_j)] (a^\dagger + a), \\ &+ \hbar \sum_{j=1}^2 \hat{g}_j [e^{i\omega_L t} (\sin(\theta_j) \cos(\theta_j) R_z^j + \cos^2(\theta_j) R_+^j - \sin^2(\theta_j) R_-^j) \\ &+ \text{H.c.}] (a^\dagger + a), \end{aligned} \quad (19)$$

The R_k^j ($k = \pm, z$) operators appearing in Eq. (19) are spin operators acting in the space of the Rabi dressed states given by

$$\begin{aligned} |2\rangle_j &= \cos(\theta_j) |e\rangle_j + \sin(\theta_j) |g\rangle_j, \\ |1\rangle_j &= -\sin(\theta_j) |e\rangle_j + \cos(\theta_j) |g\rangle_j. \end{aligned} \quad (21)$$

In the following we will assume that $\frac{\Delta_{Lj}}{\Omega_{Rj}} \ll 1$ so that $\cos(2\theta_j) \simeq 0$ and $\sin(2\theta_j) \simeq 1$. Under this condition, the Hamiltonian in Eq. (19) reduces to

$$\begin{aligned}
H^{(2)} = & \hbar \sum_{j=1}^2 \frac{\Omega_{Rj}}{2} R_z^j \\
& - \hbar \sum_{j=1}^2 \left(\frac{G_{sj}}{2} e^{-i\omega_s t} + \text{H.c.} \right) [(R_-^j + R_+^j) \sin(2\theta_j)] \\
& + \hbar \omega_c a^\dagger a - \hbar \sum_{j=1}^2 g_j \sin(2\theta_j) (R_+^j a + a^\dagger R_-^j) \\
& + \hbar \sum_{j=1}^2 \hat{g}_j \left[e^{i\omega_L t} \frac{1}{2} \sin(2\theta_j) (R_z^j + R_+^j - R_-^j) + \text{H.c.} \right] (a^\dagger + a) , \quad (22)
\end{aligned}$$

where the rotating-wave approximation has been assumed concerning the terms involving g_j . It is clearly seen from Eq. (22) that the coupling between the dressed states and the cavity mode can be described by the Jaynes-Cummings model. Even more interesting is the fact that the presence of unequal PDMs ($\vec{\mu}_{ee} \neq \vec{\mu}_{gg}$), allows an energy exchange between the QDs' dressed states and the cavity mode.

Finally, in order to remove the oscillating terms in Eq. (22), we perform the unitary transformation to a singly rotating frame which rotates at Ω_{Rj} in the laboratory frame

$$U_3 = e^{-i \left(\frac{\omega_s}{2} \sum_{j=1}^2 R_z^j + \hbar \omega_c a^\dagger a \right) t} . \quad (23)$$

Thus, Eq. (22) reduces to

$$\begin{aligned}
H^{(3)} = & \hbar \sum_{j=1}^2 \frac{(\Omega_{Rj} - \omega_s)}{2} R_z^j \\
& - \hbar \sum_{j=1}^2 \frac{G_{sj}}{2} \sin(2\theta_j) (R_-^j + R_+^j) \\
& - \hbar \sum_{j=1}^2 g_j \sin(2\theta_j) (R_+^j a + a^\dagger R_-^j) \\
& - \hbar \sum_{j=1}^2 \hat{g}_j \frac{1}{2} \sin(2\theta_j) \left(R_z^j a^\dagger e^{-i\delta' t} + R_z^j a e^{i\delta' t} \right) , \quad (24)
\end{aligned}$$

where we have assumed that $\omega_s \simeq \omega_c$, i.e., the LF field is nearly in resonance with the cavity. In addition, we have introduced magnitude $\delta' \equiv \omega_L - \omega_c$. In view of the fact that the LF is in resonance with the cavity the oscillatory terms involving $\hat{g}_j$ which appear in Eq. (24) can also be safely neglected in our analysis. We introduce the pseudo-detuning parameters Δ and Δ_s defined as $\Delta = \frac{\Omega_{R1} - \Omega_{R2}}{2}$ and $\Delta_s = \frac{\Omega_{R1} - \omega_s + \Omega_{R2} - \omega_s}{2}$, which can be modified externally through the Rabi frequencies of the optical and LF fields. The detuning Δ reflects the similarity between the QDs (i.e. for $\Delta = 0$, the transition frequencies of the QDs are the same), whereas Δ_s represents the detuning of the LF

field from the dressed states. In physical terms, the condition $\Delta_s = 0$ corresponds to resonance of the LF field with the transition frequency of the dressed levels. Note that the second term in Eq. (24) incorporates the interaction of PDMs with the LF field, which allows a coherent energy exchange between the dressed states. This is the main result of the present model. We shall see later that this term plays a crucial role in preserving quantum entanglement for a long time.

The master equation for the density operator of the system is given by

$$\frac{\partial \rho^{(3)}}{\partial t} = -\frac{i}{\hbar} [H^{(3)}, \rho^{(2)}] + L_{QD}^{(3)} \rho^{(3)} + L_c^{(3)} \rho^{(3)}, \quad (25)$$

where the Liouvillians $L_{QD}^{(3)} \rho^{(3)}$ and $L_c^{(3)} \rho^{(3)}$ have to be properly transformed and we obtain

$$\begin{aligned} L_{QD}^{(3)} \rho^{(3)} = & -\frac{\gamma_M}{2} \sum_j \left[R_+^{(j)} R_-^{(j)} \rho - 2R_-^{(j)} \rho R_+^{(j)} + \rho R_+^{(j)} R_-^{(j)} \right] \\ & -\frac{\gamma_m}{2} \sum_j \left[R_-^{(j)} R_+^{(j)} \rho - 2R_+^{(j)} \rho R_-^{(j)} + \rho R_-^{(j)} R_+^{(j)} \right] \\ & -\frac{\gamma_0}{2} \sum_j \left[R_z^{(j)} R_z^{(j)} \rho - 2R_z^{(j)} \rho R_z^{(j)} + \rho R_z^{(j)} R_z^{(j)} \right], \end{aligned} \quad (26)$$

where $\gamma_M = \frac{\gamma}{4} [1 + \cos(2\theta)]^2$, $\gamma_m = \frac{\gamma}{4} [1 - \cos(2\theta)]^2$, and $\gamma_0 = \frac{\gamma}{4} \sin^2(2\theta)$. The first term of $L_{QD}^{(3)} \rho^{(3)}$ represents incoherent pumping, the second term accounts for dipole decay of the QD excitons, whereas the third term accounts for dephasing of QD excitons. Since the unitary transformation U_3 only depends on atomic operators, the Liouvillian $L_c^{(3)} \rho^{(3)}$ remains unaltered, i.e., $L_c^{(3)} \rho^{(3)} = L_c \rho$.

In order to quantify entanglement several metrics are used such as entropy of entanglement, negativity and entanglement of formation (*EoF*), among others [88, 89]. It was proven that there is a general formula for *EoF* based on an exactly calculable quantity (concurrence C) defined for a two-qubit system as [90]

$$EoF = h \left(\frac{1 + \sqrt{1 - C^2(\rho)}}{2} \right), \quad (27)$$

where $h(x) = -x \log_2 x - (1-x) \log_2 (1-x)$ is the binary entropy function. The *EoF* takes values from 0 (no entanglement for the system) to 1 (maximum entanglement). In Eq.(2), $C^2(\rho)$ represents the concurrence defined as [90]

$$C = \max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}, \quad (28)$$

λ 's being the eigenvalues, in a decreasing order, of the matrix $\tilde{\rho} = \rho \sigma_y^P \otimes \sigma_y^T \rho^* \sigma_y^P \otimes \sigma_y^T$, where ρ^* denotes the complex conjugation of ρ , and σ_y is the y -component of the Pauli matrix. As *EoF*, the case of $C = 1$ corresponds to the existence of maximum entanglement between the two qubits, and $C = 0$ means no entanglement between the qubits. Since the two-qubit *EoF*(ρ) and the two-qubit concurrence C are monotonically related, it is very usual to characterise entanglement using only concurrence rather than *EoF*. The main advantages of concurrence are that it provides us an effective

and relatively easy approach to quantify entanglement, and more important, efficient concurrence measurement protocols based on a linear optical system or a nonlinear optical system have been proposed [91, 92, 93]. For example, it was shown that concurrence can be experimentally obtained from a simple parity measurement on two copies of a quantum state [94]. Therefore we have chosen concurrence as the parameter to quantify entanglement. It should be emphasised however that it is only *EoF* that is an entanglement measure, and that concurrence obtains its meaning via its relation to *EoF* and not viceversa.

3. Numerical results

In this section we shall investigate the entanglement creation in presence of the LF field, i.e., in a situation in which $G_{sj} \neq 0$. In order to obtain the degree of entanglement it is first necessary to solve the master equation given in Eq. (25).

