LINEAR AND NONLINEAR INTRADAY DYNAMICS BETWEEN THE EUROSTOXX-50 AND ITS FUTURES CONTRACT

Luisa Nieto
Dpt. Finanzas y Contabilidad, Universitat Jaume I
nieto@cofin.uji.es

Mª Dolores Robles
Dpt. Fundamentos de Análisis Económico II, UCM
eccua20@sis.ucm.es

Angeles Fernández
Dpt. Finanzas y Contabilidad, Universitat Jaume I
afernand@cofin.uji.es

Resumen

Nos planteamos analizar el comportamiento dinámico lineal y no lineal de los rendimientos intradía del índice bursátil Eurostoxx50 y de su contrato de futuro, los cuales debido a su relativa juventud, no han sido previamente analizados. Realizamos el estudio tanto desde la perspectiva individual como conjunta. Los resultados del contraste BDS indican que las variables no son *iid* y que la dinámica individual no lineal detectada no puede explicarse únicamente por la presencia de heteroscedasticidad condicional. Para el estudio de las relaciones dinámicas entre los precios de ambos mercados permitimos que el proceso de ajuste ante desequilibrios de la relación de cointegración a largo plazo sea no lineal. Constatamos que el Eurostoxx50 y su contrato de futuro están cointegrados y que el proceso de ajuste no es lineal. Finalmente, encontramos que los flujos de información entre mercados son bidireccionales tanto en el ámbito lineal como en el no lineal.

Abstract

We set out to analyse the linear and nonlinear dynamic behaviour of intraday returns in the Eurostoxx 50 index and its futures contract which, given their relatively recent appearance, have not yet been analysed. We shall develop our study both from an individual and from a combined approach. The results of the BDS test indicate that the variables are not *iid* and that the detected nonlinear individual dynamics cannot solely be explained by the presence of conditional heteroskedasticity. For the study of the dynamic relationships between both markets' prices, we allow the adjustment process to the imbalance of the long term cointegration relationship to be nonlinear. We find cointegration with a nonlinear adjustment process. Finally, we show that the information flow is bidirectional both in the linear as well as in the nonlinear sphere.

Key words: Nonlinearity, BDS, Nonlinear error correction mechanism, Nonlinear causality, Eurostoxx50, Index futures.

Acknowledgements:
The authors wish to thank Walter Enders for his invaluable remarks on the use of his methodology as well as Abhay Abhyankar for generously sharing Professor Craig Hiemstra's software. We also thank Stoxx Limited for providing us with the data corresponding to the Eurostoxx50 index, and Deutsche Börse for the data of the futures contract. Finally we wish to thank Fundació Caixa Castelló-Bancaixa (P1.1B 2000/17) for the provided financing. The authors remain fully responsible for any existing mistake.

1.- INTRODUCTION

The purpose of this work is to investigate the intraday relationships between the Eurostoxx50 index and its futures contract. Since its creation in 1998, this index has become one of the main indicators of the activity of the European stock markets. Our objective is twofold. On the one hand, we are interested to learn about the temporal behaviour of each of the series, on the other hand, also about the kind of existing dynamic relationships among them. Our analysis considers the possibility that the series present nonlinear dynamics both individually and in the relationship that links them.

The relationship between a stock index and its futures contract has been comprehensively studied in the financial literature, be it to verify the process of price discovery, to establish the optimum hedging strategy, or to test the fulfilment of the cost-of-carry theory and the non arbitrage condition. Most of these works imply the hypothesis that the temporal relationships between both variables are linear.

However, it has become clear over the last years that nonlinear processes can be found behind the different financial variables. Savit (1988) argues that financial markets are an example of dynamic systems which present nonlinearities, whereas Hsieh (1991) claims that price fluctuations of financial assets, which are higher than could be expected under the hypothesis of normal distribution of returns, are due to the existence of nonlinearities.

Applying different methods, many authors have analysed and detected nonlinear behaviours in financial variables. The test put forward by Brock, Dechert and Scheinkman (1987) (BDS) is no doubt one of the most widely used ones. With it one can test the null hypothesis that the series is *iid* against the alternative that it shows some kind of dynamic structure, be it linear, nonlinear or chaotic. Among those works which have used this test stand out Yang and Brorsen (1993, 1994) for futures contracts; Abhyankar (1997) with different stock indices; Hsieh (1989, 1991, 1993) for currencies, the S&P500 index and futures contracts on currencies respectively; Gao and Wang (1999) for futures contracts on currencies.

From this perspective, the first aim of this work is to study separately the linear and nonlinear dynamics of intraday returns in the European index Eurostoxx-50 and those of its futures market, considering the possibility that the nonlinear behaviour might be explained by the presence of conditional heteroskedasticity. The results seem to point at the existence of a nonlinear dynamic in both series which is not fully explained by the presence of GARCH structures on its own.

On the other hand, linear relationships between the spot and futures price of stock indices have attracted increased attention over the last decade. Without meaning to be exhaustive[1], we can highlight the works of Fleming, *et. al*. (1996) and Wahab and Lashgary (1993) which study the cointegration and the relationships of linear Granger causality between the S&P 500 and its futures contract; Grunbiechler, *et. al.* (1994) study the lead-lag linear relationships between the German DAX index and its futures contract; Booth, *et. al*. (1993) distinguish between short and long term causality between the Finnish FOX index and its futures contract. In Spain, Nieto, *et. al.* (1998). In general, they all establish a cointegration relationship between the spot and futures value of the indices. Likewise, they detect Granger causality though, in this case, the results vary slightly depending on the analysed market and the frequency of the data.

Baek and Brock (1992) revealed that the Granger causality test does not prove powerful enough to detect nonlinear relationships which may be relevant for series which individually present nonlinear behaviours. Hiemstra and Jones (1994) suggest a nonparametric method based on the work of Baek and Brock (1992) in order to analyse nonlinear causality between the industrial Dow Jones and the volume in the New York Stock Exchange. This method was applied to futures markets by Abhyankar (1998) to test the existence of nonlinear causality between the returns of the FT-SE 100 and that of its futures contract; and by Fujihara and Mougoue (1997), and Moose and Silvapulle (2000) for the price and the volume of different futures contracts on oil.

Therefore, the existence of nonlinear structures in the returns of the Eurostoxx-50 and those of its futures market leads us to consider the likely existence of nonlinear Granger causality relationships between both variables. Our results, similarly to the aforementioned works, present significant evidence of bidirectional Granger causality, both linear and nonlinear between the returns of both series.

The assumption of linearity may also be very restrictive when cointegration relationships are analysed. Pippenger and Goering (2000) argue that the effect of the transaction costs on the arbitrage activity, together with the sheer nature of the inventories control and the governmental market regulation may lead to asymmetries in the adjustment processes towards long term balance. In this line, Dwyer, *et. al.* (1996) show how the existence of nonlinear dynamics between the S&P 500 and its futures contract may be

[1] A more detailed revision can be found in Abhyankar (1998).

explained by a cost-of-carry model which adds nonlinear transaction costs for different arbitrage groups.

Over the last years, threshold cointegration models have been suggested in order to detect these sort of phenomena. In them, the linearity assumption of the adjustment towars the equilirbium is relaxed, allowing this to be asymmetrical. So, Balke and Fomby (1997) and Pippenger and Goering (2000) and, on the financial side, Enders and Granger (1998) and Enders and Silkos (2001) establish threshold cointegration between different short and long term interest rates. The main problem of this approach is the need to define explicitly the nature of the asymmetry, which in practice results in a wide variety of suggested models for whose distinction no unanimous criteria have yet been laid down. Moreover, in the cases with little information available, a priori, the estimated model may present a specification error. So as to solve these shortages, Enders and Ludlow (2000) and Ludlow and Enders (2000) developed a technique which allows to test the existence of cointegration without the need to specify the kind of nonlinear adjustment with respect to long term balance deviations. These authors suggest a modification of the Engle and Granger (1987) cointegration test, allowing long term balance deviations to follow a nonlinear process.

