# AN ERROR CORRECTION FACTOR MODEL OF TERM STRUCTURE SLOPES IN INTERNATIONAL SWAP MARKETS

Pilar Abad[1]
Alfonso Novales[2]

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## ABSTRACT

The first two principal components in the vector of term structure slopes from IRS markets in eight major currencies can be approximately identified as the slopes for the US dollar and Deutsche mark. Each of the eight slopes considered is cointegrated with these two factors. The implied Error Correction models can be very fruitful for short and medium term slope forecasting for the eight currencies. This scheme achieves a drastic reduction of dimensionality, since the eight slopes can be predicted using just univariate forecasts for the two factors. Adding more factors to the model does not lead to a significant improvement in forecasting performance, while forecasts obtained using just one factor are not as good as those from two-factor Error Correction models.

*Keywords: Factor models, term structure of interest rates, principal components, swap markets, IRS.*

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1. Departamento de Economía Aplicada. Universidade de Vigo
2. Departamento de Economía Cuantitativa, Universidad Complutense, Somosaguas, 28223 Madrid, Spain.

## 1. Introduction

Characterizing the main properties of term structure slopes has quickly become a center of attention in the analysis of fixed income markets. There are two reasons for this increased attention: on the one hand, a long line of research has accumulated robust evidence on the fact that changes in term structure slope can anticipate turning points in the business cycle [see Estrella and Hardouvelis (1991), Stock and Watson (1988), Hardouvelis (1994) and Plosser and Rouwenhorst (1994), among others]. From this research emerges the idea that fluctuations in term structure slopes contain information that is different from that contained in other, more standard indicators of production activity or demand. Besides, this type of result arises very consistently for different markets and countries. A second research line has recently been devoted to characterizing the main factors behind changes in the term structure [Litterman and Scheinkman (1988), Steeley (1990) and Knez et al. (1994)]. Work along this line has shown that term structure shifts are produced mainly by changes in the general level of interest rates, with changes in the slope and concavity of the term structure being additional factors explaining variations along the yield curve.

The importance of term structure slopes, as it emerges from these two issues, has motivated research attempting to characterize the joint fluctuations in slopes across international fixed income markets. The interest is not only in testing for robustness of the previously mentioned properties, but also in summarizing the extensive correlations observed among term structures in different currencies.

Interest rates should be expected to be correlated across currencies, since it is widely believed that monetary policy interventions in different countries are not independent from each other. Furthermore, increased monetary policy coordination, as it was the case in Europe prior to the constitution of the Euro area, led to common interest rate fluctuations among countries in the European Union. However, since monetary authorities determine very short-term interest rates, we should expect to see high correlations in the shorter end of the term structure, but not necessarily among longer-term rates. Correlations between slopes would emerge from correlations between short-term rates if interest rates at longer maturities were roughly constant (they are much less volatile than short-term rates in all countries). A view that the spread between long- and short-term interest rates is roughly constant over time would also lead from correlations among interest rates at short maturities to correlations among slopes. None of these views is fully realistic, and deviations from them will produce lower correlations among slopes than among short-term interest rates.

Two different lines have been followed to characterize co-movements among slopes in different countries. Working with monthly interest rate data on Eurodeposits on the British pound, French franc, Deutsche mark, Swiss franc, Japanese yen and *US* dollar over 1979-1998, Domínguez and Novales (2000) use linear regression models to estimate causal relationships among slopes for the different currencies, showing that the US and Deutsche mark slopes help predict future slope fluctuations in other countries.

A second approach has used factor analysis, generally in the form of principal components, to summarize such co-movements. As an example, Domínguez and Novales (2002) used a factor model approach to show that one or two factors are enough in most currencies to produce forecasts as good as those obtained from univariate models. It is quite striking that a projection by regression on just the first factor can compete with dynamic, univariate models in terms of the forecasting performance of term structure slopes, and that adding more factors does not generally lead to a significant gain in forecasting performance.

In this paper, we use the principal components technique to search for factors among a wide set of international term structure slopes from Interest Rate Swap (IRS) markets. At a difference of markets in Eurodeposits, which include maturities up to 1-year, the IRS term structure goes up to 10-year maturities, allowing for the possibility of a richer variety of changes in interest rates. After showing that fluctuations in the vector of slopes in eight different currencies can be summarized by changes in a few factors, we test for the quality of slope forecasts obtained from factor models, as compared with those obtained from univariate slope models. Using the former strategy would greatly simplify the problem of producing slope forecasts when searching for changes in economic activity across countries, since it would only be necessary to forecast a small number of factors.

Slopes are computed as the difference between 10- and 2-year rates in interest rate swap (IRS) markets in each currency. Once the most relevant factors have been characterized, we estimate projections of each slope on the common factors. Since IRS slopes turn out to be I(1) variables, these projections take the form of Error Correction models (ECM). Forecasts from these ECM models are then compared to slope forecasts from univariate models.

Section 2 describes the data and the methodology used to characterize the common factors among the set of international term structure slopes. In Section 3 we report estimates for univariate as well as Error Correction models for each slope. In Section 4 we describe the forecasting exercise, and present the obtained results. The paper closes with some conclusions.

## 2. Factor analysis

### *2.1. The data*

Data for interest rate swap (IRS) rates at 2, 3, 4, 5, 7 and 10 year maturities, for the Deutsche mark, US dollar, Japanese yen, British pound, Italian lira, Swiss franc, French franc and Spanish peseta were obtained from *DataStream* $^{TM}$, between June 26, 1991 and December 12, 1998. In this database, IRS daily data are collected at 18:00 GTM. They are the average of bid and ask quotes, as provided by Dark limited, from Intercapital Brokers Limited. We use data from each Wednesday as weekly data. When a Wednesday fell on a holiday, we take data from the previous market day. A term structure of zero coupon rates were then derived from swap rates by the *bootstrapping* method. We used the spread between 10- and 2-year zero coupon rates as term structure slope.