By solving the master Eq.(25), we can obtain the quantum correlation based on the calculation of the concurrence. To find the steady-state of the master equation (25) we make use of the Quantum Toolbox in PYTHON (QuTiP)[95, 96]. For illustration of the numerical results, we choose a realistic coupled system consisting of two quantum dots with $\mu_0 = 100$ Db, and $\omega_L \propto 10^{15}$ rad/s, a resonator with $Q = 3.6 \times 10^2$ and $\omega_c = 2\pi 6.2 \times 10^{13}$ rad/s (see Refs. [97, 98] for details). The effective dipole decay rate is selected to be $\gamma = 1$ GHz. Note that the detunings δ and δ' appearing in Eqs. (17) and (24) involve the difference between the optical frequency and that of the LF field (Terahertz range), thus both δ 's in units of γ are within the range $9 \times 10^4 \gamma$, which in turn make those oscillatory terms appearing in Eqs. (17) and (24) average to zero. In addition, we are considering quantum emitters in which the transition dipole moments (μ_{eg}) are much lower than the PDMs, thus the following condition holds $\hat{g}_j \ll g_j$, and therefore the oscillatory terms should provide a very minor correction.

We assume a symmetric spatial configuration of the two QDs with respect to the cavity center, resulting in equal coupling strengths $g_1 = g_2 = 43\gamma$ for the two QDs. The other system parameters for the simulation are chosen as $\kappa = 86.9\gamma$, $\gamma_M = \gamma_m = \gamma_0 = \gamma/4$, which in turn assumes that $\theta_1 \approx \theta_2 \approx \pi/4$. As a first step in our analysis we focus in the case of two QDs prepared in the excited state $|\Psi\rangle = |2\rangle_1|2\rangle_2$.

In what follows we shall analyze how the LF field could be used as a knob for tailoring the entanglement properties of the system. In Fig. 2(a) we plot the concurrence as a function of γt for different values of the Rabi frequency of the LF field G_{s1} , which we use as an experimental parameter to control entanglement.

For practical reasons, we need a basis in order to solve the set of differential equations obeyed by the density matrix elements which are derived from the master equation (25). One possible basis would be $|1_1 1_2\rangle$, $|1_1 2_2\rangle$, $|2_1 1_2\rangle$, and $|2_1 2_1\rangle$, where 1_i and 2_i label the ground and excited state of the i -qubit respectively. However, a more convenient basis to represent the dynamics of Eq. (25) is the so-called Dicke basis given by $|G\rangle = |1\rangle_1|1\rangle_2$, $|E\rangle = |2\rangle_1|2\rangle_2$, $|S\rangle = \frac{1}{\sqrt{2}} [|1\rangle_1|2\rangle_2 + |2\rangle_1|1\rangle_2]$, and

Figure 2. (a) Time evolution of the concurrence between the dressed states of the two qubits for different Rabi frequencies of the LF field and both QDs initially excited: $G_{s1} = G_{s2} = 0$ (solid curve), $G_{s1} = -G_{s2} = \gamma$ (dashed-dotted curve) and $G_{s1} = +G_{s2} = \gamma$ (dashed curve). (b)-(d) Time evolution of population of collective states: symmetric (solid curve) and antisymmetric (dashed curve) for the different cases considered in (a). (e) Time evolution of the average value of the number of photons in the cavity for the different cases considered in (a). The curves in (e) have been vertically displaced by 0.2 for clarity. Other parameters are: $\Delta_s = \Delta = 0$, $\kappa = 86.9\gamma$, and $g_1 = g_2 = 43\gamma$.

$|A\rangle = \frac{1}{\sqrt{2}} [|1\rangle_1 |2\rangle_2 - |2\rangle_1 |1\rangle_2]$. The $|S\rangle$ is named as symmetrical or super-radiant state, whereas $|A\rangle$ is the anti-symmetrical or sub-radiant state. As we will see later in Section

4, this basis provides a clearer understanding of the quantum correlations of the two-qubit system.

When the LF field is turned off (solid curve) entanglement between the two qubits is obtained and persists in the steady-state limit. After turning on the LF field the level of entanglement achieved is enhanced for the case in which $G_{s1} = -G_{s2} = \gamma$ (dashed-dotted curve), whereas the level of entanglement goes back to the previous one in the case in which $G_{s1} = +G_{s2} = \gamma$ (dashed curve). We can realize that at early times, the population of the sub-radiant state ρ_{AA} is lower than the superradiant one ρ_{SS} , which is detrimental for entanglement to occur. Just when population of the subradiant state is greater than the radiant state ($\rho_{AA} > \rho_{SS}$), sudden birth of entanglement takes place as explained in Ref. [32]. Here, we observe that sudden birth takes place almost at the same instant of time in the different cases analyzed as shown in Figs. 2 (b)-(d). Note that when the LF field drives the QDs in opposition of phase the level of entanglement is enhanced. Finally, the time evolution of the mean photon number ($\langle a^\dagger(t)a(t) \rangle$) is depicted in Fig. 2 (e) for the previous mentioned cases showing slight deviations among them: note that the curves have been vertically displaced by 0.2 for displaying purposes. After the initial transient, a non-null steady-state is achieved with a low value for $\langle a^\dagger(t)a(t) \rangle$ and exhibits a weak dependence on the value of G_s .

Let us analyze how the LF field influences the degree of entanglement in the antisymmetric case ($G_{s2} = -G_{s1}$): in Fig.3(a) we show the time evolution of concurrence for different Rabi frequencies of the LF field (G_{s1}). Here we observe that the level of concurrence attained in steady-state depends strongly on this external parameter that could be used as knob to tune the degree of entanglement. A better picture of this dependence is shown in Fig. 3(b): the solid curve shows that concurrence reaches a maximum value for the case $G_{s1}^{opt} = 3.95\gamma$ which we term as the optimum driving. More interestingly, the influence of the detuning Δ_s on the steady-state concurrence, which has been produced for the previously determined optimum driving G_{s1}^{opt} , is depicted with the solid curve in Fig. 3 (c). This figure reveals that when the LF field moves out of resonance from the dressed doublet, the concurrence tends to recover the value produced in the absence of the LF field. In summary, the level of entanglement can be tailored by proper values of the detuning Δ_s and of the driving amplitude G_{s1} , both of them being externally modified .

As for the symmetric case ($G_{s2} = +G_{s1}$), the time evolution of concurrence does exhibit an extremely weak dependence on the value of the driving field G_{s1} (not shown). This vanishing effect cannot be appreciated by inspecting the dashed curves in panels Fig. 3(b)-(c) while using the same vertical scale.

A better picture of the dependence of the entanglement at steady-state is obtained by varying simultaneously a couple of external parameters of the LF field. The results are presented in Fig. 4. There we can appreciate the sensitivity of $C(\infty)$ to changes of the Rabi frequency and the detuning of the LF field, confirming that when the LF drives the two quantum emitters in phase opposition ($G_{s1} = -G_{s2}$) the level of entanglement is enhanced compared to the in-phase driving ($G_{s1} = +G_{s2}$). It is also

Figure 3. (a) Time dependence of the concurrence between the two qubits for different Rabi frequencies of the LF field: $G_{s1} = \gamma$ (solid curve), $G_{s1} = 2\gamma$ (dashed-dotted curve), $G_{s1} = 5\gamma$ (dashed curve), and $G_{s1} = 10\gamma$ (dotted curve). (b) Steady-state value of the concurrence as a function of G_{s1} . (c) Steady-state value of the concurrence as a function of Δ_s for the case optimum value of the driving LF field $G_{s1}^{opt} = 3.95\gamma$. In panels (b)-(c) $G_{s2} = -G_{s1}$ (solid curve) and $G_{s2} = +G_{s1}$ (dashed curve). Other parameters as in Fig.2.

worth to mention that for the symmetric case we obtain a very weak dependence on both the Rabi frequency of the LF field and the detuning Δ_s : see the vertical scale in Fig. 4(b) compared to the one in Fig. 4(a), the latter being for the antisymmetric case. We can also appreciate that when the detunings Δ and Δ_s moves from the resonance condition, the steady-state concurrence falls as a consequence of the low excitation of the dressed states. The most relevant result is that entanglement can be obtained for a great variety of values for G_{s1} , Δ_s , and Δ , so one could get a tunable quantum correlation between the QDs by adjusting the detuning/s and the Rabi frequencies of the optical and LF field. Again we can check that for the symmetric case the steady state concurrence falls close to zero for narrow changes from the resonant condition $\Delta = 0$ and $\Delta_s = 0$: this effect is clearly seen when inspecting Fig. 4(d). In contrast to this, for the antisymmetric case we observe the emergence of a wide range of values for the detuning Δ where concurrence keeps high values. However the system exhibits a moderate sensitivity to changes of Δ_s , i.e., the closer the similarity between the two

Figure 4. (a)[(b)] Steady-state concurrence versus the Rabi frequency of the LF field G_{s1} and the detuning Δ_s for the antisymmetric[symmetric] case. (c)[(d)] Steady-state concurrence versus the detunings Δ_s and Δ for the antisymmetric[symmetric] case and $G_{s1} = 3.9\gamma$. The rest of parameters as in Fig. 2.

