A tentative model of creation and annihilation operators for neutrinos.

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We develop a formalism that extends the "invention" of Jordan and Wigner (in Pauli's words) of a certain transformation. In this way we define creation and annihilation operators with intrinsic spin and tentatively chirality for neutrinos. We also obtain their anticommutators. This method introduces a second numbering and an intermediate product.

PRELIMINARY:

We start writing the main points in the construction of the Jordan-Wigner transformation with its later use in the definition of the anticommutators for the creation and annihilation operators of electrons and positrons (this section). In the rest of the study we do the same process with an extension of the Jordan-Wigner transformation that we consider more appropriate for the neutrinos.

Let us denote as Pauli basis for the linear space of $2x2$ matrices over the complex field, the one formed by the identity and the Pauli matrices [1]:

$$\left\{ \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \right\} . \tag{1}$$

σ^x appears proportional to matrix Θ_r in [2]. After doing a simple change: $\sigma^{\pm} = \frac{1}{2}(\sigma^x \pm \mathrm{i}\, \sigma^y)$, $\overset{\wedge}{\sigma} = \frac{1}{2}(\mathbb{1} + \sigma^z)$, $\overset{\vee}{\sigma} = \frac{1}{2}(\mathbb{1} - \sigma^z)$, a new basis for the $2x2$ complex matrices, denoted as Canonical basis, is [2],[3],[4]:

$$\left\{ \overset{\wedge}{\sigma} = \sigma^+\sigma^- \equiv \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \overset{\vee}{\sigma} \equiv \sigma^-\sigma^+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}. \tag{2}$$

Matrices σ^+ and σ^- were named as 'Jordan-Wigner matrices' (see page 340 in [5]).

Consider the direct product of N of these linear spaces of 2x2 complex matrices. We implement an ordering by defining the following matrices [2],[6]:

$$\sigma^j_m \equiv \underset{\underset{1}{\uparrow}}{\mathbb{1}} \otimes \cdots \otimes \mathbb{1} \otimes \underset{\underset{m}{\uparrow}}{\sigma^j} \otimes \mathbb{1} \otimes \cdots \otimes \underset{\underset{N}{\uparrow}}{\mathbb{1}} , \tag{3}$$

with $j \in \{x, y, z, +, -, \wedge, \vee\}$. We have denoted by m a site in a linear chain with N sites. The identity is:

$$\mathbb{1}_N \equiv \mathbb{1} \otimes \cdots \otimes \mathbb{1} \otimes \mathbb{1} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} \tag{4}$$

Matrices σ^l_m satisfy: $\sigma^j_m \sigma^k_m = \delta^{jk}\mathbb{1}_{\mathbf{N}} + \mathrm{i}\,\epsilon^{jkl}\sigma^l_m$, with j, k, l in the set $\{x, y, z\}$ and m, m_1, m_2 with any of the values in $\{1, \ldots, N\}$. We have: $[\, \sigma^j_{m_1}, \sigma^k_{m_2} \,] = \mathrm{i}\, 2\delta_{m_1 m_2}\epsilon^{jkl}\sigma^l_{m_1}$ and $\{\, \sigma^j_m \,,\, \sigma^k_m \,\} = 2\delta^{jk}\mathbb{1}_{\mathbf{N}}$, with the commutator $[\, A \,,\, B \,] \equiv A\,B - B\,A$ and the anticommutator $\{A \,,\, B\} \equiv A\,B + B\,A$.

In order to obtain operators satisfying the Pauli Exclusion Principle, the matrices σ^+_m and σ^-_m are transformed into the fermion creation and annihilation operators $\mathbf{a}^\dagger_m$ and $\mathbf{a}_m$ (for particles without spin), by means of a Jordan-Wigner transformation, in the following way [2],[4]:

$$\mathbf{a}^\dagger_m \equiv \sigma^-_m \; \mathbf{V}^2, \qquad \qquad \mathbf{a}_m \equiv \mathbf{V}^2 \; \sigma^+_m, \tag{5}$$

with

$$\mathbf{V}^2 \equiv \prod_{k=1}^{m-1} \sigma^z_k, \qquad \qquad (\pm\sigma^z_k)^2 = \mathbb{1}_{\mathbf{N}}. \tag{6}$$

Or, also with: $\widetilde{\mathbf{V}}^2 \equiv \prod_{k=1}^{m-1}(-\sigma^z_k)$.

