Axial vector magnetic charge and magnetic moment.
Maxwell's equations and Lorentz force law.

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We relate the non null mass of the elementary fermions (leptons and quarks) with their spins $\frac{1}{2}$. We replace the Amperian concept in the definition of the intrinsic magnetic dipole moment for every elementary fermion by a related concept, the *axial vector magnetic charge*. In order to accommodate it to the electromagnetic interaction we define an extension of the Maxwell's equations and of the Lorentz force law. The neutrinos, electrically uncharged leptons, are a special case. We suggest: the chargeless electrical neutrinos are under the action of the electrical fields.

PRELIMINARY.

- The electron, a very small sphere with a surface charge, would rotate at $\frac{1}{\alpha} \approx 137$ times the velocity of the light over some axis of rotation of the sphere (about a hundred years ago).
- The massless photons and later the neutrinos, also massless in former times, would have an axis of rotation parallel to their direction of motion. (See Figure 1 in the Appendix G: Graphics).
- The massless neutrinos would be subjugated only by the weak interaction. Even today, most physicists working with them treat them in such way, although we admit their no null masses.

But, the Special Relativity prohibits such velocity and the Quantum Mechanics do not let massive elementary fermions with a definite axis of rotation [1], [2]. The spin [3] underneath, but without the surface, without the axis of rotation and with a 4π rotation for the returning to a previous same three dimensional position:

a 4d-geometry without geometry. What is the solution?

The content of the previous paragraphs are a common place for the physicists, clearly stated by Gerard 't Hooft in his "In search of the ultimate building blocks", published in 1997 [1]. He concludes for the spin: "an audacious idea" and "such objections are simply ignored" (for the velocity). On other side, at present time some researches are looking for "new physics" in the possible electromagnetism of the neutrinos due to their no null masses.

Photons have spin 1 without the zero projection and with zero rest mass. All the elementary fermions have spin $\frac{1}{2}$, there is no zero projection and they are massive, including the neutrinos (the neutrino solar problem behind),

could we relate the non null masses of the elementary fermions with their spins $\frac{1}{2}$? [4].

This author puzzled over this question while working on a geometric model for the elementary fermions. What follows is an attempt for answering it, with an unexpected result: an extension of the Maxwell's equations and of the Lorentz force law. In the pathway we have to enlarge the idea of an "internal space" that would contend the previous "audacious idea" of the spin (a scalar with two values) and the new idea of *characteristic axes*, not necessarily related to the three dimensional rotations, a discrete set of axial vectors, with fixed axes for every elementary fermion.

I. THE MAGNETIC MOMENT.

Faraday showed that the light under the influence of the magnetic fields changes its plane of oscillation (Faraday rotation effect, 1845). Zeeman found the appearance of splittings in the spectroscopy lines of some materials in the presence of magnetic fields (1896-97). These splittings appeared in two different forms with various materials. We know them as the "Zeeman effect" and the "anomalous Zeeman effect" [3].

At the same time (1897), Lorentz explained the first one assuming vibrating electrons and using arguments of classical electrodynamics. The role of a magnetic moment related to a mechanical angular momentum is relevant, with several approaches for its introduction. In the Bohr Sommerfeld conception of the electrons in the atoms (1913-16), the electrons "circulate" around a nucleus producing an "Amperian dipole magnetic moment". Electrons are in "orbitals" (quantization of the energy) and the orientation of these orbitals are discrete: quantization of the orbital angular momentum (a first quantization of the space).

The understanding of the anomalous Zeeman effect, the second one, took more time. It implied the revolutionary idea of Kronig and of Uhlenbeck and Goudsmit of a rotation over an "internal axis" (an "internal space"), with a half unit of angular momentum (the spin was there). The two-valuedness of something intrinsic to the electron observed in relation to the magnetic fields in the Stern Gerlach experiment. It was justified with the introduction of a fourth (angular) quantum number. Finally, the development by Pauli of a formalism of spin matrices [5]. Quantization of the space definitely established.

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The magnetic dipole moment.

Assume the definition of the vector:

$$\vec{\mu} \equiv \frac{1}{2}\,(\,\vec{r}\times\vec{j}\,) = \frac{1}{2}\,q\,\vec{r}\times\vec{v} = \frac{q}{2m}\,(\,\vec{r}\times m\vec{v}\,)\,. \tag{1}$$

This vector, the magnetic dipole moment, appears in a formula for the magnetic field produced by a stationary moving electrical charge q at a large distance. This is the Amperian model for the magnetic dipole moment.

The formula for the magnetic field (at a large distance) produced by this moment has the same structure as the one for the electric field produced by an electric dipole. This is significant for the term "dipole", visualizing it as the existence of two separated magnetic charges (North and South poles). This is the Gilbertian model. But without the existence of magnetic charges (monopoles) we can consider these magnetic dipole moments in relation to a first non vanishing term in a certain expansion which drives to the formula for the magnetic field [2], [6], [7], [8].

With $v \ll c$, the vector mechanical angular moment is: $\vec{\mathbf{L}} = \vec{r}\times\vec{p} = (\vec{r}\times m\vec{v})$ and we rewrite (1):

$$\vec{\mu} = \frac{q}{2m}\,\vec{\mathbf{L}}. \tag{2}$$

We use this for the electrons in the orbitals of the atoms, and we get the above mentioned first discrete set of values for explaining the Zeeman effect, with: [9]

$$\vec{\mu}_{orb,\,l,m} = \frac{-e}{2m_e}\,\vec{\mathbf{L}}_{l,m}\,. \tag{3}$$

In order to explain the anomalous Zeeman effect it was added a term representing "more rotation" (geometry - mechanical angular moment) in an "internal space", more magnetic moment due to another rotation, a spinning ("the spin"), with only two values:

$$\vec{\mu}_{orb} \;+\; \text{“}\,\vec{\mu}_{spin}\,\text{”}\,, \tag{4}$$

which under the action of a magnetic field $\vec{\mathbf{B}}$, in relation to the energy drives to a relation of the type:

$$\frac{e}{2m_e}\,(\,\vec{\mathbf{L}}_{\text{“orb”}} + 2\frac{g_e}{2}\,\text{“}\,\vec{\mathbf{S}}\,\text{”}\,)\;\boldsymbol{.}\;\vec{\mathbf{B}}. \tag{5}$$

With "$\vec{\mathbf{S}} \equiv \overrightarrow{\tfrac{1}{2}\hbar\sigma}$" ($\vec{\sigma}$ also a -spin- **vector**), $\mu_B \equiv \frac{e\hbar}{2m_e}$ and a gyromagnetic ratio $g_e = g_e^0 + \Delta g_e = 2 + \Delta g_e$. it is:

$$\text{“}\,\vec{\mu}_{spin} \equiv \vec{\mu}_e \equiv\; \frac{g_e}{2}\;\mu_B\,(\overrightarrow{-\sigma})\,\text{”}\,. \tag{6}$$

This expression of the (intrinsic) magnetic dipole moment has three striking features:

with (5), the energy is a scalar and $\vec{\mathbf{B}}$ is a vector. $\vec{\mathbf{L}}_{l,m}$ and $\vec{\mu}_{orb,\,l,m}$ have to be vectors, they are in a scalar product, and they are so ("quantization of the space"). Therefore, the spin beneath the internal magnetic dipole moment producing only two discrete values has to be a vector: "$\vec{\mathbf{S}}$". But, in the same way as Gerard 't Hooft writes in [1], quantum mechanics would not let us assign a direction for a rotation axis in the three dimensional space. With another gesture of recklessness (audacity?), could we assume the existence of **specific** *characteristic vectors* (spinning) in an "internal space" in a similar way as the spin (a scalar), driving to only two values?,

the velocity in the current $\vec{j}$ in (1) has disappeared in $\vec{\mu}_{spin}$ in (6),

it is proportional to the electric charge and inverse proportional to the mass (via μ_B), the constant $\hbar$ involved, although the dimensional analysis of $[\mu_B] = [e]\;L^2T^{-1}$ seems to indicate that the mass has no role. Which one is the dimension of the electric charge? See Appendix A. With the Lorentz force law, we can add in the SI of units, to include the electromagnetic phenomena, any one of the following three:

or with $[\,e\,] \equiv C$ (eventually $[\,I\,] \equiv A$), or with $[\,\vec{\mathbf{E}}\,]$, or with $[\,\vec{\mathbf{B}}\,] \equiv B$ (the **tesla**, $\hat{t}$); $([\,\vec{\mathbf{E}}\,] = L\,T^{-1}[\,\vec{\mathbf{B}}\,])$.

All the elementary fermions (leptons and quarks) have spin ($\frac{1}{2}$), but one of them has 0 electric charge. Nowadays there are researches concerning the possibility of a non zero magnetic dipole moment (electromagnetic properties) for the chargeless neutrino due to its non null mass [10], [11] and [12]. This suggests us to choose the following extension for the SI of units, as more convenient for our elementary fermions:

$$\{\,kg\;,\,m\;,\,seg\;,\,tesla \equiv \hat{t}\,\}\;\;(\{\,m,l,t,b\,\})\qquad \text{for the dimensions}\qquad \{\,M,L,T,B\,\}.$$

With this election we have for the electric charge: $[\,e\,] = MT^{-1}B^{-1}$, and for the magnetic dipole moment:

$$[\,\vec{\mu}\,] = ML^2T^{-2}B^{-1} = \;[\,mc^2\,]\;B^{-1} \;=\; [\,e\,]\;L^2T^{-1} \;=\; L\,[\,c\,]\,[\,e\,].$$

On the other hand, Planck introduced "the constant of nature h" in the study of the black body radiation [13]. The starting point for the Quantum Mechanics. This constant explaining a discretization of certain energies (levels):

$\epsilon = h\,\nu$. $\quad h$ with the dimensions of an angular momentum: $\quad [h] = ML^2T^{-1} = L\,M\,[c] = M\,[c^2]\,T$,

so that we formally write: $L = [\frac{\hbar}{c}]\,M^{-1}$ and $T = [\frac{\hbar}{c^2}]\,M^{-1}$, with the *constants of nature* $\frac{\hbar}{c}$ and $\frac{\hbar}{c^2}$ $(\hbar = \frac{h}{2\pi})$.

Proceeding in an opposite way to Planck, we **assume** for the "discrete structures of nature", our elementary fermions, the relationships: $l = \frac{\hbar}{c}\,m^{-1}$ and $t = \frac{\hbar}{c^2}\,m^{-1}$, even if we do not know what is the geometrical or physical meaning of these l and t. In a similar way to the "spin" we can take them as belonging to the "internal space". We include the neutrinos and therefore with their non null masses.

With these elements and the previous units, we write the magnetic dipole moments for the elementary fermions in the following way:

1) we associate, we define a *characteristic length* and a *characteristic time*, in terms of the *characteristic mass* of a fermion, for every elementary fermion (f):

$$\hbar \equiv c\,m_f\,l_f = c^2\,m_f\,t_f \quad \Longrightarrow \quad \Big\{ l_f = \frac{\hbar}{c}\,\frac{1}{m_f} \quad , \quad t_f = \frac{l_f}{c} = \frac{\hbar}{c^2}\,\frac{1}{m_f} \Big\}, \tag{7}$$

with the values of $\hbar = 1.05457173\ 10^{-34}\ Js$ and of $c = 2.999792\ 10^8\ m/s$. In particular for an electron:

$$m_e = 9.109382\ 10^{-31}\ Kg\,, \quad \Longrightarrow \quad \Big\{ l_e = 3.8592\ 10^{-13}\ m \quad , \quad t_e = 1.2865\ 10^{-21}\ s \Big\}, \tag{8}$$

2) the charge of an electron is, without writing the sign $(Kg\ s^{-1}\,\hat{t}^{-1} \equiv C$ - Coulomb):

$$e \ \equiv\ m_e\,t_e^{-1}\,b_e^{-1} \ =\ c\ m_e\ l_e^{-1}\,b_e^{-1} \equiv 1.60210\ 10^{-19}\ C\,, \tag{9}$$

and the Bohr magneton is:

$$\mu_B \equiv \frac{e\,\hbar}{2\,m_e} = \frac{e}{2}\,\frac{c\,m_e\,l_e}{m_e} = \frac{l_e}{2}\,c\,e = \frac{1}{2}\,m_e c^2\ \ b_e^{-1} \equiv 9.2740997\ 10^{-24}\ J\hat{t}^{-1}. \tag{10}$$

From (9) or (10) we obtain the *characteristic* b_e^{-1} for the electron (with the inverse of the magnetic field unit):

$$b_e^{-1} = \frac{e\,t_e}{m_e} = \frac{2\,\mu_B}{m_e\,c^2} = 2.2626\ 10^{-10}\ \hat{t}^{-1}, \tag{11}$$

3) we define the electric charge of any elementary fermion (up to sign), with another "constant of nature e":

$$(a)\quad \mathrm{q}_k \equiv |q_{f,k}| \ \equiv\ \frac{k}{3}\ e\ \ C \quad \text{with} \quad k \in \{0,1,2,3\}\,, \qquad (b)\quad e \equiv\ (m_f\ t_f^{-1}\ b_f^{-1})\,, \tag{12}$$

with $k = 3$ for the charged leptons (cl), $k = 2$ for the family of the u-quarks, $k = 1$ for the family of the d-quarks and $k = 0$ for the neutrinos. The factor b_ν^{-1} of the neutrinos can not be determined with formula (a)-(12).

We can write $\frac{k}{3}$ in terms of an angle: $\frac{k}{3} \equiv \frac{4}{\pi}\,\varphi_{f,k}$, obtaining: $q_{\varphi_{f,k}} \equiv |q_{f,k}| \equiv (\frac{4}{\pi}\,\varphi_{f,k})\,m_f\,t_f^{-1}\,b_f^{-1}$, with

$$\varphi_{f,k} = \frac{\pi}{12}\,k \ \in\ \Big\{ 0\Big|_{(k=0)}\,,\ \ \frac{\pi}{12}\Big|_{(k=1)}\,,\ \ \frac{\pi}{6}\Big|_{(k=2)}\,,\ \ \frac{\pi}{4}\Big|_{(k=3)} \Big\}.$$

4) Among the elementary fermions, the electrons are the best known experimentally. We take them as reference for the others. We start with the mass, the time, the inverse magnetic field and the magnetic dipole moment:

$$\begin{cases} m_f \equiv A_f\,m_e & (\text{ this defines the value of } A_f\ ) \\ t_f\ = A_f^{-1}\,t_e & (\ \hbar = m_f\ c^2\ t_f = A_f m_e\ c^2\ A_f^{-1} t_e\ ).\ \text{Similarly } l_f. \\ b_f^{-1}\ = A_f^{-2}\ b_e^{-1} & (\ q_{f,k} \overset{\frac{k}{3}}{\sim}\ e = m_f\ t_f^{-1}\ b_f^{-1} = A_f m_e\ (A_f^{-1}t_e)^{-1}\ (A_f^2\ b_e)^{-1}\ )\quad (\tfrac{k}{3}\text{ apart}) \\ \quad\text{and} & \\ \mu_f\ \overset{\frac{k}{3},g_f}{\sim}\ A_f^{-1}\,\mu_B & (\ e\,\hbar = 2m_e\,\mu_B\ \overset{\frac{k}{3},g_f}{\sim}\ 2m_f\,\mu_f\ \overset{\frac{k}{3},g_f}{\sim}\ 2A_f m_e\ A_f^{-1}\,\mu_B\ )\quad (\tfrac{k}{3}, g_f\text{ apart}) \end{cases}, \tag{13}$$

g_f a gyromagnetic ratio. We write it in the form: $g_f = g_f^0 + \Delta g_f$; in particular for an electron: $g_e^0 = 2$. Extrapolating the third line for the neutrinos we could formally write: $b_\nu^{-1} = A_\nu^{-2}\,b_e^{-1}$; but it is $k = 0$ $(q_{f,\nu} = 0)$,