We apply the methodology of Ludlow and Enders (2000) in our analysis of the cointegration relationships between the Eurostoxx50 and its futures contract. The results show nonlinearity in the error correction model which must be taken into account to model correctly the relationship between the prices of both assets. Nevertheless, there is no evidence of any significant improvement in the forecast of such prices.

The rest of the paper is structured as follows. In the next section, we analyse the linear and nonlinear individual dynamics of the series. In section 3, we analyse cointegration and causality between spots and futures of the Eurostoxx-50. Finally, in section 4 we present the main conclusions.

2.- INDIVIDUAL ANALYSIS OF THE SERIES

The data sample is made up of 7,546 intraday observations of the spot and futures prices of the Eurostoxx-50, with a fifteen-minute frequency, from 2nd Nov. 1998 to 30th Nov. 1999. The index data was provided by Stoxx Limited, and the data on the futures contract by Deutsche Börse.

In the analysis we use the first 6,342 observations. The observations of the two last months of the sample are saved for forecast exercises (1,204 observations). During the studied period of time, the futures contract was traded from 10 a.m. to 5 p.m. The trading volume during the first months of the sample is relatively low, specially during the first minutes of the session. It is for this reason that we eliminate the observations corresponding to the opening, so that the first daily observation is taken at 10.15 a.m.. This also allows us to eliminate the influence of the overnigth returns. So, each trading session is represented by 28 observations.

[Insert Table I]

Table I shows the statistics which describe the log of both series and their returns rate, calculated as $100^* \ln(S_t / S_{t-1})$.

As one can observe, both returns series are far from normal, being their kurtosis coefficients well above 3. The first order correlation coefficient suggests that the log variables are not stationary, whereas the returns are.

Two unit root tests are carried out to confirm this point: Dikey and Fuller ADF (1979) and Philips and Perron PP (1988). The results (Table II) show clearly a unit root both in the log of the Eurostoxx-50 index and in its futures contract, whereas the returns series are clearly stationary.

[Insert Table II]

In order to tes the kind of temporal dependence which the analysed series present, both linear and nonlinear temporal dependence tests will be used. The statistics of Ljung and Box (1979) will be used for the first kind of dependence. This tests the null of the absence of serial correlation with the statistics:

$$\mathrm{Q(p)} = \mathrm{T(T+2)}\sum_{\mathrm{i=1}}^{\mathrm{k}} \frac{\mathrm{r}_i^2}{\mathrm{T}-i}, \tag{1}$$

where T is the size of the sample and r_i is the simple i-order correlation coefficient. In this null (1) follows a χ^2 distribution with k degrees of freedom[2].

[2] When this test is applied to the residuals of an ARMA(p,q) model, the degrees of freedom of the χ^2 change to *k-p-q*.

The application of this test for a correlation order equalling 28 leads to the rejection of the absence of correlation for spot returns. As for futures, the null is rejected at the 5% significance level, but not at the 1%.

Two different tests, Q^2 and BDS, will be used to test the presence of nonlinear temporal dependence. The Q^2 test was suggested by McLeod and Li (1983) to detect, among others, nonlinear Garch structures. Given a temporal series x_t (t=1,...,T), the statistics is:

$$Q^2(p) = T(T+2)\sum_{i=1}^{k}\frac{R_i^2}{T-i}, \tag{2}$$

where $R_i^2 = \sum_{t=1}^{T-i}(x_t^2 - z)(x_{t-i}^2 - z) / \sum_{t=1}^{T-i}(x_t^2 - z)^2$ and $z = \frac{1}{T}\sum_{t=1}^{T} x_t^2$.

For the null of correlation absence, this statistics has a χ^2 distribution with *k* degrees of freedom[3]. The results of the application of this test to the returns series (Table III) show clearly the presence of GARCH linear dependence.

As for the BDS test, it was suggested by Brock, Dechert and Scheinkman (1987) and revised by Brock, Dechert, Scheinkman and LeBaron (1996). It allows to test when a temporal series is independent and identically distributed (*iid*). It may be used to test whether a model suits a specific temporal series, since it detects any structure in the error term, be it linear, nonlinear or chaotic.

Given the temporal series x_t (t=1,...,T), they are considered segments of the same size, called M-stories and defined as: $x(m)_t = (x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+m+1})$, where M is the dimension. From these M-stories, the integral correlation $C_m(l)$ of *m* dimension and *l* distance is defined as:

$$C_{m,T}(l) = \frac{2}{(T-m+1)(T-m)}\sum_{I \le i \le j \le T-m+1} I_l(x(m)_i, x(m)_j), \tag{3}$$

where $I_l(.)$ is the indicator function:

$$I_l(x(m)_i, x(m)_j) = \begin{cases} 1 & \text{if } \| x(m)_i, x(m)_j \| < l \\ 0 & \text{if } \| x(m)_i, x(m)_j \| \ge l \end{cases}.$$

where ||.|| indicates the maximum norm. The BDS test for an *m* fix dimension, an *l* distance and a T sample size is:

[3] See previous footnote.

$$\text{BDS}(m,l,\text{T}) = \frac{\sqrt{T}\,(\text{C}_{m,\text{T}}(l) - \text{C}_{1,\text{T}}(l)^m)}{\sigma_{m,\text{T}}(l)}\,, \tag{4}$$

where $\sigma_{m,\text{T}}(l)$ is the estimation of the asymptotic standard deviation of $\sqrt{T}\,(\text{C}_{m,\text{T}}(l) - \text{C}_{1,\text{T}}(l)^m)$ under the hypothesis that the series is *iid.*[4]

The properties of this test on finite samples were studied by Brock, Dechert and Scheinkman (1987), Brock, Dechert, Scheinkman and LeBaron (1996), Brock, Hsieh and LeBaron (1991) or Lee, White and Granger (1993), among others. Brock, Dechert and Scheinkman (1987) showed that, under the null that the series is *iid* , the statistics BDS follows a normal distribution. However, for series with unusual distributions, the distribution of the test may not be normal. For this reason, and given the high non normality of the spot and future returns of the Eurostoxx-50, p-values are calculated through bootstrap with 1000 replications.

The test is applied for *m* between 2 and 10 and *l* = 0.5, 0.75, 1, 1.25, 1.5 times the standard deviation of the series. For space limitations, we only present the results for *m*=3,6 and *l* = 1, 1.5. The hypothesis that both returns series are *iid* is clearly rejected, as it can be appreciated on Table III.

[Insert Table III]

2.1.- Linear dependence in returns

Next, we analyse the linear behaviour of the returns, considering the possible stationary effects associated to time and minute, weekday and month. We estimate the following autoregressive model:

$$x_t = \sum_{i=1}^{4} \gamma_{di} D_{jt} + \sum_{j=1}^{28} \gamma_{hj} H_{jt} + \sum_{s=1}^{11} \gamma_{ms} M_{st} + \sum_{k=1}^{p} \beta_k\, x_{t-k} + e_t \tag{5}$$

where D_j are daily dummy variables: D_1 for Monday, D_2 for Tuesday, D_3 for Thursday and D_4 for Friday. Wednesday constitutes the benchmark. H_j are dummy variables for the time and minute within the day. For instance, H_1 is a dummy for 10:15 hrs, H_2 for 10:30 hrs, etc. M_s are monthly dummy variables. For example, M1 is the variable for January, M2 for February, etc.

[Insert Table IV]

4 The expression of standard deviation, as well as a more detailed explanation of the test can be found in Brock, Dechert and Scheinkman (1987).

For the estimation of the models, we use the variance-covariance matrix suggested by Newey and West (1987, 1994) to prevent the effect of possible heteroskedasticity and residual autocorrelation. Ten lags are used for their estimation. The results obtained in the estimation (Table IV) can be summarised as follows:

After a first estimation, we decided to eliminate the monthly dummy variables for not being significant.

Not every day has a significant effect[5]. For both markets, the returns are significantly lower on Tuesdays. As for the spot market, the returns seem to be higher on Mondays (Fridays for futures).

In both markets, the returns are significantly negative between 15:45 hrs and 16:15 hrs and become significantly positive in the last half hour of the market[6].

