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Is small always beautiful? Analyzing the efficiency effects of size heterogeneity in renewable electricity auctions[☆]

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ABSTRACT

The size of bidders has been the focus of some research in the empirical literature in renewable electricity auctions, mostly related to the (negative) impact of auctions and auction design elements on the participation and award of small bidders. The main result of this literature is that small actors are discouraged from participating and being awarded in the auction and that this is a detrimental outcome of auctions. However, the impact of small bidders on the efficiency of the auction has not received a comparable degree of attention. The aim of this paper is to contribute to this literature with an analysis of the efficiency effects of bidders with different sizes in renewable electricity auctions. Our results show that a greater diversity of bidders' sizes increases the probability that the auction will not be efficient. In other words, promoting small actors with respect to large ones comes at a cost in terms of a lower allocative efficiency i.e., that the lowest-cost bidders will not be awarded. Although some governments have a goal of promoting the participation of small actors in auctions, our findings suggest that the reason for promoting actor diversity is not in the allocative efficiency of the auction, but must lie elsewhere i.e., a greater competition, mitigation of the risk of collusion, social acceptability or decentralization of renewable energy production.

1. Introduction

Actor diversity, commonly used as a synonymous of size diversity, has been a relevant, yet underresearched topic in the literature of auctions for electricity from renewable energy sources (RES-E). The size of bidders has been the focus of some analysis in the empirical literature on RES-E auctions, mostly related to the (negative) impact of auctions and auction design elements on the participation and award of small bidders. The main result of this literature is that small actors are discouraged from participating and being awarded in the auction (Grashof, 2019; IRENA, 2019; Fell, 2019; Del Río and Linares, 2014), especially in price-only auctions which do not include design elements that encourage their participation.

The literature on the impact of RES-E auctions on actor diversity is, to our knowledge, only empirical and leads to the conclusion that auctions can indeed penalize small bidders (Schenuit, 2018; Bayer et al., 2016; Bose and Sarkar, 2019; Del Río, 2017a,b). The analysis of Schenuit (2018) on auctions in Germany and the United States leads the authors to conclude that the design of auctions is complex and entails considerable transaction costs. These have a detrimental impact on the participation of small actors, competition and bid levels. The analysis of solar auctions in 2017–2018 in India shows that the awarded capacity was concentrated in only four bidders, who accounted for 60% of auctioned capacity, see Bose and Sarkar (2019). Finally, the case studies in several EU and non-EU countries carried out in the AURES project show that small actors are usually discouraged in renewable electricity auctions (Mora et al., 2017).[2]

The reasons why small bidders are discouraged from participating in auctions are diverse. It is often argued that large bidders have a higher capacity to cope with the transaction costs, the uncertainty

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[1] Both authors have equally contributed to all of the parts of this paper.
[2] See, in addition, case studies included in Wigan et al. (2016).
[3] Grashof (2019), however, suggests that, compared to large actors, smaller ones may have a higher local acceptance (which facilitates obtaining the administrative permits) and require lower levels of profitability.

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on future remuneration before the actual bidding takes place and the requested guarantees in auctions (Fell, 2019). They also have lower costs (due to economies of scale), see Dobrotkova et al. (2018), easier access to finance, more power to negotiate lower prices with equipment manufacturers, see Grashof (2019), and a greater ability to spread their risks in a large and more diverse project portfolio (Steinhilber and Soysal, 2016; Lundberg, 2019).[3] Notwithstanding, Cassetta et al. (2017) do not find a statistically significant relationship between size and bidding behavior and Bayer et al. (2016) show large variations in renewable energy market concentration in the countries considered in that study (South Africa, France, Italy and Brazil).

Therefore, according to the empirical literature of RES-E auctions, auctions seem to have detrimental impacts on the participation and awarding of small actors (see Jacobs et al. (2020) for a review). However, the key question is: why is this a problem?

It is often claimed that size (actor) diversity should be increased, i.e., smaller actors should be encouraged to participate in the auction and (eventually) be awarded (Bose and Sarkar, 2019; Bayer et al., 2016; Del Río, 2017a,b; Grashof, 2019; Schenuit, 2018). There is usually a criticism on the negative effects of auctions on the participation of small actors which is not related to allocative efficiency or competition concerns. This is related to social acceptability and other social concerns in the context of a just and inclusive energy transition (IRENA, 2019; Jacobs et al., 2020). Small and local actors are expected to be more rooted in the communities affected by renewable installations, thus making them more acceptable to the public (Lucas et al., 2013; IRENA, 2019) and (Mendonca et al., 2009).

But there might also be a competition reason on why the participation of small actors is beneficial for the functioning and outcome of auctions. There is a quantitative as well as a qualitative argument here. On the one hand, a greater number of participants would increase competition and, eventually, lead to lower bid prices. A sufficient number of actors (low market concentration) is a prerequisite for competition, free price formation and, as a result, for the lowest possible auction prices (Bayer et al., 2016). On the other hand, there is a qualitative argument. A lower diversity of actors makes agreement between them and, thus, collusion less difficult. This diversity (i.e., small together with large companies) would increase competition, reduce the likelihood of market power and collusive behavior and lead to lower bid prices.[4]

However, size diversity and smaller installations would inhibit economies of scale in RES-E generation, negatively affecting allocative efficiency. Indeed, the literature on industrial organization suggests that, although a higher level of concentration could indicate a reduction in competition, it could also suggest that the more efficient firms are more successful (Wright, 2018). As suggested by the results of econometric models (Peltzman, 1977; Brozen and Bittlingmayer, 1982; Demsetz et al., 1974), concentrated industries are more profitable than less concentrated ones. However, this might not be related to market power, but to the lower costs of large firms in terms of economies of scale. More specifically for the RES-E literature on auctions, (Hochberg and Poudineh, 2018) argue that, since large project developers have substantial financial resources and experience, these actors should be encouraged (and, consequently, actor diversity discouraged) if the goal is to have the projects built (effectiveness).

Therefore, it is difficult to tell a priori if encouraging large installations or actors instead of small ones is a negative aspect. It is explicitly assumed to be so in the specialized literature, because a model of distributed generation calls for smaller plants scattered around the territory. However, larger installations facilitate economies of scale in production.

[4] This argument is well-known in the auction-theoretic literature: a lower number of potential bidders (higher concentration) would reduce competition and make collusion more likely, leading to higher consumer prices (Ausubel and Cramton, 2011; Klemperer, 2002; Cassetta et al., 2017; Tirole, 1988).

Whatever the reasons to promote actor diversity, this is, for some governments, an end goal in itself for RES-E auctions in e.g., Germany (Anon, 2017) and Spain (MITECO, 2019). For many other countries, actor or size diversity is not an explicit goal, but the design of the auction includes eligibility criteria which may restrict participation in the auction to small projects, such as contingents (as in Greece) or size limits (as in Slovenia). Diversity is also a goal in the current Renewable Energy Directive (Directive 2001/2018)(see number 17 and article 4.4).

Despite the abundant references to actor diversity and small actors and bidders in the RES-E literature, the impact of different sizes on the efficiency of the auction has not received a comparable degree of attention. Therefore, the obvious question is: Should governments try to encourage actor diversity for an efficient functioning of auctions? In an allocative efficient auction, the bidders who can supply RES-E at the lowest costs are awarded (Haufe and Ehrhart, 2018). Thus, an auction outcome is allocative efficient if no ex-post incentives for resale exist (Ausubel and Cramton, 1999).

The aim of this paper is to analyze the efficiency effects of bidders with different sizes in renewable electricity auctions with the help of a theoretical model developed specifically for this task. Our results show that, indeed, a greater diversity of bidders' sizes increases the probability that the auction is not efficient.

As bidders' strengths are modeled by distribution functions, symmetric and asymmetric bidders differ since symmetric bidders draw their individual cost signals from the same distribution function, whereas asymmetric bidders draw them from different ones. If bidders are symmetric, all bidders have ex ante the same expected strength, where the particular signals may differ and are private information. We investigate the consequences of asymmetric bidders, i.e. bidders that ex ante have different sizes and cost expectations. Consequently, for asymmetric bidders not only the private cost signals may differ but also the expected strengths, i.e. asymmetric bidders are ex ante distinguishable from each other by their expected strengths. In order to assess bidders' strengths, we assume that different distribution functions can be ranked, i.e., a stronger bidder's cost signal is more likely to fall below a certain value than that of a weaker bidder (Haufe and Ehrhart, 2015). Or, in other words, a stronger bidder is expected to have lower costs than a weaker one. Inefficiencies may occur in a pay-as-bid auction due to asymmetries among bidders: weaker bidders submit more aggressive bids than stronger ones as they face higher competition.

Accordingly, this paper is structured as follows. The next section presents the theoretical model. Section 3 provides the analysis to characterize linear equilibria. Section 4 analyzes the efficiency of those equilibria. Section 5 concludes and discusses the policy implications of our findings.