A keystone of the Jordan and Wigner method is the trivial equality at a site: $\sigma^z\sigma^{\pm} = -\sigma^{\pm}\sigma^z$.

We get the desired commutation and anticommutation relations:

$$\text{MATRICES} \quad \overset{J.W.}{\longleftrightarrow} \quad \text{FERMION OPERATORS}$$

$$\left.\begin{array}{c}[\,\sigma^{+}_{m_1},\sigma^{+}_{m_2}\,]=[\,\sigma^{-}_{m_1},\sigma^{-}_{m_2}\,]=\ \mathbb{0}_N\\ [\,\sigma^{+}_{m_1},\sigma^{-}_{m_2}\,]=\delta_{m_1m_2}\ \sigma^{z}_{m_1}\\ \{\sigma^{+}_{m},\sigma^{-}_{m}\}=\ \mathbb{1}_N\end{array}\right\} \qquad \left\{\begin{array}{c}\{\mathbf{a}^{\dagger}_{m_1},\mathbf{a}^{\dagger}_{m_2}\}=\{\mathbf{a}_{m_1},\mathbf{a}_{m_2}\}=\mathbf{0}\\ \{\mathbf{a}^{\dagger}_{m_1},\mathbf{a}_{m_2}\}=\delta_{m_1m_2}\ \mathbf{1}\end{array}\right. \tag{7}$$

The first line, with $m_1 = m_2 = m$, can be written in either way, as commutators or as anticommutators, as it is:

$$\left(\sigma^{+}_{m}\right)^2=\left(\sigma^{-}_{m}\right)^2=\begin{pmatrix}0&0\\0&0\end{pmatrix}_N=\ \mathbb{0}_N\ ; \qquad \left(\mathbf{a}^{\dagger}_{m}\right)^2=\left(\mathbf{a}_{m}\right)^2=\ \mathbf{0}\,. \tag{8}$$

This method of Jordan and Wigner rests in a numbering, an ordering [2] [4], with implications in relation to the normal modes (see [7]). This "invention" does not include the spin up or down for the fermion. The Jordan and Wigner method proved the existence of operators obeying anticommutation rules, and as a consequence the Pauli Exclusion Principle for the particles that they define. Afterwards, this way of numbering is ignored. Finally, in the second quantization method for electrons and positrons the creation and annihilation operators were written with spins up and down and momentum (a labeling, a substitution of the previous numbering). Their corresponding anticommutation relations are [8]:

$$\left.\begin{array}{c}\{\mathbf{b}^{s_1\dagger}_{\mathbf{p}_1},\mathbf{b}^{s_2}_{\mathbf{p}_2}\}\ =\ \{\mathbf{d}^{s_1\dagger}_{\mathbf{p}_1},\mathbf{d}^{s_2}_{\mathbf{p}_2}\}\ =\ \delta_{s_1s_2}\ \delta_{\mathbf{p}_1\mathbf{p}_2}\ \mathbf{1}\\ \{\mathbf{b}^{s_1\dagger}_{\mathbf{p}_1},\mathbf{b}^{s_2\dagger}_{\mathbf{p}_2}\}\ =\ \{\mathbf{d}^{s_1\dagger}_{\mathbf{p}_1},\mathbf{d}^{s_2\dagger}_{\mathbf{p}_2}\}\ =\ \{\mathbf{b}^{s_1}_{\mathbf{p}_1},\mathbf{b}^{s_2}_{\mathbf{p}_2}\}\ =\ \{\mathbf{d}^{s_1}_{\mathbf{p}_1},\mathbf{d}^{s_2}_{\mathbf{p}_2}\}\ =\ \mathbf{0}\\ \{\mathbf{b}^{s_1\dagger}_{\mathbf{p}_1},\mathbf{d}^{s_2\dagger}_{\mathbf{p}_2}\}\ =\ \{\mathbf{b}^{s_1}_{\mathbf{p}_1},\mathbf{d}^{s_2}_{\mathbf{p}_2}\}\ =\ \{\mathbf{b}^{s_1\dagger}_{\mathbf{p}_1},\mathbf{d}^{s_2}_{\mathbf{p}_2}\}\ =\ \{\mathbf{b}^{s_1}_{\mathbf{p}_1},\mathbf{d}^{s_2\dagger}_{\mathbf{p}_2}\}\ =\ \mathbf{0}\end{array}\right\} \tag{9}$$

$\mathbf{b}^{\dagger}$ and $\mathbf{b}$ for electrons, $\mathbf{d}^{\dagger}$ and $\mathbf{d}$ for positrons, $\mathbf{p}_1$ and $\mathbf{p}_1$ for momentum, finally, s_1 and s_2 stand for spin up or down. Chirality is not included here. The chiral character forms part of the Dirac equation.