In both models, 12-order autoregressive structures are estimated. We opt to include all the lags, regardless of their significance, so as to detect the complete linear behaviour of both returns series.

Table V shows the diagnosis of temporal dependence in the residuals of the estimated linear models. Both the Q(28) statistics and the LM test for 2-order autocorrelation indicate that the existing linear structure in both returns series has been appropriately captured[7].

[Insert Table V]

Nevertheless, the remaining applied tests indicate that the returns show nonlinear temporal dependence. The $Q^2(28)$ and ARCH(5) tests [LM test to detect conditional heteroskedasticity[8], suggested by Engle(1982)] indicate that such nonlinearity may be associated to nonlinear behaviours in variance.

To confirm this last point, we apply the third moment test, suggested by Hsieh (1989). This tries to detect nonlinear behaviour in temporal series by exploiting the difference between additive and multiplicative nonlinear dependence. The first of them

---

5 Daily dummy variables with a t-ratio over a unit remain in the model.

6 All intraday variables are included regardless of their significance.

7 The LM *p*-order autocorrelation test is calculated as TR^2 of a regression of the model residuals on the explanatory variables and their *p* first lags. Under the no autocorrelation null, it is distributed as a χ^2 with *p* degrees of freedom.

8 The ARCH-LM *p*-order test is calculated as TR^2 of a regression of the squared model residuals on their *p* squared first lags Under the non conditional heteroskedasticity null, it is distributed as a χ^2 with *p* degrees of freedom. If applied to the diagnosis of a GARCH model, it is calculated for the squared standardised residuals.

makes reference to the nonlinearity found in the mean of the process, whereas with the second, that nonlinearity only enters through the variance

Starting from the filtered temporal series, u_t, i.e., the residuals of the x_t linear model, the additive nonlinearity implies that $E(u_t | x_{t-1},\ldots,x_{t-k}, | u_{t-1},\ldots,u_{t-k}) = 0$, whereas with multiplicative nonlinearity $E(u_t | x_{t-1},\ldots,x_{t-k}, | u_{t-1},\ldots,u_{t-k}) \neq 0$. Hsieh (1989) defines $\rho_{uuu}(i,j) = E(u_t u_{t-i} u_{t-j})/\sigma_u^3$, which equals zero the null hypothesis of multiplicative nonlinearity for every $i, j > 0$. $\rho_{uuu}(i,j)$ is estimated with:

$$r_{uuu}(i,j) = \left[\frac{1}{T}\sum u_t u_{t-i} u_{t-j}\right] \Big/ \left[\frac{1}{T}\sum u_t^2\right]^{3/2}, \tag{6}$$

which is asymptotically distributed under the null as a zero mean normal and $\omega(i,j)/\sigma_u^6$ variance. The asymptotic test statistic is[9]:

$$3MT = \frac{\sqrt{T}\, r_{uuu}(i,j)}{\sigma_{r_{uuu}}}, \tag{7}$$

where $\sigma_{r_{uuu}}$ is an estimation consisting in $\left(\omega(i,j)/\sigma_u^6\right)^{1/2}$.

Table VI shows the results obtained applying this test to the residuals of the previously estimated linear models. The statistics is calculated for i, j values between 1 and 5. As it can be observed, the null hypothesis of multiplicative nonlinearity is not rejected in any case, what seems to indicate that the nonlinear dependence found may be due to the presence of conditional heteroskedasticity in the series.

[Insert Table VI]

2.2.- Models for the nonlinear dependence of the returns

Given the results obtained, the following objective is to model the nonlinear behaviour detected in the series through GARCH models. Starting from the AR specification for the returns of the previous section, the residuals of both models are analysed to identify the most suitable GARCH structure[10], after which we decide to estimate the GJR-GARCH(1,1) model, suggested by Glosten *et al.* (1993). This allows to detect asymmetrical effects on the variance[11]. The final specification of the estimated models is:

---

9 For more detailed information about this test see Hsieh (1989).

10 Besides the ARCH-LM and $Q^2(28)$ tests, presented in the previous section, we also apply the test suggested by Engle and NG (1993) to detect the leverage effect on volatility, as well as the simple and partial autocorrelation functions of the squared residuals of the AR model.

11 The volatility reaction is higher when against negative rather than positive surprises.

$$x_t = \sum_{i=1}^{4} \gamma_{di} D_{it} + \sum_{j=1}^{28} \gamma_{hj} H_{jt} + \sum_{s=1}^{11} \gamma_{ms} M_{st} + \sum_{k=1}^{p} \beta_k x_{t-k} + e_t, \quad e_t \sim N(0, h_t^2) \tag{8}$$

$$h_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \delta\ S_{t-1}^{-} e_{t-1}^2 \beta_1 h_{t-1}^2 + \sum_{s=1}^{4} \gamma_{di} D_{it} + \sum_{s=1}^{11} \gamma_{hj} H_{jt} + \sum_{s=1}^{11} \gamma_{ms} M_{st}\ , \tag{9}$$

where S^{-}_t is a dummy variable that takes the value of 1 when the residual in *t* is negative and zero for another case. Variables D_i, H_j and M_s are the dummies defined in the previous section. These variables allow to consider possible day-of-the-week effects and month effects on the variance. Additionally, the intraday dummies are included to detect any possible behavioural U-shaped type patterns in the volatility.

[Insert Table VII]

The results obtained are shown on Table VII. For space limitations, we only present the model for the variance since the mean model shows few differences with the models estimated in the previous section. Given the high nonlinearity of the residuals, we opt to use the variance-covariance matrix, suggested by Bollerslev and Wooldrige (1992) and robust to non normality problems. Both models are estimated through maximum likelihood, using the BHHH algorithm by Bernd, *et al.* (1974). The main results can be summarised in the following aspects:

After an initial estimation, it was decided to eliminate those dummies which proved insignificant in the variance model, since their presence in it slows down the convergence of the estimation algorithm.

An asymmetrical effect is to be found in both models, though higher for the spot market. The response of the volatility to bad news is five times higher than the effect of good news in the spot market, and about three times higher in the futures market.

For the spot, the volatility persistence degree, calculated as $\alpha_1 + \beta_1$, is higher (0.89 against 0.62 in the futures). This result implies that the volatility has the property of mean reversion, more pronounced in the futures market.

The variance level seems to be higher on Thursdays in the spot market, whereas on Mondays it drops significantly in the futures market. Volatility follows a U-shaped type behaviour in both models, with significantly higher volatility levels at the start and end of the session, and generally lower ones in the intervening hours.

The effects associated to the month are significant in the variance, whereas none are found in the mean model. January, February and December show higher volatility levels, while this volatility drops significantly in the intervening months.

Table VIII shows the diagnosis of the temporal dependence in the residuals of the estimated GARCH models. The Q(28) test confirms that the linear behaviour of both series has been appropriately captured. The remaining tests are calculated on the standarized residuals. The $Q^2(28)$ and ARCH(1) and ARCH(10) tests indicate the presence of nonlinear temporal dependence in the standarized residual of the spot returns model, but not in the futures model.

[Insert Table VIII]

The results of the BDS test indicate that the standarized residuals of both models are not *iid* at the 5% of significance, although those of the spot GARCH model do seem to be *iid* at the 1%, when the dimension is given value 3[12]. With a 6 dimension, the rejection of the null becomes clearer.

Finally, the results of the third moment test (Table IX) indicate that, except for some *i,j* pairs, the null of multiplicative nonlinearity is rejected in both markets, with a stronger rejection in the spot series than in the futures series. All of this implies the existence of residual multiplicative nonlinear dynamic which has not been detected with the estimated GARCH models. A possible explanation of this result may be found in the existence of nonlinear relationships between both series. This dynamic, which has not been taken into account in their individual analysis, is introduced in the next section.