2. Model

Auctions, as theoretical objects, are games of incomplete information in which each bidder has, prior to bidding, some private information which is socially relevant and that can only be elicited by bidding. As for efficiency, it suffices to elicit who is the bidder with highest valuation or with lowest costs. However, eliciting information when multiple units are auctioned and some bidders have a taste for more than one unit, usually referred as multi-unit multi-bid auctions, is a delicate task.

In this paper we consider multi-unit multi-bid auctions for contracts (procurement). Each bidder (firm) observes privately her costs to fulfill the contracts and bids a price at which she commits to do it in case of being awarded one. In particular, this is the case of RES-E auctions. Contracts are allocated to the bidders who submitted the lowest prices. If those are, additionally, the bidders with lowest costs, the allocation is efficient. This paper uses a game theoretic approach to show whether the pay-as-bid auction format combined with some ex-ante asymmetry across bidders in terms of size might drive the auction mechanism away from efficiency. The intuition for the potential inefficiency that follows from our analysis is straightforward. If an ex-ante high-cost firm competes in the auction against an ex-ante low-cost one, the former, aware of her a priori disadvantage, might try to compensate it by bidding a very low price and thus get the contract despite having a higher cost. Two key assumptions in this argument are: (i) There is ex-ante an asymmetry across bidders and the higher-cost firm is aware of her disadvantage and (ii) this asymmetry has a probabilistic nature, that is, the low-cost firm cannot guarantee herself a win in the auction without making a loss.

We must start being precise about what "asymmetry" means. We assume there is a large firm and a (finite) number of small firms. The difference between both types of firms is twofold. First, the former has the capacity to fulfill all the contracts on sale, and consequently the large firm submits multiple bids, while each small firm can only fulfill one contract and, thus, bids for it. We assume two contracts are auctioned, while the number of small firms is eventually larger than two, so that the whole allocation might go to small firms. Furthermore, as it is standard in auction theory, the costs constitute the bidders' types and are assumed to be draws from random distributions. The second difference between large and small firms is that the probability distribution function that generates the costs for the large firm is to the left, in a stochastic dominance sense, of the corresponding function for the small firms. Again, details matter. While one probability distribution is to the left of the other, both are assumed to have the same support. In short, our theoretical model is such that there is not a bid from the large firm that guarantees herself a profitable contract.[5]

We consider a procurement auction in which two indivisible contracts, or units, are auctioned. The bidders are firms. A bid is a price for a single contract at which the firm commits herself to fulfill it in case of winning. We introduce some fundamental and ex-ante asymmetry among bidders by assuming that there is one large and N small firms. Each firm knows whether she is large or small and also knows the number of large and small rivals she competes with. Generally, we denote the large firm by L and a generic small firm by n, $n \in \{1, \dots, N\}$.

There are two differences between L and any n. First, L has the capacity to bid for – and fulfill in case of winning – both contracts, while n bids for just one. We assume that there are at least two small firms, that is, $N \geq 2$, since, otherwise, the large firm would have one contract for granted. Second, L's costs are expected to be lower than n's. Being more precise, each firm, either large or small, observes privately her cost(s) to fulfill the contract(s) in case of winning. Thus, L observes two costs, while each n observes just one cost. Costs are assumed to be random draws from some probability distributions which are common knowledge among all bidders. Let F and G denote the corresponding cumulative probability distributions for n and L, respectively. Thus, each n gets one draw from F, which constitutes her cost, while L gets two draws from G, which constitute L's costs. Statistical independence among all costs is assumed. For the time being, we will just assume that both probability distributions are continuous with a common and finite support, $\Omega = [\underline{w}, \overline{w}]$. Later in this section we will assume (and justify) further some specific cost distributions under which the large firm is more likely to have smaller costs in a first-order stochastic dominance sense.

Each firm, either large or small, first observes privately her cost (or costs) and then submits her bid (bids). All bids are placed simultaneously. The two lowest bids get the auctioned units. A pay-as-go format is assumed. If a firm wins a contract with a bid price p and has a cost c to fulfill it, her reward from that bid is $p - c$. The reward from any non-winning bid is obviously zero. This applies either for L or n. In particular, if L has costs d_1 and d_2 with $d_1 < d_2$ and wins exactly one contract with a bid p, then her total reward is $p - d_1$. Hereafter, c_n denotes n's cost n while the pair (d_1, d_2) denotes L's. Moreover, this definition of reward implicitly assumes that contracts are homogeneous from a firm's perspective. For instance, firm n has cost c_n regardless of which of the two auctioned contracts she is awarded with. Again, the same applies for L which, additionally, has decreasing returns to scale when she is awarded both contracts. We assume firms maximize the expected reward.

Formally, our model is a Bayesian game of incomplete information in which the types of the firms are the corresponding costs. A strategy for any small firm is a bidding function that maps costs into bids. Let β denote a generic strategy for n, which means n bids $\beta(c)$ when her cost is c, for any $c \in \Omega$. We restrict ourselves to strictly increasing and differentiable bidding functions from Ω to itself. We define the strategy space for L likewise, except for the fact that L's type and bid are both bi-dimensional. We will show that the dimension of L's strategy space can be reduced under certain conditions but, for a general presentation of the equilibrium concept, let γ denote a generic strategy for L. A Bayesian Nash equilibrium, or simply an equilibrium, is a profile formed by a pair of strategies (β, γ) such that: if each small firm plays β and L plays γ, no firm has incentives to deviate unilaterally from the profile in terms of expected reward. We will show that, even though one of the costs of L coincides with some n's, both firms might bid different prices associated to that cost at an equilibrium, which breaks the usual symmetry that holds in auction models in which all bidders are ex-ante identical.

A final question regarding the model is worth asking: Why exactly one large firm and two contracts? We aim to model competition between *ex-ante* asymmetric firms. For that, only relative magnitudes matter. If we increase the number of large firms while keeping just two contracts, presumably the small firms will have no chance to get a contract since the large ones are expected to have lower costs, thus we would essentially have competition between symmetric – *all* large – firms. On the contrary, if we increase the number of contracts while we keep only a large firm with capacity to take at most two contracts, then the small firms would not be much affected by the large firm, which again leads us to competition between symmetric – now *all* small – firms. These two variations illustrate the necessary balance the model needs in order to focus on the competition between asymmetric – large *and* small – firms. We informally claim the one-large firm two-contract model just presented is the simplest balanced setting.

3. Analysis

We split this section into three subsections. The first two characterize the best response strategies, for the large firm and a generic small firm, respectively. In the third subsection we introduce both a distributional assumption and a restriction in the strategy space which, jointly, allow us to find simple close form solutions for those best responses, which in turns allows to analyze efficiency. All proofs are in the appendix.

3.1. Best response from L

Arguably, a major challenge of our theoretical analysis is that firms whose costs are drawn from different probability distributions will, in principle, use different bidding functions. Consequently, each firm takes this heterogeneity among her rivals into account when bidding. In addition, the large firm has two costs, say d_1 and d_2 with $d_1 \leq d_2$, such that her cost is d_1 in case she is awarded just one contract and $d_1 + d_2$ in case she is awarded both contracts. That means that her type of bidding is bi-dimensional and that, eventually, the large firm might play a different bidding function for the first than for the second unit. The model allows us to address both issues separately from one another.

[5] Even in a theoretical setting in which the large firm can guarantee herself a profitable contract, it is not obvious that she will just play her sure-winning bid. However, we have opted to explore a setting in which the advantage of the large firm is less clear-cut.

Therefore, this subsection adopts the point of view of L and deals with the best response from the large firm, whose type is bi-dimensional but whose rivals are not heterogeneous. In other words, all of her rivals use the same bidding function (i.e., they are small ones).

Let us assume that all small firms are playing some strictly increasing strategy β, L bids prices p and q, where notation is such that $p \leq q$ holds, and has costs (d_1, d_2) where, without loss of generality, $d_1 \leq d_2$ holds. We sometimes will refer to p and q as L's low and high bid, respectively. The next proposition presents the problem that characterizes L's best response.

Proposition 1. *Assume all small firms play some strictly increasing strategy β. Let L's costs be denoted by $\mathbf{d} = (d_1, d_2)$, with $d_1 \leq d_2$. Then L's best response is the pair (p, q) that solves:*

$$max_{(p,q)}\{J^L(p,q;(d_1,d_2))\} \qquad s.t: \qquad p - q \leq 0, \;\; p \geq d_1, \;\; q \geq d_2$$

where:

$$J^L(p,q;\mathbf{d}) = (p-d_1)(1-F(\beta^{-1}(p)))^{N-1}(1+(N-1)F(\beta^{-1}(p))) + (q-d_2)(1-F(\beta^{-1}(q)))^N \tag{1}$$

We must notice that winning with q implies (but is not implied by) winning with p. The first term in the right hand side of (1) states that whenever L wins with p, her reward is $p - d_1$. Since p is L's lowest bid, L wins with p either when all of her rivals (all the N small firms), submit a bid higher than p, whose probability is $(1 - F(\beta^{-1}(p)))^N$, or when exactly one of them submits a bid lower than p, whose probability is $NF(\beta^{-1}(p))(1 - F(\beta^{-1}(p)))^{N-1}$. The term accompanying $p - d_1$ is the sum of both probabilities. Analogously, the second term in (1) is the reward from winning with q, which is $q - d_2$, times its probability. To compute this later probability it suffices to notice that L wins with q whenever all of the rivals submit a bid higher than q.

