

# Taxing or subsidizing factors' rents in a simple endogenous growth model with public capital\*

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This version: July 2003

## Abstract

This paper tackles the fundamental issue in public finance of whether taxing or subsidizing factor rents. In a one sector endogenous growth model with private and public capital, similar to that in Barro (1990), we find that raising taxes on factors' income as part of an optimal fiscal policy is a more pervasive result than it seems. The interaction of technological and fiscal externalities is central for this result. For instance, high enough levels of wasteful expenditures to output ratio could make positive income taxes enhance welfare. This ratio would need to be smaller, the lower the spillover externality and/or the larger the elasticities of private and public capital in the private production function.

**Keywords:** Endogenous growth, factors' rents subsidy, distorting taxes, public capital.

**JEL Classification:** E0, E6, O4

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\*Financial support from the Spanish Ministry of Education (through DGICYT grant no. PB98-0831) is gratefully acknowledged.

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# 1 Introduction

Clarifying whether factor rents should be taxed or subsidized continues to be a central issue when exploring the influence of the fiscal system on welfare. On the broader issue of taxation efficiency, the framework introduced in the seminal paper by Ramsey (1929) with infinitely-lived identical households has often been adopted, under two prevailing approximations. One line of research has examined the incidence of taxation on growth and welfare in economies where public expenditures do not influence consumers' utility or affect the production technology.<sup>1</sup> Its main purpose is to assess the optimal way to finance a given level of public expenditures by solving a second best problem. In an alternative approach, Barro (1990) considers productive public expenditures, combining endogenous growth with the public finance literature. His work was an important breakpoint regarding the study of the influence of public investment on growth and welfare.<sup>2</sup> Somewhat surprisingly, however, not much work has been done regarding the optimal simultaneous choice of public investment and a financing rule.<sup>3</sup>

We explicitly consider the simultaneous choice of public investment and the appropriate financing system in an endogenous growth framework where public capital have a positive effect on the private production process. In addition to issues that will be described below, we specifically discuss whether income from productive factors should be taxed or subsidized. We find that raising taxes on factors' income as part of an optimal fiscal policy is in this framework a more pervasive result than it seems.

In a Ramsey-type setting with no externalities, taxing production factor income will generally undermine growth and welfare. In such a setup, Judd (1985), Chamley (1986) and Lucas (1990) emphasized the negative incidence of capital income taxes on welfare. Judd (1999) also argued for a zero tax rate on physical and human capital income in the long-run. In the presence

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<sup>1</sup>Among many others, see Chamley (1986), Judd (1999), Lucas (1990), Rebelo (1990) and Jones et al. (1993,1997).

<sup>2</sup>Empirical work by Ratner (1983), Aschauer (1989) and Munnell (1990), among many others, emphasizes the positive link between public expenses and the private production process. From a theoretical point of view, Futagami et al. (1993), Glomm and Ravikumar (1994) and Turnovsky (1996, 2000) are variations on Barro (1990). In all them, public revenues come from proportional income taxes and the government chooses the welfare-maximizing ratio of productive public expenditures-to-output.

<sup>3</sup>Exceptions are, among others, Corsetti and Roubini (1996) in a two-sector endogenous growth setting and Fisher and Turnovsky (1998) in a neoclassical growth model.

of utility or technology externalities things could change, and a government trying to maximize welfare may want to subsidize capital income if there is some other less distorting instrument available when the competitive mechanism leads to under accumulation of physical capital. On the other hand, taxing capital rents might be growth and welfare-enhancing when there exists some externality in the model that induces an over-accumulation of physical capital. Often, externalities of this kind have been introduced in the production function [Fisher and Turnovsky (1998) and Corsetti and Roubini (1996)] or as a borrowing constraint in a stochastic environment [Chamley (2001)].