[Insert Table IX]

3.- DYNAMIS BETWEEN THE INDEX AND ITS FUTURES CONTRACT

We now study the dynamic behaviour of the Eurostoxx-50 stock index and that of its futures market as a whole. First, in our cointegration analysis, we allow the ECM, which eliminates the deviations of the variables from their position of long term balance, to follow a nonlinear adjustment process in the short term equations. This is possible applying the first order Fourier approach, suggested by Ludlow and Enders (2000). Secondly, besides studying the relationships of Linear Granger causality between the returns of both variables, we test

[12] It is remarkable that the null cannot be rejected for a 2 dimension, regardless of the value of *l* in any of the markets.

the existence of nonlinear causality relationships using the contrast suggested by Hiemstra and Jones (1994).

3.1.- Cointegration Analysis and Short Term Equations

Enders and Ludlow (2000) develop a technique that allows to test the existence of cointegration without having to specify the kind of nonlinear adjustment to the long term balance deviations[13]. These authors suggest an modification of the cointegration test of Engle and Granger (1987), allowing long term balance deviations to follow this kind of nonlinear process:

$$e_t = \alpha(t)\ e_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta e_{t-i} + \varepsilon_t \tag{10}$$

No specification of the functional form of $\alpha(t)$ is required, since this can be approximated by a sufficiently long Fourier series. Enders and Ludlow (2000) suggest:

$$\alpha(t) = a_0 + a_1\ \text{sen}\frac{2\ \pi\ k}{T}t + b_1\ \cos\frac{2\ \pi\ k}{T}t \tag{11}$$

where: k is a round number of the $T/2$ interval.

The key lies in that using equation (10), rather than determining a certain kind of specific nonlinear adjustment, the problem is reduced to finding the most suitable values of a_0, a_1, b_1, k.[14] These authors prove that the sufficient and necessary condition for the adjustment process not to be explosive, or, in other words, for the variables to be cointegrated is:

$$\begin{gathered} |\, a_0 \,| < 1 + r^2 / 4 \text{ for } r \leq 2 \\ r = \sqrt{a_1^2 + b_1^2} \end{gathered} \tag{12}$$

The cointegration test with nonlinear adjustment is carried out in two stages: In the first stage, we estimate the long term relationship between those variables capable of being cointegrated, which are the value of the Eurostoxx-50 index and the price of its futures contract in our analysis. The results are:

$$\begin{gathered} \hat{F}_t = 0.19 + 0.98\ C_t \\ (32.81)\ (1356.02) \end{gathered} \tag{13}$$

where: F_t is the futures price; C_t is the index value; t statistics in brackets.

---

[13] Enders and Ludlow (2000) extend the work of Ludlow and Enders (2000) on ARMA estimation with Fourier coefficients, treating cointegration relationships explicitly.

[14] The test of Engle and Granger (1987) is the specific case of a_1, b_1 equalling zero.

The following regression is estimated for all the round values of k comprised in the interval 1 to $T/2$, in order to select the most suitable k frequency

$$\Delta\hat{e}_t = \left[c + a_1 \text{ sen} \frac{2\pi\, k}{T} t + b_1 \cos \frac{2\pi\, k}{T} t\right] \hat{e}_{t-1} + \sum_{i=1}^{p-1} \delta_{t-i} \Delta\hat{e}_{t-i} + \varepsilon_t \tag{14}$$

where $\hat{e}_t$ are the residuals of equation (13), and p-1 is the number of sufficient lags to eliminate completely the autocorrelation[15].

We choose the value of k , which minimises the sum of the squared residuals. This value is called k^*, and the coefficients linked to such frequency will be c^*, a_1^*, b_1^* . By Monte Carlo methods, Enders and Ludlow (2000) obtain the critical values for the t statistics under the $c^*= 0$ null, the *F_all* statistics for the $c^*= a_1^*= b_1^*= 0$ null, and the *F_trig* statistics for the null of $a_1^*= b_1^*= 0$. In our empirical analysis k^* is 2091, p-1 = 11.

$$\Delta\hat{e}_t = \left[\underset{(-3.77)}{-0.017} + \underset{(2.43)}{0.015} \text{ sen} \frac{2\pi}{T} t + \underset{(-6.35)}{-0.039} \cos \frac{2\pi\, k}{T} t\right] \hat{e}_{t-1} + \sum_{i=1}^{p-1} \delta_{t-i} \Delta\hat{e}_{t-i} \tag{15}$$

The value of a_0^* linked to c^*= -0.017 is 0.983, which fulfils the condition for the adjustment process to be non explosive and for the variables to be cointegrated: $|a_0| < 1+r^2/4$.

All the significance restrictions of the parameters are also fulfilled (Table X). More specifically, the fact that *F_trig* is statistically significant indicates that the adjustment towards the balance is nonlinear, which implies that the use of the traditional linear error correction model is not suitable for the adjustment towards the balance between spot and futures prices. Therefore, it is possible to estimate an error correction model with Fourier adjustment.

[Insert Table X]

The model used is a VAR which was increased with ECM corresponding Fourier coefficients and with dummy variables to control the differences in the returns between different intraday intervals and different weekdays. This model was first estimated for ordinary less squares. After verifying that not all ECM coefficients were significant, the most parsimonious model was estimated as a SUR. This is expressed in equations (16-17)

[15] The estimation is carried out in differences to allow its comparison with the test of Engle and Granger (1987) for the linear case. Note that $c = a_0 - 1$.

where only ECM corresponding coefficients are presented, whereas the t-statistics appears in brackets, calculated using the Newey and West matrix (1987, 1994)[16].

Hannan-Quinn (HQC) and Schwarz (SBC) nested models criteria were calculated to establish the number of lags. Since there was no unanimity between them (the HQC selected 7 lags and the SBC selected 13 lags) we decided to choose the number of lags that totally eliminated autocorrelation following the Ljung-Box test Q(4) to Q(28), which turned out to equal 13.

$$\Delta\hat{F}_t = \underset{(-3.89)}{[-0.017]}\hat{e}_{t-1} + \sum_{i=1}^{13}\hat{\delta}_{t-i}\Delta F_{t-i} + \sum_{i=1}^{13}\hat{\delta}_{t-i}\Delta C_{t-i} + \sum_{i=1}^{4}\hat{\gamma}_{di}D_{it} + \sum_{j=1}^{28}\hat{\gamma}_{hj}H_{hj} \tag{16}$$

$$\Delta\hat{C}_t = \underset{(6.22)}{\left[0.35\cos\frac{2\pi k}{T}t\right]}\hat{e}_{t-1} + \sum_{i=1}^{13}\hat{\delta}_{t-i}\Delta F_{t-i} + \sum_{i=1}^{13}\hat{\delta}_{t-i}\Delta E_{t-i} + \sum_{i=1}^{4}\hat{\gamma}_{di}D_{it} + \sum_{j=1}^{28}\hat{\gamma}_{hj}H_{hj} \tag{17}$$

Short term equations of a model with linear ECM are estimated in order to facilitate their comparison. The results of the most parsimonious model can be summarized in equations (18 – 19):

$$\Delta\hat{F}_t = \underset{(-3.93)}{[-0.017]}\hat{e}_{t-1} + \sum_{i=1}^{13}\hat{\delta}_{t-i}\Delta F_{t-i} + \sum_{i=1}^{13}\hat{\delta}_{t-i}\Delta C_{t-i} + \sum_{i=1}^{4}\hat{\gamma}_{di}D_{it} + \sum_{j=1}^{28}\hat{\gamma}_{hj}H_{hj} \tag{18}$$

$$\Delta C_t = \sum_{i=1}^{13}\delta_{t-i}\Delta F_{t-i} + \sum_{i=1}^{13}\delta_{t-i}\Delta C_{t-i} + \sum_{i=1}^{4}\gamma_{di}D_{it} + \sum_{j=1}^{28}\gamma_{hj}H_{hj} \tag{19}$$

The main difference between both models lies in the adjustment of the price of the Eurostoxx-50 index towards long term balance. The linear model implies that only the price of the futures market reacts to the imbalances regarding long term relationships (18), while the index spot price seems to be slightly exogenous (equation 19). However, in the model with Fourier adjustment, equation (17) indicates that the spot price reacts to imbalance through the cosinus coefficient, and so, the spot price is not slightly exogenous.