### *2.2. First results*

Figure 1 shows the time behavior of term structure slopes in IRS markets for the currencies considered. Table 1 presents Dickey-Fuller and Phillips-Perron unit root tests, suggesting that, except for the Japanese yen, slopes seem to follow I(1) processes. This could give raise to spurious correlations between slopes, so appropriate statistical methods need to be used. Table 2 shows sample correlation coefficients among slopes. Slopes in European currencies display high correlations, being significant smaller for the British pound. Correlations between European slopes and that of the Japanese yen are also large and positive. Correlations between European slopes and the US slope are large, but negative. Whether the high correlations are purely spurious, due to the presence of unit roots, will be discussed later. As a first check, we present below the diagonal in Table 2 correlations between differenced slopes. Slopes in the group of European countries show again significant correlations among themselves except for Spain, but there are no other noticeable correlations. The correlations across continents which were present in level slopes all but disappear in first differences, suggesting that they were spuriously produced by the presence of trend with either the same or opposite sign in both slopes.

*2.3. Principal components among international slopes*

Principal components is a particular form of factor analysis, aimed at producing a few linear combinations of a set of variables explaining as much as possible of the fluctuations in the whole vector of variables. The principal components analysis starts from the variance-covariance matrix of the variables, in differences to their sample means. This is a semi-positive definite matrix, with non-negative eigenvalues. The eigenvector associated to the largest eigenvalue defines the linear combination of variables that explains the largest percentage of the variance in the vector of original variables. The ratio between the largest eigenvalue and the sum of all them is the percentage of variance being explained by the first principal component. Since eigenvectors corresponding to different eigenvalues are orthogonal to each other, the correlation between linear combinations defined by eigenvectors associated to successive (if different) eigenvalues display zero correlation.

Panel *a* in Table 3 shows that the first principal component explains 73.5% of the fluctuation in the vector of eight term structure slopes. The second principal component is defined through the eigenvector associated to the second largest eigenvalue of the variance-covariance matrix. It adds information which is all new, not overlapping with that contained in the fluctuations of the first principal component. The two together explain 92.0% of the fluctuation in the original variables. Adding a third component raises the percentage of explained variance to 96.4%, while a fourth one would take us to 98.4%.

The entries of a given eigenvector are not readily interpretable as the relative importance of the different slopes in each principal component, since slope levels in different currencies may be quite different from each other. To identify the components, we use linear projections of each slope on a given component. Table 3 (panel *b*) shows that the first principal component is most closely associated with European slopes, especially those in the Deutsche mark, Swiss franc and French franc. The second component is essentially the US dollar slope, while the third principal component is essentially the slope for the British pound. This interpretation may not be independent from the fact that the Euro and US dollar are the more heavily traded currencies in IRS markets. At the end of 1998, the Bank for International Settlements reported that 34,7% and 23,7%, respectively, of trades in IRS markets were made in these two currencies.

The same type of evidence can be read from panel *c* in Table 3, which presents a sequence of non-decreasing R-squared values, obtained from regressions that incorporate successive principal components as explanatory variables, having each slope as a dependent variable.

It is also useful to examine scatter diagrams in Figure 2, which suggest a similar interpretation, with the first principal component more closely related to the Deutsche mark slope than with those of any other European currency although with a negative sign, the second principal component being essentially the US dollar slope, the third component being the British pound slope again with a negative sign, and the fourth one being the slope for the Japanese yen or Italian yen.

As a last identification strategy, we computed the changes induced on each slope by a change in a given principal component. Figure 3 represents the variation induced on each slope by a two standard deviation change in a given component. These are the implied changes in each slope normalized by its standard deviation, so as to make them comparable across currencies. A change in the first component induces noticeable changes in the slopes in European currencies, as it should be expected from a change in the slope of the term structure in Deutsche marks. Changes in the second and third components affect mainly slopes for the US dollar and British pound, respectively. A change in the fourth component implies a change in slope for the Italian lira and Japanese yen.

Having a reasonable interpretation for each factor, the right panel in Table 1 shows them to be I(1) variables, not surprisingly, since we have just seen how they can be interpreted as specific slopes in most cases.

Working with data from Eurodeposits for a similar set of currencies, Domínguez and Novales (2000) find that a factor linked to slopes in European currencies explains 61% of the fluctuation in the vector of slopes. However, in their analysis, the US slope plays a minor role, revealing a significant difference between cross-currency correlations among term structures from Eurocurrency and IRS markets. This difference might be explained by a higher relative volume traded in US dollars in the IRS than in the Eurocurrency market. On the other hand, it might also reflect the fact that it is fluctuations in long maturity interest rates for the US dollar that play a leading role in influencing fluctuations in similar rates in other currencies. That would barely show in Eurocurrency markets, where just interest rates up to 1-year maturity are negotiated. Analyzing in detail these two alternative interpretations remains as an interesting issue for further research.

## 3. Models for term structure slopes

Given the evidence on non-stationarity of IRS slopes, as discussed in Section 2.2, a clear choice is to estimate univariate models in first differences. Differenced slopes display very little autocorrelation, so that short autoregressive models should be enough in most cases. On the other hand, third order autoregressions [AR(3)] in level slopes also produce stationary residuals and no significant evidence of autocorrelation [see left column for each currency in Table 4], so they could be considered acceptable for forecasting purposes. This is a simple model, the characteristic equation of the associated autoregressive polynomial having three roots, which may capture a possible cycle through two complex roots, plus a possible unit root. Since term structure slopes are all I(1) and their changes show low persistence, it is not surprising that AR(3) models in levels produce stationary, white noise residuals in all cases.