The expression in (1) suggests a route. Since J^L is additively separable in two terms, which depend on (p, d_1) and (q, d_2), the optimization problem in Proposition 1 could be de-coupled in two problems, with decision variables p and q, respectively. Yet, such decoupling is not trivial since $p \leq q$ must hold whenever $d_1 \leq d_2$ holds. The next proposition shows, as a first step, that when $d_1 = d_2$ holds, then $p = q$ also holds. The proposition states requirements for the second order conditions to hold which are generally required hereafter. Some of those requirements stem from the hazard function of the probability distribution that generates the small firms (or L's rivals) costs. We denote the hazard function as H.[6]

Proposition 2. *Assume $\beta''(c) \leq 0$ and $H_F(c)$ is non-decreasing for all $c \in (\underline{\omega}, \overline{\omega})$. If $d_1 = d_2$, then there exists a unique best response from L and it satisfies $p = q$.*

An additional element will allow us to progress in the analysis. If we consider a large (but finite) number of small firms, the de-coupled problems that characterize p and q converge both to a problem that characterizes the best response in a single-unit first-price auction with N rivals playing β, with the own bids being p in one problem and q in the other, and the costs being d_1 and d_2, respectively. Under standard conditions on β and the rivals probability distribution of costs, the solution to this latter problem is a bid that increases with the own cost, that is, $p \leq q$ holds whenever $d_1 \leq d_2$ holds. The underlying intuition for the approximation is simple. If there are many small firms, the likelihood of winning with p and with q becomes similar to one another and similar to the likelihood of submitting a bid price lower than N rivals. The next proposition summarizes the analysis.

Proposition 3. *Assume $\beta''(c) \leq 0$ and $H_F(c)$ is non-decreasing for all $c \in (\underline{\omega}, \overline{\omega})$. If $d_1 < d_2$, the best response is unique and arbitrarily close, as N grows large, to the solution of:*

$$\frac{1}{N} = (p-d)H_F(\beta^{-1}(p))\frac{d}{dp}\beta^{-1}(p) \tag{2}$$

where, taking $d = d_1$ and $d = d_2$, we obtain p and q, respectively.

In essence, Proposition 3 provides two messages. First, there exists a strictly increasing function, say γ, such that for any pair (d_1, d_2) with $d_1 < d_2$, the best response from L (under the hypothesis of the proposition) can be approximated by $(\gamma(d_1), \gamma(d_2))$, and the approximation error converges to zero as N increases. Second, γ is the best replay function of a bidder who faces N rivals in a first-price single-unit auction.

Therefore, we have shown in this subsection that the best response from the large firm requires that the hazard function of the probability distribution that generates the cost of the small firm is non-decreasing, which is a standard requirement in single unit auction models. We have identified sufficient conditions which allow us to simplify the characterization of the best response. The most important result is that, when the number of small firms is large (but finite), the best response can be approximated by a single strictly increasing bidding function. In other words, the best response from L is an strictly increasing function, say γ, such that the large firm's bids are $\gamma(d_1)$ and $\gamma(d_2)$ when her costs are d_1 and d_2. Interestingly enough, that bidding function constitutes a best response in a single unit auction model.

3.2. Best response from each small firm

We analyze the best response from small firms. Assume that all other small firms play some strictly increasing strategy β. In line with Proposition 3, L's strategy is defined by a single and strictly increasing bidding function γ. The next proposition presents the problem that characterizes n's best response.

Proposition 4. *Consider some arbitrary small firm, say n, with cost c. Assume all small firms other than n play some strictly increasing strategy β, while there is some strictly increasing bidding function γ such that L's bids are the pair $(\gamma(d_1), \gamma(d_2))$ when L's type is (d_1, d_2). Then n's best response is a scalar b that maximizes:*

$$J^n(b;c) = (b-c) \times \Big[(1-F(\beta^{-1}(b)))^{N-1}(1-G(\gamma^{-1}(b))^2) + (N-1)F(\beta^{-1}(b))(1-F(\beta^{-1}(b)))^{N-2}(1-G(\gamma^{-1}(b)))^2\Big] \tag{3}$$

The reward for firm n for winning with a bid b and a cost c is clearly $b - c$. The expected reward in (3) is the reward times the probability of winning. There are two disjoint events under which n wins: (i) when she submits the lowest bid among all small firms and her bid is smaller than the high bid from L, (ii) when she submits the second lowest bid among all the small firms and her bid is smaller than the low bid from L. The probability in (3) is the sum of the probabilities of both events. Naturally, that probability involves rivals' strategies β and γ, and the probability distributions generating the rivals types F and G for small and large firms, respectively.

In line with the analysis that characterizes the best response from L, we now focus on the solution to the problem stated in Proposition 4 when the number of small firms N is large. The basic result is presented in the next proposition.

Proposition 5. *Assume $\beta''(c) \leq 0$ and $H_F(c)$ is non-decreasing for all $c \in (\underline{\omega}, \overline{\omega})$. For a large N, the best response from firm n with cost c is given by:*

$$\frac{1}{N} = (b-c)\left(\left(\frac{N-2}{N}H_F(\beta^{-1}(b)) - \frac{f(\beta^{-1}(b))}{NF(\beta^{-1}(b))}\right)\frac{d}{db}\beta^{-1}(b) + \frac{2}{N}H_G(\gamma^{-1}(b))\frac{d}{db}\gamma^{-1}(b)\right) \tag{4}$$

[6] Referred to a cumulative continuous distribution F with density f, it is defined as $H_F(c) = \frac{f(c)}{1-F(c)}$.

In essence, Propositions 3 and 5 deliver the same message. In the former, only β is taken into account since that is the strategy played by all of the rivals. Accordingly, in Proposition 5 there is a convex combination of β and γ, with the relative weights given by the relative *abundance* of rivals playing the corresponding strategy. There is an additional term in Proposition 5 with respect to Proposition 3, since the winning conditions differ from L to n's. The requirements in terms of second order conditions are analogous in both propositions.

Therefore, in this subsection it has been shown that there is also a requirement on the hazard function of the small firm, which is similar to the large firm's problem. In addition, we have shown that, by assuming a large number of small firms, the small firm's best response can be simplified while still preserving the *ex-ante* heterogeneity of the rivals costs.

3.3. Linear GP-based equilibria

In order to address the issue of efficiency, it is necessary to further restrict the model in a way that allows us to go beyond the general properties of the previously outlined best response problems. Therefore, in this subsection we introduce two additional assumptions that will help on the journey from best responses to equilibrium. On the one hand, we assume that the probability distribution function that generates the costs, of both large and small firms, is generalized Pareto (GP hereafter) still allowing for different distributions for large than for small firms. The use of GP distributions is not new in auction theory, and in particular it is not new when studying the efficiency of multi-unit multi-bid auctions, see Ausubel et al. (2014). On the other hand, we restrict the analysis to linear best response functions. Taking again a single firm perspective, we assume all of the rivals are playing some linear strategy and we find the linear response that best approaches the firm's response. The combination of both assumptions is reasonable in the sense that, when costs are GP generated, the optimal response to a linear strategy is close to be itself linear, in a sense that will be defined in this section.

The use of Pareto distributions has a long tradition in economics to approximate some tail of a more general – eventually unknown – distribution. The statistical ground for that is the extreme value theory, which roughly states that a tail, either left or right, of *any* probability distribution that meets some regularity conditions can be in the limit approached by a Pareto distribution. A classical application in economics is to use Pareto distributions to model the right tail of the wealth's distribution over a given population. More recent uses include auction theory or the eBay market. When applied to our problem, it is reasonable to think that firms participating in the auction, either large or small, are the most efficient within their corresponding group of a larger population of potential entrants. In other words, the bidders' costs lie in the lower tail of a more general distribution of costs.