NEW CREATION AND ANNIHILATION OPERATORS

We maintain the Jordan-Wigner ordering for m, m_1, m_2, and we add a second ordering represented by the subindexes r and s taken in the set $\{1, 2, 3, 4\}$ for defining four creation operators. We propose the following definitions for our creation operators:

$$\mathbf{U}^{\dagger}_{1,m} \equiv \prod_{k=1}^{m-1}(\ \ \sigma^{z}_{k})\ (-\sigma^{x}\sigma^{+}\sigma^{x}\,)_m\ , \tag{10a}$$

$$\mathbf{U}^{\dagger}_{4,m} \equiv \prod_{k=1}^{m-1}(-\sigma^{z}_{k})\ (\ \ \sigma^{x}\sigma^{+}\sigma^{x}\,)_m\ , \tag{10b}$$

$$\mathbf{U}^{\dagger}_{3,m} \equiv \prod_{k=1}^{m-1}(\ \ \sigma^{z}_{k})\ (\ \ \sigma^{+}_{m})\ , \tag{10c}$$

$$\mathbf{U}^{\dagger}_{2,m} \equiv \prod_{k=1}^{m-1}(-\sigma^{z}_{k})\ (-\ \sigma^{+}_{m})\ . \tag{10d}$$

We have arranged the relation of the creation operators with the 'daga' ($\mathbf{U}^{\dagger}_{r,m}$) with the corresponding σ^{+}_{m}, and of the annihilation operators, their adjoints, with 'nothing' ($\mathbf{U}_{r,m}$) with the corresponding σ^{-}_{m}.

It is interesting to pay attention to the algebraic relationships:

$$\mathbf{U}^{\dagger}_{1,m} = (-1)^m\ \mathbf{U}^{\dagger}_{4,m} \ \ = \ -\,[\,(-1)^m\ \mathbf{U}_{2,m}] = -\,[\,\mathbf{U}_{3,m}]\ , \tag{11}$$

We define:

the number operators:

$$\mathbb{N}_{r,m} \equiv \mathbf{U}^{\dagger}_{r,m}\,\mathbf{U}_{r,m} = \begin{cases} (\,\sigma^x\,\sigma^+\,\sigma^-\,\sigma^x\,)_m = (\,\sigma^x\,\overset{\wedge}{\sigma}\,\sigma^x\,)_m = \overset{\vee}{\sigma}_m\,, & r=\{1,4\} \\ (\,\sigma^+\,\sigma^-\,)_m = \overset{\wedge}{\sigma}_m\,, & r=\{3,2\} \end{cases} . \tag{12}$$

They satisfy

$$\begin{cases} \mathbb{N}_{r,m}\,\mathbf{U}^{\dagger}_{r,m} = 1\,\mathbf{U}^{\dagger}_{r,m} \\ \mathbb{N}_{r,m}\,\mathbf{U}_{r,m} = 0\,\mathbf{U}_{r,m} \end{cases} , \qquad \begin{cases} \mathbf{1} - \mathbb{N}_{r,m} = \mathbf{U}_{r,m}\,\mathbf{U}^{\dagger}_{r,m} \\ \mathbf{1} - \mathbb{N}_{1,m} = \mathbf{1} - \mathbb{N}_{4,m} = \mathbb{N}_{3,m} = \mathbb{N}_{2,m} \end{cases} ,$$

the spin operator $\mathbf{S}_m \equiv \sigma^z_m$, with:

$$\mathbf{S}_m\,\mathbf{U}^{\dagger}_{r,m} \equiv \begin{cases} -1\,\mathbf{U}^{\dagger}_{r,m}\,, & \text{spin down} \quad \downarrow\,, & r=\{1,4\} \\ +1\,\mathbf{U}^{\dagger}_{r,m}\,, & \text{spin up} \quad \uparrow\,, & r=\{3,2\} \end{cases} \tag{13}$$

Equations (11) and (12) let us consider the particles associated with $r = \{1,4\}$ as antiparticles of the ones with $r = \{3,2\}$. The expressions of $\mathbf{U}^{\dagger}_{3,m}$ as a creation operator of a particle and of $\mathbf{U}_{1,m}$ as an annihilation operator of an antiparticle are formally the same, up to a sign (see (11)). Similarly with: $\mathbf{U}^{\dagger}_{4,m}$ and $\mathbf{U}_{2,m}$. This is the usual interpretation. [7].