5) the definition of the magnetic dipole moment for any elementary fermion, ruling out the neutrino family, is:

$$\mu_{f,k} \ \equiv\ \frac{g_f}{2}\,\frac{k}{3}\,\frac{1}{2}\,m_f c^2\ \ b_f^{-1} = \frac{g_f}{2}\ \frac{\mathrm{q}_k\,\hbar}{2\,m_f} = \frac{g_f}{2}\ \frac{k}{3}\ \frac{e\,\hbar}{2A_f m_e} = A_f^{-1}\ \frac{g_f}{2}\ \frac{k}{3}\,\mu_B\ \ J\hat{t}^{-1}\,, \qquad k \in \{1,2,3\}\,, \tag{14}$$

which is similar (based in) to the one obtained with the Amperian concept for the (internal) dipole magnetic moment of the electron (in formulas (4) - (6)). The electric charge involved (up to the sign). And,

6) we introduce the definition of the *axial vector magnetic charge* $\,_a\vec{J}_{mf}$:

$$ _a\vec{J}_{mf} \equiv s\, m_f\, t_f^{-1}\, b_f^{-1}\, _a\vec{\sigma}_f\ C\,, \qquad \text{with the norm:} \quad \|\, _a\vec{\sigma}_f \| = 1\,. \tag{15}$$

with $_a\vec{\sigma}_f$ a specific axial vector in the "internal space". $s \equiv s(f) = \{+, -\}$ related to the sign of the electric charge of every elementary fermion. For $k \in \{1,2,3\}$ we fix: $s \equiv \mathrm{sign}(\mathrm{q}_{ef})$, and pending of its determination for $k = 0$ (neutrinos). Look that, for example, for the electrons in a reference frame we have two possible opposite values of the helicity, one corresponding to a spin and the other one to the opposite spin ($_a\vec{\sigma}_e$ and $-\,_a\vec{\sigma}_e$). We have introduced the notation: a subindex to the *left* of a magnitude (scalar or vector) to indicate an axial (an 'a') or a polar (a 'p') character (see Appendix C). The magnitude e is an axial scalar magnitude: $_ae$. Using (b)-(12) in (15):

$$ _a\vec{J}_{mf} = s\, _ae\, _a\vec{\sigma}_f\ C = s\, _ae\, \frac{2}{\hbar}\, _a\vec{S}_f\ C, \tag{16}$$

The norm of these vectors is a fix magnitude for all the elementary fermions, the *unit of magnetic charge*:

$$\mathbf{J}_m \equiv \|\, _a\vec{J}_{mf} \| = \, _ae = \frac{2}{\hbar}\, m_e\, \mu_B = 1.60210\ 10^{-19}\ C\ \ (Kg\, s^{-1}\, \hat{t}^{-1}). \tag{17}$$

This **is not** the charge of a magnetic monopole which would not be an elementary fermion. Now, with this unit, our previous comments for $b_\nu^{-1} = A_\nu^{-2}\, b_e^{-1}$, make sense. In this way $_ae$ has a double role: it is the basic unit for all the

electro (axial scalar charge) – magnetic (axial vector charge) phenomena of the elementary fermions

We now generalize the equation (14) to account for all the elementary fermions ($k \in \{0,1,2,3\}$):

$$\mu_{f,k} \equiv \frac{g_f}{2}\, (\frac{k}{3})\, m_f^{-1}\, \frac{\hbar}{2}\, \|\vec{J}_{mf}\| = \frac{g_f}{2}\, (\frac{k}{3})\, A_f^{-1} \mu_B\,, \qquad \text{with} \qquad \mu_B = \frac{\hbar e}{2m_e} = m_e^{-1}\, \frac{\hbar}{2}\, \|\vec{J}_{m,f}\|\,. \tag{18}$$

With: $\mu_B = k\, e$ it is $k = \frac{\mu_B}{e}\, (\frac{J\hat{t}^{-1}}{Kg\, s^{-1}\hat{t}^{-1}}) = \frac{\hbar}{2\, m_e} = 0.58\ 10^{-4}\ m^2 s^{-1}$.

For the neutrinos ($k = \varphi_\nu = 0$) we have: $\mu_{0,\nu} = 0$. Later on, we will define possible vector forms with the fourth formula in (39).

II. ELECTROMAGNETIC INTERACTION OF THE ELEMENTARY FERMIONS.

A: In the way of the Maxwell's equations.

The form of the fields created by a source. We modify the Maxwell's or the Maxwell-Dirac's equations (Appendix B) to accommodate them with *an axial vector magnetic charge density* $_a\vec{j}_{mf}$ (and $[\, _a\vec{J}_{mf}] = L^3\ [\, _a\vec{j}_{mf}]$), but without a scalar magnetic charge (a monopole). See Appendix C.

For any one of the leptons or of the quarks (f fermion), the source of the fields, we propose:

$$\left\{ \begin{array}{ll} _p\nabla\cdot\, _p\vec{\mathbf{E}}_f = \frac{1}{\epsilon_0}\, _a\rho_{ef}\,, & -\,_a\partial_t\, _p\vec{\mathbf{E}}_f + c^2\, _p\nabla\times\, _a\vec{\mathbf{B}}_f = \frac{1}{\epsilon_0}\, _p\vec{j}_{ef} = \frac{1}{\epsilon_0}\, _a\rho_{ef}\, _p\vec{v}_f \\ _p\nabla\cdot\, _a\vec{\mathbf{B}}_f = \, _p0_{mf}\,, & _a\partial_t\, _a\vec{\mathbf{B}}_f + \, _p\nabla\times\, _p\vec{\mathbf{E}}_f = -\frac{s'}{\epsilon_0}\, _a\vec{j}_{mf} \end{array} \right. , \tag{19}$$

$s' \in \{1, -1\}$ still undetermined, although equations (50) and (51) suggest: $s' = 1$.

In particular for the neutrinos:

$$\left\{ \begin{array}{ll} _p\nabla\cdot\, _p\vec{\mathbf{E}}_\nu = \, _a0_{e\nu}\,, & -\,_a\partial_t\, _p\vec{\mathbf{E}}_\nu + c^2\, _p\nabla\times\, _a\vec{\mathbf{B}}_\nu = \, _p\vec{0}_{e\nu} \\ _p\nabla\cdot\, _a\vec{\mathbf{B}}_\nu = \, _p0_{m\nu}\,, & _a\partial_t\, _a\vec{\mathbf{B}}_\nu + \, _p\nabla\times\, _p\vec{\mathbf{E}}_\nu = -\frac{s'}{\epsilon_0}\, _a\vec{j}_{m\nu} \end{array} \right. . \tag{20}$$

In the third equation in (19) and in (20): if different from zero, then there would be a magnetic monopole charge, neither a lepton nor a quark. Therefore it has to be with zero polar scalar magnetic charge (the monopole).

With the massive neutrinos subject to electromagnetism and therefore ruled by possible electromagnetic Maxwell and Lorentz type equations, we need to force the fourth equation of the system, possibly like in (20). In (20) but without the term in the right hand side in the fourth equation ($_a\vec{j}_{m,\nu}$) we would be driven to the same equations that provide the electromagnetic waves in empty space. We admit this term to distinguish them, and afterwards we implement the same right term of the fourth equation in (20) for any elementary fermion (fourth equation in (19)).

The divergence in the fourth equation in (19), with $\nabla\cdot(\nabla\times\overrightarrow{\mathbf{E}})=0$ and the third equation, drive to:

$$\nabla\cdot\overrightarrow{\jmath}_{mf} \;=\; 0_{mf} \qquad\Longleftrightarrow\qquad \oiint_{\mathbb{S}} \overrightarrow{\jmath}_{mf}\cdot\overrightarrow{ds} = \oiiint_{\mathbb{V}} \nabla\cdot\overrightarrow{\jmath}_{mf}\, d^3r \;=\; 0_{mf}\ , \tag{21}$$

like with the steady currents in magnetostatics. The circles "◯" indicating a volume enclosed in a surface, or that closed surface, also a closed curve enclosing a surface.

An elementary fermion taken here as an 'analytic punctual' charge. $\delta^{(3)}$ the Dirac's delta measure in the three dimensional space, with the convention for its unit dimensions $[\delta^{(3)}]=L^{-3}$. But there are not velocities for defining ${}_a\overrightarrow{\jmath}_{mf}$ and ${}_a\overrightarrow{J}_{mf}$; there are not (scalar) magnetic monopoles. Later on we will impose velocities associated to ${}_a\overrightarrow{J}_{mf}$. Under this assumption ('essentially - analytical punctual' charges) we should read:

$$\begin{aligned}
&a1)\quad {}_a\!\iiint_{\mathbb{V}} {}_a\rho_{ef}(\overrightarrow{\mathbf{r}}')\, d^3r' = {}_a\!\iiint_{\mathbb{V}} {}_a\mathrm{q}_{ef}(\overrightarrow{\mathbf{r}}')\ \delta^{(3)}(\overrightarrow{\mathbf{r}}'-\overrightarrow{\mathbf{r}})\, d^3r' \;=\; {}_a\mathrm{q}_{ef}(\overrightarrow{\mathbf{r}})\ ,\\
&a2)\quad {}_a\!\iiint_{\mathbb{V}} {}_p\overrightarrow{\jmath}_{ef}(\overrightarrow{\mathbf{r}}')\, d^3r' = {}_a\!\iiint_{\mathbb{V}} {}_a\mathrm{q}_{ef}(\overrightarrow{\mathbf{r}}')\,{}_p\overrightarrow{\mathrm{v}}_f(\overrightarrow{\mathbf{r}}')\delta^{(3)}(\overrightarrow{\mathbf{r}}'-\overrightarrow{\mathbf{r}})\, d^3r' = {}_a\mathrm{q}_{ef}(\overrightarrow{\mathbf{r}})\,{}_p\overrightarrow{\mathrm{v}}_f(\overrightarrow{\mathbf{r}})\ ,\\
\text{but:}\quad &b\,)\quad {}_a\!\iiint_{\mathbb{V}} {}_a\overrightarrow{\jmath}_{mf}(\overrightarrow{\mathbf{r}}')\, d^3r' = {}_a\!\iiint_{\mathbb{V}} {}_a\overrightarrow{J}_{mf}(\overrightarrow{\mathbf{r}}')\ \delta^{(3)}(\overrightarrow{\mathbf{r}}'-\overrightarrow{\mathbf{r}})\, d^3r' \;=\; {}_a\overrightarrow{J}_{mf}(\overrightarrow{\mathbf{r}})\ ,
\end{aligned} \tag{22}$$

we set: ${}_a\mathrm{q}_{ef}\,{}_p\overrightarrow{\mathrm{v}}_f = {}_p\overrightarrow{J}_{ef}$. The extra term ${}_a\overrightarrow{\jmath}_{mf}$ makes advisable further research (conservation laws). [14] [15] [16] [17]

We establish for the fermions and quarks, as a source for the electromagnetic fields:

an axial vector magnetic charge (${}_a\overrightarrow{J}_{mf}$), and without a polar scalar magnetic charge (a monopole),

We write equations (19), with the 'punctual' sources of the fields (the elementary fermions), in integral form:

$$\left\{\begin{array}{ll}
{}_p\!\oiint_{\mathbb{S}} {}_p\overrightarrow{\mathbf{E}}_f\cdot\overrightarrow{ds}' = \frac{1}{\epsilon_0}\,{}_a\mathrm{q}_{ef}, & -\,{}_a\partial_t\ {}_a\!\iint_A {}_p\overrightarrow{\mathbf{E}}_f\cdot\overrightarrow{da}' + c^2\,{}_p\!\oint_{\mathbb{C}} {}_a\overrightarrow{\mathbf{B}}_f\cdot\overrightarrow{dl}' = \frac{1}{\epsilon_0}\,{}_a\mathrm{q}_{ef}\ {}_a\!\iint_A \delta^{(3)}(\overrightarrow{\mathbf{r}}'-\overrightarrow{\mathbf{r}})\,[\,{}_p\overrightarrow{\mathrm{v}}_f\cdot\overrightarrow{da}'\,]\\
{}_p\!\oiint_{\mathbb{S}} {}_a\overrightarrow{\mathbf{B}}_f\cdot\overrightarrow{ds}' = {}_p0_{mf}, & {}_a\partial_t\ {}_a\!\iint_A {}_a\overrightarrow{\mathbf{B}}_f\cdot\overrightarrow{da}' + {}_p\!\oint_{\mathbb{C}} {}_p\overrightarrow{\mathbf{E}}_f\cdot\overrightarrow{dl}' = -\frac{s'}{\epsilon_0}\ {}_a\!\iint_A \delta^{(3)}(\overrightarrow{\mathbf{r}}'-\overrightarrow{\mathbf{r}})\,[\,{}_a\overrightarrow{J}_{mf}\cdot\overrightarrow{da}'\,]
\end{array}\right. , \tag{23}$$

the last terms in the second equation (relate it to the intensity of a current) and especially the one in the fourth equation require more research (relate it to (21)). For charged leptons, if we choose a surface with $\overrightarrow{da}'$ parallel to ${}_p\overrightarrow{\mathrm{v}}_f$ it is $[\,{}_a\overrightarrow{J}_{mf}\cdot\overrightarrow{da}'\,]=0$ (see later (39) and figure 1 in Appendix G).

An interesting direct consequence of (19) in the equations:

$$\left\{\begin{array}{l}
[\,-\,\partial_{tt}+c^2\nabla^2\,]\,\overrightarrow{\mathbf{E}}_f = \frac{1}{\epsilon_0}\left(\ \partial_t\overrightarrow{\jmath}_{ef} + s'\,c^2\ \nabla\times\overrightarrow{\jmath}_{mf}\ +\ c^2\nabla\,\rho_{ef}\ \right)\\
[\,-\,\partial_{tt}+c^2\nabla^2\,]\,\overrightarrow{\mathbf{B}}_f = \frac{1}{\epsilon_0}\left(\,s'\,\partial_t\overrightarrow{\jmath}_{mf}\ -\ \nabla\times\overrightarrow{\jmath}_{ef}\,\right)
\end{array}\right. , \tag{24}$$

Also, with the standard Maxwell's equations we have: $\nabla\times(\overrightarrow{\mathbf{E}}+\partial_t\overrightarrow{\mathbf{A}})=\overrightarrow{\mathbf{0}} \quad\Longrightarrow\quad \overrightarrow{\mathbf{E}}_f+\partial_t\overrightarrow{\mathbf{A}}_f=-\nabla\varphi$, but now it is: $\nabla\times(\overrightarrow{\mathbf{E}}_f+\partial_t\overrightarrow{\mathbf{A}}_f)=-\frac{s'}{\epsilon_0}\,\overrightarrow{\jmath}_{mf}$. At the macroscopic level this can have important implications if the directions of the $\overrightarrow{\jmath}_{mf}$ are not randomly distributed, for example with the ferromagnetism.

We have obtained the previous modifications in a straightforward way. This is not the case for the following ones (next subsection), leaving open various possibilities, and therefore the necessity of an even deeper research. Obviously we require experimental verification or either disapproval, by looking at already known experimental results.

B: In the way of the Lorentz force law.

We are interested in the force exerted by the electromagnetic fields over an elementary fermion. We propose:

generalized formula for the electromagnetic force over a lepton or a quark:

$${}_p\overrightarrow{\mathbf{F}}_f \equiv \Big[\,{}_p\overrightarrow{\mathbf{F}}_{elec} + \Big\{\,{}_p\overrightarrow{\mathbf{F}}_{amp}\,\Big] + {}_p\overrightarrow{\mathbf{F}}_{spin}\ \Big\} + {}_p\overrightarrow{\mathbf{F}}_{m,E} \equiv \Big[\,{}_p\overrightarrow{\mathbf{F}}_{\overrightarrow{E},q_e} + \Big\{\,{}_p\overrightarrow{\mathbf{F}}_{\overrightarrow{B},q_e}\Big] + {}_p\overrightarrow{\mathbf{F}}_{\overrightarrow{B},\overrightarrow{J}_m}\ \Big\} + {}_p\overrightarrow{\mathbf{F}}_{\overrightarrow{E},\overrightarrow{J}_m}\,. \tag{25}$$

The square brackets indicating the classical Lorentz force law. The curly brackets: the previous form of the action of the magnetic field over the electrical currents (here denoted Amperian) and the new one due to that action, but now over the axial vector magnetic charge (the spin) via the "intrinsic" magnetic dipole moment. The fourth term, the action of electrical fields over the axial vector magnetic charge.