Generalized Pareto are three-parameter distributions, usually written $GP(\mu, \sigma, \xi)$, where the arguments are location, scale and shape, respectively. Recall that we have denoted the distribution of small and large firms by F and G, respectively, and we have restricted them to have common support $[\underline{w}, \overline{w}]$. In terms of that notation, we consider $GP(\underline{w}, \sigma, -\sigma/\kappa)$, where we denote $\kappa := \overline{w} - \underline{w}$ and $\sigma > 0$. Common support implies κ common to F and G, while σ might differ. For the small firms, the cumulative probability distribution function is:

$$F(c) = 1 - \left(1 - \frac{1}{\kappa}(c - \underline{w})\right)^{\kappa/\sigma_F}$$

The expression for G is identical with σ_F replaced by σ_G. Since we will interpret F and G as the lower tail of a general cost distribution, the scale parameter, σ, should be one such that the densities are both increasing. For that, it can be shown that it suffices to assume $\sigma > \kappa$. Additionally, F stochastically dominates G whenever $\sigma_G < \sigma_F$ holds. Finally, for any σ, the corresponding hazard functions are not decreasing, as required by the best response results in the previous sections. Moreover, the GP distributions have linear inverse hazard functions. Reversely, a linear inverse hazard function implies that the corresponding distribution is GP. The Appendix contains a lemma with the proof of these statements.

Our second assumption in this section is to assume linear bidding functions. Specifically, let us write

$$\beta(c) = \overline{w} + \beta_0(c - \overline{w}); \qquad \gamma(c) = \overline{w} + \gamma_0(c - \overline{w}) \tag{5}$$

where β_0 and γ_0 are parameters to be determined by the equilibrium conditions and are restricted to lie in[7] $(0, 1]$. Notice that (5) imposes $\beta(\overline{w}) = \gamma(\overline{w}) = \overline{w}$. In short, any firm (whether large or small) with the highest cost, $c = \overline{w}$, is bound to have no reward, defined as price minus cost, in case of winning. Apart from that, linearity assumes that the reward decreases linearly with cost.

Proposition 6. *Assume F and G are $GP(\underline{w}, \sigma_F, -\sigma_F/\kappa)$ and $GP(\underline{w}, \sigma_G, -\sigma_G/\kappa)$, respectively, in propositions 3 and 5 all firms play linear strategies given by (5). Consider additionally equilibrium conditions by which all the small firms play the same strategy. Then β_0 and γ_0 satisfy:*

$$1 = \frac{1-\beta_0}{\beta_0}\kappa\left(\left(N - 1 - \frac{1}{F(c)}\right)\frac{1}{\sigma_F} + \frac{2}{\sigma_G}\right) \tag{6}$$

and

$$1 = \frac{1-\gamma_0}{\gamma_0}\kappa \cdot N\frac{1}{\sigma_F} \tag{7}$$

respectively, for any $c \in [\underline{w}, \overline{w}]$.

Now we must notice the difference between the Eqs. (6) and (7) in Proposition 6. The latter equation does not depend on c, consequently there exists a unique value of γ_0 which is valid for any c. In other words, if all of the rivals of firm L are playing a linear strategy, L's best response is linear as well. In contrast, in Eq. (6) there is a term which depends on c, which means that value of β_0 which solves the equation changes with c. That implies that the best response from a small firm when all other rivals play a linear strategy is non-linear.

Consequently, the remaining problem is to approximate Eq. (6) in Proposition 6 with an expression that does not depend on realizations of the cost. A usual approach in economics is to take expectations on both sides of Eq. (6), which could be interpreted as taking a *representative* cost realization. Unfortunately, taking expectations in Eq. (6) implies to compute $E\{F(c)^{-1}\}$, which is not finite, as it is shown in a lemma in the supplementary material. Our approach is to replace that expectation by the median value. In short, among the small firms, we take as representative the median firm with respect to $F(c)^{-1}$.

Using the median value of $F(c)^{-1}$, we approximate Eq. (6) in Proposition 6 with:

$$1 = \frac{1-\beta_0}{\beta_0}\kappa\left((N-3)\frac{1}{\sigma_F} + \frac{2}{\sigma_G}\right) \tag{8}$$

The solution of β_0 in Eq. (8) and the solution of γ_0 in Eq. (7) in Proposition 6 are, respectively:

$$\beta_0^* = \frac{\theta_\beta}{1+\theta_\beta}; \qquad \gamma_0^* = \frac{\theta_\gamma}{1+\theta_\gamma} \tag{9}$$

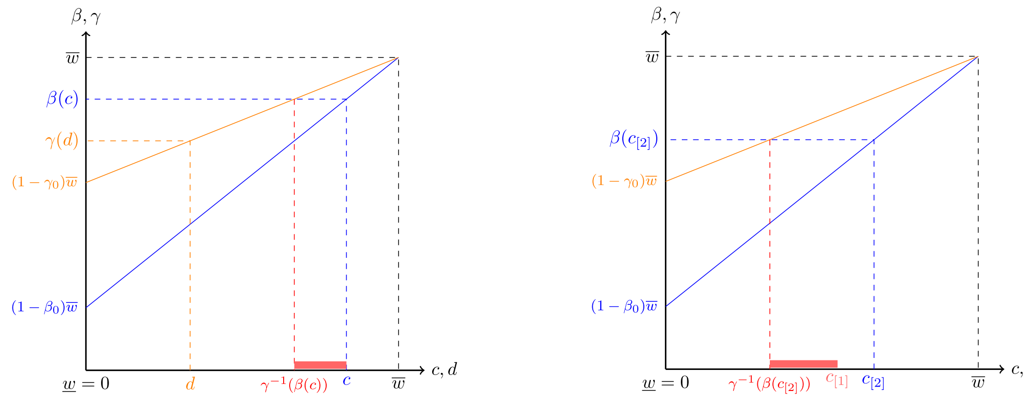
where we have denoted

$$\theta_\beta = \kappa\left((N-3)\frac{1}{\sigma_F} + \frac{2}{\sigma_G}\right); \qquad \theta_\gamma = \kappa\frac{N}{\sigma_F} \tag{10}$$

The comparison between β_0^* and γ_0^* is straightforward. Notice that expression $\theta/(1+\theta)$ is strictly increasing in θ. Thus:

$$\beta_0^* > \gamma_0^* \iff \theta_\beta > \theta_\gamma \iff \sigma_F > \frac{3}{2}\sigma_G$$

[7] $\beta_0 > 0$ simply states that β is increasing. $\beta_0 \le 1$ is equivalent to $\beta(c) \ge c$, that is, the firm does not operate at a loss. The analogous statements are valid for γ_0.



(a) Inefficiency for one contract. (b) Inefficiency defined by (12).

Fig. 1. Inefficient allocation in a one-contract setting (a) and in a two-contract setting when the small firms win both contracts (b). In both panels, the bidding function for the large and small firms are represented by the orange and blue lines, respectively. Each bidding function maps cost realizations (horizontal axis) into bids (vertical axis). The figure illustrates the relative position of the bidding functions under Eq. (11). The dashed lines are potential cost realizations and their corresponding bids. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Recall that, whenever both distributions are GP distributed over a common and finite support, F stochastically dominates G if and only if $\sigma_F > \sigma_G$. In addition, $\beta_0^* > \gamma_0^*$ can be interpreted as the small firms being more aggressive than the large one, that is, they submit lower prices. Putting both things together: if the cost distribution of the large firm, G, is sufficiently *to the left* of the cost distribution of the small firm, F, then the small firms bid more aggressively than the large one. This statement has also an *only if* part. For instance, if $\sigma_F = \sigma_G$, that is, both distribution of costs are equal to one another, then the large firm is more aggressive. We must bear in mind that the successive approximations in which β_0^* and γ_0^* are based assume a large number of small firms. Some straightforward additional properties of the linear equilibrium obtained above are omitted.

4. Results on allocative efficiency

This section uses the obtained equilibrium to analyze allocative efficiency, which for simplicity we simply refer to as efficiency in the rest of this section. We use simulations to compute the likelihood of having inefficient allocations at an equilibrium under different configurations of parameter values. We explore the likelihood of an inefficient allocation under linear equilibria and GP-distributed costs in which small firms are more aggressive than the large one, that is, the following holds:

$$\beta_0 > \gamma_0 \tag{11}$$

The basic idea is illustrated in Fig. 1. Consider first panel (a) which, for simplicity, assumes a single-contract procurement auction between a large and just a small firm. The panel depicts two cost realizations, for the large firm (orange, denoted by d) and the small firm (blue, denoted by c), respectively, such that the auction allocation is efficient: the large firm has lower costs than the small one and wins the auction. However, if keeping c fixed, the realization d lies in the red interval drawn in the cost axis, which is $(\gamma^{-1}(\beta(c)), c)$, then the corresponding allocation would be inefficient: L has lower cost but the small firm gets the contract. Panel (b) considers two contracts and N small firms. Perhaps the most extreme case of inefficiency is that in which: (i) small firms win both contracts, and (ii) even the highest cost of the large firm is smaller than the lowest cost of the small firms. Formally, (i) and (ii) occur if and only if:

$$\gamma^{-1}(\beta(c_{[2]})) \le d_{[1]} \le d_{[2]} \le c_{[1]} \le c_{[2]} \tag{12}$$

The panel (b) of Fig. 1 depicts the logic of the previous inequalities. If both costs realizations of the large firm, $d_{[1]}$ and $d_{[2]}$ lie in the red interval, the statements (i) and (ii) above hold.