Also, particles with $r = 3$ and with $r = 4$ can be taken as antiparticles of each other, now differentiating in a $(-1)^{m-1}$ sign (a minus or nothing). To properly explain this we need to understand the difference between $\mathbf{U}^{\dagger}_{1,m}$ and of $\mathbf{U}^{\dagger}_{4,m}$, both creation operators of the same type of particle (including spin), but with a $(-1)^m$ factor. The discussion is cumbersome and is postponed to the conclusions.

The $\overset{\mathbf{I}}{\bullet}$*-product*:

$$r = s\,,\ \begin{cases} m_1 = m_2 \\ m_1 \neq m_2 \end{cases} \qquad \mathbf{U}^{*}_{r,m_1} \overset{\mathbf{I}}{\bullet} \mathbf{U}^{**}_{r,m_2} \equiv \mathbf{U}^{*}_{r,m_1}\,\mathbf{U}^{**}_{r,m_2} \tag{14a}$$

$$r \neq s\,, \quad m_1 \neq m_2\,, \qquad \mathbf{U}^{*}_{r,m_1} \overset{\mathbf{I}}{\bullet} \mathbf{U}^{**}_{s,m_2} \equiv \mathbf{U}^{*}_{r,m_1}\,\mathbf{U}^{**}_{s,m_2} \tag{14b}$$

$$r \neq s\,, \quad m_1 = m_2 = m\,, \qquad \mathbf{U}^{*}_{r,m} \overset{\mathbf{I}}{\bullet} \mathbf{U}^{**}_{s,m} \equiv \mathbf{U}^{*}_{r,m}\,\mathbf{I}_{rs,m}\,\mathbf{U}^{**}_{s,m} \tag{14c}$$

where:

$$\mathbf{I}_{rs,m} = \mathrm{sign}(r-s)\,\sigma^x_m = -\,\mathbf{I}_{sr,m}\,, \tag{15}$$

and * or ** indicate either creation or annihilation operators ('daga' or 'nothing'). It is the usual product, except for the case (14c) ($r \neq s$, $m_1 = m_2 = m$).

In view of the definitions (14) with (15) and (10), we are interested in the following products of the corresponding σ^j_l at sites k and m (see Appendix A):

$$\begin{array}{lll} (r = s \text{ or } r \neq s) & 1) & \sigma^z_k\ \sigma^z_k = (-\,\sigma^z_k)\,(-\,\sigma^z_k) = \mathbb{1}_N\,, \qquad \sigma^z_k\,(-\,\sigma^z_k) = -\,\mathbb{1}_N \\ & 2) & \{\sigma^{\pm}_m \overset{\mathbf{I}}{;} \sigma^z_m\} \equiv \{\sigma^{\pm}_m\,,\,\sigma^z_m\} = \sigma^{\pm}_m\,\sigma^z_m + \sigma^z_m\,\sigma^{\pm}_m = \mp\sigma^{\pm}_m \pm \sigma^{\pm}_m = \mathbb{0}_N \\ (r = s) & 3) & \{\sigma^+_m\,,\,\sigma^-_m\} = \mathbb{1}_N\,, \qquad \{\sigma^+_m\,,\,\sigma^+_m\} = \{\sigma^-_m\,,\,\sigma^-_m\} = \mathbb{0}_N\ . \\ (r \neq s) & 4) & \{\sigma^{\pm}_m \overset{\mathbf{I}}{;} \sigma^{\mp}_m\} = \sigma^{\pm}_m\,\mathbf{I}_{rs,m}\,\sigma^{\mp}_m + \sigma^{\mp}_m\,\mathbf{I}_{sr,m}\,\sigma^{\pm}_m = \mathrm{sign}(r-s)\,(\mathbf{0}-\mathbf{0})_N = \mathbb{0}_N \\ & & \{\sigma^{\pm}_m \overset{\mathbf{I}}{;} \sigma^{\pm}_m\} = \sigma^{\pm}_m\,\mathbf{I}_{rs,m}\,\sigma^{\pm}_m + \sigma^{\pm}_m\,\mathbf{I}_{sr,m}\,\sigma^{\pm}_m = \mathbb{0}_N \end{array} \tag{16}$$

2) represents the above mentioned keystone of the Jordan-Wigner method, 3) related to the number operators and to the Pauli Exclusion Principle, and 4) is meant for the evaluation of formula (14c). It is the novelty of this method.