QDs the better as shown in Fig. 4(c).

We have also analyzed whether the initial state influences the steady-state value of concurrence. To this end, we have chosen the cases in which the two-qubit are prepared in different Bell states: $|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|2\rangle_2 \pm |2\rangle_1|1\rangle_2)$, and $|\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|1\rangle_1|1\rangle_2 \pm |2\rangle_1|2\rangle_2)$. The parameters used for the simulation were selected as the ones used to produce Fig. 3 except for the initial condition, and the results are presented in Fig. 5(a) for the case with $G_{s1} = -G_{s2}$ and in Fig. 5(b) for the in-phase driving ($G_{s1} = +G_{s2}$). Here we can appreciate that for the four cases considered the concurrence exhibits a convergence to the same final value, whereas at the initial stage of the time evolution we observe the death and revival for the cases $|\Psi_{+}\rangle$ and $|\Phi_{\pm}\rangle$. In the case of the antisymmetric initial state the concurrence exhibits an oscillatory behavior.

In order to explain the results obtained with the full Hamiltonian, in the next Section we are going to derive an effective model suitable for weakly driven dissipative systems.

4. Discussion: effective Hamiltonian in the weak qubit-cavity coupling regime

To gain insight and better understanding of the above results, we proceed to derive an effective model describing the two-qubit subsystem in the weak qubit-cavity regime. The main advantage of an effective Hamiltonian approach in that weak regime is

Figure 5. Time dependence of the concurrence between the two qubits for different initial state: $|\Psi_+\rangle$ (solid curve), $|\Psi_-\rangle$ (dashed-dotted curve), $|\Phi_+\rangle$ (dashed curve) and $|\Phi_-\rangle$ (dotted curve). (a)[(b)] The Rabi frequency of the LF field is set to $G_{s1} = -G_{s2} = 3.9\gamma$ [$G_{s1} = +G_{s2} = 3.9\gamma$] and the rest of parameters as in Fig. 3.

that, apart from simplifying the calculations, it provides us with an intuitive insight into the processes that are responsible for the evolution of the system, in particular entanglement-generation mechanisms. Since our conditions deviate from the mentioned weak coupling regime, the effective approach becomes approximate and eventually could give wrong results. We are in fact in an intermediate regime and nevertheless the effective Hamiltonian gives results very close to those of the exact calculation, as we shall see below, which tells us that the effective Hamiltonian can capture the main behavior of the system.

We assume a symmetric spatial configuration of the two QDs with respect to the cavity center, resulting in equal coupling strengths $g_1 = g_2 \equiv g$ for the two QDs. The key idea is to eliminate the field operators of the cavity mode in Eq. (24). To this end we assume that the atom-cavity coupling is relatively weak, so that $\kappa \gg g \gg \gamma_j$, ($j = M, m, 0$), whereas g/κ is finite. These conditions imply that the cavity mode response to the reservoir is much faster than that produced by its interaction with the QDs, which means that the emitted photons cannot react back with the QDs, i.e., the Born approximation is valid. The condition $\kappa \gg g \gg \gamma$ also indicates that the two-atom system has a short memory, i.e., the Markoff approximation holds. We assume that at $t = 0$ the density matrix $\rho(0)$ can be factorized into a product of atoms and field density operators. In this case, we can adiabatically eliminate the field operator of the cavity mode in Eqs. (25), giving rise to a master equation for the atomic variables only. According to adiabatic elimination, the Hamiltonian for the present regime is transformed through the unitary operator $U_4 = e^{\lambda_D}$ where

$$\lambda_D = \sum_{j=1}^2 \left(\lambda_j R_+^{(j)} a - \lambda_j^* a^\dagger R_-^{(j)} \right), \quad (29)$$

λ_j being

$$\lambda_j = -i \frac{g_j}{\kappa/2 + i(\Omega_{Rj} - \omega_s)}. \quad (30)$$

Using the Baker-Campbell-Hausdorff relation, the transformed Hamiltonian could be expanded as

$$H^{(4)} = H^{(3)} + [\lambda_D, H^{(3)}] + \frac{1}{2!} [\lambda_D, [\lambda_D, H^{(3)}]] + \dots \quad (31)$$

The expansion in Eq. (31) indicates that the third and higher-order terms in λ_j involve coupling between photons and excitons, whereas the second-order term includes the coupling between the two QDs. As far as $\lambda_j \ll 1$, we can safely ignore the third and higher order terms and trace over the photon bath operators taking into account the bath properties

$$\begin{aligned} \langle a^\dagger(\omega) \rangle &= \langle a(\omega) \rangle = 0, \\ \langle a^\dagger(\omega') a(\omega) \rangle &= 0, \\ \langle a(\omega') a^\dagger(\omega) \rangle &= \delta(\omega' - \omega). \end{aligned} \quad (32)$$

In doing that we arrive at the master equation

$$\frac{\partial \rho_{QD}^{(4)}}{\partial t} = -\frac{i}{\hbar} [H_{eff}, \rho_{QD}^{(3)}] + L_{QD}^{(4)} \rho_{QD}^{(4)} + L_c^{(4)} \rho_{QD}^{(4)}, \quad (33)$$

where the effective Hamiltonian reads

$$\begin{aligned} H_{eff} &= \frac{\hbar}{2} \sum_{j=1}^2 \omega_{eff}^{(j)} R_z^j - \hbar \sum_{j=1}^2 \frac{G_{sj}}{2} (R_-^j + R_+^j) \\ &\quad + \hbar \left(J_{eff} R_+^{(1)} R_-^{(2)} + J_{eff}^* R_-^{(1)} R_+^{(2)} \right), \end{aligned} \quad (34)$$

with

$$\omega_{eff}^{(j)} = \Omega_{Rj} - \omega_s - \frac{g_j^2 (\Omega_{Rj} - \omega_s)}{\kappa^2/4 + (\Omega_{Rj} - \omega_s)^2}, \quad (35)$$

$$J_{eff} = g_2 \lambda_1 + g_2 \lambda_1^*. \quad (36)$$

In addition, the unitary transformation $U^{(4)}$ induce new decay terms between the dresses states, namely

$$\begin{aligned} L_c^{(4)} \rho &= -\frac{\Gamma_j^M}{2} \sum_j \left[R_+^{(j)} R_-^{(j)} \rho - 2 R_-^{(j)} \rho R_+^{(j)} + \rho R_+^{(j)} R_-^{(j)} \right] \\ &\quad - \frac{\Gamma_{ij}^M}{2} \sum_{i \neq j} \left[R_+^{(i)} R_-^{(j)} \rho - 2 R_-^{(j)} \rho R_+^{(i)} + \rho R_+^{(i)} R_-^{(j)} \right], \end{aligned}$$

where the coefficients read

$$\begin{aligned} \Gamma_j^M &= \frac{\kappa_j |\lambda_j|^2}{2} = \frac{\kappa_j}{2} \frac{g_i^2}{\kappa^2/4 + (\Omega_{Rj} - \omega_s)^2}, \\ \Gamma_{ij}^M &= \frac{\kappa_j \lambda_i \lambda_j^*}{2}, \end{aligned}$$

and finally $L_{QD}^{(4)} \rho_{QD}^{(4)} = U_4^\dagger L_{QD}^{(3)} \rho_{QD}^{(3)} U_4$.

The different coefficients appearing in Eq. (34) have a clear interpretation, namely: (i) the interaction with the cavity mode shifts the transition frequency of the QDs according to Eq. (35); and (ii) the cavity mode generates a dipole-dipole interaction J_{eff} (as given by Eq. (36)) between the two qubits and creates coherence between the two qubits which is essential for the generation of entanglement in this system. Note that the effective decay dynamics for the QDs is also modified by the cavity as indicated by Eq. (37). In addition, the decay rate of the excited state of the j -th qubit is significantly enhanced, whereas new crossing decay rates Γ_j^M and Γ_{ij}^M emerge in the effective model as a consequence of the QD-cavity interaction, which is absent in the dynamics equations for a vacuum-mediated two-QD system.