Once both models have been estimated, we carry out the analysis of their forecasting capacity to determine whether the variables forecast can be improved by taking into account the detected nonlinearity in the relationships of both series. To that aim, we forecast a period ahead with the last 1,204, reserved to that purpose and which correspond to October and November 1999.

[16] For space limitations, the complete results of the estimations of models (16–17) and (18–19) have not been included. These models only include those dummy variables which are significant. These results are available from the authors on request.

We calculate the mean squared error in percentages (RECMP)[17] to compare the forecasting capacity. The results of this analysis indicate that, as far as forecast is concerned, this does not improve, being the RECMP of both models identical. (0.1844 for spot and 0.2014 for futures).

3.2.- Linear Causality

When two or more variables are cointegrated, it is necessary to distinguish between short and long term Granger causality. Long term causality [Granger 1986)] is the result of including all variables lagged by one period in the ECM. This causality will always occur at least in one direction since, according to Granger representation theorem, if two variables are cointegrated, at least one of them must respond to the deviations of the long term balance relationship. In other words, the ECM must be significant in, at least, one of both short term equations. Long term causality will be linear when the ECM is linear and nonlinear should the ECM include any nonlinear expression.

In the case of the Eurostoxx-50, the index value causes its futures contract returns linearly in the long term, since the index enters linearly in the ECM of the futures returns equation (equation 16). On the other hand, the price of the futures contract causes the long term index returns nonlinearly, since the coefficient associated to the cosinus in the ECM of the index returns equation is significant. (equation 17).

Linear Granger causality (1969), also known as short term linear causality, analyses the temporal information flows in a linear context. Using a more formal definition, a variable is said to cause another, if the introduction of the lags of the causal variable in the model of the caused variable improves the forecast of the caused variable.

To test the existence of short term causality, we start from a VAR model and carry out a combined significance test of the lags of the causal variable in the equation of the caused variable. The null to test "X does not cause Y in the short term", is equivalent to testing that the coefficients associated to the lags of Y in the equation of X equal zero.

[Insert Table XI]

Table XI shows the results of the short term linear causality tests for both models estimated in the previous section: panel A for the model with linear ECM (equations 18-19), and panel B for the model with nonlinear ECM (equations 16-17). In both cases, the results of the Wald test for a χ^2 with 13 degrees of freedom show the existence of bidirectional linear causality, i.e., the information flows from the spot market towards the futures market and from the futures market towards the spot market. Nevertheless, the statistics associated to the spot returns causality on futures returns is much higher than the statistics associated to the futures returns causality on spot returns. This seems to indicate that the spot causality on futures is higher. In fact, the lags of the spot returns are significant for more than three hours, while the futures causality on the spot lasts one hour (13 and 4 lags respectively).

3.3.- Nonlinear Causality

Traditionally, lead-lag relationships have been mostly analysed in a linear sphere. However, recent research into nonlinear dependencies on different financial variables in general, and on index returns in particular [Abhyankar, *et. al.* (1997)] points out the possibility of more complex causality relationships between stock index returns and those of its futures contract.

Baek and Brook (1992a,b) suggest a nonparametric method to detect the existence of nonlinear causality relationships, using integral correlation. Hiemstra and Jones (1994) modify the method by Baek and Brook, and obtain a statistics with N(0,1) distribution under certain conditions.

Offering an intuitive definition[18]: be $\{X_t\},\{Y_t\}$ t=1,2,... two strictly stationary and weakly dependent time series; be X_t^m the vector of *m* leads of X_t, Y_t^m the vector of *m* leads of Y_t; and be $X_{t-Lx}^{Lx} = \left(X_{t-Lx}, X_{t-Lx+1},..., X_{t-1}\right)$ and $Y_{t-Ly}^{Ly} = \left(Y_{t-Ly}, Y_{t-Ly+1},..., Y_{t-1}\right)$ the vectors of *Lx* and *Ly* lags of X_t, Y_t respectively.

For some given values of *m*, *Lx*, *Ly* ≥ 1 and for *e* > 0, we say that *Y* does not strictly cause *X* in Granger terms if:

$$\begin{aligned} &\Pr(\| X_t^m - X_s^m \| < e \mid \| X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx} \| < e , \| Y_{t-Ly}^{Ly} - Y_{s-Ly}^{Ly} \| < e) \\ &= \Pr(\| X_t^m - X_s^m \| < e \mid \| X_{t-Lx}^{Lx} - X_{s-Lx}^{Lx} \| < e) \end{aligned} \tag{20}$$

where *Pr*(.) means the probability and ||.|| the maximum norm.

---

17 The expression of this statistics is: $RECMP = 100\left(\frac{1}{N} \sum_{s=1}^{N} \left[(r_s^p - r_s) / r_{t+s}\right]^2\right)^{1/2}$ where r is a variable to forecast, N is the number of forecasts carried out.

18 A detailed description of all the mathematical calculations is included in Hiemstra and Jones (1994).

The modified causality test by Baek and Brook determines whether the conditional probability that two arbitrary vectors of *m* length remain within an *e* distance (given that the corresponding vectors of *Lx* lags are within that distance) is influenced by the corresponding lags vector *Ly*. In other words, testing if the lagged values of *Y* are capable of forecasting the present value of *X*.

In practice, expression (20) is implemented using integral correlation. This 'counts' the number of times that both vectors are one within a specific distance of the other. Hiemstra and Jones (1994) argue that a positive and significant value of the statistics suggests that the lagged values of *Y* help to forecast *X*, whereas a negative and significant value of the statistics suggests that the knowledge of the lagged values of *Y* blurs the forecast of *X*. For this reason, they maintain that the critical values corresponding to a right-tail must be used.

Besides, Hiemstra and Jones (1994) argue that, when carrying out the nonlinear Granger causality test between two variables, it is advisable to eliminate the possible linear forecasting power of the variables by means of a VAR model. In this way, any increase in the forecasting power of a residuals series on the other can be regarded as nonlinear forecasting power. For this reason, in our empirical analysis of the nonlinear causality test between the returns of the Eurostoxx-50 and those of its futures market, we have used the standarized residuals of the VAR models estimated in section 3.1 (equations 16-17 and 18-19)[19].

The modified Baek and Brook test requires from the researcher to choose the *m* lead values, the *Lx Ly* lags and the scale parameter *e*. Since no references are made in the existing literature on how to select the optimum values, we follow Hiemstra and Jones (1994), and fix $m = 1$, $e = 1.5$ times the standard deviation (equalling 1 in this case, as they are standarized series) and the same number of lags for both series so that $Lx=Ly$ takes values between 1 and 10, which allows us to test the existence of nonlinear causality during a maximum interval of two and a half hours[20].

[Insert Table XII]

Table XII shows the results of the modified Baek and Brook test, applied to the residuals of the VAR models corresponding to the returns of the Eurostoxx-50 index and the

[19] Linear causality tests on these residuals confirm the absence of causality between both series.
[20] It was also calculated for e=1 and $e = 0.5$, obtaining similar results.

returns of its futures contract: panel A for the model with linear ECM (equations 18-19) and panel B for the model with nonlinear ECM (equations 16-17). Unanimously, and regardless of the path taken in the adjustment of imbalances regarding the long term relationship, all statistics indicate the existence of bidirectional short term nonlinear causality between the returns of both markets. This result is repeated for all the lags considered. None of the normalized statistics is below 6.56, which seems to be a very strong piece of evidence in favour of the existence of nonlinear causality in both directions.

For short term nonlinear causality, we cannot observe that either market leads the other, unlike the results for linear causality. These results agree with those obtained by Abhyankar (1998) for the FT-SE 100 index and its futures market.

4.- CONCLUSIONS

The main objective of this study has been to analyse the individual and combined behaviour of the Eurostoxx-50 and its futures contract. It has become evident that nonlinear dynamics exist in the returns which cannot solely be explained by the presence of conditional heteroskedasticity.

First of all, we carried out a BDS test both on the returns series linearly filtered considering intraday stationarities, weekday and month, and on the returns series filtered by GARCH effects. The results of such tests indicate that the variables are not *iid* in both cases This result coincides with the existing literature on the behaviour of several financial series.