The last two equations (4) in (16)) are specially interesting, as the $\mathbb{O}_N$ result could be obtained also as a commutator, by the algebraic usage of (15), once suppressed the sign$(r-s)$, but in this formulation it is essential to be to the right or to the left in the product defined in (14.c) (see remark A) in the summary).

ANTICOMMUTATORS

With the product defined in (14), the associated anticommutators are:

$$\{\, \mathbf{U}^{*}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{**}_{s,m_2} \,\} \equiv\; \mathbf{U}^{*}_{r,m_1} \overset{\mathrm{I}}{\bullet} \mathbf{U}^{**}_{s,m_2} \;+\; \mathbf{U}^{**}_{s,m_2} \overset{\mathrm{I}}{\bullet} \mathbf{U}^{*}_{r,m_1}. \tag{17}$$

In particular:

for $\;r = s\;$ $(\,m_1 = m_2\;$ or $\;m_1 \neq m_2\,)$:

$$\{\, \mathbf{U}^{*}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{**}_{r,m_2} \,\} \equiv\; \mathbf{U}^{*}_{r,m_1}\, \mathbf{U}^{**}_{r,m_2} + \mathbf{U}^{**}_{r,m_2}\, \mathbf{U}^{*}_{r,m_1} \;=\; \{\, \mathbf{U}^{*}_{r,m_1} \,,\, \mathbf{U}^{**}_{r,m_2} \},\tag{18}$$

for $\;r \neq s\;$ and $\;m_1 \neq m_2$:

$$\{\, \mathbf{U}^{*}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{**}_{s,m_2} \,\} \equiv\; \mathbf{U}^{*}_{r,m_1}\, \mathbf{U}^{**}_{s,m_2} + \mathbf{U}^{**}_{s,m_2}\, \mathbf{U}^{*}_{r,m_1} \;=\; \{\, \mathbf{U}^{*}_{r,m_1} \,,\, \mathbf{U}^{**}_{s,m_2} \},\tag{19}$$

for $\;r \neq s\;$ and $\;m_1 = m_2 = m\;$, we also have an actual anticommutator:

$$\{\, \mathbf{U}^{*}_{r,m} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{**}_{s,m} \,\} \equiv\; \mathbf{U}^{*}_{r,m} \overset{\mathrm{I}}{\bullet} \mathbf{U}^{**}_{s,m} + \mathbf{U}^{**}_{s,m} \overset{\mathrm{I}}{\bullet} \mathbf{U}^{*}_{r,m} \;=\; \{\, \mathbf{U}^{**}_{s,m} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{*}_{r,m} \,\}.\tag{20}$$

The case $r \neq s,\; m_1 = m_2 = m$ requires the application of 4) in formulas (16). Other cases are represented by standard anticommutators.

It remains to demonstrate that relations similar to the ones in (9) are satisfied by the anticommutators defined by (17), with the creation operators defined in (10) and also with their adjoints, the annihilation operators:

assume $\;m1 < m2\;$ (similarly with $\;m1 > m2$):

$$\{\, \mathbf{U}^{*}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{**}_{r,m_2} \,\} \sim\; \{\sigma^{\pm}_{m_1} \,,\, \sigma^{z}_{m_1}\} \,(\, \prod_{k=m_1+1}^{m_2-1} \sigma^{z}_{k} \,)\, \sigma^{\hat{**}}_{m_2} =\; \mathbf{0},\tag{21}$$

assume $\;m_1 = m_2 = m\;,\quad r = s$:

$$\{\, \mathbf{U}^{*}_{r,m} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{*}_{r,m} \,\} \;=\; \{\, \mathbf{U}^{*}_{r,m} \,,\, \mathbf{U}^{*}_{r,m} \,\} \;\sim\; (\sigma^{\pm}_{m})^2 \;\;=\; \mathbf{0},\tag{22}$$

$$\{\, \mathbf{U}^{*}_{r,m} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{\bar{*}}_{r,m} \,\} \;=\; \{\, \mathbf{U}^{*}_{r,m} \,,\, \mathbf{U}^{\bar{*}}_{r,m} \,\} \;=\; \{\sigma^{+}_{m} \,,\, \sigma^{-}_{m}\} \;=\; \mathbf{1},\tag{23}$$