In order to see that the effective model explains the features of Figs. 2-4, we derive the optical Bloch equations for the density matrix elements $\rho_{nm}(t) = \text{Tr}_S(\rho(t)R_{mn})$, according to Eq. (33). The common basis to obtain the different matrix density elements is given by $|11\rangle$, $|12\rangle$, $|21\rangle$, and $|22\rangle$. However, the effective Hamiltonian (34) takes a more transparent form in the so-called Dicke basis given by $|G\rangle = |1\rangle_1|1\rangle_2$, $|E\rangle = |2\rangle_1|2\rangle_2$, $|S\rangle = \frac{1}{\sqrt{2}}[|1\rangle_1|2\rangle_2 + |2\rangle_1|1\rangle_2]$, and $|A\rangle = \frac{1}{\sqrt{2}}[|1\rangle_1|2\rangle_2 - |2\rangle_1|1\rangle_2]$. In this basis, the effective Hamiltonian reduces to

$$\begin{aligned} H_{eff} = & \hbar \sum_{j=E,S,A,G} \omega^{(j)} |j\rangle\langle j| + \Delta (|A\rangle\langle S| + |S\rangle\langle A|) \\ & + \frac{1}{\sqrt{2}}(G_{s2} + G_{s1})(|S\rangle\langle G| + |E\rangle\langle S|) \\ & + \frac{1}{\sqrt{2}}(G_{s2} - G_{s1})(|A\rangle\langle G| - |E\rangle\langle A|) + \text{H.c.} , \end{aligned} \quad (37)$$

where

$$\begin{aligned} \omega_E &= 2\Delta_s , \\ \omega_s &= \Delta_s + J_{eff} , \\ \omega_a &= \Delta_s - J_{eff} , \end{aligned} \quad (38)$$

and

$$\begin{aligned} \Delta_s &= \frac{\omega_{eff}^{(1)} + \omega_{eff}^{(2)}}{2} , \\ \Delta &= \frac{\omega_{eff}^{(1)} - \omega_{eff}^{(2)}}{2} . \end{aligned} \quad (39)$$

The population dynamics can be obtained from the density matrix equations which are fully reproduced in Appendix 5. In the case that $\Delta_s = \Delta = 0$, which is considered in Figs. 2-4, we obtain that $\Gamma_1^M = \Gamma_2^M = \Gamma^M$, and $\Gamma_{12}^M = \Gamma_{21}^M$, while the time evolution of the symmetrical and antisymmetrical populations are given by

$$\begin{aligned} \dot{\rho}_{SS}(t) = & -(\Gamma^M + \Gamma_{12}^M + \gamma_M + 2\gamma_m + 2\gamma_0) \rho_{SS} + (\Gamma^M + \Gamma_{12}^M + \gamma_M) \rho_{EE} \\ & - \frac{i}{\sqrt{2}}(G_{s1} + G_{s2})(\rho_{GS} - \rho_{SG} + \rho_{ES} - \rho_{SE}) + \gamma_m \rho_{GG} , \end{aligned}$$

$$\begin{aligned}
\dot{\rho}_{AA}(t) = & -(\Gamma^M - \Gamma_{12}^M + \gamma_M + \gamma_m + 2\gamma_0) \rho_{AA} + (\Gamma^M - \Gamma_{12}^M + \gamma_M) \rho_{EE} \\
& - \frac{i}{\sqrt{2}}(G_{s2} - G_{s1})(\rho_{GA} - \rho_{AG} + \rho_{AE} - \rho_{EA}) \\
& + 2\gamma_0 \rho_{SS} + \gamma_m \rho_{GG} .
\end{aligned} \tag{40}$$

In order to verify the validity of the effective Hamiltonian, we represent in Figs. 6(a) and 6(c) the values of the different elements of the steady-state density matrix elements obtained from the the full Hamiltonian and with the effective Hamiltonian (Figs. 6(b) and 6(d)), for one of the cases considered in Fig. 3(a).

Figure 6. Density matrix elements at steady-state divided in real elements (a) and (b) and imaginary parts (c) and (d). The full solution is obtained from the Hamiltonian in Eq. (24), whereas the other solution is obtained from the effective Hamiltonian in Eq. (34). The Rabi frequency of the LF field is set to $G_{s1} = -G_{s2} = 2\gamma$ and the rest of parameters as in Fig. 3(a).

It can be seen that there is a close match between the full and the effective solutions for both the real and imaginary parts of the density matrix elements. In addition, when using the effective Hamiltonian we have obtained very similar results for the concurrence as the ones obtained with the full model in Fig. 3 and Fig. 4, so we conclude that the effective model is appropriate in the regime considered in this work.

On the other hand, the derived Hamiltonian in the Dicke basis Eq. (37) in conjunction with Eq. (40) provide us with physical intuition on the dynamics of the atomic system: energy shifts, coherent couplings and decaying processes. All these magnitudes are indicated in Fig. 7.

It can be seen in Fig.(7) that an energy shift appears between the symmetric and antisymmetric state given by J_{eff} , and a coherent coupling between the symmetric and antisymmetric states takes place in the case when $\Delta \neq 0$, i.e., when the transition frequencies of the QDs are different. In addition, the ground state $|G\rangle$ can be connected with excited state $|E\rangle$ via two different ways, depending on an effective coherent coupling given by $\frac{1}{\sqrt{2}}(G_{s2} + G_{s1})$ or $\frac{1}{\sqrt{2}}(G_{s2} - G_{s1})$, respectively, as expressed

Figure 7. The two-qubit cooperative states $|i\rangle$ where $i = E, S, A, G$, stand for excited, symmetrical, antisymmetrical and ground Dicke states, respectively. (a) Energy shifts J_{eff} , interaction Δ , and coherent couplings are indicated. (b) Superradiant and subradiant dissipation channels: $\gamma_S = \Gamma^M + \Gamma_{12}^M + \gamma_M + 2\gamma_m + 2\gamma_0$, and $\gamma_{\text{sub}} = \Gamma^M - \Gamma_{12}^M + \gamma_M + \gamma_m + 2\gamma_0$.

in Eq. (40). In Fig. 7(b) we depict the two decay channels from the excited state: a superradiant channel via the symmetric to the ground state, with decay rate $\gamma_S = \Gamma^M + \Gamma_{12}^M + \gamma_M + 2\gamma_m + 2\gamma_0$; and a subradiant channel via the antisymmetric state with decay rate $\gamma_{\text{sub}} = \Gamma^M - \Gamma_{12}^M + \gamma_M + \gamma_m + 2\gamma_0$.

In order to explain the numerical results displayed in Fig. 2-3, an approximate expression for the concurrence can be obtained in the effective picture in the case of a weak optical driving field. Following Ref. [32] the concurrence reads

$$C \simeq \sqrt{(\rho_{SS} - \rho_{AA})^2 + 4\text{Im}(\rho_{AS})} - 2\sqrt{\rho_{GG} - \rho_{EE}}. \quad (41)$$

Equation (41) indicates us that the entanglement can attain large values when the two-qubit system is driven into either the symmetric or antisymmetric state with a high probability, and simultaneously the probability of the excited state $|E\rangle$ remains low. Thus, it is evident that depending on whether $G_{s2} = -G_{s1}$ or $G_{s2} = G_{s1}$, the concurrence will take very different values. In the former case, the coherent transition $|G\rangle \rightarrow |S\rangle \rightarrow |E\rangle$ vanishes, and only the direct transition $|G\rangle \rightarrow |A\rangle \rightarrow |E\rangle$ is allowed. In the present work, the superradiant decay rate is $\gamma_S \simeq 40\gamma_{\text{sub}}$, so the antisymmetric state is a quasi-nondecaying state. This in turn leads the accumulation of population into state $|A\rangle$ in the course of time. For the case with $\Delta = 0$, a direct transition between state $|A\rangle$ to symmetric state $|S\rangle$ is forbidden, so the steady-state population ρ_{EE} is nearly zero. This explains the results depicted in Fig. 2(a)-(c). Moreover, by inspecting Eq. (40) we can see that if $G_{s1} = -G_{s2}$, the time evolution of the population of the symmetric state ρ_{SS} does not depend on the Rabi frequency of the LF field, and the dominant term is the decaying term (proportional to ρ_{SS}), which explains the fact that ρ_{SS} does not change significantly in time and is near zero at steady-state. On the other hand, this population depends on the driving LF field through the term $\frac{2i}{\sqrt{2}}(G_{s1})(\rho_{GA} - \rho_{AG} + \rho_{AE} - \rho_{EA})$. The antisymmetric state is populated more efficiently than the symmetric one, which explains the fact that the population of the $|A\rangle$ state reaches a high value at steady-state. This can be clearly seen in Fig.8(a)

Figure 8. Steady state population of the symmetric, antisymmetric, and ground states as a function of the Rabi frequency of the LF field. (a) $G_{s2} = -G_{s1}$, and (b) $G_{s2} = G_{s1}$. The rest of parameters as in Fig. 3.

where steady-state population of the symmetric and antisymmetric states are shown as a function of the Rabi frequency of the LF field, which are obtained by solving the corresponding equations derived for the effective model. We notice that for low values of G_{s1} the population ρ_{AA} increases monotonically until it reaches a maximum, while the population of the symmetric state remains close to zero. This is due to the fact that only the direct transition $|G\rangle \rightarrow |A\rangle \rightarrow |E\rangle$ is effective and, simultaneously, the state $|E\rangle$ depopulates because of the high value of γ_S . In the case considered, $\Gamma^M - \Gamma_{12}^M = 0$, thus, if $\frac{1}{2\sqrt{2}}G_{s1} > \gamma_M + \gamma_m + 2\gamma_0$, the population of ρ_{AA} saturates. Since $\gamma_M = \gamma_m = \gamma_0 = \frac{\gamma}{4}$, the above condition results in $G_{s1} > 3.5\gamma$, which nearly corresponds to the value obtained in the numerical simulations. It is worth noting that the behavior of the population ρ_{aa} in Fig. 8(a) is similar that of the concurrence displayed in Fig. 3(b). In fact, this behavior follows from Eq.(41), where $C \simeq |\rho_{AA} - \rho_{SS}|$ since ρ_{EE} is nearly zero (see Fig.6(b)).