However, these results rely upon the type of taxes available to finance public expenditures. Indeed, Jones et al. (1997) and Milesi-Ferretti and Roubini (1998) extend the zero tax rate result to labor and consumption taxes in models with human capital in the absence of externalities. In this setup, Jones et al. (1997) points out that certain *public revenue constraints, based on informational and political constraints which are not explicitly modeled*, could imply that taxing productive factors in the long-run might be optimal in a second-best sense.

Our setting links the Ramsey and Barro frameworks by looking at optimal public investment and taxation in an endogenous growth economy. In this framework, we consider some *structural externalities*: (i) public capital enters as an input in the available production technology, (ii) there exists a spillover effect of aggregate private capital in the production process [as in Romer (1986)]; (iii) there are three types of public expenses: public investment, which affects production through the stock of public capital available every period, public consumption, that directly enhances welfare and some kind of wasteful public expenditure which does not influence production or welfare; we also consider a *fiscal externality*: (iv) wasteful public expenditures are a constant share of total public expenditures and exogenously given. The need to finance every period the constant proportion of wasteful public expenditures can be seen as a constraint of the kind suggested by Jones et al. (1997). We allow for lump-sum as well as proportional income taxes, but exclude the possibility of issuing any debt.<sup>4</sup>

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<sup>4</sup>Certain activities of the public sector can directly affect the private production process while others influence welfare, as in our model economy: public expenditures in infrastructures, as well as those securing property rights, investment by public firms, expenditures on education, etc., can be all considered as part of a broad concept of public investment, since they enhance one way or another the private production process. The provision of public goods, such as parks, sport centers, libraries, recreational areas, hospitals, muse-

In this framework, we characterize the welfare-maximizing levels of public investment and consumption as fractions of output, as well as the optimal way to split the required public resources between lump-sum and income taxes. To address the issue of whether subsidizing factor rents is desirable, we allow for negative lump-sum or income tax rates. We show that subsidizing factor rents is not by any means a definite result, since it depends in a non-trivial manner on the interaction between technology and fiscal externalities. In fact, we will see that there are many instances in which taxing factors' income is optimal. We pay special attention to the role of technological externalities and the influence of the constant wasteful expenditures ratio in determining whether subsidizing or taxing factor rents is welfare enhancing as well as on determining the optimal tax/subsidy rate. As we will see, the interaction of technological and fiscal externalities is going to be central for taxing factors' income to be a second best optimal policy.

Since private agents do not internalize the spillover effect, they will tend to underaccumulate physical capital, and a subsidy might move them in the right direction. The presence of public capital in aggregate technology constitutes a second externality. The fact that the typical consumer ignores the positive effect of public capital on the productivity of private capital when making decisions will also lead her to invest below the socially optimal level. In principle, these two technological externalities would suggest subsidizing private investment. In this framework, it would be sound fiscal policy to raise lump-sum taxes to finance subsidies on investment.

But there are also an important externality producing distortions on the revenue side: the constraint to pay for a given amount of wasteful expenditures produces a distortion in the form of a lower bound for required government resources. The main result of the paper could be interpreted as follows: if the government is efficient, in the sense that wasteful public expenditures

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ums, the environmental budget, among many others, may have a positive and significant effect on welfare. On the other hand, bureaucratic and administrative costs or interest payments do not enhance welfare or production, so they can be considered as some kind of wasteful government expenditure. A first-best resource allocation would reduce the level of wasteful spending down to zero immediately, keeping it at that level forever. However, wasteful public expenditures are far from zero in actual economies, being significant not only in absolute values, but also as a percentage of total expenditures or total output. Such a fraction could be seen as a measure of inefficiency in the public sector. With this in mind, the main goal of the paper is to characterize a second-best allocation, conditioned by the existence of an exogenous path of wasteful public expenditures, as described in our fiscal externality.

are *relatively* low, subsidizing factors' income is optimal when aiming to maximize welfare; on the other hand, it would be optimal to raise resources from factors income. This fiscal externality distorts the consumption-saving choice. So long as more resources are needed to finance wasteful expenditures, less resources are left to consume and save. Moreover, whenever raising revenues from lump-sum taxes, private consumption, an argument in the utility function, suffers from most of the crowding-out effect. Thus, above certain levels of wasteful expenditures, collecting additional resources from distortionary taxation turns optimal in order to reduce this excess of crowding-out.