assume $\;m1 = m2 = m\;,\quad r \neq s$:

$$\{\, \mathbf{U}^{*}_{r,m} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{**}_{r,m} \,\} \sim \{\sigma^{\hat{*}}_{m} \overset{\mathrm{I}}{\underset{,}{\bullet}} \sigma^{\hat{**}}_{m}\} = \sigma^{\hat{*}}_{m}\, \mathbf{I}_{rs,m}\, \sigma^{\hat{**}}_{m} + \sigma^{\hat{**}}_{m}\, \mathbf{I}_{sr,m}\, \sigma^{\hat{*}}_{m} \;=\; \mathbf{0}.\tag{24}$$

The $\bar{*}$ means that if $*$ denotes a creation operator ('daga'), then $\bar{*}$ denotes an annihilation operator ('nothing', no 'daga'), or also the opposite way. And the $\sigma^{\bar{*}}_{m}$ means σ^{-} instead of σ^{+} or σ^{+} instead of σ^{-}. Finally: $\sigma^{\hat{*}}_{m}$ denotes the following: $\begin{cases} \sigma^{\hat{*}}_{m} = \sigma^{*}_{m}, & \text{for } \; r = \{3,2\} \\ \sigma^{\hat{*}}_{m} = \sigma^{\bar{*}}_{m}, & \text{for } \; r = \{1,4\} \end{cases}$, and similarly with $\sigma^{\hat{**}}_{m}$. This last convention is in line with the operations in 4) (16) and with the matrix equality: $\sigma^{x}\sigma^{\pm}\sigma^{x} = \sigma^{\mp}$ (also, this is beneath equations (11)). See Appendix A.

As a result, these anticommutators verify similar equations to the ones in (9):

$$\left.\begin{aligned} \{\, \mathbf{U}^{\dagger}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}_{s,m_2} \,\} \;&=\; \delta_{m_1 m_2}\, \delta_{rs}\, \mathbf{1} \\ \{\, \mathbf{U}^{\dagger}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}^{\dagger}_{s,m_2} \,\} = \{\, \mathbf{U}_{r,m_1} \overset{\mathrm{I}}{\underset{,}{\bullet}} \mathbf{U}_{s,m_2} \,\} \;&=\; \mathbf{0} \end{aligned}\right\}\tag{25}$$

SUMMARY

In view of the anticommutator relationships satisfied by the operators in (25) and (9) we would be tempted of establishing the relationship of $\mathbf{U}_{1,m}^{\dagger}$ and $\mathbf{U}_{4,m}^{\dagger}$ with the two $\mathbf{b}^{\dagger}$ operators (could be $\mathbf{d}^{\dagger}$), and of $\mathbf{U}_{2,m}^{\dagger}$ and $\mathbf{U}_{3,m}^{\dagger}$ with the two $\mathbf{d}^{\dagger}$ operators (could be $\mathbf{b}^{\dagger}$), i.e. the ones with r or s equal to $\{1\,\mathrm{or}\,4\}$ with a type of particle and the ones with subindexes in $\{3,2\}$ with the opposite electrically charged type of particle (its antiparticle).

This is not the case. According to (13), the ones with $\{1,4\}$ have exclusively spin down, and the ones with $\{3,2\}$ have exclusively spin up. Another distinction consist in: the pair $\mathbf{U}_{1,m}^{\dagger}$ and $\mathbf{U}_{3,m}^{\dagger}$ with a $+\sigma^z$ in the productorial part, meanwhile $\mathbf{U}_{2,m}^{\dagger}$ and $\mathbf{U}_{4,m}^{\dagger}$ have a $-\sigma^z$. Therefore, we admit the non standard proposal:

1 we establish the following correspondences:

$\prod_{k=1}^{m-1} \sigma^z_k$ with a left chirality for the neutrino, and a right chirality for the antineutrino.

$\prod_{k=1}^{m-1} (-\sigma^z)_k$ with a right chirality for the neutrino, and a left chirality for the antineutrino.

2 according to this and to (13), the operators

with the pair $\{1,4\}$ correspond to operators for a spin down neutrino with {left-right} chirality; and

with the pair $\{3,2\}$ correspond to operators for a spin up antineutrino with {right-left} chirality.

It is the definition of the $\overset{\mathbf{I}}{\bullet}$ -product what makes significant to differentiate the various r,s values.