In Fig. 8(b) we consider the case $G_{s1} = G_{s1}$. This situation is completely different: only the transitions $|G\rangle \rightarrow |S\rangle \rightarrow |E\rangle$ is coupled to the LF field, whereas the faster decaying processes are governed by the superradiant component, which affects the same transitions. This means that the other transition $|G\rangle \rightarrow |A\rangle \rightarrow |E\rangle$ is decoupled. This explain why the population ρ_{AA} remains nearly constant for all values of G_{s1} . On the other hand, for low values of G_{s1} , the population accumulates in the lower state $|G\rangle$ (see Fig. 8(b)). In any case it can be seen that the difference $|\rho_{AA} - \rho_{SS}|$ keeps always lower values than in the previous case and approaches zero for high values of G_s . This fact explains the lower values of the concurrence shown in Fig. 2(a). Finally, the result shown in Fig. 3(c) explores the influence of the parameter Δ on the concurrence. We note that the maximum for the concurrence is obtained in the case with $\Delta = 0$. This result can be explained by considering the fact that when $\Delta \neq 0$ a non-vanishing coupling between the $|S\rangle$ and $|A\rangle$ states appears which in turn results into an equalization of their populations, thus a decrease in the difference $|\rho_{AA} - \rho_{SS}|$ is produced and, consequently, the concurrence takes lower values than in the previous case. Here we want to highlight that the effective Hamiltonian not only provides us with physical explanations of the numerical findings but gives similar numerical results

in the considered range of parameters.

5. Conclusions

In this work we have proposed a scheme for preserving and controlling entanglement in the steady state between a pair of QDs with broken inversion symmetry coupled to a photonic cavity and driven by a bichromatic field. Due to the broken inversion symmetry, not only transverse coupling but also longitudinal coupling with external fields takes place. The complete master equation is solved numerically. We have analyzed the effects on entanglement of different parameters that can be experimentally controlled. It is found that concurrence can be tailored by changing the amplitude of the LF field, and by changing the energy difference of the dressed states via the Rabi frequency of the strong optical field. An effective Hamiltonian for the atomic subsystem is derived, which allows us to explain the obtained features. It is worth mentioning that the system considered in this work avoid the extremely precise control of positioning of the quantum emitters which is required in systems where entanglement is obtained through the engineering of long-range dipole-dipole interaction.

Finally, it should be stated that, although speculative, the hybrid system here investigated can be implemented in realistic QD-cavity systems. Nitride based QDs may exhibit pronounced piezoelectric effect with strengths in the order of several MV/cm which results in a value of $\mu_0 \approx 200$ Debye for a typical 4.1 nm in height QD [97]. Values of μ_0 up to 20 Debye has been reported in self-assembled InAs-GaAs [98]. Another possible realization involves the use of gammaglobulin macromolecules where values of $\mu_0 \approx 100$ Debye have been considered [99]. On the other hand, the QD-cavity interaction strength is proportional to the value of μ_0 as given in Eq. (6), thus the values for the rest of the parameters used in our simulations can be achieved by making use of cavities with quality factor in the order of $Q = 10^4$ like the one used in Ref. [100]. In addition, it is worth mentioning that the possibility of achieving the double dressing of a QD using a bichromatic field has been experimentally observed in self-assembled InGaAs QDs embedded in a low-Q planar microcavity [101]. Thus, the structure modeled here could be fabricated using available nano-technologies for the growth and the precise positioning of the involved elements.

The results predicted in this work may have a potential application in the field of quantum information processing and control of quantum entanglement in cavity QED experiments.

Appendix Density matrix equations

From Eqs.(34), it is straightforward to obtain the equations of evolution for the elements of the density matrix in the Dicke basis:

$$\dot{\rho}_{EE}(t) = - [\Gamma_1^M + \Gamma_2^M + 2\gamma_M] \rho_{EE} + \gamma_m \rho_{SS} + \gamma_m \rho_{AA}$$

$$\begin{aligned}
& -\frac{i}{\sqrt{2}}[(G_{s1} + G_{s2})\rho_{SE} - (G_{s2} - G_{s1})\rho_{AE}] \\
& +\frac{i}{\sqrt{2}}[(G_{s1} + G_{s2})\rho_{ES} - (G_{s2} - G_{s1})\rho_{EA}] , \\
\dot{\rho}_{SS}(t) = & -\frac{1}{2}[\Gamma_1^M + \Gamma_2^M + \Gamma_{12}^M + \Gamma_{21}^M + 2\gamma_M + 2\gamma_m]\rho_{SS} \\
& +\frac{1}{2}(\Gamma_1^M + \Gamma_2^M + \Gamma_{12}^M + \Gamma_{21}^M + 2\gamma_M)\rho_{EE} + \gamma_m\rho_{GG} \\
& +\frac{1}{4}[\Gamma_1^M - \Gamma_2^M + \Gamma_{12}^M - \Gamma_{21}^M]\rho_{SA} \\
& +\frac{1}{4}[\Gamma_1^M - \Gamma_2^M - \Gamma_{12}^M + \Gamma_{21}^M]\rho_{AS} \\
& -i\frac{J - J^*}{2}(\rho_{AS} + \rho_{SA}) + i\Delta(\rho_{AS} - \rho_{SA}) \\
& -\frac{i}{\sqrt{2}}(G_{s1} + G_{s2})(\rho_{GS} - \rho_{SE}) \\
& +\frac{i}{\sqrt{2}}(G_{s1} + G_{s2})(\rho_{SG} - \rho_{ES}) - 2\gamma_0(\rho_{SS} - \rho_{AA}) , \\
\dot{\rho}_{AA}(t) = & -\frac{1}{2}[\Gamma_1^M + \Gamma_2^M - \Gamma_{12}^M - \Gamma_{21}^M + \Gamma_1^m + 2\gamma_M + 2\gamma_m]\rho_{AA} \\
& +\frac{1}{2}(\Gamma_1^M + \Gamma_2^M - \Gamma_{12}^M - \Gamma_{21}^M + 2\gamma_M)\rho_{EE} + \gamma_m\rho_{GG} \\
& +\frac{1}{4}[\Gamma_1^M - \Gamma_2^M + \Gamma_{12}^M - \Gamma_{21}^M]\rho_{SA} \\
& +\frac{1}{4}[\Gamma_1^M - \Gamma_2^M - \Gamma_{12}^M + \Gamma_{21}^M]\rho_{AS} \\
& -i\Delta(\rho_{AS} - \rho_{SA}) + i\frac{J - J^*}{2}(\rho_{AS} + \rho_{SA}) \\
& -\frac{i}{\sqrt{2}}(G_{s2} - G_{s1})(\rho_{GA} + \rho_{AE}) \\
& +\frac{i}{\sqrt{2}}(G_{s2} - G_{s1})(\rho_{AG} + \rho_{EA}) + 2\gamma_0(\rho_{SS} - \rho_{AA}) , \tag{A.1}
\end{aligned}$$

and the coherences

$$\begin{aligned}
\dot{\rho}_{AS}(t) = & -\frac{1}{2}[\Gamma_1^M + \Gamma_2^M + 2\gamma_M + 2\gamma_m - 2i(J + J^*)]\rho_{AS} \\
& +i\frac{(J - J^*)}{2}(\rho_{SS} - \rho_{AA}) + i\Delta(\rho_{SS} - \rho_{AA}) \\
& +\frac{1}{2}(\Gamma_{21}^M - \Gamma_{12}^M)\rho_{EE} + \frac{1}{4}(\Gamma_1^M - \Gamma_2^M + \Gamma_{12}^M - \Gamma_{21}^M)\rho_{SS} \\
& -\frac{1}{2}(\Gamma_1^M + \Gamma_2^M + 2\gamma_M + 2\gamma_m)\rho_{AS} + \frac{i}{\sqrt{2}}(G_{s2} + G_{s1})\rho_{AE} \\
& -\frac{i}{\sqrt{2}}(G_{s2} - G_{s1})\rho_{GS} + \frac{i}{\sqrt{2}}(G_{s2} - G_{s1})\rho_{ES} \\
& +\frac{i}{\sqrt{2}}(G_{s2} + G_{s1})\rho_{AG} - 2\gamma_0(\rho_{AS} - \rho_{SA}) ,
\end{aligned}$$