We write the Lorentz force law $\overrightarrow{\mathbf{F}} = \mathrm{q}\,(\,\overrightarrow{\mathbf{E}}+\overrightarrow{\mathrm{V}}\times\overrightarrow{\mathbf{B}}\,)$, for a localized continuous distributions of charge with electric charge density ρ and with a current density $\overrightarrow{\jmath}_e=\rho\,\overrightarrow{\mathrm{V}}$, in a spatial volume V, due to the electromagnetic fields $\overrightarrow{E}$ and $\overrightarrow{B}$ in the following form: [18]

$$\overrightarrow{\mathbf{F}}(\mathbb{V}(r),t) = \oiiint_{\mathbb{V}} \Big\{\,\rho\,\Big[\overrightarrow{E}+\overrightarrow{\mathrm{V}}\times\overrightarrow{B}\,\Big]\Big\}(\overrightarrow{\mathbf{r}}',t)\,d^3r' \equiv \overrightarrow{\mathbf{F}}_e+\overrightarrow{\mathbf{F}}_b\,. \tag{26}$$

We consider a region of the space where the magnetic field is a smooth function of the space, with a Taylor series:

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}') = \vec{\mathbf{B}}(\vec{\mathbf{r}}')\Big|_{\vec{\mathbf{r}}'=\vec{\mathbf{r}}} + (\vec{\mathbf{r}}'-\vec{\mathbf{r}})\boldsymbol{\cdot}\Big(\nabla_{\vec{\mathbf{r}}''}\,\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}}\Big) + \dots\ , \tag{27}$$

for every point $\vec{\mathbf{r}}$ which eventually can be taken $\vec{\mathbf{r}} = \vec{\mathbf{0}}$.

For the 'essentially punctual' charges ($\vec{\mathbf{P}}$) of the previous subsection we write (26) in the standard Lorentz form:

$$_p\vec{\mathbf{F}}_{ef} = {}_a\mathrm{q}_{ef}\ {}_p\vec{E}\ , \qquad {}_p\vec{\mathbf{F}}_{bf} = {}_a\mathrm{q}_{ef}\ {}_p\vec{\mathrm{v}}_f \times {}_a\vec{B} \qquad \Big({}_a\vec{B} = \vec{\mathbf{B}}(\vec{\mathbf{r}}')\Big|_{\vec{\mathbf{r}}'=\vec{\mathbf{r}}=\vec{\mathbf{P}}}\Big)\ . \tag{28}$$

But, $\iiint_{\mathbb{V}} \vec{j}_e \times \big[(\vec{\mathbf{r}}'-\vec{\mathbf{r}})\boldsymbol{\cdot}\big(\nabla_{\vec{\mathbf{r}}''}\,\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}}\big)\big]\,d^3r'$ lets us the description of another type of force: the force on a magnetic moment (problem 6.22 in [2], [8] and equations (2)-(15) in [19]). We now impose localized and steady currents, obtaining:

$$a)\ \nabla\boldsymbol{\cdot}\vec{j}_e = 0 \qquad \text{and} \qquad b)\ \iiint_{\mathbb{V}} \vec{j}_e\, d^3r' = 0 \implies \Big(\iiint_{\mathbb{V}} \vec{j}_e\, d^3r'\Big)\times\Big(\vec{\mathbf{B}}(\vec{\mathbf{r}}')\Big|_{\vec{\mathbf{r}}'=\vec{\mathbf{r}}}\Big) = 0\ , \tag{29}$$

and driving to:

$$\begin{aligned} {}_a\vec{\mu}_{amp} &\equiv \tfrac{1}{2}\iiint_{\mathbb{V}} {}_p\vec{\mathbf{r}}'\times {}_p\vec{j}_e(\vec{\mathbf{r}}')\,d^3r' \\ {}_a\mathbf{U} &\equiv -\,{}_a\vec{\mu}_{amp}\boldsymbol{\cdot}\,{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'') \qquad , \\ {}_p\vec{\mathbf{F}}^{\,\mu}_{b} &= -\,{}_p\nabla_{\vec{\mathbf{r}}''}\,{}_a\mathbf{U}\Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}} \end{aligned} \tag{30}$$

This result relies in two conditions: continuity of the mathematical functions, up to a certain border for $\vec{j}_e$ and steady currents. It works for macroscopic media, for example, currents in wires. But, with the elementary fermions (point charges here), which belong to the domain of quantum mechanics, we can write in the same sense expressed by Griffiths (first paragraph in page 224 in [2]): in what way could a point charge because of its movement give rise to a steady current, ... ? See also, in relation to the magnetic field, the concept of "point magnetic dipole" in: subsection 11.2.3 and chapter 13 in [18] and also [8]. Even though its works, an example in the quantization of the angular momentum of the orbitals in the atoms (see formula (3)). Naively, imposing ${}_p\vec{j}_{ef}(\vec{\mathbf{r}}') = {}_a\mathrm{q}_{ef}\,{}_p\vec{\mathrm{v}}_f\,\delta^{(3)}(\vec{\mathbf{r}}') = {}_p\vec{J}_{ef}(\vec{\mathbf{r}}')\,\delta^{(3)}(\vec{\mathbf{r}}')$, we can infer from formula (30) for the "punctual elementary fermions":

$${}_p\vec{\mathbf{F}}^{\,\mu}_{bf} = {}_p\nabla_{\vec{\mathbf{r}}''}\big[\,{}_a\vec{\mu}_{ef}\boldsymbol{\cdot}\,{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big]\Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}}\ ; \qquad {}_a\vec{\mu}_{ef} = \frac{1}{2}\,{}_p\vec{\mathbf{r}}\times{}_p\vec{J}_{ef}(\vec{\mathbf{r}})\ ; \qquad {}_p\vec{J}_{ef} = {}_a\mathrm{q}_{ef}\ {}_p\vec{\mathrm{v}}_f\ . \tag{31}$$

In a first step, tentatively, we propose the followings third and fourth forms for the electromagnetic force over the elementary fermions, trying to follow closely the forms of the first and second ones:

$$\left.\begin{array}{lll}
(1)\quad {}_p\vec{\mathbf{F}}_{elec}: & {}_p\vec{\mathbf{F}}_{ef} = {}_a\mathrm{q}_{ef}\ {}_p\vec{\mathbf{E}} = & (\alpha\,{}_a e)\qquad {}_p\vec{\mathbf{E}} \\
(2)\quad {}_p\vec{\mathbf{F}}_{amp}: & \nearrow\ {}_p\vec{\mathbf{F}}_{bf} = {}_p\vec{J}_{ef}\times{}_a\vec{\mathbf{B}} = [\,{}_a\mathrm{q}_{ef}\ {}_p\vec{\mathrm{v}}_f\,]\times{}_a\vec{\mathbf{B}} & = [\,{}_p\vec{\mathrm{u}}_{\mathrm{v,f}}\quad(\beta\quad\alpha\,{}_a e\ )\,]\times(c\,{}_a\vec{\mathbf{B}}) \\
 & \searrow\ {}_p\vec{\mathbf{F}}^{\,\mu}_{bf} = {}_p\nabla_{\vec{\mathbf{r}}''}\big[\big(\tfrac{1}{2}\,{}_p\vec{\mathbf{r}}\times{}_p\vec{J}_{ef}(\vec{\mathbf{r}})\big)\boldsymbol{\cdot}\,{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big]\Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}} & \text{Amperian} \\
(3)\quad {}_p\vec{\mathbf{F}}_{spin}: & \left\{\begin{array}{l}\text{Amperian}\qquad {}_p\vec{\mathbf{F}}^{\text{Amp}}_{spin} \\ \text{Gilbertian}\qquad {}_p\vec{\mathbf{F}}^{\text{Gilb}}_{spin}\end{array}\right. & \begin{array}{l} = [\,{}_p\vec{\mathrm{u}}_{\mathrm{v,f}}\times(\ \frac{g^0_f}{2}\,{}_a\vec{J}_{mf})]\times(c\,{}_a\vec{\mathbf{B}}) \\ = [\,{}_p\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}}\boldsymbol{\cdot}(\ \frac{g^0_f}{2}\,{}_a\vec{J}_{mf})]\quad(c\,{}_a\vec{\mathbf{B}})\end{array}
\end{array}\right\} \tag{32}$$

$$(4)\quad {}_p\vec{\mathbf{F}}_{m,E}: \qquad \left\{\begin{array}{ll} (a) & -\,(\beta)\ s''s\ \{\ {}_a\vec{\mathrm{u}}_{\mathrm{v,f}}\boldsymbol{\cdot}(\ \frac{g^0_f}{2}\,{}_a\vec{J}_{mf}\ )\}\qquad {}_p\vec{\mathbf{E}} \\ (b) & -\,(\beta)\ s''s\ \{\ {}_a\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}}\times(\ \frac{g^0_f}{2}\,{}_a\vec{J}_{mf}\ )\}\times{}_p\vec{\mathbf{E}} \end{array}\right.$$

$\alpha \equiv \frac{\mathrm{k}}{3}s = \frac{\mathrm{k}}{3}\,\mathrm{sign}(\mathrm{q}_{ef})$. $\|\,{}_p\vec{\mathrm{v}}_f\,\| = \beta\,c$ and ${}_p\vec{\mathrm{v}}_f \equiv \beta\,c\,{}_p\vec{\mathrm{u}}_{\mathrm{v,f}}$. ${}_p\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}}$ in the plane containing ${}_a\vec{J}_{mf}$ and ${}_p\vec{\mathrm{u}}_{\mathrm{v,f}}$. $s''\in\{1,-1\}$ pending of an experimental fixing. ${}_a\vec{\mathrm{u}}_{\mathrm{v,f}}$ the corresponding axial unit vector (same triad of coordinates).

${}_p\vec{\mathrm{v}}_f$ the velocity of the fermion in the laboratory frame, where we measure the electric and magnetic fields. The axial vector magnetic charge ${}_a\vec{J}_{mf}$ and the unitary polar vector ${}_p\vec{\mathrm{u}}_{\mathrm{v,f}}$ in general in different directions (see Figures 1 and 2). For the neutrinos we set ${}_a\vec{J}_{m,\nu}$ parallel to ${}_p\vec{\mathrm{u}}_{\mathrm{v},\nu}$, even being massive, one axial and polar the other.

- The *first term* $\quad {}_p\overrightarrow{\mathbf{F}}_{elec} \equiv {}_p\overrightarrow{\mathbf{F}}_{\vec{E},q_e}$. In (28). Nothing new.

- The *second term* ${}_p\overrightarrow{\mathbf{F}}_{amp} \equiv {}_p\overrightarrow{\mathbf{F}}_{\vec{B},q_e}$.

For ${}_p\overrightarrow{\mathbf{F}}_{bf}$: directly a force over an electric charge due to a magnetic field. In (28). Nothing new.

For ${}_p\overrightarrow{\mathbf{F}}^{\mu}_{bf}$: and also, we consider a global null electrical charge and we do account for electrical currents (currents in neutrally charged electrical wires). Besides, these results applies to one electron in an orbital in an atom, bounded due to ${}_p\overrightarrow{\mathbf{F}}_{elec}$. We apply it exclusively to the charged leptons with the Amperian concept for current loops. It does not account for the interpretation of the spin in the sense of producing an "intrinsic" dipole magnetic moment. Let us look at it in detail for the magnetic moment (see [2], [6], [8], [18]), and here (31):

$$ {}_p\overrightarrow{\mathbf{F}}_{amp} = -{}_p\nabla_{\vec{\mathbf{r}}''}\Big(-{}_a\vec{\mu}_{amp} \cdot {}_a\vec{\mathbf{B}} \Big)\Big|_{\vec{\mathbf{r}}''=0} \equiv -{}_p\nabla_{\vec{\mathbf{r}}''}\, {}_a\mathbf{U}\Big|_{\vec{\mathbf{r}}''=0} , \qquad \Big({}_a\vec{\mu}_{amp} \equiv \frac{1}{2}\int {}_p\vec{\mathbf{r}} \times {}_p\vec{\mathcal{J}}(\vec{\mathbf{r}})\, d^3r \Big). \tag{33} $$

In particular, we write for an electron ($k=3$) in an orbital in an atom, still without spin (formula (3)):

$$ {}_a\vec{\mu}_{amp} \equiv \frac{1}{2}\, {}_p\vec{r} \times \Big(\mathrm{q}_{f,k}\ {}_p\vec{\mathrm{v}}_f \Big)\Big|_{k=3}^{f=e} = \mathrm{sign}(\mathrm{q}_{f,k})\frac{k}{3}\,\mu_B\,\frac{1}{\hbar}\, {}_a\overrightarrow{\mathbf{L}}_{\text{“orb”}}\Big|_{k=3}^{f=e} , \qquad \Big(\underset{\text{quantified}}{{}_a\overrightarrow{\mathbf{L}}_{\text{“orb”}} \equiv {}_p\vec{r} \times (m_e\ {}_p\vec{\mathrm{v}}_e)} \Big). \tag{34} $$

– From the *second term* ${}_p\overrightarrow{\mathbf{F}}^{\mu}_{bf}$ to the *third term* ${}_p\overrightarrow{\mathbf{F}}_{spin} \equiv {}_p\overrightarrow{\mathbf{F}}_{\vec{B},\vec{\mathcal{J}}_m}$. Pay attention to equations (2)-(6). In these equations we add a term in the following form, the Amperian concept of the intrinsic magnetic dipole moment, with only two possible values:

$$ \mu_B \Big(\frac{1}{\hbar}\, {}_a\overrightarrow{\mathbf{L}}_{\text{“orb”}} + \text{“}\, \frac{1}{\hbar}\frac{g_e^0}{2}\, {}_a\overrightarrow{\mathbf{L}}_{e,spin}\text{”} \Big) , \qquad \Big(\text{“}\overrightarrow{\mathbf{L}}_{e,spin} \sim \vec{\vec{r}} \times (m_e \vec{\vec{\mathrm{V}}})\text{” ?} \Big) . \tag{35} $$

But, neither $\vec{\vec{r}}$ nor $\vec{\vec{\mathrm{V}}}$ seems to have any physical or geometrical meaning; they could be related with what it was denoted an "internal space" (unknown). In this study, the unit space vector ${}_a\vec{\mathrm{u}}_{\mathrm{r}}$ also in the "internal space".

The term 'spin' associated to some kind of rotation, and used with different connotations:

as a 'scalar' with two assigned values, 'up' or 'down', intrinsic, without specific three dimensional orientation. Once an electron with one of the values, such value does not change, without minding in what way (direction) the electron could be moving, while not interacting,

also, with a g_e^0 [20], when subject to a magnetic field it makes its appearance through the "internal" vector magnetic dipole moment for its interaction with such external magnetic field. This vector magnetic dipole moment with an orientation in the standard three dimensional space: the mixture of equation (33) with equation (35). See later, with the Stern Gerlach experiment (only 2 values), the positions of Einstein, Ehrenfest, Schwinger and Feynman,

with a g_e, in Quantum Electrodynamics (QED) there appear spin matrices representing directions in relation to the calculation of the form factors, with $(p_\nu' - p_\nu)\,\sigma^{\mu\nu}$. It is illustrative to look at equations from (6.35) to (6.37) in Peskin [21], particularly the two previous to (6.37). Here, equations (5) and (6).

In our equations, the *characteristic vector* ("internal") corresponds to the spin, but it avoids the concept of a three dimensional axis of rotation. Remind the very illustrative paragraphs in Gerard 't Hooft's book [1] and the preliminary section.