Theoretically, a second order autoregression in first differences should be equivalent to a third order autoregression in levels, but it is unclear that they perform equally well in practice. In fact, it is often the case working with interest rate data that level models produce better forecasts than models in differences [Abad and Novales (2002)]. A preliminary exploration of slope forecasts with both types of models suggested that models in levels produce in our data set forecasts at least as good as those from models in differences, so we proceeded with the analysis of level slopes, although paying special attention to the possible presence of unit roots in the residuals in each estimated model.

The right column for each currency in Table 4 display two sets of estimates: the upper panel presents estimations of a long-run relationship between each slope and the corresponding first two principal components,

$$S_t = \hat{\alpha}_0 + \hat{\alpha}_1 F_{1,t} + \hat{\alpha}_2 F_{2,t} + \hat{u}_t \qquad (1)$$

Being a static model, residuals display extensive autocorrelation, but they turn out to be stationary in all cases, so this equation is a cointegrating relationship between each slope and the first two common factors, which we associated in the previous section to the European and US term structure slopes. This relationship can be interpreted as a long-term equilibrium relationship between the three variables. The lower panel then contains estimates of the Error Correction Model (ECM) for each slope. In this equation, the differenced slope is projected on one lag of itself, the first lagged difference of the two principal components, and the lagged error-correction term $\hat{u}_t$, the residual from cointegrating relationship (1),

$$\Delta S_t = \hat{\beta}_1 \hat{u}_{t-1} + \hat{\beta}_2 \Delta S_{t-1} + \hat{\beta}_3 \Delta F_{1,t-1} + \hat{\beta}_4 \Delta F_{2,t-1} + \hat{\varepsilon}_t \qquad (2)$$

Even though this is a single-equation exercise, it should be interpreted as the corresponding equation from the vector ECM. R-squared and Standard Error of Estimate (SEE) in the right column for each currency in Table 4 correspond to the ECM equation. It is interesting to see that the SEE is essentially the same for both models.

Estimated coefficients are statistically significant in the cointegrating relationship (1). The first factor, which is negatively related to the slope in European markets, enters with a negative sign in the long-run relationships for those countries. The second factor, which is positively related to the US slope, enters with a positive sign in the cointegrating equation for that country. In ECM estimates, the error correction term is statistically significant except for the Swiss franc, and the associated coefficient has the right sign in all cases except the Deutsche mark. There is no significant evidence of residual autocorrelation, so the model seems to appropriate capture the dynamics in slope fluctuations.

## 4. Forecasting with slope models

As mentioned in the Introduction, our final goal is to analyze the extent to which a few common factors for a set of international term structure slopes are able to provide good predictions of future fluctuations in the set of eight slopes considered. By good forecasts we understand forecasts at least as good as those that could be obtained from univariate models for each slope.

If our search is successful, we could reduce the problem of forecasting a potentially large number of slopes to that of forecasting a reduced number of factors. This possibility is far from trivial, since principal components are designed to fit the data, but not to capture the autocorrelation in the data, and a better fit does not necessarily come together with an improvement in forecasting ability. Hence, it is important that we discuss next the extent to which the in-sample explanatory power of the principal components for term structure slopes across countries can actually be used to improve upon more complex forecasting models.

### *4.1. The forecasting exercise*

We computed *static* and *dynamic* forecasts over the last six months in our sample, July to December 1998, from both estimated models for each slope. Six months should be a long enough period so that results are not contaminated by any particular event. *Dynamic forecasts* are obtained with models estimated using data until the end of June1998. These are once-and-for all predictions over the 27 weeks of the second semester of 1998. We also computed *static forecasts*. For them, we estimated the previous models with data until the end of June 1998, and obtained forecasts for the first week in July 1998. We then repeatedly estimated the models adding one data point at a time, and producing forecasts for the next week each time. That way, we have a sequence of 27 one-step ahead forecasts for each slope. Except for the first forecasting week, dynamic forecasting always produces larger forecast errors than static forecasting, since the former exercise projects over the whole forecasting period using only data up to the last week of June 1998.

As a first option, we computed static and dynamic forecasts from autoregressive slope models. As discussed above, level slopes produce forecasts at least as good as differenced slopes, in spite of the general presence of a unit root. AR(3) models in Table 4 seem statistically reasonable and, as explained, produce stationary residuals, so they can be used for forecasting purposes. As an alternative, we obtained forecasts from estimated ECM in Table 4.

As we ran out of actual data for the lagged slope in dynamic forecasting from univariate models, we use previously obtained forecasts. The same applies to the use of ECM, in which forecasts for the principal components need to be obtained previously to computing slope forecasts. To do so, we need to start by computing forecasts for the principal components. In consistency with the autoregressive models for slopes themselves, forecasts for the principal components were also obtained from AR(3) models in levels for each factor. This is quite reasonable, since we have shown in Section 2.3 how principal components can be interpreted as an approximation to specific slopes.

In addition to being satisfactory from the point of view of estimation and forecasting, the choice of a common AR(3) univariate structure for each slope and principal component avoids the possibility that model searching could bias the results of our forecasting tests. A variety of trials run on this data set suggest that such a structure can be routinely used, without need of investing time resources in searching for the best dynamic specification each time the forecasting system is updated.