$$\begin{aligned}
\dot{\rho}_{SE}(t) = & -\frac{1}{2} \left[\Gamma_1^M + \Gamma_2^M + \frac{1}{2}(\Gamma_{12}^M + \Gamma_{21}^M) + 2\gamma_M + \gamma_m - i(2\Delta_s - (J + J^*)) \right] \rho_{SE} \\
& - \gamma_m \rho_{GS} - \frac{1}{4} [\Gamma_{21}^M - \Gamma_{12}^M] \rho_{AE} + i\Delta \rho_{AE} \\
& - \frac{i}{2} (J - J^*) \rho_{AE} + \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) (\rho_{SS} - \rho_{EE}) \\
& - \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) \rho_{SA} - \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) \rho_{GE} - 2\gamma_0 \rho_{SE} , \\
\dot{\rho}_{AE}(t) = & -\frac{1}{2} \left[(\Gamma_1^M + \Gamma_2^M) - \frac{\Gamma_{12}^M + \Gamma_{21}^M}{2} + 2\gamma_M + 2\gamma_m - i(2\Delta_s + J + J^*) \right] \rho_{AE} \\
& - \gamma_m \rho_{GA} + i \frac{(J - J^*)}{2} \rho_{SE} + i\Delta \rho_{SE} - \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) (\rho_{AA} - \rho_{EE}) \\
& + \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) \rho_{AS} - \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) \rho_{GE} - 2\gamma_0 \rho_{AE} , \\
\dot{\rho}_{GS}(t) = & -\frac{1}{2} \left[(\Gamma_1^M + \frac{\Gamma_1^M + \Gamma_2^M}{2} + 2\gamma_M + 3\gamma_m - i(2\Delta_s + J + J^*)) \right] \rho_{GS} \\
& + \frac{1}{2} [\Gamma_1^M + \Gamma_2^M + \Gamma_{12}^M + \Gamma_{21}^M + 2\gamma_M] \rho_{SE} \\
& + \frac{1}{2} [\Gamma_2^M - \Gamma_1^M + \Gamma_{12}^M - \Gamma_{21}^M] \rho_{AE} \\
& + \frac{1}{2} \left[+\frac{1}{2}(\Gamma_{12}^M - \Gamma_{21}^M) \right] \rho_{GA} + i \frac{J^* - J}{2} \rho_{GA} - i\Delta \rho_{GA} \\
& - \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) (\rho_{SS} - \rho_{GG}) - \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) \rho_{AS} \\
& + \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) \rho_{GE} - 2\gamma_0 \rho_{GS} , \\
\dot{\rho}_{GA}(t) = & -\frac{1}{2} \left[(\Gamma_1^M - \frac{\Gamma_{12}^M + \Gamma_{21}^M}{2} + \gamma_M + 3\gamma_m - i(2\Delta_s + J + J^*)) \right] \rho_{GA} \\
& + \frac{1}{2} [\Gamma_1^M - \Gamma_2^M + \Gamma_{12}^M - \Gamma_{21}^M] \rho_{SE} \\
& - \frac{1}{2} [\Gamma_1^M + \Gamma_2^M - \Gamma_{12}^M - \Gamma_{21}^M + 2\gamma_M] \rho_{AE} \\
& + \frac{1}{2} \left[-\frac{1}{2}(\Gamma_{12}^M - \Gamma_{21}^M) \right] \rho_{GS} + i \frac{J - J^*}{2} \rho_{GS} - i\Delta \rho_{GS} \\
& - \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) (\rho_{AA} - \rho_{GG}) - \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) \rho_{GE} \\
& - \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) \rho_{SA} - 2\gamma_0 \rho_{GA} , \\
\dot{\rho}_{EG}(t) = & -\frac{1}{2} (\Gamma_1^M + \Gamma_2^M + 2\gamma_M + 2\gamma_m + 4i\Delta_s) \rho_{EG} \\
& + \frac{i}{\sqrt{2}} (G_{s2} + G_{s1}) (\rho_{ES} - \rho_{SG}) \\
& + \frac{i}{\sqrt{2}} (G_{s2} - G_{s1}) (\rho_{AG} + \rho_{EA}) - 4\gamma_0 \rho_{EG} , \tag{A.2}
\end{aligned}$$

with $\rho_{GG}(t) + \rho_{EE}(t) + \rho_{SS}(t) + \rho_{AA}(t) = 1$ and $\rho_{nm}(t) = \rho_{mn}^*(t)$. In addition, the detunings Δ and Δ_s become tuning parameters which can be modified externally through the Rabi frequencies of the optical and LF fields, namely, $\Delta = \frac{\Omega_{R1} - \Omega_{R2}}{2}$ and $\Delta_s = \frac{\Omega_{R1} + \Omega_{R2}}{2} - \omega_s$. The detuning Δ reflects the similarity between the QDs (for $\Delta = 0$, the transition frequencies of the QDs are the same), while Δ_s represents the detuning of the LF field from the dressed states. Another important fact is that the decay rates Γ_{ij}^M take complex values when $\Delta \neq 0$, so the imaginary part of these parameters introduces additional detunings.

For the case $\Delta = \Delta_s = 0$, Eqs. in (40) are obtained. It worth noting that if we make $\gamma_M = \gamma_m = \gamma_0 = \Gamma_m = \Gamma_{12}^m = 0$, and $G_{1s} = G_{2s} = 0$ we recover the equations presented in Ref. [32].

Acknowledgments

The authors acknowledge funding through project FIS2017-87360-P (MICINN). I. Gonzalo also acknowledges the support from project FIS2016-76110-P (MICINN).

References

- [1] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74**(20) 4091–4094 URL <https://link.aps.org/doi/10.1103/PhysRevLett.74.4091>
- [2] Cirac J I and Zoller P 2000 *Nature* **404** 579–581 ISSN 1476-4687 URL <https://doi.org/10.1038/35007021>
- [3] Mermin N D 1985 *Phys. Today* **38** 38 URL <https://doi.org/10.1063/1.880968>
- [4] Bennett C H and DiVincenzo D P 2000 *Nature* **404** 247–255 ISSN 1476-4687 URL <https://doi.org/10.1038/35005001>
- [5] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70**(13) 1895–1899 URL <https://link.aps.org/doi/10.1103/PhysRevLett.70.1895>
- [6] Gisin N, Ribordy G, Tittel W and Zbinden H 2002 *Rev. Mod. Phys.* **74**(1) 145–195 URL <https://link.aps.org/doi/10.1103/RevModPhys.74.145>
- [7] Paz JP Z W 2002 Environment-induced decoherence and the transition from quantum to classical *Fundamentals of Quantum Information. Lecture Notes in Physics* ed Heiss D (Berlin Heidelberg: Springer) chap 587, pp 77–148
- [8] Shor P W 1995 *Phys. Rev. A* **52**(4) R2493–R2496 URL <https://link.aps.org/doi/10.1103/PhysRevA.52.R2493>
- [9] Zanardi P and Rasetti M 1997 *Phys. Rev. Lett.* **79**(17) 3306–3309 URL <https://link.aps.org/doi/10.1103/PhysRevLett.79.3306>
- [10] Tombesi P and Vitali D 1995 *Phys. Rev. A* **51**(6) 4913–4917 URL <https://link.aps.org/doi/10.1103/PhysRevA.51.4913>
- [11] Ban M 1998 *J. Mod. Opt.* **45** 2315–2325 ISSN 0950-0340 URL <https://doi.org/10.1080/09500349808231241>
- [12] Maniscalco S, Francica F, Zaffino R L, Lo Gullo N and Plastina F 2008 *Phys. Rev. Lett.* **100**(9) 090503 URL <https://link.aps.org/doi/10.1103/PhysRevLett.100.090503>
- [13] Das S and Agarwal G S 2009 *J. Phys. B: At. Mol. Opt. Phys.* **42** 141003
- [14] Mundarain D and Orszag M 2009 *Phys. Rev. A* **79**(5) 052333 URL <https://link.aps.org/doi/10.1103/PhysRevA.79.052333>
- [15] Kastoryano M J, Reiter F and Sørensen A S 2011 *Phys. Rev. Lett.* **106**(9) 090502 URL <https://link.aps.org/doi/10.1103/PhysRevLett.106.090502>