The fact that the neutrinos are exclusively down spin particles and, up to present day, experimentally ultra-relativistic, could explain why they are detected only as left helicity particles. According to the standard model we have that for ultra-relativistic particles, left helicity implies (approximately) left chirality. Therefore, the question is: why we do not see the right chiral part?, or perhaps, why we do not infer the right chiral part? (flipping of the chiralities and the Dirac equation). Corresponding statements for antineutrinos.

We remark three points:

A) It is intrinsic to the definitions of the anticommutators (20) and (24), and so forth to the antisymmetry, the presence of the sign and of the intermediary matrices (15) in (14). For general operators A and B, the anticommutator $\{A \overset{\mathbf{I}}{\bullet}; B\}$ is not necessarily 0 or 1. With our definitions, the ordering r,s is intrinsic to the product of the operators A and B. In other words, what it is stated in (20) is trivial, what is meaningful is that for the kind of operators treated in this study, thanks to these definitions the values of these anticommutators for the operators defined in (10) are zero, and therefore the operators considered here are antisymmetric with this product.

B) The Pauli Exclusion Principle is satisfied. Now, with the up or down spin and with the left or right chirality included in the construction-definition of the creation and annihilation operators.

C) There is a translational property along the chain (the productorial part). It should be searched the pertinent of the concept of chiral particle, and in relation to this of the weak interactions.

To properly work out the content of points A) and C) drives to a profound revision of the concept of time-space. Also, to demonstrate the statements above requires to further develop the model here proposed. An attempt to answer these questions is performed in a set of *studies* (see Appendix B).

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APPENDIX A. SKELETON OF THE METHOD.

$$
\begin{array}{l}
\qquad\qquad\qquad 1 \qquad\qquad\qquad\qquad m_1 \qquad\qquad\qquad\qquad\qquad\qquad m_2 \qquad\qquad N \\
\qquad\qquad\qquad \downarrow \qquad\qquad\qquad\qquad \downarrow \qquad\qquad\qquad\qquad\qquad\qquad \downarrow \qquad\qquad \downarrow \\
\text{For} \quad m_1 < m_2 \ , \qquad \{\mathbf{U}^{*}_{r,m_1} \overset{\bullet}{;} \mathbf{U}^{**}_{s,m_2}\} = \{\mathbf{U}^{*}_{r,m_1} \ , \ \mathbf{U}^{**}_{s,m-2}\} \sim \\
\sim \{ \ \epsilon\sigma^{z} \otimes \cdots \otimes \epsilon\sigma^{z} \otimes \quad \rho\,\sigma^{\pm} \quad \otimes \ \mathbb{1} \ \otimes ..... \otimes \ \mathbb{1} \ \otimes \ \mathbb{1} \ \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} \ \overset{\bullet}{;} \\
\qquad \tilde{\epsilon}\sigma^{z} \otimes \cdots \otimes \tilde{\epsilon}\sigma^{z} \otimes \quad \tilde{\epsilon}\sigma^{z} \quad \otimes \ \tilde{\epsilon}\sigma^{z} \otimes ..... \otimes \ \tilde{\epsilon}\sigma^{z} \otimes \ \tilde{\rho}\,\sigma^{\hat{**}} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} \ \} \ = \\
= \ \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \cdots \otimes \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \quad \rho\tilde{\epsilon}\ \sigma^{\pm}\ \sigma^{z} \quad \otimes \ \tilde{\epsilon}\sigma^{z} \otimes ..... \otimes \ \tilde{\epsilon}\sigma^{z} \otimes \tilde{\rho}\,\sigma^{\hat{**}} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} \ + \\
+ \ \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \cdots \otimes \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \quad \rho\tilde{\epsilon}\ \sigma^{z}\ \sigma^{\pm} \quad \otimes \ \tilde{\epsilon}\sigma^{z} \otimes ..... \otimes \ \tilde{\epsilon}\sigma^{z} \otimes \tilde{\rho}\,\sigma^{\hat{**}} \otimes \mathbb{1} \otimes \cdots \otimes \ \mathbb{1} \ = \\
= \ \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \cdots \otimes \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \ \ \rho\tilde{\epsilon}\{\ \sigma^{\pm} \ , \ \sigma^{z} \ \} \ \otimes \ \tilde{\epsilon}\sigma^{z} \otimes ..... \otimes \ \tilde{\epsilon}\sigma^{z} \otimes \tilde{\rho}\,\sigma^{\hat{**}} \otimes \mathbb{1} \otimes \cdots \otimes \ \mathbb{1} \ = \ \mathbf{0}.
\end{array}
\qquad ((21))
$$

Similarly with $m_1 > m_2$, it is an anticommutator.