In each case, we computed four indicators of forecasting performance: mean absolute error, median absolute error, root mean squared error, and the U-statistic proposed by Theil. Percent Root Mean Square Errors are not advisable in this forecasting exercise, since the slope often becomes small in absolute value, to the point that even acceptable forecast errors might produce huge percent errors for a single period, dominating the value of any time aggregate forecasting performance indicator. Hence, we will use their versions in absolute terms. To compare the forecasting performance of both models, we calculated the ratio between comparable criteria for both models. Since we have four forecasting error criteria, and compute static as well as dynamic forecasts, we have 64 comparisons in total.

### 4.2. *Forecasting results*

Below each currency code in Table 5, we show the sample average absolute value of each slope over the forecasting horizon, the reference against which forecast statistics should be compared to evaluate forecast performance. Bold figures highlight cases in which the principal components model outperforms the univariate slope model in forecasting.

Statistics in Table 5 show that:

1) median one-step-ahead forecast errors from AR(3) models oscillate between 3% and 5% of the sample mean absolute slope for all currencies except the US dollar and British pound, for which they reach levels of 11,2% and 10,3%, respectively. Hence, autoregressive slope models in levels produce acceptable one-step ahead forecasts in most currencies,
2) quite strikingly, in spite of the need to predict the principal components, ECM with factors produce slope forecasts which are in some cases even better than those from univariate models. In 35 out of the 64 forecasting performance indicators, the use of principal components leads to better slope forecasts than univariate models. In fact, even the long-run cointegrating relationship between slope and principal components leads to better forecast than the autoregressive models in 20 of the 64 comparisons,
3) only two principal components are enough to produce this forecasting performance. In fact, even though there is additional explanatory power in further components (as shown in Table 3), adding them to the least-squares projections does not significantly improve forecasting performance. On the contrary, we could not proceed along with just one factor, the factor ECM then leading to better forecasts in just 25 of the 64 comparisons.
4) Even though the first two components reflect fluctuations in Deutsche mark and US dollar slopes, future slope values for the Japanese yen and British pound can be satisfactorily predicted by the proposed error correction factor model. Even though the third and fourth principal components can be approximately interpreted as slopes in these two currencies, the explanatory power of the first two components seems to be important enough so that the information in the next two factors can be safely ignored for forecasting purposes.

These results are quite striking because slope forecasts from ECM rest on forecasts from autoregressive models for the factors. That way, we add to the sampling error in estimating ECM, that from estimating autoregressive models for the factors. Yet, in spite of this double estimation process, factor models often predict better than autoregressive models.

The practical implication is that to forecast the set of eight international IRS market slopes, we only need to forecast the two common factors. Applying estimates from cointegrating relationships (1) to new slope data will provide us with the error correction residual. Adding univariate forecasts for the factors, we can easily compute forecasts for the vector of eight slopes from estimates of ECM (2).

## 5. Conclusions.

We have shown that the first two principal components can explain 92% of the fluctuation in a vector of term structure slopes from the IRS market in eight major currencies, the two factors being closely related to the slopes for the Deutsche mark and US dollar. We have also found strong evidence that reducing the dimensionality of the vector of slopes by using these two factors can be very fruitful for short and medium term slope forecasting in all these currencies. The reduction in dimensionality leads to a very simple forecasting scheme for term structure slopes, in the form of an Error Correction model between each slope and two common factors. Adding more factors to the model does not lead to a significant improvement in forecasting performance, while forecasts obtained using just one factor are not as good as those from two-factor Error Correction models.

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**Table 1. Unit root tests**

| | Slopes | | | | | | | | Principal Components | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF | First | Second | Third | Fourth |
| **Augmented Dikey-Fuller** | | | | | | | | | | | | |
| **Level** | -1.596 | -3.026 | -1.431 | -0.846 | -1.798 | -1.778 | -1.876 | -1.456 | -1.847 | -0.571 | -2.167 | -2.656 |
| **Differences** | -7.734 | -8.128 | -10.695 | -8.399 | -8.137 | -8.431 | -7.678 | -7.798 | -8.191 | -7.989 | -7.699 | -8.772 |
| **Phillips-Perron statistic** | | | | | | | | | | | | |
| **Level** | -1.544 | -3.399 | -1.853 | -0.806 | -1.484 | -2.012 | -1.870 | -1.439 | -1.843 | -0.540 | -1.803 | -2.851 |
| **Differences** | -20.246 | -17.846 | -28.492 | -18.195 | -19.620 | -20.719 | -19.327 | -20.216 | -18.168 | -23.056 | -19.243 | -22.056 |

Note: Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root statistics. Critical values for both statistics: -3.45 (1%), -2.87 (5%).

**Table 2. Contemporaneous correlation coefficients between slopes**

| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF |
|---|---|---|---|---|---|---|---|---|
| **DEM** | 1.000 | 0.799 | 0.833 | -0.703 | 0.344 | 0.687 | 0.914 | 0.939 |
| **JPY** | 0.134 | 1.000 | 0.495 | -0.393 | 0.422 | 0.279 | 0.701 | 0.664 |
| **ESP** | 0.117 | 0.071 | 1.000 | -0.813 | 0.059 | 0.814 | 0.769 | 0.887 |
| **USD** | 0.169 | 0.092 | 0.063 | 1.000 | 0.304 | -0.601 | -0.780 | -0.728 |
| **GBP** | 0.286 | 0.003 | -0.250 | 0.127 | 1.000 | 0.210 | 0.099 | 0.180 |
| **ITL** | 0.201 | -0.038 | -0.098 | 0.051 | 0.345 | 1.000 | 0.651 | 0.755 |
| **CHF** | 0.423 | -0.043 | -0.004 | 0.133 | 0.140 | 0.040 | 1.000 | 0.877 |
| **FRF** | 0.637 | -0.010 | 0.041 | 0.178 | 0.272 | 0.317 | 0.231 | 1.000 |

Note: The upper triangular matrix contains correlation coefficients between level slopes. The lower triangular matrix contains correlation coefficients between first differenced slopes.