Another type of inefficiency occurs when small firms win exactly one unit, thus the large firm wins the other, while the losing cost of the large firm is smaller than the winning cost of the small one. The chain of inequalities necessary and sufficient for that is:

$$d_{[1]} \le \gamma^{-1}(\beta(c_{[2]})), \qquad \gamma^{-1}(\beta(c_{[1]})) \le d_{[2]} \le c_{[1]} \tag{13}$$

The first inequality in (13) ensures that the large firm wins with $\gamma(d_{[1]})$, whereas the second chain of inequalities ensures that the large firm does not win with $\gamma(d_{[2]})$ though $d_{[2]} \le c_{[1]}$ holds. This inefficiency is depicted in Fig. 2.

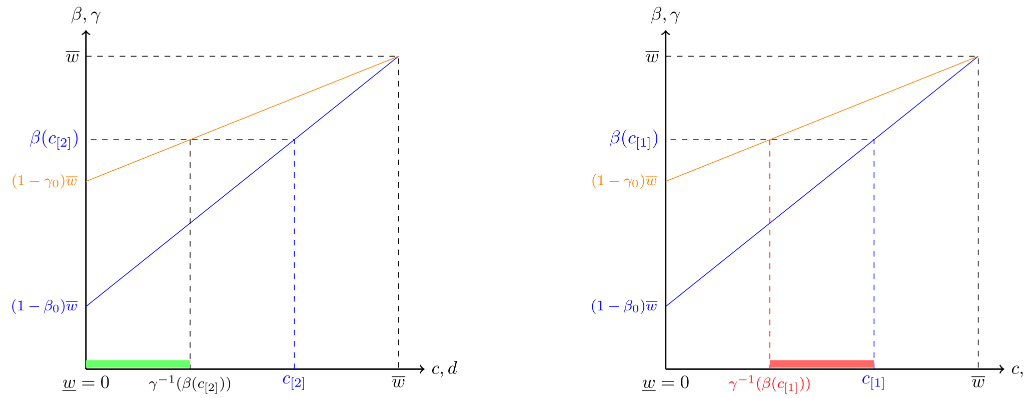
While it is possible to obtain closed-form expressions for the probabilities of both kinds of inefficiency presented above, the comparative statics analysis is rather complex. Hence, we have carried out that exploration using Monte Carlo simulations.[8] In order to select parameter values, recall that we interpret GP distributions as approximations of the left tail of a more general distribution, which, as mentioned above, implies that both σ_F and σ_G are greater than κ. In addition, we want to consider situations in which (11) holds. As shown in the previous section, this implies that $\sigma_F > \frac{3}{2}\sigma_G$.

Table 1 shows the combinations of parameters. The first row just fixes the support of both F and G to be the interval $[0, 1]$. Shifts in these values are qualitatively irrelevant. The fundamental parameters of the model are σ_F, σ_G and N. The next three rows fix σ_F, while considering variations both in σ_G and N. Each value for σ_G is combined with each value for N, which leads to 14×3 combinations of parameters. For each combination of parameters, the last row indicates that 1000 repetitions of the auctions are carried out. Each repetition is a Monte Carlo (MC) run. For each MC run, draws from F and G are generated, then bids take place and the auction allocation is computed. The random generator seed is fixed and common to every combination of parameters, in order to set homogeneous realizations across combinations.[9]

The results of the simulations are depicted in Fig. 3. First, we must notice that, for any value of the ratio σ_G/σ_F and for any value of N, the auction is efficient in a large percentage of cases for the selected parameter values. The vertical axis shows that the lowest percentage of efficiency is roughly 88%. Whenever there is no efficiency, the

[8] The simulations have been programmed in Python 3.6.5. The basic source code is available from the corresponding author upon request.

[9] The computation time increases linearly with the number of parameter combinations. For all of the combinations in the table, the total computation time was aprox. 1250 s in a MacBook Pro with processor 2.4 GHz Intel Core i5.



(a) $d_{[1]}$ must lie in the green interval. (b) $d_{[2]}$ must lie in the red interval.

Fig. 2. Inefficient allocation when a small firm wins one unit. The notations and coloring are as in Fig. 1. Panels (a) and (b) represent the first and the second chain of inequalities in (13), respectively. Inefficiency is only in the red interval. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 1
Parameter values used in the simulations.

Notation	Description	Value (s)
$\underline{w}$, κ	Lower bound, range both for F and G	0,1
σ_F	Scale of F	6
σ_G	Scale of G	$\{1.2, 1.4, \ldots, 3.8\}$
N	Number of small firms	$\{10, 15, 20\}$
MC	Number of MC runs	1000

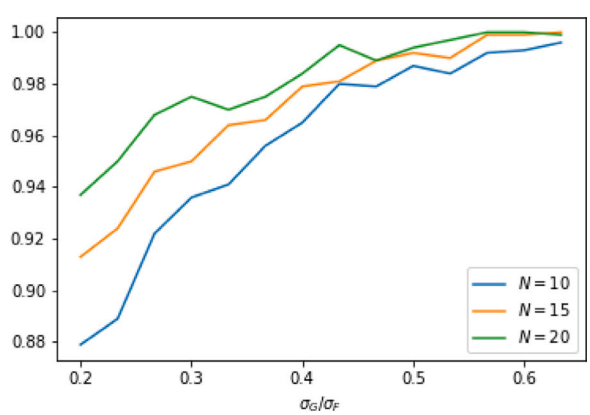


Fig. 3. Percentage of MC runs with efficient allocations. The vertical axis shows the percentage of MC runs in which the auction allocation is efficient for a given parameter combination. The horizontal axis runs along the ratio σ_G/σ_F, while the different lines are for different values of N, as indicated in the legend.

inefficiency corresponds most of the times to the one characterized in (13).[10] Second, given a fixed value for N, the percentage of efficient allocations increases with the ratio σ_G/σ_F. Third, given a fixed value of the ratio σ_G/σ_F, the percentage of efficient allocations increases with N.

The variation with N is quite natural: as the number of small firms increases, any winner among them must necessarily have a relatively low cost. However, the variation with the ratio σ_G/σ_F casts some doubts on the auction as an efficiency-inducing mechanism. As that ratio decreases, that is, as we move from right to left in Fig. 3, two effects take place. It is helpful to consider them in the following order. First, it increases the *ex-ante* difference between large and small firms' technologies (recall that σ is the only parameter in which F and G differ from one another). Second, the small firms, which are aware of that *ex-ante* comparative disadvantage, bid more aggressively, that is, bid lower prices for each possible cost realization.[11] The first effect seems to make efficiency more likely: a larger *ex-ante* difference between large and small firms should make it easier to elicit which are the ones with lower cost. However, the second effect pushes in the opposite direction. Our simulation exercise illustrates a very simple setting in which the second effect dominates, even though the number of small firms pushes strongly towards large percentages of efficiency.

5. Conclusion

This paper has analyzed the efficiency effects of bidders with different sizes in renewable electricity (i.e., procurement) auctions. There is a widespread debate in countries organizing auctions on the impact of this instrument on actor diversity and whether small actors should either be exempted from participating in this instrument (and be eligible to alternative administratively-set support) or whether the adoption of some design elements could encourage the participation (and awarding) of these types of actors in auctions through, e.g. contingents or special rules for them (such as longer realization periods, different pricing rules or correction factors in the merit order). This is an important topic because it is often argued that small actors are key in a just and fair energy transition (Grashof, 2019; IRENA, 2019; Fell, 2019; MITECO, 2019) and, indeed, some countries have the explicit goal of supporting the participation of small actors in auctions, including Germany and Spain.

However, there has been much less focus in the renewable electricity auction literature on the pros and cons of encouraging such diversity, and particularly small actors, on the allocative efficiency of the auction, understood as auctions in which the lowest-cost bidders are awarded contracts. Some authors argue that promoting the participation of small actors may be positive for the efficient outcome of the auction due to a greater competition and lower concentration and risk of collusion.

The literature on renewable energy auctions often presumes that "small is beautiful" and that "large is ugly", but usually disregards the idea that large actors may have certain advantages in terms of costs and/or capacity to deal with risks. If larger actors have greater economies of scale, lower transaction costs, more ability to cope with

[10] Not reported in the paper.