As for the dynamic analysis of the relationships between spot and futures markets, we highlight the existence of cointegration between the prices of the Eurostoxx 50 and its futures contract. Following Enders and Ludlow (2000), we have proved that the adjustment process is nonlinear in the model of the adjustments against imbalances in the long term relationship. The results of this model show that the traditional linear ECM, more restrictive that the Fourier adjustment, is not more suitable as, in this case, the spot price seems to be a slightly exogenous variable, whereas the use of a more flexible Fourier adjustment demonstrates that the spot price responds to imbalance by means of the cosinus coefficient. This result indicates that the model with linear ECM fails to explain the adjustment process of both markets in all its complexity.

Finally, in the study of causal relationships, and following Hiemstra and Jones (1994), we distinguish between linear and nonlinear Granger causality, establishing that the information flows are bidirectional both in the linear and in the nonlinear sphere. Remarkably, as for short term nonlinear causality, neither market seems to lead the other which could be due to the low levels of futures volume due to the youth of the futures contract. These results as a whole prove the importance of considering the existence of nonlinear relationships between the financial variables studied. Nevertheless, we are aware that the econometric tests applied show the existence of nonlinear dynamics empirically but fail to explain theoretically and formally the nature of such dynamics.

REFERENCES

Abhyankar, A. (1998): "Linear and nonlinear Granger causality: Evidence from the UK stock index futures market", *The Journal of Futures Market*, 18: 519-540.

Abhyankar, A. (1992): , "A nonparametric test for independence of multivariate time series", *Statistica Sinica*, 2: 137-156.

Abhyankar, A.; Copeland, L. S.; Wong, W. (1997): "Uncovering nonlinear structure in real-time stock market indexes: The S&P 500, the DAX, the Nikkei 225 and the FT-SE 100", *Journal of Business Economics and Statistics*, 15: 1-14.

Baek, E.; Brook, W. (1992): "A general test for nonlinear Granger causality: Bivariate model", Working Paper, Iowa State Unversity and University of Wisconsin, Madison

Balke, N. S.; Fomby, T. B. (1997): "Threshold cointegration", *International Economic Review*, 38: 627-645.

Berndt, E. K.; Hall, B. H.; Hall, R. E.; Haussman, J. A. (1974): " Estimation and inference in nonlinear structure models", *Annals of Economic and Social Measurement*, 4: 653-665.

Blasco, N.; Santamaria, R. (1996): "Testing memory patterns in the Spanis stock market", *Applied Financial Economics*, 6: 401-411.

Bollerslev, T.; Wooldridge J. M. (1992): "Quasi-Maximum likelihood estimation and inference in dynamic models with time varying covariances", *Econometric Reviews*, 11: 143-172.

Booth, G. G.; Martikainen, T.; Puttonen, V. (1993): "The international lead-lag effect between market returns: Comparison of stock index futures and cash markets", *Journal of International Financial Markets, Institutions and Money*, 3: 59-71.

Brock, W. A.; Hsieh, D. A.; LeBaron, B. (1991): *Nonlinear dynamics, chaos and instability*, Cambridge, MA: The MIT press.

Brock, W., W. Dechert and J. Scheinkman, (1987) "A test for independence based on the correlation dimension", Working Paper, University of Wisconsin at Madison, University of Houston, and University of Chicago.

Brock, W.; Dechert, D.; Sheinkman, J.; LeBaron, B. (1996): "A test for independence based on the correlation dimension" *Econometric Reviews*, 15: 197-235.

Dickey, D A; Fuller W A (1979): "Distribution of the estimators for autoregressive time series with a unit root", *Journal of the American Statistical Association*, 74: 427-431.

Dwyer, G. P.; Locke, P.; Yu, W. (1996): "Index arbitrage and nonlinear dynamics between the S&P500 futures and cash", *Review of Financial Studies*, 9: 353-387.

Enders, W.; Granger, C.W. (1998): "Unit root test and asymmetric adjustment with an exemple using the term structure of interest rates", *Journal of Business and Economic Statistics*, 16: 304-311.

Enders, W.; Loudlow, J. (2000): "Non-linear decay: Tests for an attractor using a Fourier approximation", Iowa State University, (Mimeo)

Enders. W.; Silkos, P. L. (2001): "Cointegration and threshold adjustment", *Journal of Business and Economic and Statistics*, 19: 166-176.

Engle, R. F. (1982): “Autoregressive conditional heteroskedasticity with estimates of the variance of U.K. inflation”, *Econometrica*, 50: 987-1008.

Engle, R. F.; Granger, C. W. J. (1987): “Cointegration and error correction: Representation, estimation and testing”, *Econometrica*, 55: 251-276.

Engle, R. F.; Ng V. K. (1993): “Measuring and testing the impact of news on volatility”, *Journal of Finance*, 48: 1022-1082.

Fleming, J.; Ostdiek, B: ; Whaley, R. (1996): “Trading costs and the relative rates of price discovery in stock, futures and option markets”, *The Journal of Futures Markets*, 16: 353-387.

Fujihara, R.; Mougoue, M. (1997): “An examination of linear and nonlinear causal relationships betwenn price variability and volume in petroleum futures markets”, *Journal of Futures Markets*, 17: 385-416.

Gao, A. H.; Wang, G. H. K. (1999):“Modelling non linear dynamics of daily futures prices change”, *The Journal of Futures Markets*, 19: 325-351.

Glosten, L. R.; Jaganathan, R.; Runkle, D. (1993): “On the relation between the expected value and the volatility of the normal excess return on stocks”, *Journal of Finance*, 48: 1779-1801.

Granger , C. W. J. (1969): “Investigating causal relations by econometric models and cross-spectral methods”, *Econometrica*, 37: 424-438.

Granger , C. W. J. (1986): “Some recent developments in a concept of causality”, *Journal of Econometrics*, 39: 199-211.

Grunbichler, A.; Longstaff, F.; West, E. (1987): “Electronic screen trading and the transmission of information: An empirical examination”, *Journal of Financial Intermediation*, 3: 166-187.

Hiemstra, C.; Jones, J. D. (1994): “Testing for linear and nonlinear Granger causality in the stock price-volume relation”, *The Journal of Finance*, 49 (5): 1639-1664.

Hsieh, D. A. (1989): “Testing for nonlinear dependence in daily foreign exchange rate”, *Journal of Business*, 62: 339-368.

Hsieh, D. A. (1991), “Chaos and nonlinear dynamics: Application to financial markets” *Journal of Finance*, 46: 1839-1876.

Hsieh, D. A. (1993), “Implications of nonlinear dynamics of financial risk management”, *Journal of Finance and Quantitative Analysis*, 28: 41-64.

Lee, T.; White, H.; Granger, C. (1993): “Testing for neglected nonlinearity in time series models: A comparison of neural network method and alternative tests” *Journal of Econometrics*, 56: 269-290.

Ljung, G.; Box, G. (1979): “On a measure of lack of fit in time series models” *Biometrika*, 66: 265-270.

Ludlow, J.; Enders, W. (2000): “Estimating non-linear ARMA models using a Fourier coefficients”, *International Journal of Forecasting*, 16: 333-347.

MacKinnon, J. G. (1991): “Critical values for cointegration tests” en *Long-run Economic Relationships: Readings in Cointegration*, editado por R F Engle y C W J Granger, Oxford University Press.

McLeod, A I; Li, W K (1983).”Diagnostic checking ARMA time series models using squared-residual autocorrelations” *Journal of Time Series Analysis*, 4: 269-273.

Mossa, I. A.; Silvapulle, P. (2000): “The price-volume relationship in the crude oil futures market: Some results based on linear and nonlinear causality testing”, *International Review of Economics and Finance*, 9: 11-30.

Newey, W.; West, K. (1987): “Hypothesis testing with efficient method of moments estimation”, *International Economic Review*, 28: 777-787.

Newey, W.; West, K. (1994): “Automatic lag selection in covariance matrix estimation”, *Review of Economic Studies*, 61: 631-653.

Nieto, L.; Fernández, A; Muñoz, M.J. (1998): “Market efficiency in the Spanish derivatives markets: An empirical analysis”, *International Advances in Economic Research*, 4: 349-355.