**Table 3. Principal Components in IRS slopes**

| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF |
|---|---|---|---|---|---|---|---|---|
| **Panel a: Variance-covariance matrix of interest rates** | | | | | | | | |
| **Eigenvalues** | 4026.4 | 1012.5 | 238.7 | 110.3 | 44.5 | 25.2 | 13.0 | 5.8 |
| **First eigenvector** | -0.4922 | -0.4811 | -0.2456 | -0.2577 | -0.2632 | -0.1936 | -0.3819 | -0.3835 |
| **Second eigenvector** | -0.251 | 0.2312 | -0.3445 | 0.7604 | 0.2992 | -0.139 | -0.2032 | -0.191 |
| **Third eigenvector** | -0.1378 | 0.312 | -0.0322 | 0.2451 | -0.8708 | -0.1113 | 0.2113 | 0.0848 |
| **Fourth eigenvector** | 0.2668 | 0.2721 | -0.4833 | -0.3403 | 0.1062 | -0.6208 | 0.2762 | -0.1802 |
| **Panel b: R-squared coefficients on individual principal components** | | | | | | | | |
| **On first component** | 0.9644 | 0.674 | 0.6526 | 0.3554 | 0.2113 | 0.5032 | 0.7796 | 0.8604 |
| **On second component** | 0.6257 | 0.1911 | 0.8163 | 0.9349 | 0.0530 | 0.5141 | 0.7223 | 0.7216 |
| **On third component** | 0.1370 | 0.0977 | 0.0328 | 0.0329 | 0.9295 | 0.1287 | 0.0113 | 0.0515 |
| **On fourth component** | 0.0530 | 0.3246 | 0.0525 | 2.87E-03 | 0.0209 | 0.2105 | 0.0801 | 9.31E-05 |
| **Panel c: R-squared coefficients on subsets principal components** | | | | | | | | |
| **On first component** | 0.9644 | 0.674 | 0.6526 | 0.3554 | 0.2113 | 0.5032 | 0.7796 | 0.8604 |
| **On first two comp.** | 0.9823 | 0.7164 | 0.8724 | 0.9511 | 0.8339 | 0.5952 | 0.8803 | 0.9343 |
| **On first three comp.** | 0.9824 | 0.8257 | 0.9000 | 0.9537 | 0.9957 | 0.7242 | 0.9096 | 0.9374 |
| **On first four comp.** | 0.9915 | 0.8932 | 0.9492 | 0.9904 | 0.9984 | 0.9008 | 0.931 | 0.9574 |