In brief, the magnetism is related to two different physical entities: first, the Amperian type (currents in wires) which is applicable to orbital electrons in atoms, here expressed in equations (33) - (34) (after (2)-(32)), and second, the spin type. Customarily this second one represented also as Amperian in the way of equations in (35) with (6), with the definition of the Bohr magneton μ_B and only two possible values. A guidance in the Stern Gerlach experiment.

For us:

- the *third term* ${}_p\overrightarrow{\mathbf{F}}_{spin} \equiv {}_p\overrightarrow{\mathbf{F}}_{\vec{B},\vec{\mathcal{J}}_m}$. Instead of the usual interpretation, in this study we propose for the magnetic part of the electromagnetic force over an elementary fermion, due to its axial vector magnetic charge, the two possibilities in (3)-(32). They are simple formulations for this type of force, once assumed that the elementary fermions can not be monopoles (see Appendix B and equations (19)) or their combination, and after including the axial vector magnetic charges. This procedure entails the inclusion of the neutrinos as it avoids the usage of the electrical charge concept. The geometric arguments play an essential role.

We assume for the elementary fermions a point-like structure in the sense that they do not have any further "physical structure", they are not composed of other "even more elementary particles", although we conjecture a "geometry".

In order to write (3)-(32) we have established the following correspondences with previous expressions in (32):

$$ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}}\ (\beta\ \alpha\ {}_a e\ )\ \text{ in (2)-(32)} \quad \begin{array}{lll} \nearrow & \left\{ [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times (\ \frac{g_f^0}{2}\ {}_a\vec{\jmath}_{mf})] \times \right\} & \text{Amperian, like in (2)-(32) a vector} \\ \searrow & \left\{ [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}}^{\ \perp} \cdot (\ \frac{g_f^0}{2}\ {}_a\vec{\jmath}_{mf})] \right\} & \text{Gilbertian, like in (1)-(32) a scalar} \end{array} .$$

We have suppressed the speed factor (β) to express the no dependence in the speed, like with the electric charge in (1)-(32) and also like in equation (6).

We start with the Amperian form. In a similar way to the one used to define (26), we define:

$$ \vec{\mathbf{F}}^{\mathrm{Amp}}_{spin}(\mathbb{V}(r),t) = \iiint_{\mathbb{V}} \left\{ [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times (\frac{g_f^0}{2}\ {}_a\vec{\jmath}_{mf})] \times (\ c\, {}_a\vec{\mathbf{B}}\ ) \right\}(\vec{\mathbf{r}}', t)\ d^3r' \ . \tag{36} $$

Again, we proceed as in [8] and in (2)-(15) in [19], with the implementation of the Taylor series in (27), and we express: $\vec{\mathbf{F}}^{\mathrm{Amp}}_{spin}(V(r),t) = \vec{\mathbf{F}}_{11} + \vec{\mathbf{F}}_{12} + \ldots$:

$$ \left\{ \begin{array}{l} \vec{\mathbf{F}}_{11} = \iiint_{\mathbb{V}} \left\{ [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times (\ \frac{g_f^0}{2}\ {}_a\vec{\jmath}_{mf})] \times \Big(c\ \vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}} \Big) \right\}(\vec{\mathbf{r}}', t)\ d^3r' \\ \vec{\mathbf{F}}_{12} = \iiint_{\mathbb{V}} \left\{ [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times (\ \frac{g_f^0}{2}\ {}_a\vec{\jmath}_{mf})] \times \Big(c\ (\vec{\mathbf{r}}' - \vec{\mathbf{r}}) \cdot (\nabla_{\vec{\mathbf{r}}''}\ \vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}}) \Big) \right\}(\vec{\mathbf{r}}', t)\ d^3r' \end{array} \right. . \tag{37} $$

We are able to write $\vec{\mathbf{F}}_{12}$ like the usual one ($\vec{\mathbf{F}} = \nabla(\vec{\mu} \cdot \vec{\mathbf{B}})$), the Amperian one like (15) in [19] and here (30), driving to (31) for the punctual charge, with the substitution ${}_p\vec{\jmath}_{ef}(\vec{\mathbf{r}}) \ \longrightarrow\ [c\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times (\frac{g_f^0}{2}\ {}_a\vec{\jmath}_{mf})]$:

$$ \begin{aligned} \vec{\mathbf{F}}_{12} =\ & {}_p\nabla_{\vec{\mathbf{r}}''} \Big[\Big\{ \tfrac{1}{2}\ {}_p\vec{\mathrm{r}}_f \times \big(c\ \frac{g_f^0}{2}\ s\ {}_a e\ [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times {}_a\vec{\sigma}_f\ ] \big) \Big\} \cdot {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'') \Big] \Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} = \\ =\ & {}_p\nabla_{\vec{\mathbf{r}}''} \Big[\{\ s\ \frac{g_f^0}{2}\ \mu_B\ \frac{{}_p\vec{\mathrm{r}}_f}{l_e} \times [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times {}_a\vec{\sigma}_f\ ] \} \cdot {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'') \Big] \Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} = \\ =\ & {}_p\nabla_{\vec{\mathbf{r}}''} \Big[\{\ s\ \frac{g_f^0}{2}\ A_f^{-1} \mu_B\ \ {}_p\vec{\mathrm{u}}_{\mathrm{r,f}} \times [\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times {}_a\vec{\sigma}_f\ ] \} \cdot {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'') \Big] \Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} = \\ =\ & {}_p\nabla_{\vec{\mathbf{r}}''} \Big[\ s\ \frac{g_f^0}{2}\ A_f^{-1} \mu_B \sin(2\varphi_{f,k})\ \ {}_a\vec{\mathrm{u}}_{\mu_\varphi} \cdot {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'') \Big] \Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} =\ {}_p\nabla_{\vec{\mathbf{r}}''} \Big[\ {}_a\vec{\mu}_{\varphi,f} \cdot {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'') \Big] \Big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} . \end{aligned} \tag{38} $$

We have used (10) $c\,e = \mu_B \frac{2}{l_e} = \mu_B A_f^{-1} \frac{2}{l_f}$. All the $''\vec{\mathrm{u}}\text{'s}''$ indicating unit vectors.

We have imposed and defined the following geometrical conjectures:

$$ \begin{array}{ll} {}_p\vec{\mathrm{r}}_f \equiv l_f\ {}_p\vec{\mathrm{u}}_{\mathrm{r,f}} = A_f^{-1} l_e\ {}_p\vec{\mathrm{u}}_{\mathrm{r,f}}\ , & {}_p\vec{\mathrm{u}}_{\mathrm{r,f}}\ =\ {}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \qquad \text{(both imposed)} \\ {}_p\vec{\mathrm{u}}_{\mathrm{r,f}} \times [\,{}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times {}_a\vec{\sigma}_f] \equiv \sin(2\varphi_{f,k})\ \ {}_a\vec{\mathrm{u}}_{\mu_\varphi} & {}_a\vec{\mu}_{\varphi,f}\ \equiv\ s\ \frac{g_f^0}{2}\ A_f^{-1}\mu_B\ \sin(2\varphi_{f,k})\ \ {}_a\vec{\mathrm{u}}_{\mu_\varphi} \\ \qquad {}_p\vec{\mathrm{u}}_{\mathrm{v,f}}^{\ \perp} = {}_p\vec{\mathrm{u}}_{\mathrm{r,f}}^{\ \perp}\ \parallel\ {}_a\vec{\mathrm{u}}_{\mu_\varphi} & \end{array} . \tag{39} $$

Where ${}_p\vec{\mathrm{r}}_f$ (${}_p\vec{\mathrm{u}}_{\mathrm{r,f}}$) are in the “internal space”; their meaning different to the one imposed for the angular momentum.

In favor of this geometrical interpretation the essentially orthogonal polarizations of electrons and neutrinos, The spin vectors (the *characteristic axes*) for the neutrinos 'almost' in the direction of their velocities due to its 'almost' zero mass, and accordingly to the polarizations, the spin vectors (the *characteristic axes*) of the electrons in a plane 'almost' perpendicular to their velocities.

For the neutrinos it is: $\varphi_{k=0} = 0$, and therefore ${}_a\vec{\sigma}_\nu$ is parallel to ${}_p\vec{\mathrm{u}}_{\mathrm{v},\nu}$. This implies: ${}_a\vec{\mu}_{\varphi=0} = {}_a\vec{0}$. In this way, the neutrinos are included, obtaining for them: ${}_a\vec{\mu}_\nu = {}_a\vec{0}$.

For the other elementary fermions: $\varphi_{k=\{1,2,3\}} \in \left\{ \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4} \right\}$. For all the values of k it is ${}_a\vec{\mathrm{u}}_{\mu_\varphi}$ perpendicular to ${}_p\vec{\mathrm{u}}_{\mathrm{v,f}}$, with the first one axial and the second one polar. In the definition of the magnetic dipole moment in equation (6), ${}_a\vec{\mu}_e$ replaces the right hand side term (${}_a\vec{\mathrm{u}}_{\mu_{\frac{\pi}{4}}}$ equal to plus or minus $\vec{\sigma}$). See the graphic in Figure 1.

With (36)-(39) we express: ${}_p\vec{\mathbf{F}}^{\mathrm{Amp}}_{spin} = \vec{\mathbf{F}}_{11} - {}_p\nabla\, {}_aU_f + \ldots$ with ${}_aU_f \equiv -\,{}_a\vec{\mu}_{\varphi,f} \cdot {}_a\vec{\mathbf{B}}$

Let us now inquire about $\vec{\mathbf{F}}_{11}$. We have to do $\vec{\mathbf{F}}_{11} = \vec{\mathbf{0}}$. The parallel result to (29) is:

$$ \begin{array}{ll} & a)\ \ \nabla \cdot \vec{\jmath}_{mf} = 0 \qquad \text{in (21)}, \\ \text{but we need to prove here:} & b)\ \ \Big(\iiint_{\mathbb{V}} [\,{}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times \vec{\jmath}_{mf}]\ d^3r' \Big) \times \Big(\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} \Big) = \vec{\mathbf{0}} \end{array} . \tag{40} $$

We have not proven b). In a preliminary step for the proof we could use: $\nabla_{\vec{\mathbf{r}}'} \cdot \{\vec{\mathrm{u}}_{\mathrm{r}'} \times [r' \vec{\mathbf{F}}(\vec{\mathbf{r}}')]\} = 0$, where $\vec{\mathbf{r}}'$ is a position vector and $\vec{\mathbf{F}}$ is a vector function. For the neutrinos it is: $[\,{}_p\vec{\mathrm{u}}_{\mathrm{v},\nu} \times \vec{j}_{m\nu}] = \vec{\mathbf{0}}$; and for the charged leptons it is: $\vec{j}_{m,cl}$ perpendicular to ${}_p\vec{\mathrm{u}}_{\mathrm{v,cl}} = {}_p\vec{\mathrm{u}}_{\mathrm{r,cl}}$.

In a different way, acting like in (28), which inherits the 'essentially' punctual character of the particles:

$$\begin{aligned} \vec{\mathbf{F}}_{11} &= s\,\frac{g_f^0}{2}\, c\; {}_a e\; [\,{}_p\vec{\mathrm{u}}_{\mathrm{v,f}} \times {}_a\vec{\sigma}_f\,] \times {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} = \\ &= s\,\frac{g_f^0}{2}\, c\; {}_a e\; \sin(2\varphi_{f,k})\; {}_p\vec{\vec{\mathrm{u}}}_{\mathrm{v},\sigma,\mathrm{f}} \times {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} = \\ &= s\,\frac{2}{l_f}\, \|{}_a\vec{\mu}_{\varphi,f}\|\; {}_p\vec{\vec{\mathrm{u}}}_{\mathrm{v},\sigma,\mathrm{f}} \times {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\vec{\mathbf{r}}_f} \;. \end{aligned} \tag{41}$$

For electrons with perpendicular trajectories to the magnetic field and with $\vec{\sigma}_e$ parallel to $\vec{\mathbf{B}}$, we have the behavior: $\|\vec{\mathbf{F}}_{11}\| = \dfrac{2}{l_e}\; \dfrac{\|\vec{\mathbf{B}}\|}{\|\nabla(\vec{\mathrm{u}}_{\mu_e} \cdot \vec{\mathbf{B}})\|}\, \|\vec{\mathbf{F}}_{12}\| \sim\; 10^{13}\, \|\vec{\mathbf{F}}_{12}\|$. The action of the magnetic field would produce a very large positive or negative acceleration of the particle in the tangent of the trajectory.

The mathematical treatment involved in differentiating (40) from (41) have to be clarified. Should we include a Dirac δ to cancel the $\vec{\mathbf{F}}_{11}$ term? But, for us, the elementary fermions have geometrical structure.

For the second expression in (3)-(22):

$$ {}_p\vec{\mathbf{F}}^{\,\mathrm{Gilb}}_{spin} = \vec{\mathbf{F}}_{22} + \vec{\mathbf{F}}_{21} + \ldots \qquad \begin{cases} \vec{\mathbf{F}}_{21} = c\,\dfrac{g_f^0}{2}\, s\; {}_a e\; [\,{}_p\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}} \cdot {}_a\vec{\sigma}_f]\; \Big({}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f}\Big) \\ \vec{\mathbf{F}}_{22} = c\,\dfrac{g_f^0}{2}\, s\; {}_a e\; [\,{}_p\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}} \cdot {}_a\vec{\sigma}_f]\; \Big((\,{}_p\tilde{\vec{\mathbf{r}}}_f \cdot {}_p\nabla_{\vec{\mathbf{r}}''})\,{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f}\Big) \end{cases} . \tag{42}$$

Our handling of the Gilbertian term follows similar steps, now the similitude with (1)-(22). For $\vec{\mathbf{F}}_{22}$:

$$\begin{aligned} \vec{\mathbf{F}}_{22} &= s\,\frac{g_f^0}{2}\,\mu_B A_f^{-1}\frac{2}{l_f}\,\cos(\tfrac{\pi}{2} - 2\varphi_{f,k})\,(\,{}_p\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}} \cdot {}_a\vec{\mathrm{u}}_{\mu_\varphi})\,(\,{}_p\tilde{\vec{\mathbf{r}}}_f \cdot {}_p\nabla_{\vec{\mathbf{r}}''})\,{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f} = \\ &= \big(s\,\frac{g_f^0}{2}\,A_f^{-1}\mu_B\,\sin(2\varphi_{f,k})\;{}_a\tilde{\vec{\mathrm{u}}}_{\mathrm{r,f}} \cdot {}_p\nabla_{\vec{\mathbf{r}}''}\big)\,{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f} = \big(\,{}_a\vec{\mu}_{\varphi,f} \cdot {}_p\nabla_{\vec{\mathbf{r}}''}\big)\;{}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f} \;, \end{aligned} \tag{43}$$

with ${}_p\vec{\mathrm{u}}^{\,\perp}_{\mathrm{v,f}} \cdot {}_a\vec{\mathrm{u}}_{\mu_\varphi} = {}_p1$ and ${}_a\tilde{\vec{\mathrm{u}}}_{\mathrm{r,f}} \equiv {}_p1\,\dfrac{2}{l_f}\,{}_p\tilde{\vec{\mathbf{r}}}_f \equiv {}_a\vec{\mathrm{u}}_{\mu_\varphi}$ (imposed).