Phillips, P. C. B.; Perron, P. (1988): “Testing for a unit root in time series regression”, *Biometrika*, 75: 335-346.

Pippenger, M. K.; Goering, G. E. (2000): “Additional results on the power of unit root and cointegration tests under threshold processes”, *Applied Economic Letters*¸7: 641-644.

Savit, R. (1988): “When random is not random: An introduction to chaos in market prices”, *Journal of Futures Markets*, 8: 271-290.

Wahab, M.; Lashigari, M. (1993): “Price discovery and error correction in stock index and stock index futures markets: A cointegration approach”, *The Journal of Futures Markets*, 13: 711-42.

Yang, S. R.; Brorsen, B. W. (1993):“Nonlinear dynamics of daily futures prices: conditional heteroskedasticity or chaos?”, *The Journal of Futures Markets,* 13: 175-191.

Yang, S. R.; Brorsen, B. W. (1994):“Daily futures price changes and nonlinear dynamics”, *Structural Change and Economic Dynamics*, 5: 111-132.

Table I: Descriptive statistics

	Mean	Max.	Min.	SD	Skew.	Kurt.	Jarque-Bera	ρ_1
Ln spot	8.174	8.290	7.972	0.077	-0.967	3.132	993.18 [a]	0.999
Ln future	8.175	8.294	7.978	0.075	-0.946	3.164	952.02 [a]	0.999
Spot returns	0.003	2.210	-2.368	0.236	0.300	17.942	59084.96 [a]	0.057
Future returns	0.003	2.566	-2.764	0.266	0.026	17.491	55481.40[a]	0.004

[a] Denotes the rejection of the null hypothesis of normality at 1% of significance level.

Table II: Unit root test

	Ln spot	Ln future	Spot returns	Future returns
ADF	-2.413	-2.425	-22.078[a]	-22.449[a]
PP	-2.459	-2.411	-75.616[a]	-79.312[a]

[a] Denotes the rejection of the null hypothesis of non stationarity at 1% of significance level. Critical values at 1% and 5% are respectively -3.4345 and –2.8625 [Mckinon (1991)]

Table III. Temporal dependence tests

	BDS [a]					
	m=3, l=1	m=3, l=1.5	m=6, l=1	m=6, l=1.5	Q(28)	$Q^2(28)$
Ln spot	--	--	--	--	166826[c]	172408 [c]
Ln future	--	--	--	--	168667 [c]	172017 [c]
Spot returns	17.947 (0.000)	14.605 (0.000)	26.834 (0.000)	19.257 (0.000)	98.063 [c]	481.53 [c]
Future returns	19.243 (0.000)	17.058 (0.000)	27.506 (0.000)	21.818 (0.000)	42.490 [b]	255.88 [c]

[a] p-values in parentheses. These p-values have been simulated by *bootstraping* with 1000 replications
[b,c] Denotes respectively the rejection of the null hypothesis of non serial correlation at 5% and 1% of significance levels.

Table IV: Linear models[a]

	Spot returns			Future returns		
	Coefficient	t-ratio	p-value	Coefficient	t-ratio	p-value
D1	0.009	1.009	0.313	--	--	--
D2	-0.018	-2.036	0.042	-0.022	-2.587	0.010
D4	--	--	--	0.015	1.466	0.143
H1015	0.074	1.410	0.159	0.058	1.082	0.279
H1030	-0.015	-0.926	0.355	-0.034	-1.236	0.217
H1045	-0.015	-1.087	0.277	0.008	0.454	0.650
H1100	0.010	0.787	0.432	-0.013	-0.995	0.320
H1115	-0.036	-2.642	0.008	-0.033	-2.047	0.041
H1130	-0.001	-0.054	0.957	-0.008	-0.593	0.554
H1145	-0.002	-0.215	0.830	-0.015	-1.191	0.234
H1200	0.007	0.623	0.533	0.011	0.859	0.390
H1215	0.014	1.367	0.172	0.003	0.232	0.817
H1230	0.006	0.545	0.586	0.005	0.478	0.633
H1245	0.017	1.758	0.079	0.020	1.747	0.081
H1300	0.000	0.027	0.979	-0.005	-0.429	0.668
H1315	0.009	0.870	0.384	0.003	0.240	0.811
H1330	0.006	0.570	0.569	0.018	1.460	0.144
H1345	-0.009	-0.869	0.385	0.000	0.022	0.983
H1400	0.000	-0.023	0.982	-0.012	-1.180	0.238
H1415	0.025	2.070	0.039	0.021	1.646	0.100
H1430	0.011	0.953	0.341	0.005	0.390	0.697
H1445	0.037	2.159	0.031	0.036	1.949	0.051
H1500	0.004	0.404	0.686	0.025	1.845	0.065
H1515	0.008	0.789	0.430	0.000	0.037	0.970
H1530	0.014	1.410	0.159	0.020	1.977	0.048
H1545	-0.060	-4.235	0.000	-0.055	-3.813	0.000
H1600	-0.020	-1.249	0.212	-0.029	-1.567	0.117
H1615	-0.027	-1.480	0.139	-0.042	-2.296	0.022
H1630	0.007	0.508	0.611	-0.002	-0.140	0.889
H1645	0.030	2.317	0.021	0.058	3.212	0.001
H1700	0.054	2.794	0.005	0.103	4.890	0.000
β_1	0.048	3.067	0.002	-0.003	-0.156	0.876
β_2	0.034	2.339	0.019	0.010	0.629	0.529
β_3	0.038	2.636	0.008	0.017	1.055	0.292
β_4	-0.007	-0.449	0.654	0.016	1.083	0.279
β_5	-0.019	-1.302	0.193	-0.020	-1.558	0.119
β_6	0.002	0.154	0.878	0.003	0.251	0.802
β_7	0.052	4.140	0.000	0.046	3.384	0.001
β_8	0.019	1.735	0.083	0.006	0.506	0.613
β_9	-0.002	-0.137	0.891	0.006	0.446	0.656
β_{10}	0.002	0.108	0.914	-0.005	-0.362	0.718
β_{11}	0.009	0.730	0.466	0.024	1.987	0.047
β_{12}	0.028	1.948	0.052	0.010	0.666	0.506
R^2 Adjusted	0.016			0.014		
σ	0.234			0.264		
Log Likelyhood	230.260			-531.790		
Jarque-Bera	57752 [b]			55417[b]		

[a] Newey-West standard errors and covariances (10 lags).

[b] Denotes the rejection of the null hypothesis of normality at 1% of significance level.

Table V: Lineal model: Temporal dependence test over residuals

	BDS[a]							
	m=3, l=1	m=3, l=1.5	m=6, l=1	m=6, l=1.5	LM(2)	ARCH(5)	Q(28)	$Q^2(28)$
Spot	18.497 (0.000)	14.949 (0.000)	27.347 (0.000)	19.667 (0.000)	1.392 (0.000)	22.801 [b]	27.485	526.540[c]
Future	18.774 (0.000)	16.631 (0.000)	26.318 (0.000)	21.100 (0.000)	3.581 (0.000)	39.954 [b]	16.054	251.370[c]

[a] p-values in parentheses. These p-values have been simulated by *bootstraping* with 1000 replications.
[b] Denotes the rejection of the null hypothesis of homoskedasticity at 1% of significance level.
[c] Denotes the rejection of the null hypothesis of non serial correlation at 1% of significance level.

Table VI. Linear model: Third moment test over residuals

		Spot		Future	
i	j	$r_{uuu}(i,j)$ [a]	$3MT$	$r_{uuu}(i,j)$ [a]	$3MT$
1	1	-0.0108	-0.899	0.0091	0.360
1	2	-0.0145	-1.145	-0.0041	-0.151
1	3	-0.0147	-1.114	-0.0075	-0.267
1	4	-0.0137	-1.011	-0.0074	-0.248
1	5	-0.0070	-0.507	0.0068	0.224
2	2	-0.0217	-1.283	-0.0336	-0.963
2	3	-0.0274	-1.549	-0.0505	-1.375
2	4	-0.0258	-1.413	-0.0611	-1.576
2	5	-0.0203	-1.080	-0.0472	-1.173
3	3	-0.0254	-1.261	-0.052	-1.217
3	4	-0.0194	-0.926	-0.0581	-1.277
3	5	-0.0078	-0.362	-0.0311	-0.658
4	4	0.0014	0.059	-0.0388	-0.726
4	5	0.0103	0.426	-0.0202	-0.363
5	5	0.0166	0.629	0.0103	0.175

[a] Values have been multiplied by 1000.