**Table 4. Estimated Models**

| | DEM | | JPY | | ESP | | USD | | GBP | | ITL | | CHF | | FRF | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Own lag** | | | | | | | | | | | | | | | | |
| **i=1** | 0.967 | | 1.059 | | 0.675 | | 1.076 | | 1.002 | | 0.942 | | 1.009 | | 0.970 | |
| | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | |
| **i=2** | 0.049 | | -0.153 | | 0.362 | | 0.034 | | 0.044 | | 0.007 | | 0.040 | | 0.062 | |
| | (0.071) | | (0.074) | | (0.058) | | (0.074) | | (0.072) | | (0.070) | | (0.073) | | (0.071) | |
| **I=3** | -0.022 | | 0.070 | | -0.059 | | -0.114 | | -0.058 | | 0.028 | | -0.057 | | -0.040 | |
| | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | | (0.051) | |
| **Constant** | | -0.418 | | 0.509 | | 0.232 | | 0.621 | | -1.701 | | 0.159 | | 0.279 | | 0.141 |
| | | (0.024) | | (0.045) | | (0.049) | | (0.028) | | (0.058) | | (0.059) | | (0.047) | | (0.034) |
| **First component** | | -0.604 | | -0.344 | | -0.183 | | -0.091 | | -0.720 | | -0.151 | | -0.307 | | -0.346 |
| | | (0.007) | | (0.013) | | (0.014) | | (0.008) | | (0.017) | | (0.017) | | (0.014) | | (0.010) |
| **Second component** | | -0.150 | | 0.108 | | -0.401 | | 0.610 | | 0.711 | | -0.178 | | -0.271 | | -0.225 |
| | | (0.008) | | (0.014) | | (0.015) | | (0.009) | | (0.019) | | (0.019) | | (0.015) | | (0.011) |
| **Error correction (t-1)** | | 0.066 | | -0.031 | | -0.152 | | -0.058 | | -0.058 | | -0.054 | | -0.008 | | -0.074 |
| | | (0.031) | | (0.013) | | (0.033) | | (0.022) | | (0.018) | | (0.017) | | (0.014) | | (0.022) |
| **Slope (t-1)** | | -0.117 | | 0.115 | | -0.096 | | 0.118 | | -0.067 | | -0.093 | | -0.032 | | -0.131 |
| | | (0.085) | | (0.056) | | (0.071) | | (0.068) | | (0.068) | | (0.054) | | (0.058) | | (0.068) |
| **First comp (t-1)** | | -0.037 | | 0.025 | | 0.030 | | 0.026 | | -0.212 | | -0.153 | | -0.064 | | -0.085 |
| | | (0.053) | | (0.030) | | (0.078) | | (0.034) | | (0.063) | | (0.053) | | (0.035) | | (0.048) |
| **Second comp (t-1)** | | -0.071 | | -0.013 | | 0.459 | | 0.001 | | -0.064 | | -0.170 | | -0.036 | | -0.133 |
| | | (0.042) | | (0.036) | | (0.127) | | (0.045) | | (0.077) | | (0.060) | | (0.039) | | (0.044) |
| **R-squared** | 0.994 | 0.008 | 0.980 | 0.011 | 0.946 | 0.177 | 0.991 | 0.019 | 0.978 | 0.072 | 0.950 | 0.077 | 0.991 | 0.003 | 0.987 | 0.066 |
| **SEE** | 0.091 | 0.090 | 0.075 | 0.076 | 0.207 | 0.199 | 0.077 | 0.077 | 0.139 | 0.134 | 0.137 | 0.131 | 0.085 | 0.084 | 0.099 | 0.096 |
| **Q(3)** | 6.16 | 3.02 | 0.96 | 1.23 | 11.96 | 7.48 | 0.96 | 5.88 | 8.94 | 0.81 | 11.21 | 1.17 | 11.06 | 5.30 | 2.69 | 1.40 |
| | [0.29] | [0.39] | [0.97] | [0.75] | [0.04] | [0.06] | [0.97] | [0.12] | [0.11] | [0.85] | [0.05] | [0.76] | [0.05] | [0.15] | [0.75] | [0.71] |
| **Q(10)** | 8.50 | 7.74 | 2.71 | 4.86 | 13.76 | 14.92 | 8.19 | 14.89 | 16.88 | 20.22 | 27.61 | 24.30 | 16.91 | 17.17 | 7.12 | 8.60 |
| | [0.58] | [0.65] | [0.99] | [0.90] | [0.18] | [0.14] | [0.61] | [0.14] | [0.08] | [0.03] | [0.00] | [0.01] | [0.08] | [0.07] | [0.71] | [0.57] |
| **LM(1)** | 2.27 | 3.99 | 0.44 | 0.30 | 0.89 | 4.79 | 0.38 | 4.23 | 1.21 | 0.33 | 0.23 | 2.49 | 4.56 | 4.83 | 0.34 | 0.00 |
| | [0.13] | [0.05] | [0.51] | [0.59] | [0.35] | [0.03] | [0.54] | [0.04] | [0.27] | [0.57] | [0.63] | [0.11] | [0.03] | [0.03] | [0.56] | [1.00] |
| **LM(4)** | 3.89 | 7.49 | 4.40 | 1.76 | 12.89 | 9.12 | 1.74 | 6.55 | 7.68 | 7.25 | 3.82 | 6.64 | 11.02 | 19.55 | 0.80 | 1.76 |