We easily observe the similarities with classical electromagnetism. In order to obtain the same magnetic field, at least at large enough distances of the magnetic dipole (moments), the $\mathbf{r}$ in $\mathbf{m} = \frac{1}{2}\int(\mathbf{r} \times \mathbf{J})d^3\mathrm{r}$ and the $\tilde{\mathbf{r}}$ in $\mathbf{m} = (+g - (-g))\tilde{\mathbf{r}}$ (two opposite magnetic monopoles) are perpendiculars to each other. See equations (12) and (19) in [19] and the graphic representations of the fields of a "pure" and of a "physical" dipole in Figure 5.55 in [2] (page 255). And the term $\vec{\mathbf{F}}_{21}$:

$$\vec{\mathbf{F}}_{21} = s\,\frac{g_f^0}{2}\, c\; {}_a e\; \sin(2\varphi_{f,k})\; {}_p1\; {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f} = {}_p\{s\,\frac{2}{l_f}\,\|{}_a\vec{\mu}_{\varphi,f}\|\}\; {}_a\vec{\mathbf{B}}(\vec{\mathbf{r}}'')\big|_{\vec{\mathbf{r}}''=\tilde{\vec{\mathbf{r}}}_f} \;, \tag{44}$$

Using a similar argument to the one after (41), this force would have a very large action in the direction of the magnetic field, which we know it is not the case. This can be an argument against the Gilbertinan interpretation of the magnetic moment. Also, experiments with neutrons (Section VII in [19] and [22]) suggest the Amperian character for the force ${}_p\vec{\mathbf{F}}_{spin}$. Therefore we study ${}_p\vec{\mathbf{F}}_{spin} = {}_p\vec{\mathbf{F}}^{\,\mathrm{Amp}}_{spin}$, even without electric charge.

We will not further discuss these questions. One way to avoid them consists in establishing, instead of the first expression in (3)-(32), a fundamental law for this force, also valid for neutrinos, in the form:

$${}_p\vec{\mathbf{F}}_{spin} = {}_p\vec{\mathbf{F}}^{\,\mathrm{Amp}}_{spin} = -\,{}_p\nabla\,{}_aU_f \qquad \text{with} \qquad {}_aU_f \equiv -\,{}_a\vec{\mu}_{\varphi,f} \cdot {}_a\vec{\mathbf{B}} \qquad \text{(equation (38)).}$$

Griffiths [2] (page 378) writes about a term in the Lorentz force law with such form ($\nabla(\mathbf{m}.\mathbf{B})$):

"in any event it is *not* classical electrodynamics". Obviously this is the case: the spin is not classical physics.

We can compare the formula (14), prior to the radiative corrections $(\Delta g_f = 0)$, with the last formula in (39), the absolute values and the norms (see Appendix D):

$$\frac{|\, \mu_{f,k\,|_{\Delta g_f=0}} \,|}{\| \,_{a}\vec{\mu}_{\varphi,f} \,\|} = \frac{\frac{k}{3}}{\sin(2\varphi_{f,k})} = \begin{cases} \frac{1/3}{\sin(\pi/6)} = \frac{2}{3} = 0.66 & \text{d-quarks} \\ \frac{2/3}{\sin(\pi/3)} = \frac{4}{3\sqrt{3}} = 0.77 & \text{u-quarks} \\ \frac{1}{\sin(\pi/2)} = 1 & \text{charged leptons} \end{cases} . \quad (45)$$

$$_{a}\vec{\mu}_{\nu} = \vec{0} \qquad \text{neutrinos}$$

Let us interpret these equations for the elementary fermions:

a) *for the quarks.* Unluckily, the results of the equation in (38) and particularly (45) tell us their irrelevancy for the quarks: at the experimental level and at least at present day, the quarks under the strong force are always confined and we also do not know their masses with enough precision. This argument applies to the fourth term (4)-(32) too. Therefore we are not going to consider them any further (see Appendix D),

b) *for the neutrinos.* With $_{a}\vec{\sigma}_{\nu}$ parallel to $_{p}\vec{\mathrm{u}}_{\mathrm{v},\nu}$, which implies $\varphi_{k=0} = 0$ and therefore $\sin(2\varphi_{k=0}) = \sin(0) = 0$, it is: $_{p}\vec{\mathbf{F}}_{spin}\Big|_{\nu} = {}_{p}\vec{\mathbf{0}}$ *the magnetic fields do not have action over the neutrinos*,

c) *for the electrically charged leptons (cl):* $2\varphi_{cl,3} = \frac{\pi}{2} \implies \sin(2\varphi_{cl,3}) = \sin(\frac{\pi}{2}) = 1$, with $\| \,_{a}\vec{\mu}_{\frac{\pi}{4},f} \,\| = \frac{g^0_{cl}}{2} A^{-1}_{cl} \mu_B$, and therefore $_{a}\vec{\mu}_{\frac{\pi}{4},cl} = s \, \| \,_{a}\vec{\mu}_{\frac{\pi}{4},cl} \,\| \; _{a}\vec{\mathrm{u}}_{\mu_{\frac{\pi}{4}}} = s \, \mu_{cl,3} \; _{a}\vec{\mathrm{u}}_{\mu_{\frac{\pi}{4}}}$, with $\mu_{cl,3}$ in (14).

We now apply these results to the classical Stern Gerlach type experiments. We initially consider electrons, with their trajectories perpendicular to the magnetic field and taking apart the cyclotron effects due to the force $_{p}\vec{\mathbf{F}}_{elec}$. The Lorentz force law drove for the magnetic dipoles to a force of the form $\nabla(\vec{\mu} \cdot \vec{\mathbf{B}})$. This is the result obtained with (38). But, already in (1922) Einstein, Ehrenfest and later Schwinger and Feynman pointed out the difficulties for explaining the only two specific values [23] , [24]. The question relies on whether the dominance of: (a) the rotation of the magnetic dipole moment direction around the lines of the magnetic field due to a torque (dynamical) with the Larmor frequency, or (b) over a different rotation that tends to align the magnetic dipole with the magnetic field, likewise a compass in the magnetic field of the Earth. The role of the measurement process is treated in [25], [26] with a spin relaxation and in a different way in [27], [28] (with the wave function).

There is a noticeable difference in our treatment: we have imposed the inclusion of the *characteristic axis* in a plane perpendicular to the velocity of the charged lepton. We write some first approach classical qualitative comments.

In the experiment, the magnetic field varies in a small region from zero to the values inside the magnet. At first the magnetic field is zero and there is not a “Larmor rotation”. As the particle moves to enter in the magnet, the magnetic field increases and it starts the alignment competing with the “Larmor rotation”, but this rotation would force a change in the direction of the velocity due to our definition of $_{a}\vec{\mathrm{u}}_{\mu_e}$, always perpendicular to $_{p}\vec{\mathrm{u}}_{\mathrm{v,e}}$. This change could be ′arbitrary′ large depending in the ′arbitrary′ directions of the dipole magnetic moment. Could we conclude the predominance of the alignment over the “Larmor rotation”?

In what way do we have the “Larmor rotation”? We can introduce it via QED with the action of the radiative corrections. We assume a modification of the vector magnetic dipole moment by adding a very small vector in any direction. Experimental results with the anomalous magnetic moments tells us of a small modification of the electromagnetic vector magnetic dipole moment given by the way in which the anomalous $\sigma_{\mu\nu}$ couples to the electromagnetism alone [21]. The result of this modification can be in any close direction to the original one. Once the particles enter the magnetic field and with the predominance of the alignment described above, we have the “Larmor rotation” of a slightly modified axis (of $_{a}\vec{\mu}_{\tilde{\varphi},cl}$) around the lines of the magnetic field, with a very small angle between them. This one is the fact that permits this “Larmor rotation”, which is independent of the value of the angle.

Schematically: with $\tilde{\varphi} = \frac{\pi}{2} + \Delta\varphi$ we have a “Larmor rotating” vector $_{a}\vec{\mu}_{\tilde{\varphi},cl}$. Bellow equation (13) we wrote $g_f = g^0_f + \Delta g_f$; we now write: $_{a}\vec{\mu}_{\tilde{\varphi},cl} = {}_{a}\vec{\mu}_{\frac{\pi}{4},cl} + \Delta \, _{a}\vec{\mu}_{\Delta\varphi,cl}$ with $_{a}\vec{\mu}_{\frac{\pi}{4},cl}$ parallel to $_{a}\vec{\mathbf{B}}$. The condition of the orthogonality of the vector magnetic dipole and the velocity forces a helical trajectory for the charged lepton. The helix with an irregular but periodic pitch and a very small radius. Perhaps this reminds of the old concept ‘Zitterbewegung’ [29].

The above explanation would also justify the results of Rabi (1929) [23] (pages 24, 25) and [24] (pages 58, 59) using a homogeneous magnetic field but with the bean in a non perpendicular direction to the magnetic field.

Original experiments with neutral atoms, complex systems, still require to add some more explanations.

- The *fourth term* $_{p}\vec{\mathbf{F}}_{m,E} \equiv {}_{p}\vec{\mathbf{F}}_{\vec{E},\vec{J}_m}$. Now the guidance is with $_{p}\vec{\mathbf{F}}_{elec} = (\alpha \, _{a}e) \, _{p}\vec{\mathbf{E}}$ (the first one in (1)-(32)), and we write it in the form:

$$_{p}\vec{\mathbf{F}}_{m,E} = -(\beta) \, s'' \frac{g^0_f}{2} \, _{a}e \, [\, _{a}\vec{\mathrm{u}}_{\mathrm{v,f}} \cdot {}_{a}\vec{\sigma}_f] \; _{p}\vec{\mathbf{E}} = -(\beta) \, s'' \frac{g^0_f}{2} \frac{2}{c \, l_e} \mu_B \, [\, _{a}\vec{\mathrm{u}}_{\mathrm{v,f}} \cdot {}_{a}\vec{\sigma}_f] \; _{p}\vec{\mathbf{E}} \, . \quad (46)$$

It is not clear the β dependence; in any case, experiments would decide. A possible experiment in Appendix F.

For the charged leptons it is: $\quad {}_p\vec{\mathbf{F}}_{m,E}\big|_{cl} = {}_p\vec{\mathbf{0}}$, due to the perpendicularity of ${}_a\vec{\sigma}_{cl}$ with ${}_a\vec{\mathrm{u}}_{\mathrm{v,cl}}$.

And for the neutrinos, due to the parallelism of ${}_a\vec{\sigma}_\nu$ with ${}_a\vec{\mathrm{u}}_{\mathrm{v},\nu}$, we obtain:

generalized formula for the electromagnetic force over a neutrino:

$$ {}_p\vec{\mathbf{F}}_\nu = -(\beta)\, s'' \, \frac{g_\nu^0}{2} \, {}_a e \; {}_p\vec{\mathbf{E}} \, . \tag{47} $$

The second form with: $\quad {}_p\vec{\mathbf{F}}_{m,E} = -(\beta)\, s'' \frac{g_\nu^0}{2} \, {}_a e \, [\, {}_a\vec{\mathrm{u}}_{\mathrm{v},\nu}{}^{\perp} \times {}_a\vec{\sigma}_\nu] \times {}_p\vec{\mathbf{E}} \quad$ (in (4)-(32) (b)). This one towards the form of the Lorentz-Dirac force law (equation (51)), although without monopoles. We need some other rule to fix ${}_a\vec{\mathrm{u}}_{\mathrm{v},\nu}{}^{\perp}$.

Three more simple possibilities with:

$$ (4) \quad {}_p\vec{\mathbf{F}}_{m,E} \; : \; \begin{cases} (c) & -(\beta)\, s''s\, \{ \, {}_a\vec{\mathrm{u}}_{\mathrm{v,f}}{}^{\perp} \cdot (\frac{g_f^0}{2} \, {}_a\vec{J}_{mf}) \} \quad {}_p\vec{\mathbf{E}} \\ (d) & -(\beta)\, s''s\, \{ \, {}_a\vec{\mathrm{u}}_{\mathrm{v,f}} \times (\frac{g_f^0}{2} \, {}_a\vec{J}_{mf}) \} \times {}_p\vec{\mathbf{E}} \\ (e) & -(\beta)\, s''s\, \{ \qquad (\frac{g_f^0}{2} \, {}_a\vec{J}_{mf}) \} \times {}_p\vec{\mathbf{E}} \end{cases} $$

(c) and (d) in order to follow the forms expressed in (3)-(32) and (e) again with the Lorentz-Dirac form in (51).

We summarize: charged leptons, neutrinos

$$ {}_p\vec{\mathbf{F}}_{m,E} \; : \; -s''(\beta) \frac{g_f^0}{2} \, {}_a e \begin{cases} \{ {}_a\vec{\mathrm{u}}_{\mathrm{v,f}} \cdot {}_a\vec{\sigma}_f \} \; {}_p\vec{\mathbf{E}} \longrightarrow & {}_p\vec{\mathbf{0}} \quad ,, & {}_p\vec{\mathbf{E}} & (a) \\ [{}_a\vec{\mathrm{u}}_{\mathrm{v,f}}{}^{\perp} \times {}_a\vec{\sigma}_f] \times {}_p\vec{\mathbf{E}} \longrightarrow & {}_p\vec{\mathbf{0}} \quad ,, & [{}_a\vec{\mathrm{u}}_{\mathrm{v},\nu}{}^{\perp} \times {}_a\vec{\sigma}_\nu] \times {}_p\vec{\mathbf{E}} & (b) \\ \{ {}_a\vec{\mathrm{u}}_{\mathrm{v,f}}{}^{\perp} \cdot {}_a\vec{\sigma}_f \} \; {}_p\vec{\mathbf{E}} \longrightarrow & {}_p\vec{\mathbf{E}} \quad ,, & {}_p\vec{\mathbf{0}} & (c) \\ [{}_a\vec{\mathrm{u}}_{\mathrm{v,f}} \times {}_a\vec{\sigma}_f] \times {}_p\vec{\mathbf{E}} \longrightarrow & [{}_a\vec{\mathrm{u}}_{\mathrm{v,cl}} \times {}_a\vec{\sigma}_{cl}] \times {}_p\vec{\mathbf{E}} \quad ,, & {}_p\vec{\mathbf{0}} & (d) \\ {}_a\vec{\sigma}_f \times {}_p\vec{\mathbf{E}} \longrightarrow & {}_a\vec{\sigma}_{cl} \times {}_p\vec{\mathbf{E}} \quad ,, & {}_a\vec{\sigma}_\nu \times {}_p\vec{\mathbf{E}} & (e) \end{cases} . $$

There are reasons against the last three: electrons would be notoriously affected in arbitrary electric fields (apart from (1)-(22)). On other side, (c) and (d) would represent a null force on the neutrinos.

In relation to our proposed generalization of the Lorentz force equation it deserves a mention two different types of researches. First, some researchers studying the force on magnetic dipoles, using internal forces and "hidden momentum", add a term and they write for the force the formula: $\vec{\mathbf{F}} = \nabla(\vec{\mu} \cdot \vec{\mathbf{B}}) - \frac{d}{dt}(c^{-2}\vec{\mu} \times \vec{\mathbf{E}})$ [14], [15], [16]. Second, the model proposed by Rashba (1957) for the action of electric fields over spin-orbit couplings, [30], [31].

The following commentaries are pertinent for the neutrinos, [32], [33], [34]

i) an important consequence of the massive neutrinos is their possible electromagnetic interaction, "a new physics". At present time, experimental results are negative: there is not a magnetic dipole moment up to $10^{-11}\mu_B$ or $10^{-12}\mu_B$. The frontier value for not having this "new physics " in the order of $10^{-19}\mu_B$, still far from reachable in any experiment [32]. Let us pay attention to μ_B. It was obtained in relation to the electric charge and mass of an electron and used for the muon and quarks (in the way of (13)). Continuing that process, the value for the μ_ν should be 0 as we would write $k = 0$ in (14) (electrically chargeless neutrino) even with A_ν^{-1} very large, a consequence of a mass of, at most, a few electron-volts for a neutrino. This means that μ_B is taken exclusively as a unit of measurement. In brief: $A_\nu^{-1} k \, \mu_B$ $(k = 0)$ or $\frac{q_\nu \hbar}{2m_\nu}$ $(q_\nu = 0)$ either are 0 or have no meaning for neutrinos,

ii) in this research, the neutrinos have an *axial vector magnetic charge* ${}_a\vec{J}_{m,\nu}$ with norm $\mathbf{J}_{m,\nu} = e \neq 0$ but with null magnetic moment ${}_a\vec{\mu}_\nu = {}_a\vec{0}$ (see figure 1). This is due to the *characteristic axis* (spin) always in the direction of the velocity and both under a vector product (equations in (39)). Afterwards (perhaps) it could be added a magnetic moment, due to virtual processes, with: $\mu_\nu = (\frac{g_\nu}{2})\, \mu_B < (10^{-11})\, 10^{-23} = 10^{-34}\, J\hat{t}^{-1}$.

iii) the neutrinos are under the action of electrical fields ${}_p\vec{\mathbf{F}}_\nu$ (in (4)-(32)), although their electrical charge is 0, but they are not affected by magnetic fields $({}_p\vec{\mathbf{F}}_{\vec{B}, \vec{J}_{m,\nu}} = {}_p\vec{0})$, although their magnetic charge $(\mathbf{J}_{m,\nu} = e)$ is not 0. This is so with the ruling equation (47), up to the possible effect of virtual processes (previous comment).