Table VII. GARCH modelsa

	Spot returns			Future returns		
	Coefficient	z-ratio	p-value	Coefficient	z-ratio	p-value
α_0	0.0011	4.586	0.000	0.0115	7.304	0.000
α_1	0.0088	3.180	0.002	0.0187	1.882	0.060
δ	0.0429	5.016	0.000	0.0519	3.116	0.002
β_1	0.8816	58.957	0.000	0.6039	13.392	0.000
D1	--	--	--	-0.0014	-2.758	0.006
D3	0.0005	2.502	0.012	--	--	--
H1015	0.5291	8.604	0.000	0.3766	7.288	0.000
H1030	-0.5040	-9.594	0.000	--	--	--
H1045	--	--	--	-0.1437	-6.062	0.000
H1100	--	--	--	-0.0077	-1.504	0.133
H1130	-0.0075	-3.761	0.000	--	--	--
H1200	--	--	--	-0.0080	-2.559	0.011
H1215	--	--	--	-0.0080	-3.207	0.001
H1230	-0.0051	-3.645	0.000	--	--	--
H1245	--	--	--	-0.0055	-1.961	0.050
H1300	--	--	--	-0.0093	-4.381	0.000
H1330	--	--	--	-0.0097	-4.392	0.000
H1445	0.0455	2.980	0.003	--	--	--
H1500	-0.0369	-2.725	0.006	--	--	--
H1530	0.0039	1.932	0.053	--	--	--
H1600	0.0257	7.712	0.000	0.0296	4.624	0.000
H1645	-0.0092	-2.581	0.010	0.0141	1.966	0.049
H1700	0.0507	2.389	0.017	0.0339	2.673	0.008
M1	0.0028	3.544	0.000	0.0261	5.534	0.000
M2	0.0010	2.584	0.010	0.0057	3.538	0.000
M4	--	--	--	-0.0016	-2.019	0.044
M6	-0.0004	-2.724	0.006	-0.0028	-3.221	0.001
M7	--	--	--	-0.0029	-3.314	0.001
M8	--	--	--	-0.0018	-1.610	0.108
M9	-0.0004	-2.674	0.008	--	--	--
M12	0.0009	3.028	0.003	0.0100	3.782	0.000
R^2 Adjusted		0.006			0.007	
σ		0.235			0.265	
Log Likelyhood		2185.7			1151.3	
Jarque-Bera		1457.3[b]			4646.7[b]	

[a] ML estimation with BHHH algorithm. Bollerslev-Wooldrige standard errors and covariances.
[b] Denotes the rejection of the null hypothesis of normality at 1% of significance level.

Table VIII: GARCH model: Temporal dependence test over residuals

	BDS[a]							
	m=3, l=1	m=3, l=1.5	m=6, l=1	m=6, l=1.5	ARCH(1)	ARCH(10)	Q(28)	$Q^2(28)$
Spot	2.104 (0.036)	2.390 (0.030)	3.919 (0.000)	3.755 (0.000)	6.394[b]	23.647[c]	24.301	56.808[d]
Future	2.050 (0.014)	2.366 (0.016)	4.252 (0.000)	4.977 (0.000)	0.455	10.484	19.395	19.395

[a] p-values in parentheses. These p-values have been simulated by *bootstraping* with 1000 replications.
[b] Denotes the rejection of the null hypothesis of homoskedasticity at 5% of significance level
[c] Denotes the rejection of the null hypothesis of homoskedasticity at 1% of significance level
[d] Denotes the rejection of the null hypotheses of non serial correlation at 1% of significance level

Table IX. GARCH model: Third moment test over residuals

i	j	Spot $r_{uuu}(i,j)$	Spot $3MT$	Future $r_{uuu}(i,j)$	Future $3MT$
1	1	-0.054	-1.324	-0.029	-0.848
1	2	-0.070	-1.596	-0.044	-1.178
1	3	-0.105	-2.200[a]	-0.076	-1.902
1	4	-0.111	-2.245[a]	-0.084	-1.981[a]
1	5	-0.093	-1.837	-0.082	-1.844
2	2	-0.104	-1.874	-0.074	-1.494
2	3	-0.157	-2.603[b]	-0.125	-2.341
2	4	-0.165	-2.620[b]	-0.143	-2.506[a]
2	5	-0.164	-2.500[a]	-0.144	-2.391[a]
3	3	-0.193	-2.728[b]	-0.142	-2.101[a]
3	4	-0.183	-2.458[a]	-0.170	-2.356[a]
3	5	-0.171	-2.189[a]	-0.141	-1.865
4	4	-0.159	-1.944	-0.140	-1.626
4	5	-0.158	-1.830	-0.132	-1.465
5	5	-0.149	-1.567	-0.147	-1.490

[a, b] Denotes respectively the rejection of the null hypothesis at 5% and at 1% of significance levels.

Table XX. Significance restrictions

	Statistic	Critical V. 5%	Critical V. 1%
F_all	20.52[b]	8.08	9.74
F_trig	23.29[b]	8.03	9.74
$c^* = 0$	-3.77[a]	-3.44	-4.13

[a, b] Denotes respectively the rejection of the null hypothesis at 5% and at 1% of significance levels.

Table XXI. Linear Granger causality test

Panel A: Linear ECM	Statistic	Prob.
Future returns does not cause spot returns	23.877[a]	0.0323
Spot returns does not cause future returns	975.454[b]	0.0000
Panel B: Non-linear ECM		
Future returns does not cause spot returns	23.443[a]	0.0367
Spot returns does not cause future returns	976.165[b]	0.0000

[a, b] Denote respectively the rejection of the null hypothesis at 5% and at 1% of significance levels, Newey-West standard errors and covariances.

Table XXII. Non-linear Granger causality test

Panel A: Linear ECM					
Future returns does not cause spot returns			Spot returns does not cause future returns		
Lags	Spread	Statistic	Lags	Spread	Statistic
1	0.0087	7.419[a]	1	0.0084	7.081[a]
2	0.0134	8.872[a]	2	0.0140	8.899[a]
3	0.0153	9.529[a]	3	0.0151	9.607[a]
4	0.0157	9.487[a]	4	0.0153	9.626[a]
5	0.0156	9.114[a]	5	0.0157	9.588[a]
6	0.0157	8.821[a]	6	0.0149	8.904[a]
7	0.0160	8.449[a]	7	0.0148	8.471[a]
8	0.0162	7.964[a]	8	0.0140	7.864[a]
9	0.0159	7.344[a]	9	0.0139	7.295[a]
10	0.0153	6.651[a]	10	0.0132	6.664[a]
Panel B: Non-linear ECM					
1	0.0086	7.377[a]	1	0.0084	7.014[a]
2	0.0133	8.857[a]	2	0.0139	8.898[a]
3	0.0152	9.499[a]	3	0.0152	9.632[a]
4	0.0156	9.444[a]	4	0.0153	9.683[a]
5	0.0155	9.048[a]	5	0.0158	9.662[a]
6	0.0156	8.746[a]	6	0.0151	8.984[a]
7	0.0160	8.424[a]	7	0.0149	8.536[a]
8	0.0161	7.894[a]	8	0.0142	7.929[a]
9	0.0157	7.260[a]	9	0.0141	7.383[a]
10	0.0151	6.560[a]	10	0.0136	6.785[a]

Lag=Lx=Ly. In all cases we set m=1. Spread is the difference of conditional probabilities in expression (20).

[a] Denotes the rejection of the null hypothesis of absence of nonlinear causality at 1% of significance level.