| | [0.42] | [0.11] | [0.36] | [0.78] | [0.01] | [0.06] | [0.78] | [0.16] | [0.10] | [0.12] | [0.43] | [0.16] | [0.03] | [0.00] | [0.94] | [0.78] |
| **ADF** | -7.82 | -7.98 | -8.10 | -8.03 | -9.79 | -8.41 | -8.92 | -8.33 | -8.17 | -8.39 | -7.96 | -8.12 | -7.76 | -7.74 | -7.87 | -7.83 |
| **PP** | -19.70 | -19.52 | -19.47 | -19.38 | -19.67 | -19.01 | -19.73 | -19.99 | -19.76 | -19.89 | -19.71 | -20.09 | -19.67 | -19.41 | -19.68 | -19.65 |

Note: The left column in each currency contains autorregressive models that were estimated using levels of interest rates. A (generally non-significant) constant was included in all models. The right column in each currency contains estimates of a possible cointegration relationship between each slope and the corresponding first two principal components, as well as estimates of the implied error correction model, when it applies. Standard deviations in parentheses. Statistics shown for each regression include: adjusted R-squared, standard error of estimate (SEE), Ljung-Box autocorrelation statistics of orders 3 and 10 (Q(3), Q(10)), Breusch-Godfrey autocorrelation statistics of orders 1 and 4 (LM(1), LM(4)), and Augmented Dickey-Fuller and Phillips-Perron statistics to test for unit root in the residuals. *p*-values are included in square brackets.

**Table 5. Forecasting performance indicators**

| | DEM | | JPY | | ESP | | USD | | GBP | | ITL | | CHF | | FRF | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Sample absolute mean values: 7/1/1998 - 12/30/1998** | | | | | | | | | | | | | | | |
| | 1.0034 | | 1.1044 | | 0.9304 | | 0.4476 | | 0.6012 | | 0.8745 | | 1.4941 | | 1.0013 | |
| | **Forecasting model** | | | | | | | | | | | | | | | |
| | **AR model** | **Factor models** | **AR model** | **Factor models** | **AR model** | **Factor models** | **AR model** | **Factor models** | **AR model** | **Factor models** | **AR model** | **Factor models** | **AR model** | **Factor models** | **AR model** | **Factor models** |
| | **Static forecasts** | | | | | | | | | | | | | | | |
| **Mean** | 0.0507 | **0.0376** | 0.0678 | **0.0658** | 0.0554 | 0.0593 | 0.0631 | **0.0617** | 0.0856 | **0.0823** | 0.0511 | 0.0553 | 0.0669 | 0.0700 | 0.0515 | 0.0525 |
| **Median** | 0.0383 | **0.0376** | 0.0326 | 0.0450 | 0.0389 | **0.0323** | 0.0502 | **0.0384** | 0.0619 | **0.0542** | 0.0431 | 0.0433 | 0.0634 | 0.0685 | 0.0416 | 0.0463 |
| **RMSE** | 0.0640 | **0.0376** | 0.0989 | **0.0973** | 0.0745 | 0.0812 | 0.0942 | 0.0960 | 0.1202 | **0.1142** | 0.0670 | 0.0676 | 0.0856 | 0.0868 | 0.0647 | **0.0627** |
| **U-Theil** | 0.9892 | **0.9069** | 0.9720 | 0.9731 | 0.9825 | **0.9804** | 0.9287 | **0.9235** | 0.9611 | 0.9650 | 0.9854 | 0.9858 | 0.9867 | 0.9868 | 0.9890 | 0.9904 |
| | **Dynamic forecasts** | | | | | | | | | | | | | | | |
| **Mean** | 0.0611 | 0.2756 | 0.2953 | **0.1853** | 0.2045 | **0.0923** | 0.2750 | **0.2721** | 0.5448 | **0.4716** | 0.2730 | **0.2549** | 0.1525 | 0.1698 | 0.0880 | **0.0544** |
| **Median** | 0.0527 | 0.2284 | 0.2252 | **0.1090** | 0.1956 | **0.0906** | 0.2796 | **0.2677** | 0.5637 | **0.4852** | 0.2316 | **0.2187** | 0.1088 | 0.1352 | 0.0726 | **0.0509** |
| **RMSE** | 0.0760 | 0.3403 | 0.3672 | **0.2393** | 0.2473 | **0.1096** | 0.3364 | **0.3343** | 0.6497 | **0.5664** | 0.3224 | **0.3023** | 0.1957 | 0.2165 | 0.1051 | **0.0660** |
| **U-Theil** | 0.9857 | **0.7550** | 0.7816 | 0.8891 | 0.8518 | 0.9687 | 0.5160 | 0.5176 | 0.7908 | 0.8002 | 0.7486 | 0.7742 | 0.9301 | **0.9162** | 0.9724 | 0.9882 |