III. DISCUSSION. CONCLUSIONS.

We can adopt one of these two positions, to keep some of the defined magnitudes in an "internal space" (unknown) in a similar way as in the Amperian model, or to implement a model which would explain these magnitudes. This last is the position of this author as mentioned above envisaging a geometrical model for the elementary fermions [35].

In order to answer the question in the Preliminary we proceeded to the construction associated to equations (7)-(18). We have started with the conjecture in (7). This first conjecture suggests the non null masses of every elementary fermion, including the neutrinos. And we also obtain unexpected results: there are hints for some features of the electromagnetic phenomena that were not accounted previously. Is this a possible "new physics", beyond the Standard Model? The initial key point is in the concept of an axial vector magnetic charge, our second conjecture (in (15)).

The conjecture in (7) jointly with our definition of electric charge (9) and (b)-(12) involve the definition of four characteristic magnitudes $m_f\,, l_f\,, t_f\,, b_f,$ corresponding to $\{\,M\,, L\,, T\,, B\,\},$ for every elementary fermion. A question lays in: what are the actual values and the physical or geometrical significance of these four magnitudes? We have seen the results for the electrons (charged leptons), hardly for the quarks, and nothing for the neutrinos (we do not know their masses); there are not enough experimental data. The significance of m_f is essentially clear; which is not the case for l_f , t_f and b_f.

We have been guided by the following idea: there is a *vector magnetic charge* (spin vector) which we conjecture in equation (15), and an electric charge (scalar), both axial sources and independent of each other, although deeply interconnected in the physical phenomena: the electromagnetic fields. The neutrinos are singular cases: they do not have electric charge and therefore they can not produce an electric current (${}_a\overrightarrow{J}_{e,\nu} = {}_a\overrightarrow{0}$). The definition of an internal magnetic dipole moment for them as an Amperian concept is questionable. The pathway is as follows.

The elementary fermions have four different values of the electric charge, without taking account of the sign, which we relate with four different values of a constant (and of an angle variable) to a unit of electric charge ${}_a e$.

But, they have only one value of the spin with two projections (as scalars). Its measurement involves the intrinsic dipole magnetic moment, customarily considered as Amperian, which implies a non null electric charge, therefore it should not be valid for the neutrinos. In order to solve this problem, it is in our aim to avoid the electric charge as the source for the intrinsic dipole magnetic moments. This motivates the definitions (15)-(18) in a first step, with a vector character for these sources.

Previous comment, at once with the form of the equations and the dimensional analysis in the formulas in (19), suggest the vector character for the basic magnitude representing the internal magnetic property (the spin) of the elementary fermions. This vector in the definition (15), in terms of the characteristics mass, length - time and magnetic field; and in (16)-(17) with the same unit: the electric-magnetic charge ${}_a e$. But there are various specific (discrete) values of this vector, with the characteristic axes $\overrightarrow{\sigma}_f(\varphi_k)$, behind the definition: $\frac{k}{3} \equiv \frac{4}{\pi}\,\varphi_{f,k}$, up to a cylindrical symmetry. They are graphically represented in the Figures 1 and 2 in Appendix G. Our analysis forbids the inclusion of polar scalar magnetic charges (monopoles) in the definition of the elementary fermions.

Let us return to the intrinsic magnetic dipole moment $\mu_{f,k}$. In a first approach we define it in terms of the parameter imposed for the electric charge, k, in formula (18); this is coincident with the results of the Amperian treatment. Later, by using geometrical argument we propose its vector definition in equations (39), this time in terms of a new parameter, an angle $\varphi_{f,k}$. The relation between them, thanks to the vector character: $\frac{k}{3} \longleftrightarrow \sin\varphi_{f,k}$. We write it in the form ${}_a\overrightarrow{\mu}_{\varphi,f}$. Up to this point our arguments are based only in the electromagnetism with a generalization of the Maxwell's equations and of the Lorentz force law.

Afterwards, the necessity for a rectification of this value due to the inclusion of the radiative corrections, with implications for the value of the gyromagnetic ratio. The radiative corrections permit slights vector corrections of the internal dipole magnetic moment in any close direction to the exclusively electromagnetic one with an average $\langle\, {}_a\overrightarrow{\mu}_{\varphi,f}(g_f)\,\rangle$ assumed in the original direction ${}_a\overrightarrow{\mu}_{\varphi,f}(g_f^0)$; except, perhaps, for the neutrinos (due to the zero value). The vector aspect implies:

$$\|\,{}_a\overrightarrow{\mu}_{\varphi,f}(g_f^0)\,\| - \|\,\Delta\,{}_a\overrightarrow{\mu}_{\varphi,f}(\Delta g_f)\,\| \;\le\; \|\,{}_a\overrightarrow{\mu}_{\varphi,f}(g_f)\,\| \;\le\; \|\,{}_a\overrightarrow{\mu}_{\varphi,f}(g_f^0)\,\| + \|\,\Delta\,{}_a\overrightarrow{\mu}_{\varphi,f}(\Delta g_f)\,\|\,.$$

For the QED treatment for the electrons see the chapters 6 and 7 in [21]. In particular we are interested in equations from (6.33) to (6.35) and (7.46). At the present stage it is not clear the relationship of $\sigma^{\mu\nu}\,q_\nu$ and of our geometrical hypothesis. Could this be relevant in the specific calculation of the g-factor and its concomitant α? (apply it to the g-2 muon problem).

With the excuse of studying the relationship of the masses of the elementary fermions and their spins, we have been driven to a generalization of the classical electromagnetism. The spin is a quantum mechanical concept. For us it is a bridge from the classical to the quantum. We need it in order to explain the magnetism (the Amperian apart) but we can not understand it classically: the obscure concept of "rotations in an internal space". In this study we have based the (internal) magnetism upon the concept of a *characteristic axis*, avoiding any reference to rotations, although its defining direction is expressed in terms of the Pauli matrices, which are deeply involved with rotations. We do not need the three dimensional rotations any more for this magnetism (the spin).

We have tried to show up the study as simple and far reaching as possible, with two important drawbacks:

the anomalous magnetic moment; we need to study it with the techniques of the QED which is out of scope at this level, motivating many of the questions without answer. Here we have posted a possible answer to the reticence expressed by Einstein, Ehrenfest, Schwinger and Feynman; and

a deeper justification of the geometrical foundations and also of their analytical consequences. In this sense, it reminds the "obscure internal space" concept. An exposition of the geometrical structure of the elementary fermions is in another study [35]. In addition, the metrical structure of the time space would be very relevant.

In brief, the main points of this study are:

the conjecture in (7) with the characteristic mass, length, time and subsequently the magnetic part,

the conjecture in (15) for a discrete axial magnetic vector charge; discrete in two aspects: a magnitude and a direction in space, an axial characteristic axis (with the ${}_a\vec{\sigma}$ vectors),

the generalizations: straight ahead for the Maxwell's equations and complex for the Lorentz force law,

the definition of a vector dipole magnetic moment in terms of an axial vector magnetic charge and unit vector velocities,

a special consequence, the unique behavior of the neutrinos: they have magnetic charge but they do not respond to magnetic fields, up to a possible anomalous magnetic moment; although they do not have electric charge they are under the action of electric fields, with a suggestion for an experimental verification. See Appendices F and G (figure 4).

The exposition raises many questions along the text. Even more we have to search that we are not driven to erroneous answers to what it is already known. Finally, with Griffisth, after the equation (5.2) in the page 212 in [2], we write:

a theory needs to be validated with experiments.

The author of these pages is conscious of the unusual of the proposed model. Even if it is wrong or there are mistakes, the motivation to present it is based in the hope that there are enough ideas for a better development by other researchers.

IV. ACKNOWLEDGMENTS

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APPENDIX A: PHYSICAL CONSTANTS AND DIMENSIONS. [2] [36]

$$\begin{cases} c = 2.999\,792\,458\ \ 10^{8}\ ms^{-1}\ , & [\,c\,] = L\,T^{-1} \\ \hbar = 1.054\,571\,817\ 10^{-34} J\,s\ , & [\,\hbar\,] = ML^2T^{-1} \\ e = 1.602\,176\,634\ 10^{-19}\,Kg\,s^{-1}\,\hat{t}^{-1}\ \ (C)\ , & [\,e\,] = MT^{-1}B^{-1} \end{cases} .$$

$$1\ MeV = 1.602\,176\,634\ 10^{-13}\ J\ , \qquad 1\ J = 6.241\,509\,074\ 10^{12}\ MeV\ .$$

$$m_e = 9.109\,383\,701\ 10^{-31} Kg\ , \qquad m_e\,(c^2) = 0.510\,998\,950\,MeV\ .$$

$$[\ {}_a\vec{\mathbf{B}}\,] \equiv B\quad , \qquad [\ {}_p\vec{\mathbf{E}}\,] = [\ {}_p\vec{v} \times {}_a\vec{\mathbf{B}}\,] = LT^{-1}B\ .$$

$$\epsilon_0 = 8.85\,10^{-12}\ \ C^2N^{-1}m^{-2}\ , \qquad [\,\epsilon_0\,] = [\,C^2N^{-1}m^{-2}\,] = ML^{-3}B^{-2}\ .$$

$$c^2 = (\epsilon_0\ \mu_0)^{-1}.$$

$\mu_0 = 4\pi\, 10^{-7}\ NA^{-2} \quad (NC^{-2}T^2)\ ,\qquad\qquad [\,\mu_0\,] = [\,c^{-2}\,\epsilon_0^{-1}\,] = M^{-1}L\,T^2B^2\ .$

$$\left\{\begin{array}{ll} [\,\overrightarrow{L}\,] = [\,\overrightarrow{r}\times\overrightarrow{p}\,] = [\,m\ \overrightarrow{r}\times\overrightarrow{v}\,] = ML^2T^{-1} = [\,\hbar\,]\,, & \text{Mech. ang. moment} \\ [\,\overrightarrow{\mu}\,] = [\,\frac{e}{2}\ \overrightarrow{r}\times\overrightarrow{v}\,] = ML^2T^{-2}B^{-1} = M^{-1}\,[\,e\,]\,[\,\hbar\,]\,, & \text{Magnetic moment} \end{array}\right. .$$

$$\left\{\begin{array}{ll} 2 = g_e^0 \qquad g-\text{factor (gyromagnetic ratio)}, & g_e = 2.002\,319\,304\,361\,53 \\ \mu_B = \dfrac{e\,\hbar}{2m_e} = 9.274\,099\,7\ 10^{-24}\ J\hat{t}^{-1}\ \ (m^2A), & \hat{t}\equiv \text{Tesla}\,,\quad [\,\hat{t}\,] = B \\ \mu_e = -928.476\,43\ 10^{-26}\ J\hat{t}^{-1} & \\ \mu_\mu = -\quad 4.490\,44\ 10^{-26}\ J\hat{t}^{-1} \approx \frac{1}{206.768}\ \mu_e & m_\mu = 206.768\ \ m_e \\ \mu_p = \qquad 1.410\,606\ 10^{-26}\ J\hat{t}^{-1} \approx -\frac{1}{658}\ \mu_e & \end{array}\right. .$$

$$[\,{}_p\overrightarrow{j}_e\,] = \left\{\begin{array}{c} [\mu_0^{-1}\ {}_p\nabla\times{}_a\overrightarrow{\mathbf{B}}] \\ [\ {}_a\rho_e\ {}_p\overrightarrow{v}_\rho\ ] \end{array}\right\} = ML^{-2}T^{-2}B^{-1}\,, \qquad \text{in}\left\{\begin{array}{c}\text{Maxwell's equations}\\ \text{current density def.}\end{array}\right\}\ .$$

$$[\ {}_p\overrightarrow{J}_e\,] = [\ {}_a\mathrm{q}_e\ \ {}_p\overrightarrow{v}\ ] = MLT^{-2}B^{-1}\ = L^3\ [\ {}_p\overrightarrow{j}_e\,] = [\,e\,]\ [\,c\,]\ , \qquad \text{Lorentz equation}\ .$$

$$[\ {}_a\overrightarrow{j}_m\,] = [\,\epsilon_0\ {}_a\partial_t\ \ {}_a\overrightarrow{\mathbf{B}}\,] = ML^{-3}T^{-1}B^{-1} = [\ \frac{1}{c}\ {}_p\overrightarrow{j}_e\,], \qquad \text{in generalized Maxwell's equations (19)}\ .$$

$$[\ {}_a\overrightarrow{J}_m\,] = MT^{-1}B^{-1} = L^3\,[\ {}_a\overrightarrow{j}_m\,] = [\,e\,]\ , \qquad \text{in generalized Lorentz equation (22)}\ .$$

$$[\,{}_a\overrightarrow{\mu}\,] = ML^2T^{-2}B^{-1} = [mc^2]\,B^{-1} = [\,{}_a\overrightarrow{J}_m]\,T\,[\,c^2\,] \qquad (\,{}_a\overrightarrow{\mu} = I\overrightarrow{\int da}\ ,\quad U = -\,{}_a\overrightarrow{\mu}\cdot{}_a\overrightarrow{\mathbf{B}})\ .$$

$$[\,\overrightarrow{d_e}\,] = [\,\overrightarrow{q\,r}\,] = MLT^{-1}B^{-1} = [\ {}_p\overrightarrow{J}_e\,]\ T\ , \qquad \text{Electric dipole moment}\ .$$

$$[\ {}_a\overrightarrow{\mu}\ ] = [\ \overrightarrow{d_e}\,]\,[\,c\,]\ , \qquad \text{Magnetic dipole moment}\ .$$

APPENDIX B: MAXWELL AND LORENTZ TYPE EQUATIONS. [2] [6] [37]

Maxwell's equations:

$$\left\{\begin{array}{ll} {}_p\nabla\cdot{}_p\overrightarrow{\mathbf{E}} = \frac{1}{\epsilon_0}\ {}_a\rho_e\ , & -\,{}_a\partial_t\ {}_p\overrightarrow{\mathbf{E}}\ +\ c^2\ {}_p\nabla\times{}_a\overrightarrow{\mathbf{B}}\ =\ \frac{1}{\epsilon_0}\ {}_p\overrightarrow{j}_e \\ {}_p\nabla\cdot{}_a\overrightarrow{\mathbf{B}}\ =\ \ {}_p0_m\ , & {}_a\partial_t\ {}_a\overrightarrow{\mathbf{B}}\ +\ \ {}_p\nabla\times{}_p\overrightarrow{\mathbf{E}}\ =\ \ {}_a\overrightarrow{0}_m \end{array}\right. . \tag{48}$$

Lorentz force law:

$${}_p\overrightarrow{\mathbf{F}}_e\ =\ {}_a\mathrm{q}_e\,(\,{}_p\overrightarrow{\mathbf{E}}\ +\ {}_p\overrightarrow{\mathrm{V}}\times{}_a\overrightarrow{\mathbf{B}}\,)\ , \qquad {}_a\mathrm{q}_e \equiv \iiint {}_a\rho_e\ dV\ , \qquad \left\{\begin{array}{l} {}_p\overrightarrow{j}_e \equiv\ {}_a\rho_e\ {}_p\overrightarrow{\mathrm{V}} \\ {}_a\mathrm{q}_e\ {}_p\overrightarrow{\mathrm{V}} \equiv\ {}_p\overrightarrow{J}_e \\ [\ {}_a\mathrm{q}_e\,] = MT^{-1}B^{-1} \end{array}\right. . \tag{49}$$