For $m_1 = m_2 = m$,

$$
\begin{array}{l}
\text{with} \quad r = s \ , \qquad \{\mathbf{U}^{\hat{*}}_{r,m} \overset{\bullet}{;} \mathbf{U}^{\hat{**}}_{r,m}\} = \{\mathbf{U}^{\hat{*}}_{r,m} \ , \ \mathbf{U}^{\hat{**}}_{r,m}\} \sim \\
\sim \{ \ \epsilon\sigma^{z} \otimes \cdots \otimes \ \epsilon\sigma^{z} \ \otimes \quad \rho\,\sigma^{\hat{*}} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \overset{\bullet}{;} \\
\qquad \epsilon\sigma^{z} \otimes \cdots \otimes \ \epsilon\sigma^{z} \ \otimes \quad \rho\,\sigma^{\hat{**}} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \} \ = \\
= \quad \mathbb{1} \otimes \cdots \otimes \ \ \mathbb{1} \ \ \otimes \quad \sigma^{\hat{*}}\sigma^{\hat{**}} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ + \\
+ \quad \mathbb{1} \otimes \cdots \otimes \ \ \mathbb{1} \ \ \otimes \quad \sigma^{\hat{**}}\sigma^{\hat{*}} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \ = \\
= \quad \mathbb{1} \otimes \cdots \otimes \ \ \mathbb{1} \ \ \otimes \quad \{\sigma^{\hat{*}} \ , \ \sigma^{\hat{**}}\} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \ = \ \{\mathbf{0} \, , \, \mathbf{1}\}.
\end{array}
\quad \begin{cases} ((22)) \\ ((23)) \end{cases}
$$

$$
\begin{array}{l}
\text{with} \quad r \neq s \ , \qquad \{\mathbf{U}^{*}_{r,m} \overset{\bullet}{;} \mathbf{U}^{**}_{s,m}\} \ \sim \\
\sim \{ \ \epsilon\sigma^{z} \otimes \cdots \otimes \ \epsilon\sigma^{z} \ \otimes \quad \rho\,\sigma^{\hat{*}} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \overset{\bullet}{;} \\
\qquad \tilde{\epsilon}\sigma^{z} \otimes \cdots \otimes \ \tilde{\epsilon}\sigma^{z} \ \otimes \quad \tilde{\rho}\,\sigma^{\hat{**}} \quad \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \} \ = \\
= \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \cdots \otimes \ \epsilon\tilde{\epsilon}\,\mathbb{1} \ \otimes \ \ \rho\tilde{\rho}\ \sigma^{\hat{*}}\mathbf{I}_{rs,m}\sigma^{\hat{**}} \ \ \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ + \\
+ \ \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \cdots \otimes \ \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \ \ \rho\tilde{\rho}\ \sigma^{\hat{**}}\mathbf{I}_{ss,m}\sigma^{\hat{*}} \ \ \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \ = \\
= \epsilon\tilde{\epsilon}\,\mathbb{1} \otimes \cdots \otimes \ \epsilon\tilde{\epsilon}\,\mathbb{1} \ \otimes \ \ \rho\tilde{\rho}\ \{ \ \sigma^{\hat{*}} \overset{\bullet}{;} \sigma^{\hat{**}} \ \} \ \ \otimes \ \mathbb{1} \ \otimes \quad \ldots \quad \otimes \ \ \mathbb{1} \ \ \otimes \ \ \ldots \ \ \otimes \mathbb{1} \ \ = \ \mathbf{0} \ .
\end{array}
\qquad ((24))
$$

In order to obtain these results we have applied (16), with $\epsilon, \tilde{\epsilon}, \rho, \tilde{\rho}$ equal to 1 or -1, and the covenants after (24).

APPENDIX B. PROGRAM OF THE *STUDIES* CONTAINING THIS RESEARCH:

0) Creation and annihilation operators for neutrinos (the skeleton of a method). *(This study).*
Study I,1) Geometry: Euler. 3- and 4- dimensional vectors in polar exponential form.
Study I,2) Geometry: Vectors.
Study I) Geometry of the time and the space.
Study II) Spin matrices and symmetries in the time and space.
Study III) Physics: creation - annihilation operators for elementary fermions.
Study IV) Magnetic charge.
Study V) Addenda.

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