- [16] Marr C, Beige A and Rempe G 2003 *Phys. Rev. A* **68**(3) 033817 URL <https://link.aps.org/doi/10.1103/PhysRevA.68.033817>
- [17] Reiter F, Kastoryano M J and Sørensen A S 2012 *New J. Phys.* **14** 053022 URL <https://doi.org/10.1088%2F1367-2630%2F14%2F5%2F053022>
- [18] Man Z X, Xia Y J and Lo Franco R 2015 *Sci. Rep.* **5** 13843 EP – article URL <https://doi.org/10.1038/srep13843>
- [19] Majer J, Chow J M, Gambetta J M, Koch J, Johnson B R, Schreier J A, Frunzio L, Schuster D I, Houck A A, Wallraff A, Blais A, Devoret M H, Girvin S M and Schoelkopf R J 2007 *Nature* **449** 443 EP – URL <https://doi.org/10.1038/nature06184>
- [20] Yönaç M, Yu T and Eberly J H 2006 *J. Phys. B: At. Mol. Opt. Phys.* **39** S621–S625 URL <https://doi.org/10.1088%2F0953-4075%2F39%2F15%2Fs09>
- [21] Al-Amri M, Li G x, Tan R and Zubairy M S 2009 *Phys. Rev. A* **80**(2) 022314 URL <https://link.aps.org/doi/10.1103/PhysRevA.80.022314>
- [22] Mazzola L, Maniscalco S, Piilo J, Suominen K A and Garraway B M 2009 *Phys. Rev. A* **79**(4) 042302 URL <https://link.aps.org/doi/10.1103/PhysRevA.79.042302>
- [23] Clark S G and Parkins A S 2003 *Phys. Rev. Lett.* **90**(4) 047905 URL <https://link.aps.org/doi/10.1103/PhysRevLett.90.047905>
- [24] Duan L M and Kimble H J 2003 *Phys. Rev. Lett.* **90**(25) 253601 URL <https://link.aps.org/doi/10.1103/PhysRevLett.90.253601>
- [25] Plenio M B and Huelga S F 2002 *Phys. Rev. Lett.* **88**(19) 197901 URL <https://link.aps.org/doi/10.1103/PhysRevLett.88.197901>
- [26] Xu J B and Li S B 2005 *New J. Phys.* **7** 72–72 URL <https://doi.org/10.1088%2F1367-2630%2F7%2F1%2F072>
- [27] Malinovsky V S and Sola I R 2004 *Phys. Rev. Lett.* **93**(19) 190502 URL <https://link.aps.org/doi/10.1103/PhysRevLett.93.190502>
- [28] Chen Z, Zhou Y, and Shen J T 2016 *Opt. Lett.* **41**(14) 3313 URL <https://doi.org/10.1364/OL.41.003313>
- [29] Chen Z, Zhou Y, and Shen J T 2017 *Phys. Rev. A* **96** (5) 053805 URL <https://doi.org/10.1103/PhysRevA.96.053805>
- [30] Chen Z, Zhou Y, and Shen J T 2017 *Opt. Lett.* **42**(4) 887 URL <https://doi.org/10.1364/OL.42.000887>
- [31] Chen Z, Zhou Y, and Shen J T 2018 *Phys. Rev. A* **98** (5) 053830 URL <https://doi.org/10.1103/PhysRevA.98.053830>
- [32] Ficek Z and Tanaś R 2008 *Phys. Rev. A* **77**(5) 054301 URL <https://link.aps.org/doi/10.1103/PhysRevA.77.054301>
- [33] Almeida M P, de Melo F, Hor-Meyll M, Salles A, Walborn S P, Ribeiro P H S and Davidovich L 2007 *Science* **316** 579–582 ISSN 0036-8075 URL <https://science.sciencemag.org/content/316/5824/579>
- [34] Yu T and Eberly J H 2009 *Science* **323** 598–601 ISSN 0036-8075 URL <https://science.sciencemag.org/content/323/5914/598>
- [35] Bellomo B, Lo Franco R and Compagno G 2007 *Phys. Rev. Lett.* **99**(16) 160502 URL <https://link.aps.org/doi/10.1103/PhysRevLett.99.160502>
- [36] Kok P, Munro W J, Nemoto K, Ralph T C, Dowling J P and Milburn G J 2007 *Rev. Mod. Phys.* **79**(1) 135–174 URL <https://link.aps.org/doi/10.1103/RevModPhys.79.135>
- [37] Milburn G J 2009 *Phys. Scr.* **T137** 014003 URL <https://doi.org/10.1088%2F0031-8949%2F2009%2Ft137%2F014003>
- [38] Bloch I 2008 *Nature* **453** 1016 EP – URL <https://doi.org/10.1038/nature07126>
- [39] Lukin M D 2003 *Rev. Mod. Phys.* **75**(2) 457–472 URL <https://link.aps.org/doi/10.1103/RevModPhys.75.457>
- [40] Blatt R and Wineland D 2008 *Nature* **453** 1008 EP – URL <https://doi.org/10.1038/nature07125>
- [41] Hanson R and Awschalom D D 2008 *Nature* **453** 1043 EP – URL

- <https://doi.org/10.1038/nature07129>
- [42] Wrachtrup J and Jelezko F 2006 *J. Phys.: Cond. Matter* **18** S807–S824 URL <https://doi.org/10.1088%2F0953-8984%2F18%2F21%2Fs08>
 - [43] Doherty M W, Dolde F, Fedder H, Jelezko F, Wrachtrup J, Manson N B and Hollenberg L C L 2012 *Phys. Rev. B* **85**(20) 205203 URL <https://link.aps.org/doi/10.1103/PhysRevB.85.205203>
 - [44] Galiatdinov A, Korotkov A N and Martinis J M 2012 *Phys. Rev. A* **85**(4) 042321 URL <https://link.aps.org/doi/10.1103/PhysRevA.85.042321>
 - [45] Harris R, Berkley A J, Johnson M W, Bunyk P, Govorkov S, Thom M C, Uchaikin S, Wilson A B, Chung J, Holtham E, Biamonte J D, Smirnov A Y, Amin M H S and Maassen van den Brink A 2007 *Phys. Rev. Lett.* **98**(17) 177001 URL <https://link.aps.org/doi/10.1103/PhysRevLett.98.177001>
 - [46] Iliopoulos N, Terzis A F, Yannopapas V and Paspalakis E 2016 *Ann. Phys.* **365** 38 – 53 ISSN 0003-4916 URL <http://www.sciencedirect.com/science/article/pii/S0003491615003930>
 - [47] Loss D and DiVincenzo D P 1998 *Phys. Rev. A* **57**(1) 120–126 URL <https://link.aps.org/doi/10.1103/PhysRevA.57.120>
 - [48] Hanson R, Kouwenhoven L P, Petta J R, Tarucha S and Vandersypen L M K 2007 *Rev. Mod. Phys.* **79**(4) 1217–1265 URL <https://link.aps.org/doi/10.1103/RevModPhys.79.1217>
 - [49] Borges H S, Sanz L, Villas-Bôas J M, Diniz Neto O O and Alcalde A M 2012 *Phys. Rev. B* **85**(11) 115425 URL <https://link.aps.org/doi/10.1103/PhysRevB.85.115425>
 - [50] Temirov R, Green M F B, Friedrich N, Leinen P, Esat T, Chmielniak P, Sarwar S, Rawson J, Kögerler P, Wagner C, Rohlfing M and Tautz F S 2018 *Phys. Rev. Lett.* **120**(20) 206801 URL <https://link.aps.org/doi/10.1103/PhysRevLett.120.206801>
 - [51] Gambetta J, Blais A, Boissonneault M, Houck A A, Schuster D I and Girvin S M 2008 *Phys. Rev. A* **77**(1) 012112 URL <https://link.aps.org/doi/10.1103/PhysRevA.77.012112>
 - [52] Fry P W, Itskevich I E, Mowbray D J, Skolnick M S, Finley J J, Barker J A, O'Reilly E P, Wilson L R, Larkin I A, Maksym P A, Hopkinson M, Al-Khafaji M, David J P R, Cullis A G, Hill G and Clark J C 2000 *Phys. Rev. Lett.* **84**(4) 733–736 URL <https://link.aps.org/doi/10.1103/PhysRevLett.84.733>
 - [53] Patanè A, Levin A, Polimeni A, Schindler F, Main P C, Eaves L and Henini M 2000 *Appl. Phys. Lett.* **77** 2979–2981 URL <https://doi.org/10.1063/1.1322631>
 - [54] Warburton R J, Schulhauser C, Haft D, Schäfflein C, Karrai K, Garcia J M, Schoenfeld W and Petroff P M 2002 *Phys. Rev. B* **65**(11) 113303 URL <https://link.aps.org/doi/10.1103/PhysRevB.65.113303>
 - [55] Li L s and Alivisatos A P 2003 *Phys. Rev. Lett.* **90**(9) 097402 URL <https://link.aps.org/doi/10.1103/PhysRevLett.90.097402>
 - [56] Schulhauser C, Haft D, Schäfflein C, Karrai K, Warburton R J, Garcia J, Schoenfeld W and Petroff P M 2002 *Physica E: Low-dimensional Systems and Nanostructures* **13** 161 – 164 ISSN 1386-9477 URL <http://www.sciencedirect.com/science/article/pii/S1386947701005100>
 - [57] Sugisaki M, Ren H W, Nair S V, Nishi K and Masumoto Y 2002 *Phys. Rev. B* **66**(23) 235309 URL <https://link.aps.org/doi/10.1103/PhysRevB.66.235309>
 - [58] Ostapenko I A, Hönig G, Kindel C, Rodt S, Strittmatter A, Hoffmann A and Bimberg D 2010 *Applied Physics Letters* **97** 063103 URL <https://doi.org/10.1063/1.3477952>
 - [59] Bavli R and Band Y B 1991 *Phys. Rev. A* **43**(1) 507–514 URL <https://link.aps.org/doi/10.1103/PhysRevA.43.507>
 - [60] Antón M A and Gonzalo I 1991 *J. Opt. Soc. Am. B* **8** 1035–1041 URL <http://josab.osa.org/abstract.cfm?URI=josab-8-5-1035>
 - [61] Gavrilenko V P and Oks E 2000 *J. Phys. B: At. Mol. Opt. Phys.* **33** 1629–1643 URL <https://doi.org/10.1088%2F0953-4075%2F33%2F8%2F312>
 - [62] Kmetić M A and Meath W J 1985 *Phys. Lett. A* **108** 340 – 343 ISSN 0375-9601 URL <http://www.sciencedirect.com/science/article/pii/0375960185901100>
 - [63] Brown A, Meath W J and Tran P 2002 *Phys. Rev. A* **65**(6) 063401 URL