Maxwell-Dirac equations (polar scalar magnetic charge, monopole: ${}_p\rho_m,\ {}_p\mathrm{q}_m$*):*

$$\left\{\begin{array}{ll} {}_p\nabla\cdot{}_p\overrightarrow{\mathbf{E}} = \frac{1}{\epsilon_0}\ {}_a\rho_e\ , & -\,{}_a\partial_t\ {}_p\overrightarrow{\mathbf{E}}\ + c^2\ {}_p\nabla\times{}_a\overrightarrow{\mathbf{B}}\ =\ \frac{1}{\epsilon_0}\ {}_p\overrightarrow{j}_e \\ {}_p\nabla\cdot{}_a\overrightarrow{\mathbf{B}} = \mu_0\ {}_p\rho_m\ , & {}_a\partial_t\ {}_a\overrightarrow{\mathbf{B}}\ +\ \ {}_p\nabla\times{}_p\overrightarrow{\mathbf{E}}\ = -\mu_0\ {}_p\rho_m\ \ {}_p\overrightarrow{v_{\rho_m}} \end{array}\right. . \tag{50}$$

Formula for a Lorentz-Dirac force law:

$${}_p\overrightarrow{\mathbf{F}} = \begin{array}{c} {}_a\mathrm{q}_e\ [\ {}_p\overrightarrow{\mathbf{E}}\ +\ \beta\ {}_p\overrightarrow{\mathrm{U}}_v\times(c\,{}_a\overrightarrow{\mathbf{B}})\,]\,+ \\ +\,\frac{{}_p\mathrm{q}_m}{c}\,(c\,{}_a\overrightarrow{\mathbf{B}} - \beta\ {}_p\overrightarrow{\mathrm{U}}_v\times\ {}_p\overrightarrow{\mathbf{E}}\ ) \end{array}, \qquad \left\{\begin{array}{l} c\ \beta\ \equiv\ \|{}_p\overrightarrow{\mathrm{V}}\|\ \ ,,\quad {}_p\overrightarrow{\mathrm{V}}\equiv\ c\ \beta\ \ {}_p\overrightarrow{\mathrm{U}}_v \\ [\ {}_p\mathrm{q}_m\,] = c\,[\ {}_a\mathrm{q}_e\,] = MLT^{-2}B^{-1} \end{array}\right. . \tag{51}$$

APPENDIX C: A DIGRESSION: ELECTRO (SCALAR) - MAGNETIC (VECTOR) CHARGES FOR THE ELEMENTARY FERMIONS HAVE AN AXIAL CHARACTER.

Some naive considerations about geometry (in $\mathbb{R}^3$) and physics.

We start situating two observers $\mathbf{O_1}$ and $\mathbf{O_2}$ in symmetrical positions with respect to a central point, an origin for a couple of coordinate systems $\mathbf{S_1}$ and $\mathbf{S_2}$, also the point for our test magnitudes. Both coordinate systems related by a parity transformation, opposition of the space coordinates. Associate $\mathbf{O_1}$ with $\mathbf{S_1}$ and $\mathbf{O_2}$ with $\mathbf{S_2}$.

1) We *kick* (a force) a mass-point at the origin. This mass-point moves in any direction. We represent the force and the movement with arrows starting in the origin, we denote them as *polar vectors*. Both observers *see* a vector, but the second observer *sees* it in an opposite way to the first one. By using the term *see* we mean: the vector "$\longrightarrow$", which is represented by $\overrightarrow{\mathrm{v}}\big|_{S_1} = (x_1, x_2, x_3)\big|_{S_1}$ and also by $\overrightarrow{\mathrm{v}'}\big|_{S_2} = (x'_1, x'_2, x'_3)\big|_{S_2}$ with $\{\, x'_1 = -x_1,\ x'_2 = -x_2,\ x'_3 = -x_3 \,\}$. The electric field ${}_p\overrightarrow{E}$ belongs to this kind of vectors.

2) Consider the temperature at any point, and in particular also in the origin. We associate to these points numerical values, we denote them as *axial scalars*. Both observers *see* a value at a point, and in both coordinate systems we have a unique value $v\big|_{S_1} = v\big|_{S_2}$. The $'$punctual$'$ electric charge ${}_a q$ belongs to this kind of scalars.

The work ${}_aW = {}_p\overrightarrow{\mathbf{F}} \cdot {}_p\overrightarrow{\mathbf{d}}$ is an axial scalar. The electric current vector ${}_p\overrightarrow{J}_e = {}_a\mathrm{q}_e\ {}_p\overrightarrow{\mathrm{v}}$ is a polar vector.

3) We define the mechanical angular moment ${}_a\overrightarrow{\mathbf{L}} = {}_p\overrightarrow{r} \times {}_p\overrightarrow{p}$. A product of two polar vectors, therefore it has only one representation in the two coordinate systems, there is not a change of sign (a mathematical operation). We denote these type of vectors, *axial vectors*.

Two polar vectors, one rotated of the other one, have as reference for the rotation an axis of rotation and an angle, with a rule for doing it (usually the righthand rule). We can fix this axis with a vector. Both observers *see* a unique line (the axis of rotation), and in both coordinate systems we have a unique vector represented by $\overrightarrow{a}\big|_{S_1} = (a_1, a_2, a_3)\big|_{S_1}$ and also by $\overrightarrow{a'}\big|_{S_2} = (a'_1, a'_2, a'_3)\big|_{S_2}$ with $\{\, a'_1 = a_1\,,\ a'_2 = a_2\,,\ a'_3 = a_3 \,\}$ $(\ {}_a\overrightarrow{a} = {}_p\alpha\ {}_p\overrightarrow{n} = (-{}_p\alpha)\,(-{}_p\overrightarrow{n})\ (= {}_a\alpha\ {}_a\overrightarrow{n}))$. ${}_p\alpha$ a *polar scalar* (see -4)- below). In this sense it is of interest the reading of Klein and Sommerfeld [38].

The spin ${}_a\overrightarrow{\sigma}$ and the magnetic field ${}_a\overrightarrow{B}$ belong to this kind of vectors.

In electromagnetism the concept of charge connotes a non zero divergence of the corresponding field. This is depicted for the electric charge and the field in the left part of the figure 3 (Appendix G). For the electrical charge density there is a continuity equation ${}_p\nabla \cdot {}_p\overrightarrow{j}_e + {}_a\partial_t\ {}_a\rho_e = {}_a0_e$.

Customarily we establish a zero magnetic charge with the zero divergence of the magnetic field. This can be exemplified with a limestone. For any closed surface with a limestone inside, the balance of the incoming and outgoing lines of the magnetic field is zero (a total null flux). Now, accept the axial vector magnetic charge as a source for a magnetic field and relate it to a defining axial characteristic vector (the spin). This is depicted in the right part of the figure 3 (Appendix G). With these assumptions we tentatively establish:

the axial vector ${}_a\overrightarrow{J}_{mf}$ *(therefore the related spin) is the vector magnetic charge and a source for a magnetic field.*

We obtain magnetic fields in two different ways (two origins): in an Amperian way with a moving electrical charge, and in a "spin" way (intrinsic) with this vector magnetic charge (see formulas in (19)).

We have suppressed the divergence requirement for the related field in the condition for a charge; different to the first Maxwell's equation in (19) with the electric field and the electric charge. We maintain the null divergence of the magnetic field. And we can verify a $'$continuity$'$ equation with null divergence for the axial vector magnetic charge density ${}_p\nabla \cdot {}_a\overrightarrow{j}_{mf} = {}_p0_m$.

Even for the Amperian magnetic dipole moment we can consider an axial vector, the area vector related to the enclosed surface of a loop circuit ($\overrightarrow{a} = \frac{1}{2}\oint \overrightarrow{r} \times \overrightarrow{dl}$). The problem with this is that it looses its meaning for the punctual charge. (See in Griffitsh [2] equation (1.107) and pages 255 and 269). Actually, this is a question involved under the named "rotations in an internal space". Might this suggests a relation between an axial vector and a null divergence?

4) The Dirac monopole: a *polar scalar.* Mathematically there is not a question. But, what does it mean here to *see*? With an analogy similar to that one of the temperature, $\mathbf{O_1}$ would obtain a value and $\mathbf{O_2}$ the opposite value, at once in a dimensionless point. What would other observers obtain? Nonetheless, the monopoles are not ordinary matter (elementary fermions).

Strange enough, several decades ago it was observed in some physical reactions (weak interactions) a lack of symmetry in relation to space, which was explained with a polar - axial vector character of the interaction (*V-A*). Is this only a characteristic of the interaction or is it telling us something intrinsic of the local time space of a particle?

Could we conclude from this type of observations the axial character of the charges, either scalar or vector?

We suggest: *the electromagnetic charges have exclusively axial character, whatsoever scalar or vector like, scalar for the electric part and vector for the magnetic part, giving rise to polar electric fields and to axial magnetic fields.*

APPENDIX D: A NOTE, THE ROUGHLY ESTIMATIONS WITH QUARKS.

Experimental values:

$$\left.\begin{array}{lll} m_p = 938.272\ \frac{MeV}{c^2}\,, & \mu_p = \ \ 1.410606\ 10^{-26}\, J\hat{t}^{-1} \ \approx & 1.521\ 10^{-3}\,\mu_B\ J\hat{t}^{-1} \\ m_n = 939.565\ \frac{MeV}{c^2}\,, & \mu_n = -0.966236\ 10^{-26}\, J\hat{t}^{-1} \ \approx & -1.042\ 10^{-3}\,\mu_B\ J\hat{t}^{-1} \end{array}\right\}.$$

Theoretical values of the magnetic moments for the u-quark and d-quark (after a model):

$$\left.\begin{array}{l} \mu_p = \frac{4}{3}\tilde{\mu}_u - \frac{1}{3}\tilde{\mu}_d \\ \mu_n = \frac{4}{3}\tilde{\mu}_d - \frac{1}{3}\tilde{\mu}_u \end{array}\right\}, \qquad \left.\begin{array}{l} \tilde{\mu}_u = \frac{1}{5}(4\mu_p + \mu_n) = \ \ 0.935238\ 10^{-26} \approx \ \ \frac{1}{991.192}\,\mu_B\, J\hat{t}^{-1} \\ \tilde{\mu}_d = \frac{1}{5}(\mu_p + 4\mu_n) = -0.490868\ 10^{-26} \approx -\frac{1}{1888.49}\,\mu_B\, J\hat{t}^{-1} \end{array}\right\}.$$

This accounts for various symmetries in a baryonic wave function with three particles, the "static" quarks, the only structure considered. This was relevant in the origins, for the constituent quark model. This construction goes from the protons and neutrons to the quarks, ignoring most of the complexities of the structure of the proton or of the neutron.

Amperian definition of the intrinsic magnetic moments and related "effective" masses:

$$\mu = \frac{q\,\hbar}{2\,m} = \mu_B\ \frac{q}{e}\ \frac{m_e}{m} \qquad \Longrightarrow \qquad m = \frac{\mu_B}{\mu}\ \frac{q}{e}\ m_e \quad \text{(ignoring a gyromagnetic ratio).}$$

$$\left.\begin{array}{l} \tilde{m}_u = \dfrac{\mu_B}{\tilde{\mu}_u}\ \dfrac{q_u}{e}\ m_e = \ \ 991.19\ \dfrac{2}{3}\ m_e \approx 666\ m_e \approx 338\ \frac{MeV}{c^2} \\ \tilde{m}_d = \dfrac{\mu_B}{|\,\tilde{\mu}_d\,|}\ \dfrac{q_d}{e}\ m_e = 1888.49\ \dfrac{1}{3}\ m_e \approx 639\ m_e \approx 322\ \frac{MeV}{c^2} \end{array}\right\}.$$

A very simplified model:
$$\left.\begin{array}{l} m_p \approx 2\bar{m}_u + \bar{m}_d \\ m_n \approx \bar{m}_u + 2\bar{m}_d \end{array}\right\} \qquad \left.\begin{array}{l} \bar{m}_u \approx \dfrac{1}{3}(2m_p - m_n) \approx 312\ \frac{MeV}{c^2} \\ \bar{m}_d \approx \dfrac{1}{3}(2m_n - m_p) \approx 313\ \frac{MeV}{c^2} \end{array}\right\}.$$

We can see: with $\bar{m}_u \approx \bar{m}_d \approx \tilde{m}_u \approx \tilde{m}_d$ and $q_u = -2\,q_d$ it is $\tilde{\mu}_u \approx -2\,\tilde{\mu}_d$.

Relation of the nucleon magnetic model as a "bare" proton (m_p) and the previous value:

$$\mu_N = \frac{q\,\hbar}{2\,m_p} = \mu_B\ \frac{m_e}{m_p} = 5.05\ 10^{-27}\ J\hat{t}^{-1} \qquad \text{we have:} \quad \left\{\begin{array}{l} \mu_p = \gamma_p\ \mu_N \\ \gamma_p = 2.7918 \end{array}\right. ,$$

written in a similar form to the one with the gyromagnetic ratio. Is it a lack of knowledge of the structure of the proton?

In our model, with $\sin(2\varphi) = \left(\frac{\sin(2\varphi)}{k/3}\right)\frac{k}{3}$ we have: $\sin(2\varphi_u) \equiv \sin\frac{\pi}{3} \approx (1.3)\ q_u$, $\sin(2\varphi_d) \equiv \sin\frac{\pi}{6} = (1.5)\ |q_d|$,

so that $\hat{m}_u \approx 1.3\ \tilde{m}_u \approx 439\ \frac{MeV}{c^2}$, $\hat{m}_d \approx 1.5\ \tilde{m}_d \approx 482\ \frac{MeV}{c^2}$,

are too large values for the resulting masses of a proton or a neutron in a "static" quark model.

To introduce the values of the masses of the quarks provided by the Particle Data Group [36] is significant in the opposite way, i.e. from quarks to protons and neutrons. Unlikely we do not have enough knowledge of the structure of the protons and neutrons. And this is so even for the spin concept: "What we know and what we don't know about the proton spin after 30 years" [39]. Besides, their masses are not well precised, specifically the ones of the u-quark and d-quark [36] [40], which in the Standard model are the main components of protons and neutrons in their simplest formulations. Therefore our results are of the theoretical kind that can help in an explanation.

With the values of the masses in the table in Appendix E, provided by the Particle Data Group [36]:

$$m_u \approx 2.29\ \frac{MeV}{c^2}\ , \qquad m_d \approx 4.59\ \frac{MeV}{c^2}\ : \qquad q_u\ m_u^{-1} \approx 4\,|q_d|\,m_d^{-1} \quad \Longrightarrow \quad \mu_{u,1} \approx 4\ \mu_{d,1}\ ,$$

and of $\sin(2\varphi_d) = \frac{1}{2}$, $\sin(2\varphi_u) = \frac{\sqrt{3}}{2} = \sqrt{3}\ \sin(2\varphi_d)$, we obtain the magnetic moments (absolute values) in the last column $\mu_{f,2}\ (\varphi)$ in the table related to the values in the previous column $\mu_{f,1}\ (k)$:

$$\left.\begin{array}{l} \mu_{d,2} = \left(\frac{1/2}{1/3}\right)\ \mu_{d,1} = \ 1.5\ \ \mu_{d,1} = \ \ 1.5\ \ A_d^{-1}\frac{1}{3}\,\mu_B = \qquad \frac{1}{18}\,\mu_B \qquad\qquad\qquad\qquad \approx \ \ 36\ \mu_p \\ \mu_{u,2} = \left(\frac{\sqrt{3}/2}{2/3}\right)\mu_{u,1} = \ 1.3\ \ \mu_{u,1} = \ \ 1.3\ \ A_u^{-1}\frac{2}{3}\,\mu_B = \frac{1}{3\sqrt{3}}\,\mu_B \approx \frac{1}{5.2}\,\mu_B \approx 2\sqrt{3}\,\mu_{d,2} \ \ \approx \ 125\ \mu_p \end{array}\right\} J\hat{t}^{-1}.$$

The lack of knowledge of these two orders of magnitude are a consequence of our ignorance of a detailed model for the proton. And in relation to the masses (values of A_f), in what way is it: $10^2\ (m_d + 2m_u) \approx m_p$?