[11] An equivalent statement in terms of large firm's behavior is as follows. The large firm, which is aware of her *ex-ante* comparative advantage, bids less aggressively, as she perceives herself as almost a sure winner.

the uncertainty on future remuneration in the planning stage and the high upfront costs and requested guarantees, easier access to finance and a greater possibility to pay lower prices for RES-E equipment, then their participation in auctions would be particularly attractive for governments which prioritize the efficiency goal. Our results suggest that, if allocative efficiency is the government's goal, then small may not be so beautiful. They show that the greater the heterogeneity between firms in terms of diversity of size, the greater the probability of an inefficient outcome. Therefore, the policy implications are straightforward: promoting actor diversity in auctions comes at a cost in terms of lower efficiency levels. It might certainly be justified for other reasons, but not if what we want is to build renewable energy projects at the lowest costs. Costs, in this context, refer to direct generation costs (LCOE) and they do not necessarily include indirect costs (profile and grid costs, see Breitschopf and Held (2014)). Therefore our findings do not necessarily mean that "large is beautiful" when considering system costs (direct plus indirect generation costs). But it certainly means that, if the aim is to have the lowest direct generation costs in the short-term, then encouraging small bidders should not be the priority.

However, support cost efficiency, defined as low award prices (Ehrhart et al., 2019) would benefit from a higher competition in auctions. There is a lower probability of collusive behavior with a greater actor diversity, i.e., a higher participation of small actors. Thus, discouraging small actors would tend to lead to a decline in competition.

The reason for having actor diversity is related to the greater level of competition in the auction but it also falls under the broad umbrella of the benefits it provides in terms of a just and fair energy transition, including decentralization of renewable energy production. Diversity of bidders (investment companies, energy suppliers, project developers and private investors) is a potential important aspect for public acceptance. A greater diversity (and number) of bidders reduces the risk of not meeting the RES-E targets due to the non-compliance of a single bidder.

The above suggests that it is very difficult to achieve all the policy goals simultaneously and that trade-offs and conflicts are unavoidable. Thus, achieving actor diversity by encouraging smaller actors may be done at the expense of worsening other policy goals such as efficiency (in terms of direct generation costs). It also suggests that, if actor diversity is a policy goal, then special rules for small actors in auctions (e.g., contingents or less stringent prequalification requirements) or ensuring their participation in the energy transition through a different support scheme (such as administratively-set feed-in tariffs or premiums) would be justified.

Appendix A. Proofs

Proof of Proposition 1. Some additional notation on order statistics will be helpful. Let the set of costs of small firms be denoted by C, that is $C := \{c_1, c_2, \dots, c_N\}$. The set of order statistics, denoted $\{c_{[1]}, c_{[2]}, \dots, c_{[N]}\}$, is an enumeration of C such that:

$$c_{[1]} \leq c_{[2]} \leq \cdots \leq c_{[N]}$$

Similarly, $d_{[1]}$ and $d_{[2]}$, satisfying $d_{[1]} \leq d_{[2]}$, denote the order statistics of $\boldsymbol{d} = \{d_1, d_2\}$. Let $F_{[j]}$ denote the cumulative distribution of $c_{[j]}$ and, similarly, we use $G_{[j]}$ for $d_{[j]}$.

Let $\mathcal{Z}_j$ denote the event in which L is assigned exactly j contracts, with $j \in \{1, 2\}$. Thus, the expected reward for L of bidding (p, q) is:

$$J^L(p, q; \boldsymbol{d}) = (p - d_1)\, Pr(\mathcal{Z}_1 \mid p, q) + (p + q - d_1 - d_2)\, Pr(\mathcal{Z}_2 \mid p, q) \quad (14)$$

Notice that $\mathcal{Z}_1$ occurs if and only if $\beta(c_{[1]}) \leq q$ and $p \leq \beta(c_{[2]})$ or, equivalently, $c_{[1]} \leq \beta^{-1}(q)$ and $\beta^{-1}(p) \leq c_{[2]}$. Thus,

$$Pr(\mathcal{Z}_1 \mid p, q) = Pr(c_{[1]} \leq \beta^{-1}(q), \beta^{-1}(p) \leq c_{[2]})$$

Since $p \leq q$ holds and β is strictly increasing, it is $\beta^{-1}(p) \leq \beta^{-1}(q)$. Using standard results on order statistics, we have:

$$Pr(\mathcal{Z}_1 \mid p, q) = (1 - F(\beta^{-1}(p)))^{N-1}(1 + (N-1)F(\beta^{-1}(p))) - (1 - F(\beta^{-1}(q)))^N \quad (15)$$

In addition, $\mathcal{Z}_2$ occurs if and only if $q \leq \beta(c_{[1]})$ or, equivalently, $\beta^{-1}(q) \leq c_{[1]}$. Thus,

$$Pr(\mathcal{Z}_2 \mid p, q) = Pr(\beta^{-1}(q) \leq c_{[1]}) = 1 - F_{[1]}(\beta^{-1}(q))$$

Using the standard expression for $F_{[1]}$, we have:

$$Pr(\mathcal{Z}_2 \mid p, q) = (1 - F(\beta^{-1}(q)))^N \quad (16)$$

Eqs. (14) to (16) define L's expected reward (J^L) in terms of fundamentals, the own bids (p and q) and the small firms' strategy (β). Combining these equations, we have the statement of the proposition.

Lemma 7. *Whenever L selects $p < q$, p and q are exclusively a function of d_1 and d_2, respectively.*

Proof of Lemma 7. In the sequel, we omit the arguments of J^L where it leads to no ambiguity. Let us define the Lagrangian

$$\mathcal{L}^L = J^L(p, q; \boldsymbol{d}) - \lambda(p - q)$$

where λ is the Lagrange multiplier. The Kuhn–Tucker conditions are:

$$\frac{\partial}{\partial p} J^L = \lambda; \quad \frac{\partial}{\partial q} J^L = -\lambda; \quad \lambda(p - q) = 0; \quad \lambda \geq 0 \quad (17)$$

From the Kuhn–Tucker conditions, in (17), whenever $p < q$ holds, it has to be:

$$\frac{\partial}{\partial p} J^L = 0; \quad \frac{\partial}{\partial q} J^L = 0; \quad (18)$$

Notice in (1) that J^L is additively separable in p and q, that is, it can be written as:

$$J^L = h_1(p; d_1) + h_2(q; d_2)$$

where h_1 and h_2 correspond to the first and second term in (1), respectively, that is:

$$h_1(p; d_1) = (p - d_1)(1 - F(\beta^{-1}(p)))^{N-1}(1 + (N-1)F(\beta^{-1}(p))); \; h_2(q; d_2) = (q - d_2)(1 - F(\beta^{-1}(q)))^N$$

Thus, (18) is equivalent to:

$$\frac{d}{dp} h_1(p; d_1) = 0 \quad \frac{d}{dq} h_2(q; d_2) = 0 \quad (19)$$

Proof of Proposition 2. We make use of h_1 and h_2, defined in the proof of Lemma 7. In order to simplify the exposition, we omit arguments from the functions where it leads to ambiguity. Consider first h_1. It is:

$$h_1 = (p - d_1)\left((1 - F)^N + NF(1 - F)^{N-1}\right)$$

Its derivative is:

$$\frac{d}{dp} h_1 = (1 - F)^N \left(1 + N\frac{F}{1 - F}\left(1 - (N-1)(p - d_1)\frac{f}{1 - F}\frac{d}{dp}\beta^{-1}\right)\right)$$

Consider now h_2. It is:

$$h_2 = (q - d_2)(1 - F)^N$$

Its derivative is

$$\frac{d}{dq} h_2 = (1 - F)^N \left(1 - N(q - d_2)\frac{f}{1 - F}\frac{d}{dq}\beta^{-1}\right)$$

The rest of the proof is organized in two steps.

Step 1, we prove that h_1 is concave. For any p satisfying $0 < F(\beta^{-1}(p)) < 1$, it is:

$$\frac{d}{dp} h_1 \geq 0 \iff 1 - (N-1)(p - d_1) H_F \frac{d}{dp}\beta^{-1} \geq \frac{F - 1}{NF}$$

If $\beta''(c) \leq 0$ and $H_F(c)$ is non-decreasing, the left hand side of the latter inequality is strictly decreasing, whereas the right hand side is strictly increasing. In addition, the previous implication holds true if we replace the inequality sign with an equality one. Thus, there is at most one critical point of h_1, in which its derivative is zero, and its derivative is positive only to the left of the critical point.