- <https://link.aps.org/doi/10.1103/PhysRevA.65.063401>
- [64] Gibson G N 2002 *Phys. Rev. Lett.* **89**(26) 263001 URL <https://link.aps.org/doi/10.1103/PhysRevLett.89.263001>
- [65] Avetissian H K, Brown A and Mkrtchian G F 2009 *Phys. Rev. A* **80**(3) 033413 URL <https://link.aps.org/doi/10.1103/PhysRevA.80.033413>
- [66] Paspalakis E, Boviatsis J and Baskoutas S 2013 *Journal of Applied Physics* **114** 153107 URL <https://doi.org/10.1063/1.4825320>
- [67] Oster F, Keitel C H and Macovei M 2012 *Phys. Rev. A* **85**(6) 063814 URL <https://link.aps.org/doi/10.1103/PhysRevA.85.063814>
- [68] Greenberg Y S 2007 *Phys. Rev. B* **76**(10) 104520 URL <https://link.aps.org/doi/10.1103/PhysRevB.76.104520>
- [69] Oelsner G, Macha P, Astafiev O V, Il'ichev E, Grajcar M, Hübner U, Ivanov B I, Neilinger P and Meyer H G 2013 *Phys. Rev. Lett.* **110**(5) 053602 URL <https://link.aps.org/doi/10.1103/PhysRevLett.110.053602>
- [70] Lange W, Walther H and Agarwal G S 1994 *Phys. Rev. A* **50**(5) R3593–R3596 URL <https://link.aps.org/doi/10.1103/PhysRevA.50.R3593>
- [71] Viola L and Lloyd S 1998 *Phys. Rev. A* **58**(4) 2733–2744 URL <https://link.aps.org/doi/10.1103/PhysRevA.58.2733>
- [72] Akram U, Evers J and Keitel C H 2005 *J. Phys. B: At. Mol. Opt. Phys.* **38** L69–L77 URL <https://doi.org/10.1088%2F0953-4075%2F38%2F4%2F101>
- [73] Evers J and Keitel C H 2004 *Phys. Rev. Lett.* **92**(15) 159303 URL <https://link.aps.org/doi/10.1103/PhysRevLett.92.159303>
- [74] Jaynes E T and Cummings F W 1963 *IEEE Proc.* **51** 89–109 URL <https://doi.org/10.1109/PROC.1963.1664>
- [75] Kibis O V, Slepian G Y, Maksimenko S A and Hoffmann A 2009 *Phys. Rev. Lett.* **102**(2) 023601 URL <https://link.aps.org/doi/10.1103/PhysRevLett.102.023601>
- [76] Kavokin K V, Kaliteevski M A, Abram R A, Kavokin A V, Sharkova S and Shelykh I A 2010 *Applied Physics Letters* **97** 201111 URL <https://doi.org/10.1063/1.3519978>
- [77] Savenko I G, Kibis O V and Shelykh I A 2012 *Phys. Rev. A* **85**(5) 053818 URL <https://link.aps.org/doi/10.1103/PhysRevA.85.053818>
- [78] Saiko A P and Fedoruk G G 2008 *JETP Letters* **87** 128–132 ISSN 1090-6487 URL <https://doi.org/10.1134/S002136400803003X>
- [79] Carr L D, DeMille D, Kreams R V and Ye J 2009 *New J. Phys.* **11** 055049 URL <https://doi.org/10.1088%2F1367-2630%2F11%2F5%2F055049>
- [80] Pérez-Ríos J, Herrera F and Kreams R V 2010 *New J. Phys.* **12** 103007 URL <https://doi.org/10.1088%2F1367-2630%2F12%2F10%2F103007>
- [81] Laucht A, Villas-Bôas J M, Stobbe S, Hauke N, Hofbauer F, Böhm G, Lodahl P, Amann M C, Kaniber M and Finley J J 2010 *Phys. Rev. B* **82**(7) 075305 URL <https://link.aps.org/doi/10.1103/PhysRevB.82.075305>
- [82] Kim H, Sridharan D, Shen T C, Solomon G S and Waks E 2011 *Opt. Express* **19** 2589–2598 URL <http://www.opticsexpress.org/abstract.cfm?URI=oe-19-3-2589>
- [83] Majumdar A, Bajcsy M, Rundquist A, Kim E and Vučković J 2012 *Phys. Rev. B* **85**(19) 195301 URL <https://link.aps.org/doi/10.1103/PhysRevB.85.195301>
- [84] Albert F, Sivalertporn K, Kasprzak J, Strauß M, Schneider C, Höfling S, Kamp M, Forchel A, Reitzenstein S, Muljarov E A and Langbein W 2013 *Nature Comm.* **4** 1747 EP – article URL <https://doi.org/10.1038/ncomms2764>
- [85] Reimann R, Alt W, Kampschulte T, Macha T, Ratschbacher L, Thau N, Yoon S and Meschede D 2015 *Phys. Rev. Lett.* **114**(2) 023601 URL <https://link.aps.org/doi/10.1103/PhysRevLett.114.023601>
- [86] Scully M O and Zubairy M S 1997 *Quantum Optics* (Cambridge: Cambridge University Press) ISBN 0524235953

- [87] Klimov A B, Sainz I and Saavedra C 2004 *Journal of Optics B: Quantum and Semiclassical Optics* **6** 448–453 URL <https://doi.org/10.1088%2F1464-4266%2F6%2F11%2F004>
- [88] Bennet C H, Divincenze D P, Smolin J A and Wootters W K 1996 *Phys. Rev. A.* **54**(2) 3824 URL <https://link.aps.org/doi/10.1103/PhysRevLett.%102.023601>
- [89] Nielsen M A and Chuang I L 2010 *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press)
- [90] Wootters W K 1998 *Phys. Rev. Lett.* **80**(10) 2245–2248 URL <https://link.aps.org/doi/10.1103/PhysRevLett.80.2245>
- [91] Walborn S P, Souto Ribeiro P H, Davidovich L, Mintert, F, and Buchleitner, A 2006 *Nature* **440** 1022 URL <https://doi.org/10.1038/nature04627>
- [92] Zhang L H, Yang Q, Yang M, Song W, Cao, Z L 2013 *Phys. Rev. A* **88** 062342 URL <https://doi.org/10.1103/PhysRevA.88.062342>
- [93] Romero, G., López, C E, Lastra, F., Solano, E., Retamal, J C 2007 *Phys. Rev. A* **75** 032303 URL <https://doi.org/10.1103/PhysRevA.75.032303>
- [94] Schmid C, Kiesel N, Wiecezorek W and Weinfurter H 2008 *Phys. Rev. Lett.* **101** 260505 URL <https://doi.org/10.1103/PhysRevLett.101.260505>
- [95] Johansson J, Nation P and Nori F 2012 *Comput. Phys. Commun.* **183** 1760 – 1772 ISSN 0010-4655 URL <http://www.sciencedirect.com/science/article/pii/S0010465512000835>
- [96] Johansson J, Nation P and Nori F 2013 *Comput. Phys. Commun.* **184** 1234–1240 ISSN 0010-4655 URL <http://www.sciencedirect.com/science/article/pii/S0010465512003955>
- [97] Widmann F, Simon J, Daudin B, Feuillet G, Rouvière J L, Pelekanos N T and Fishman G 1998 *Phys. Rev. B* **58**(24) R15989–R15992 URL <https://link.aps.org/doi/10.1103/PhysRevB.58.R15989>
- [98] Jin P, Li C M, Zhang Z Y, Liu F Q, Chen Y H, Ye X L, Xu B and Wang Z G 2004 *Applied Physics Letters* **85** 2791–2793 URL <https://doi.org/10.1063/1.1801678>
- [99] Kovarskii V A and Prepelitsa O B 2001 *Opt. Spec.* **90** 351–355 ISSN 1562-6911 URL <https://doi.org/10.1134/1.1358440>
- [100] Englund D, Faraon A, Fushman I, Stoltz N, Petroff P and Vuckovic J 2007 *Nature* **450** 857–861 URL <https://doi.org/10.1038/nature06234>
- [101] He Y, He Y M, Liu J, Wei Y J, Ramírez H Y, Atatüre M, Schneider C, Kamp M, Höfling S, Lu C Y and Pan J W 2015 *Phys. Rev. Lett.* **114**(9) 097402 URL <https://link.aps.org/doi/10.1103/PhysRevLett.114.097402>