Therefore, what does it happen with the protons and neutrons? Even for just the spin? (see [39]).

We know for an electron: $\Delta\mu_e \equiv \mu_e - \mu_B \approx 10^{-26}\, J\hat{t}^{-1}$, this value obtained after considering virtual processes. We also know for protons and neutrons: $\mu_{p,n} \approx 10^{-26}\, J\hat{t}^{-1} \approx \Delta\mu_e \approx 10^{-3}\mu_e$.

The baryons have 0, ± 1 electrical charges, and only a few have ± 2. The baryons with $\frac{3}{2}$-spin are extremely unstable, and the ones with $\frac{1}{2}$-spin (a minimum) are also unstable (some of them not so extremely), except the proton (stable) and the neutron ('almost' stable). These two latter are the baryons with the smallest masses.

An analogy. Let us consider **a kind of "bag" model** for the protons and the neutrons:

first: the electric charge (a scalar). The proton with $q = +1$ and the neutron with $q = 0$. These charges are the sum of the charges of the component quarks. With the electric fields we have a superposition principle. We are accustomed, in a first step, to simply add the values of the electric charges (at least for far enough distances). Multiple pole developments appear in later stages. The value of the electric charge does not seem to have any relationship with the stability of the baryons,

second: the spin (a scalar). In a similar way as with the electric charge we can obtain *the spin scalar charge* $(\frac{1}{2})$ in a proton or in a neutron summing projections of the spins (scalars) of the individual confined quarks. This was assumed in $\mu_p = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$, $\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$. Is the spin (the scalar, the 'up' and 'down') related with the stability of the baryon? Perhaps just a little bit, protons and neutrons have a $\frac{1}{2}$-spin (a minimum with three quarks), but also many other baryons. All the baryons with a $\frac{3}{2}$-spin seem to be even less stable.

In the framework presented here it remains:

third: the spin (a vector), the axial vector magnetic charge. In a similar way as for the electric charge and the spin (a scalar) we sum the vectors, defining:

$$\overrightarrow{{}_aJ_p} \equiv \overrightarrow{{}_aJ_u} + \overrightarrow{{}_aJ_{u'}} + \overrightarrow{{}_aJ_d} \qquad \text{and} \qquad \overrightarrow{{}_aJ_n} \equiv \overrightarrow{{}_aJ_u} + \overrightarrow{{}_aJ_{d'}} + \overrightarrow{{}_aJ_d}.$$

Let us look for a minimum for the magnetic moment, perhaps for a proton or a neutron. With the conditions defined in (39) applied to the three quarks, it is easy to obtain vectors which would have the exclusive global direction ${}_p\overrightarrow{\mathrm{u}}_{\mathrm{r}} \equiv {}_p\overrightarrow{\mathrm{u}}_{\mathrm{v}}$, which would imply a zero magnetic moment for a proton or a neutron $\mu_{p,n}\,(g^0_{d,u}) = 0$, like with the neutrinos. We are summing the axial vector magnetic charges $(\overrightarrow{J})$, not the magnetic moments $(\overrightarrow{\mu})$, and having assumed that this sum has the same direction as a resultant ${}_p\overrightarrow{\mathrm{u}}_{\mathrm{r}}$ of the proton or of the neutron.

Is it possible to consider, in a similar way as with the virtual processes in the electrons and the muons, that:

$$|\Delta\mu_{p,n}| = |\mu_{p,n} - 0| \approx 10^{-26}\, J\hat{t}^{-1} \approx 10^{-3}\mu_B?, \qquad \text{(see bellow for neutrinos).}$$

This time the {QED, Week, QCD} virtual processes are more significant. Could this be the reason for $10^2(m_d{+}2m_u) \approx m_p$? Perhaps the concept of "flipping" for the quarks is relevant, not only of a "flipp".

Could the combination of these rules be good enough, as a first step, for justifying the order of magnitude of the magnetic moments of the protons and neutrons? Are there other minimums? and, is this related to the stability of the particle?

We have seen already, with the charged leptons ($k = 3$ and $\varphi = \frac{\pi}{4}$), simple relationships, as for them it is $\frac{3}{3} = \sin\frac{\pi}{2} = 1$ $(\frac{\mathrm{k}}{3} = \sin 2\varphi)$. And, finally, the neutrinos. Present day experiments suggest that if the neutrinos have a magnetic moment (in this research anomalous), this moment satisfies

$$\| \overrightarrow{\mu}_\nu(g_f) \| = \| \overrightarrow{\mu}_\nu(g^0_f) + \Delta\overrightarrow{\mu}_\nu(\Delta g_f) \| = \| \Delta\overrightarrow{\mu}_\nu(\Delta g_f) \| < 10^{-11}\mu_B \sim 10^{-34}\, J\hat{t}^{-1}.$$

Would we have virtual processes with neutrinos as to obtain it? As a vector, what direction would it have? Could there be a precession of the Larmor's type? Would we need the electromagnetic fields in the direction of the movement for such precession?

Analogies?, or wanderings? Let us borrow the title of a Norton lecture (IV) at Harvard (1973) by Leonard Bernstein:

"The unanswered question. The Delights and Dangers of Ambiguity ".

Too many questions, and we need answers. This is, we need a model and experimental data. Once more, we submit this research with the intention of signaling a different pathway for the approaching to a model for the elementary fermions.

APPENDIX E: TABLE.

This table shows the values of the magnitudes we have been handling for the elementary fermions [36] [40]. The signs of the charges are shiwn only in q_k . We have chosen the masses of the quarks inside their ranks, in such a way as to get afterwards simple results. The numbers written for these fermions (first column) have to be multiplied by the numbers in the third line (e), after the fifth column onwards. The formulas of t_f, l_f, b^{-1} were mentioned in (13). Last two columns up to $g_f/2$ and related in (45).

-	k	q_k C	$m_f c^2$ MeV	A_f	l_f m	t_f seg	b_f^{-1} tesla$^{-1} \equiv \hat{t}^{-1}$	$\mu_{f,k}$ $J\hat{t}^{-1}$	$\mu_{f,\varphi}$ $J\hat{t}^{-1}$
e	$(\frac{1}{3})$	$(1.6\ 10^{-19})$	-	(0.51^{-1})	$3.86\ 10^{-13}$	$1.29\ 10^{-21}$	$2.26\ 10^{-10}$	$9.27\ 10^{-24}$	
neutrinos	0	0	?	?	?		?	0	
electron	3	-1	0.51	1	1		1	1	
u	2	$\frac{2}{3}$	2.29	$\frac{9}{2}$	$\frac{2}{9}$		$\frac{2^2}{9^2}$	0.148	0.192
d	1	$-\frac{1}{3}$	4.59	9	$\frac{1}{9}$		$\frac{1}{9^2}$	0.037	0.055
muon	3	-1	105.66	207	$\frac{1}{207}$		$\frac{1}{207^2}$	$4.8\ 10^{-3}$	
s	1	$-\frac{1}{3}$	102	200	$5\ 10^{-3}$		$2.5\ 10^{-5}$	$1.7\ 10^{-3}$	$2.5\ 10^{-3}$
c	2	$\frac{2}{3}$	1275	2500	$4\ 10^{-4}$		$1.6\ 10^{-7}$	$2.7\ 10^{-4}$	$3.5\ 10^{-4}$
tau	3	-1	1777	3484	$\frac{1}{3484}$		$\frac{1}{3484^2}$	$2.9\ 10^{-4}$	
b	1	$-\frac{1}{3}$	4250	8333	$1.2\ 10^{-4}$		$2.4\ 10^{-8}$	$4\ \ 10^{-5}$	$6\ \ 10^{-5}$
t	2	$\frac{2}{3}$	170000	333333	$3\ \ 10^{-6}$		$9\ 10^{-12}$	$2\ \ 10^{-6}$	$2.6\ \ 10^{-6}$

TABLE I. Various magnitudes for the elementary fermions.

APPENDIX F: A PROPOSAL FOR AN EXPERIMENT WITH NEUTRINOS. FIGURE 4.

In the Fermilab website [41] we read about the following possibility: to " 'see' the sun with neutrinos ".

Could we 'see' the trajectories of the neutrinos? Consider neutrinos with the appropriate energy as to extract electrons from their atoms in a detector and with the property: the angles of the trajectories of the neutrino and of the extracted electron coincide. The question is: can a detector measure that outgoing electron angle and discriminate the neutrinos?

The experimental device would consist of three parts. The first part, the source of the neutrinos (in principle neutrinos from the sun). A second part, for the possible deflection of the neutrinos going through an electrical field. The electrical field being null outside of this device. The third part, the detector.

With the source of neutrinos far enough we have neutrinos arriving to the deflection mechanism with parallel directions and with the direction of the electrical field prepared perpendicularly to them. The equation (47) presented in this research suggests for the neutrinos a parallel behavior to the one of the electrons in the Stern Gerlach experiment. Therefore, substituting the magnetic field and the Amperian internal dipole magnetic moment (electric charge current) by the electric field and the axial vector magnetic charge (the spin as a vector with a unit of charge).

The neutrinos are ultrarelativistic, we do not know their masses but we can know their range of energies and we can select a wide range of electric fields, in correspondence.

There are three different settings. Without electrical field ($\overrightarrow{\mathbf{E}} = \overrightarrow{\mathbf{0}}$), with electrical field ($\overrightarrow{\mathbf{E}} \neq \overrightarrow{\mathbf{0}}$) and with electrical field in the opposite direction ($-\overrightarrow{\mathbf{E}} \neq \overrightarrow{\mathbf{0}}$). We could verify equation (47) if the three different types of results shown in the Appendix G figure 4 (three different distributions of neutrinos), were differentiated.

Antineutrinos and neutrinos have opposite spins. With antineutrinos we should have similar results, but inverting the senses of the electrical fields with respect to the neutrino case.

APPENDIX G: GRAPHICS.

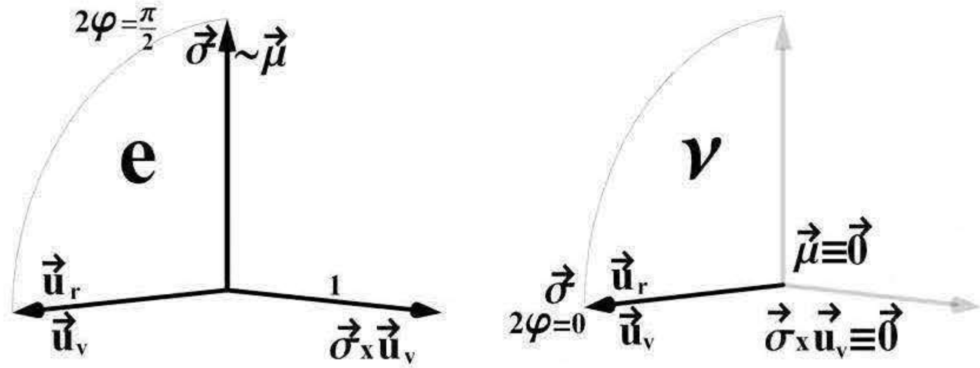

FIG. 1: Leptons

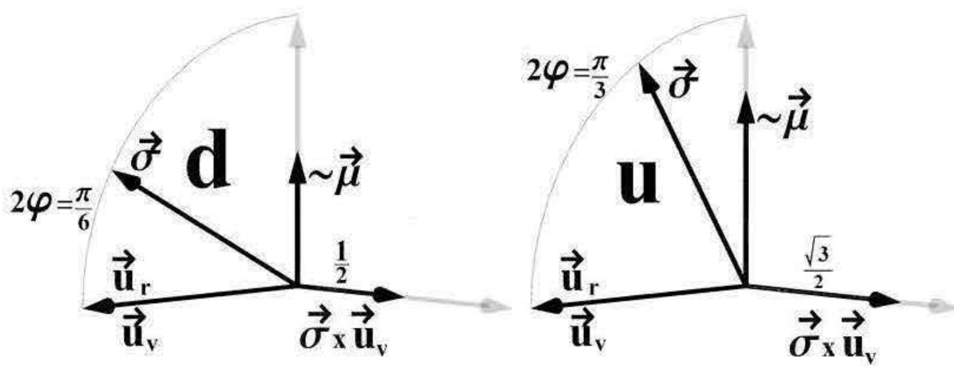

FIG. 2: Quarks

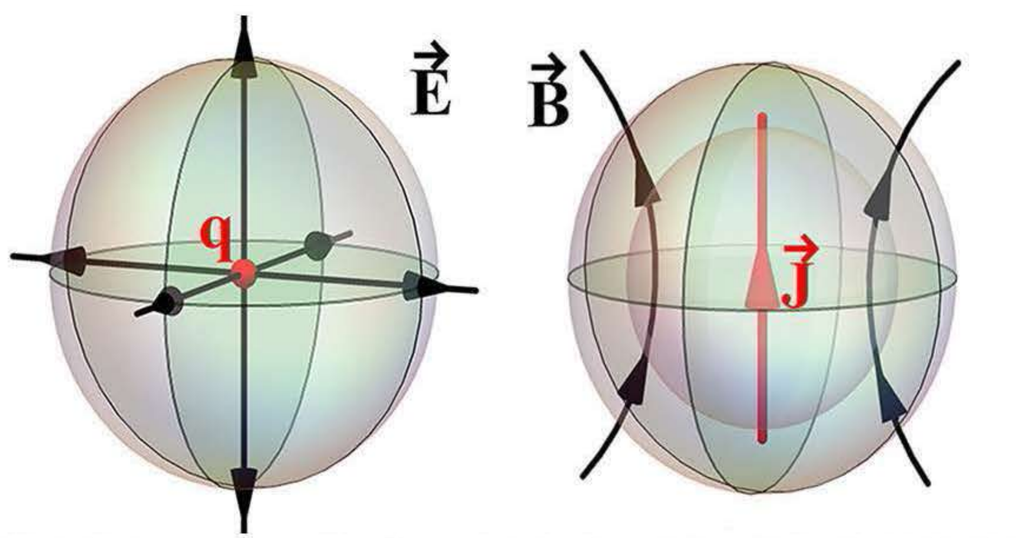

FIG. 3: Electromagnetism. The charges in red color and the field lines in black color.

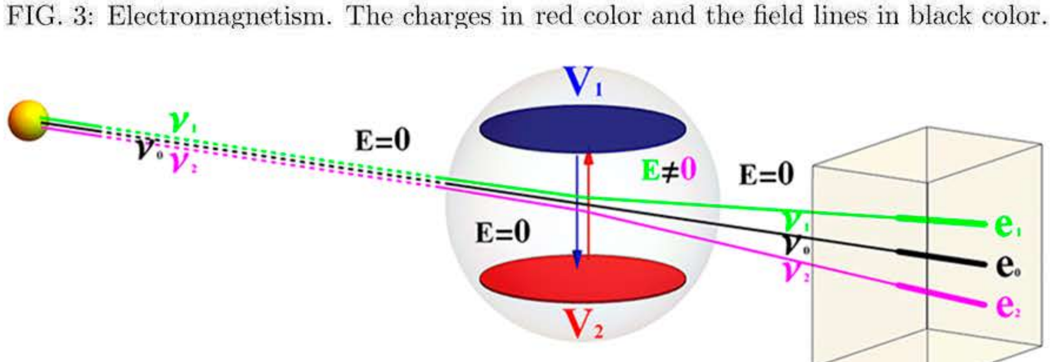

FIG. 4: An experiment with neutrinos in an electric field.

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