Step 2. Let $d_1 = d_2 = d$. Assume a value of q, say q^0, such that:

$$\frac{d}{dq}h_2|_{q^0} = 0$$

That value of q^0 satisfies (at any interior solution):

$$1 - N(q^0 - d)\frac{f}{1-F}\frac{d}{dq}\beta^{-1}|_{q^0} = 0$$

Now, if we set $p = q^0$ in the derivative of h_1 we obtain:

$$\frac{d}{dp}h_1|_{q^0} = (1-F)^N\left(1 + N\frac{F}{1-F}(q^0-d)\frac{f}{1-F}\frac{d}{dp}\beta^{-1}|_{q^0}\right) > 0$$

Thus, under $d_1 = d_2 = d$, any q^0 satisfying $\frac{d}{dq}h_2|_{q^0} = 0$ implies $\frac{d}{dp}h_1|_{q^0} > 0$. From Kuhn–Tucker conditions, $p < d$ implies (19). Under concavity of h_1, proved in step 1, since $\frac{d}{dp}h_1|_{q^0} > 0$, the value of p that makes zero $\frac{d}{dp}h_1$ is greater than q^0, which contradicts $p < d$. Thus, it must be $p = d$.

Proof of Proposition 3. For any interior solution, it is:

$$\frac{d}{dp}h_1|_{q^0} = 0 \iff 1 + N\frac{F}{1-F}\left(1 - (N-1)(p-d_1)\frac{f}{1-F}\frac{d}{dp}\beta^{-1}\right) = 0$$
$$\iff \frac{1}{N} + \frac{1}{FN(N-1)} = (p-d_1)H_F\frac{d}{dp}\beta^{-1}$$

For N large enough, the second term in the left hand side of the previous equality converges to zero faster than the first, that is:

$$\frac{1}{N} + \frac{1}{FN(N-1)} = \frac{1}{N} + O\left(\frac{1}{N^2}\right)$$

Which leads to approximate the previous equality as

$$\frac{d}{dp}h_1|_{q^0} = 0 \iff \frac{1}{N} = (p-d_1)H_F\frac{d}{dp}\beta^{-1}$$

The analogous expression for the derivative of h_2 is straightforward, just replacing d_1 with d_2 (and for h_2 that first order condition is exact). Furthermore, if $\beta''(c) \leq 0$ and $H_F(c)$ is non-decreasing, the right hand side of the latter equality is strictly increasing in p, such that: (i) there is at most one solution to the latter equality, (ii) the solution for d_1 is strictly smaller than the solution for d_2 and (iii) the second order condition holds. Notice that the term $O\left(\frac{1}{N^2}\right)$ is strictly positive.

Proof of Proposition 4. Some additional notation will be helpful. Let $\mathcal{N}_{-n}$ denote the set of the small rivals of n. We denote the order statistics of the types of firms in $\mathcal{N}_{-n}$ as:

$$c_{[1]} \leq c_{[2]} \leq \cdots \leq c_{[N-1]}$$

We will denote $F_{[1]}$ the cumulative probability distribution of $c_{[1]}$ and $G_{[j]}$ the corresponding cumulative probability distribution of $d_{[j]}$, for $j \in \{1,2\}$. Similarly, $d_{[1]}$ and $d_{[2]}$ denote the order statistics for L's costs.

Assume now that firm n bids b. The firm n wins one contract if and only if any of the following two events occur. The first event, which we denote by $\mathcal{E}_A$, is defined by $b \leq \beta(c_{[1]})$ and $b \leq \gamma(d_{[2]})$, or equivalently, $\beta^{-1}(b) \leq c_{[1]}$ and $\gamma^{-1}(b) \leq d_{[2]}$. Since c's and d's are mutually independent, we have:

$$Pr(\mathcal{E}_A) = Pr(\beta^{-1}(b) \leq c_{[1]}, \gamma^{-1}(b) \leq d_{[2]}) = \left(1 - F_{[1]}(\beta^{-1}(b))\right) \times \left(1 - G_{[2]}(\gamma^{-1}(b))\right) \quad (20)$$

The first term in the right hand side of Eq. (20) can be computed from the standard results on order statistics. In turn, the second term is in fact the second minimum within a pair of random draws. Therefore, we have:

$$1 - F_{[1]}(\beta^{-1}(b)) = (1 - F(\beta^{-1}(b)))^{N-1} \quad \text{and} \quad G_{[2]}(\gamma^{-1}(b)) = G(\gamma^{-1}(b))^2 \quad (21)$$

The second event under which n wins one contract is denoted by $\mathcal{E}_B$ and it is defined by $\beta(c_{[1]}) \leq b \leq \beta(c_{[2]})$ and $b \leq \gamma(d_{[1]})$, or equivalently, $c_{[1]} \leq \beta^{-1}(b) \leq c_{[2]}$ and $\gamma^{-1}(b) \leq d_{[1]}$. We have:

$$Pr(\mathcal{E}_B) = Pr(c_{[1]} \leq \beta^{-1}(b) \leq c_{[2]}, \gamma^{-1}(b) \leq d_{[1]})$$
$$= Pr(c_{[1]} \leq \beta^{-1}(b) \leq c_{[2]}) \times \left(1 - G_{[1]}(\gamma^{-1}(b))\right) \quad (22)$$

The first term in the right hand side of Eq. (22) can be computed as follows:

$$Pr(c_{[1]} \leq \beta^{-1}(b) \leq c_{[2]}) = (N-1)F(\beta^{-1}(b))(1 - F(\beta^{-1}(b)))^{N-2} \quad (23)$$

The second term in the right hand side of Eq. (22) is the minimum of a pair of random draws. It is:

$$1 - G_{[1]}(\gamma^{-1}(b)) = \left(1 - G(\gamma^{-1}(b))\right)^2 \quad (24)$$

Using Eqs. (20) to (24) and noting that events are $\mathcal{E}_A$ and $\mathcal{E}_B$ are disjoint, we can write firm n expected reward as in the statement of the proposition. The term multiplying $b - c$ in the statement of the proposition is $Pr(\mathcal{E}_A) + Pr(\mathcal{E}_B)$.

Lemma 8. *Let $J^n_N(b,c)$ denote $J^n(b,c)$ in (3) for N small firms. Fix any interior point b, that is $F \equiv F(\beta^{-1}(b)) \in (0,1)$ and $G \equiv G(\gamma^{-1}(b)) \in (0,1)$. Fixed b, the sequence of real numbers $\{J^n_2, J^n_3, J^n_4, \dots\}$ converges to $J^n_* := (b-c)B$.*

Proof. In the sequel, we simplify the arguments of the functions where it leads to no ambiguity. Thus, we write:

$$J^n = (b-c) \times \left[(1-F)^{N-1}(1-G^2) + (N-1)F(1-F)^{N-2}(1-G)^2\right]$$

Define $A := (1-F)^{N-1}(1-G^2)$ and $B := (N-1)F(1-F)^{N-2}(1-G)^2$. Thus,

$$J^n_N = (b-c)(A+B)$$

The proof has two steps. In the first step we show that, given b interior, it is $\lim_{N\to\infty} \frac{A}{B} = 0$. In effect,

$$\lim_{N\to\infty} \frac{A}{B} = \lim_{N\to\infty} \frac{1}{N-1}\frac{1-F}{F}\frac{1+G}{1-G} = 0$$

Second step. Fixed any b interior, it is:

$$\frac{J^n_N}{B} = (b-c)\left(\frac{A}{B} + 1\right)$$

Therefore

$$\lim_{N\to\infty} \frac{J^n_N}{B} = b - c = \lim_{N\to\infty} \frac{J^n_*}{B}$$

Using standard properties of the limit operator, $\lim_{N\to\infty} J^n_N = \lim_{N\to\infty} J^n_*$ follows.

The underlying economic intuition for Lemma 8 is simple. Notice that A and B are $Pr(\mathcal{E}_A)$ and $Pr(\mathcal{E}_B)$, respectively. Consider the point view of firm n. As N grows large, it increases the likelihood that some *other* small firm wins one contract or, equivalently, the likelihood A, under which firm n is the best among all small firms, becomes negligible as compared to B.

Proof of Proposition 5. We consider N large and approximate n's firm objective function by J^n_*. For N large, we use Lemma 8 to write the problem for firm n as

$$max_b\{(b-c)F(1-F)^{N-2}(1-G)^2\}$$

where arguments and constants have been omitted. The first order condition is

$$\frac{d}{db}J_*^n = 0 \iff F(1-F)^{N-2}(1-G)^2 + (b-c)\left(\left(f(1-F)^{N-2}(1-G)^2 - fF(N-2)(1-F)^{N-3}(1-G)^2\right)\frac{d}{db}\beta^{-1} - 2F(1-F)^{N-2}g(1-G)\frac{d}{db}\gamma^{-1}\right) = 0$$
$$\iff 1 + (b-c)\left(\left(\frac{f}{F} - (N-2)H_F\right)\frac{d}{db}\beta^{-1} - 2H_G\frac{d}{db}\gamma^{-1}\right) = 0$$

where in the last line we have divided by $F(1-F)^{N-2}(1-G)^2$, which implicitly assumes we are only considering interior solutions. The expression in the proposition follows from the last line straightforwardly. For N large enough, we can write:

$$\frac{d}{db}J_*^n = -(b-c)H_F\frac{d}{db}\beta^{-1}$$

Thus, if $\beta''(c) \le 0$ and $H_F(c)$ is non-decreasing for all $c \in (\underline{\omega}, \overline{\omega})$, the second order condition holds.