Note: Forecasts obtained from estimated models in Table 4. Mean and Median are the mean and median absolute values of the forecasting errors. RMSE denotes the Root Mean Square Error, while U-Theil denotes Theil´s statistic. Boldface figures denote cases when factors models forecast better than univariate models.

**Figure 1.** Time behavior of term structure slopes in IRS markets.

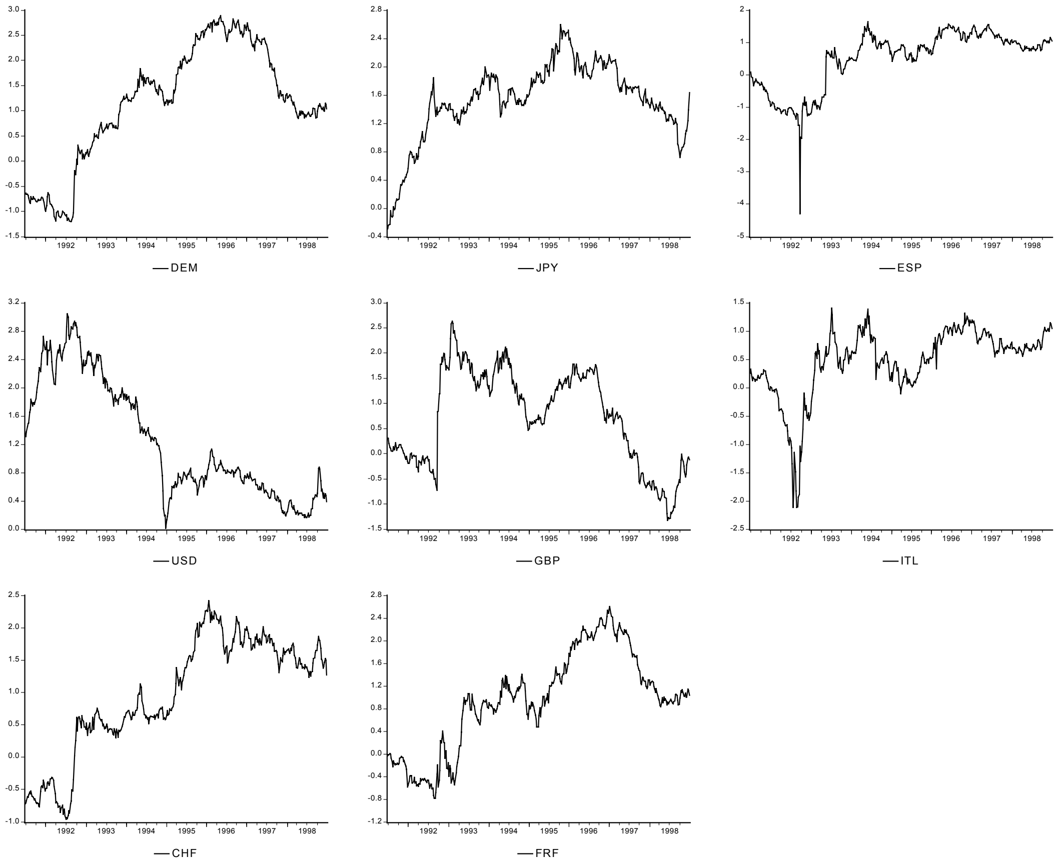
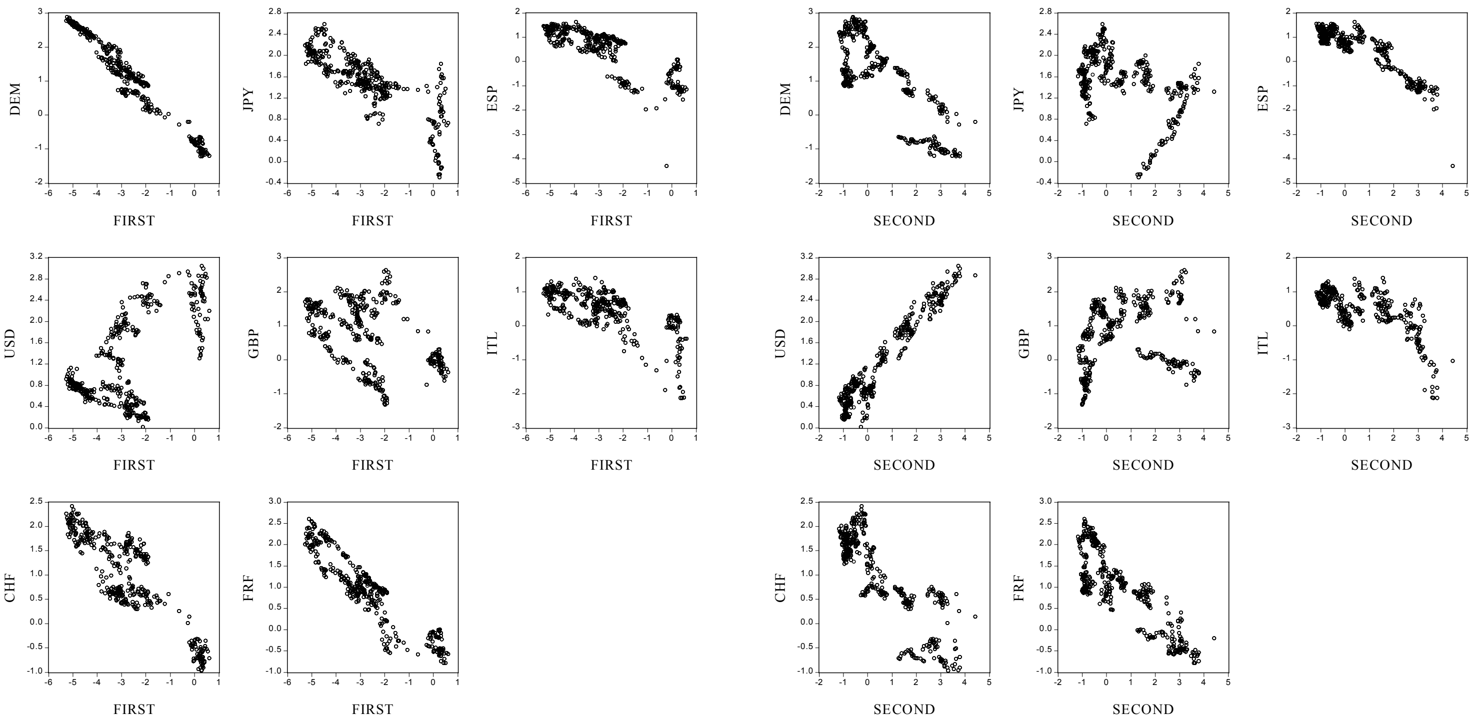
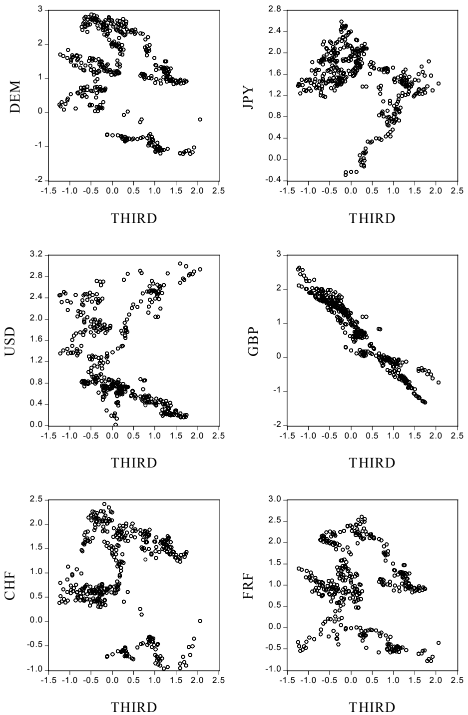


**Figure 2.** Projections on principal components

**a)** on first component **b)** on second component



**c)** on third component




**d)** on fourth component

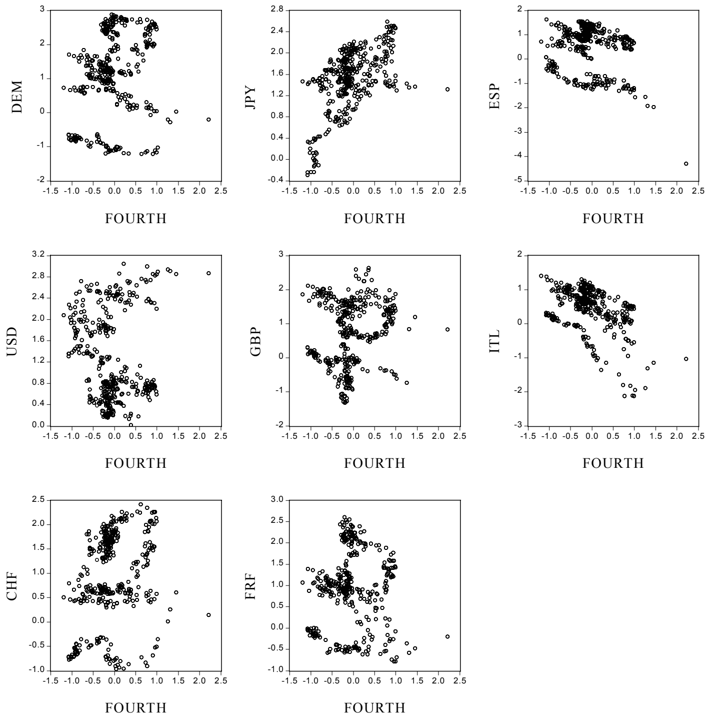


**Figure 3.** Changes induced on each slope by a change in a given principal component