Generalized Pareto. The generalized Pareto distribution with finite support has the cumulative probability distribution:

$$F(z) = 1 - \left(1 + \frac{\xi(z-\mu)}{\sigma}\right)^{-1/\xi}$$

for $\mu \le z \le \mu - \frac{\sigma}{\xi}$, where $\sigma > 0$ and $\xi < 0$. We denote the distribution by $GP(\mu, \sigma, \xi)$, where the parameters listed in parenthesis are location, scale and shape, respectively.

Stochastic dominance. F first-order stochastically dominates G (FOSD) whenever $F(c) \le G(c)$ for any $c \in [\underline{w}, \overline{w}]$. F (HRSD) hazard-rate stochastically dominates G whenever $H_F(c) \le H_G(c)$ for any $c \in [\underline{w}, \overline{w}]$. F (LRSD) likelihood-ratio dominates G whenever $\frac{f(c)}{g(c)} \le \frac{f(c')}{g(c')}$ for any $\underline{w} \le c \le c' \le \overline{w}$.

Lemma 9. *Assume F and G are GP distributed. Then:*

1. *F and G common support $[\underline{w}, \overline{w}]$ whenever F and G are $GP(\underline{w}, \sigma_F, -\sigma_F/\kappa)$ and $GP(\underline{w}, \sigma_G, -\sigma_G/\kappa)$, respectively, where $\kappa := \overline{w} - \underline{w}$.*
2. *The hazard function of F is*
$$H_F(z) = \frac{1}{\sigma} \times \frac{1}{1 - \kappa^{-1}(z - \underline{w})} = \frac{1}{\sigma_F} \times \frac{\kappa}{\overline{w} - z}$$
3. *Any hazard function such that its inverse is linear corresponds to a GP distribution.*
4. *The density f is strictly increasing whenever $\sigma_F > \kappa$ holds.*
5. *If F and G have common and finite support on $[\underline{w}, \overline{w}]$, F FOSD, HRSD and LRSD dominates G whenever $\sigma_G \le \sigma_F$ holds.*

Proof of Lemma 9. Part 1. Take $\underline{w} = \mu$, $\overline{w} = \mu - \frac{\sigma}{\xi}$ in the definition of $GP(\mu, \sigma, \xi)$. Notice than then $F(\underline{w}) = 0$ and $F(\overline{w}) = 1$ for any σ. Part 2. It follows from applying the definition of H. Part 3. It follows from integrating along $[\underline{w}, c]$, for any c in the support, on both sides at

$$\frac{f(z)}{1 - F(z)} = \frac{1}{\sigma} \times \frac{\kappa}{\overline{w} - z}$$

Part 4. It follows from taking the derivative of f. Part 5. It follows from algebra.

Proof of Proposition 6. We divide the proof in two parts, dealing with small and large firms, respectively.

Par 1. Proposition 5 delivers the best response from n when its cost is c all other small firms are playing β and L plays γ. Now, at an equilibrium, the best response is precisely to follow rivals' strategy, or, equivalently, $b = \beta(c)$. Using this equilibrium condition, the equation in Proposition 5 is:

$$\frac{1}{N} = (\beta(c) - c)\left(\left(\frac{N-2}{N}H_F(c) - \frac{f(c)}{NF(c)}\right)\frac{1}{\beta'(c)} + \frac{2}{N}H_G(\gamma^{-1}(\beta(c)))\frac{d}{db}\gamma^{-1}(\beta(c))\right) \tag{25}$$

where we have used that $\frac{d}{db}\beta^{-1}(\beta(c)) = \frac{1}{\beta'(c)}$. In addition, we must notice that L and the small firms might play different strategies, that is, $\gamma^{-1}(\beta(c)) \ne c$.

Next, impose F and G are $GP(\underline{w}, \sigma_F, -\sigma_F/\kappa)$ and $GP(\underline{w}, \sigma_G, -\sigma_G/\kappa)$. Notice that $\gamma^{-1}(z) = \overline{w} + \frac{1}{\gamma_0}(z - \overline{w})$, thus $\gamma^{-1}(\beta(c)) = \overline{w} + \frac{\beta_0}{\gamma_0}(c - \overline{w})$. Therefore, using Lemma 9, we have

$$H_G(\gamma^{-1}(\beta(c))) = \frac{1}{\sigma_G} \times \frac{\kappa}{\overline{w} - \gamma^{-1}(\beta(c))} = \frac{\gamma_0\kappa}{\beta_0\sigma_G} \times \frac{1}{\overline{w} - c}$$

In addition, since:

$$\frac{d}{db}\gamma^{-1}(\beta(c)) = \frac{1}{\gamma_0}$$

we have:

$$H_G(\gamma^{-1}(\beta(c)))\frac{d}{db}\gamma^{-1}(\beta(c)) = \frac{\kappa}{\beta_0\sigma_G} \times \frac{1}{\overline{w} - c} \tag{26}$$

Also, it is:

$$\beta(c) - c = (\overline{w} - c)(1 - \beta_0) \tag{27}$$

Using (26) and (27) into (25), leads to

$$\frac{1}{N} = (\overline{w} - c)(1 - \beta_0)\left(\left(\frac{N-2}{N}\frac{\kappa}{\sigma_F}\frac{1}{\overline{w} - c} - \frac{f(c)}{NF(c)}\right)\frac{1}{\beta_0} + \frac{2}{N}\frac{\kappa}{\beta_0\sigma_G} \times \frac{1}{\overline{w} - c}\right)$$

which, for any $c \in (\underline{w}, \overline{w})$, which characterizes interior solutions, the previous equality is equivalent to:

$$\frac{1}{N} = \frac{1 - \beta_0}{\beta_0}\kappa\left(\frac{N-2}{N}\frac{1}{\sigma_F} + \frac{2}{N}\frac{1}{\sigma_G} - \frac{f(c)(\overline{w} - c)}{N\kappa F(c)}\right) \tag{28}$$

It is:

$$\frac{f(c)(\overline{w} - c)}{\kappa} = \frac{1}{\sigma_F} \times (1 - F(c))$$

which, substituted in (28), multiplying by N and re-arranging, leads to the expression in the proposition.

Part 2. Consider the large firm. Essentially, we impose the equilibrium condition $p = \gamma(d)$ in the equation in Proposition 3. At an equilibrium that equation is:

$$\frac{1}{N} = (\gamma(d) - d)H_F\left(\beta^{-1}(\gamma(d))\right) \cdot \frac{d}{dp}\beta^{-1}(\gamma(d)) \tag{29}$$

We proceed similarly to the small firms case. It is:

$$\beta^{-1}(\gamma(d)) = \overline{w} + \frac{\gamma_0}{\beta_0}(d - \overline{w})$$

Thus:

$$H_F\left(\beta^{-1}(\gamma(d))\right) = \frac{1}{\sigma_F} \times \frac{\kappa}{\overline{w} - \beta^{-1}(\gamma(d))} = \frac{\beta_0\kappa}{\gamma_0\sigma_F} \times \frac{1}{\overline{w} - c}$$

Also

$$H_F\left(\beta^{-1}(\gamma(d))\right) \cdot \frac{d}{dp}\beta^{-1}(\gamma(d)) = \frac{\kappa}{\gamma_0\sigma_F} \times \frac{1}{\overline{w} - c}$$

In addition,

$$\gamma(d) - d = (\overline{w} - d)(1 - \gamma_0)$$

Substituting the latter two equalities into (29) and re-arranging, we obtain the expression in the proposition.

Lemma 10. *Let X be a uniform distribution in [0,1] and $Y := \frac{1}{X}$, then: (i) $E\{Y\}$ is not finite, (ii) the median of Y is 2.*

Proof of Lemma 10. Clearly, the support of Y is $[1, \infty)$. For any $y \in [1, \infty)$. The cumulative probability distribution is:

$$F_Y(y) := Pr(Y \leq y) = Pr\left(\frac{1}{X} \leq y\right) = Pr\left(\frac{1}{y} \leq X\right) = 1 - Pr\left(X \leq \frac{1}{y}\right)$$
$$= 1 - \frac{1}{y}$$

where we have used that $Pr(X \leq x) = x$ for any $x \in [0, 1]$. (i) The density function is the derivative of F, that is $f_Y(y) = \frac{1}{y^2}$. The expectation is

$$E\{Y\} = \int_1^\infty y f_Y(y) dy = \int_1^\infty \frac{dy}{y} = [lny]_1^\infty = \infty$$

(ii) The median is the value $\tilde{y}$ which satisfies $F_Y(\tilde{y}) = \frac{1}{2}$. Using the expression for F_Y, $\tilde{y} = 2$ follows straightforwardly.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2021.105698.

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