The absence of the spillover externality eliminates the main reason to subsidize private investment. Hence, subsidies are then never optimal, no matter how important the externality of public capital may be. This result shows, first, that it is not necessary for optimality of factor income taxes that the government be constrained by a high level of wasteful public expenditures. Secondly, that the presence of a public capital externality is not, by itself, enough to imply the optimality of income taxes.

The paper is organized as follows. In section 2 the basic framework is described. In section 3 the competitive equilibrium and the long-run equilibrium path are characterized. In sections 4 the growth- and welfare-maximizing public investment policies under income and lump-sum taxes are characterized. Section 5 drives the interpretation of the results in terms of the interaction between technological and fiscal externalities. Finally, section 6 ends with main conclusions and extensions.

## 2 The economy

The model draws on work by Barro (1990), Futagami et al. (1993), Glomm and Ravikumar (1994) and Marrero and Novales (2003). The economy consists of a continuum of identical firms, a fiscal authority and a representative household.

### 2.1 Firms

Firms are identical, they rent the same amount of physical capital  $k_t$  and labor  $l_t$  from households, and produce  $y_t$  units of the consumption commodity. The capital stock used on the aggregate by all firms,  $K_t$ , is taken as a

proxy for the index of knowledge available to each single firm [Romer (1986)]. Finally, public capital,  $K_t^g$ , is exogenous to the private production process and affects all individual firms in the same way. Except for these two *technological externalities*,  $y_t$  is produced according to a standard *Cobb-Douglas* function,

$$y_t = f(l_t, k_t, K_t, K_t^g) = Al_t^{1-\alpha} k_t^\alpha K_t^\phi (K_t^g)^\theta, \quad \alpha, \theta \in (0, 1), \phi \in [0, 1], \quad (1)$$

where  $\alpha$  is the share of private capital in output,  $\theta$  and  $\phi$  are the constant elasticities of output with respect to public capital and the knowledge index, and  $A$  is a technological scale.

Since all firms are identical, we can aggregate on (1) to obtain total output,  $Y_t$ ,

$$Y_t = AL_t^{1-\alpha} K_t^{(\alpha+\phi+\theta)} \left(\frac{K_t^g}{K_t}\right)^\theta, \quad (2)$$

where  $L_t$ ,  $K_t$  are the total amounts of labor and physical capital used by all firms in the economy. During period  $t$ , each firm pays the competitive-determined wage  $w_t$  on the labor it hires and the rate  $r_t$  on the capital it rents and thus, the profit maximizing problem of the firm turns out to be static, leading to the usual marginal product conditions,

$$r_t = f'_{k_t} = \alpha Al_t^{1-\alpha} k_t^{\alpha-1} K_t^\phi (K_t^g)^\theta = \alpha \frac{y_t}{k_t} = \alpha \frac{Y_t}{K_t}, \quad (3)$$

$$w_t = f'_{l_t} = (1-\alpha) Al_t^{-\alpha} k_t^\alpha K_t^\phi (K_t^g)^\theta = (1-\alpha) \frac{y_t}{l_t} = (1-\alpha) \frac{Y_t}{L_t}, \quad (4)$$

where we have used the fact that each firm treats its own contribution to the aggregate capital stock as given. From these optimally conditions we have the standard result on income distribution,

$$Y_t = r_t K_t + w_t L_t. \quad (5)$$

## 2.2 Public sector and fiscal policy

At any period  $t$ , the public sector collects taxes,  $T_t$ , to finance its total expenditures,  $G_t$ , with  $G_t \leq Y_t$  for all period  $t$ . Expenditures are made up by public investment,  $I_t^g$ , public services,  $C_t^g$ , and some kind of wasteful public expenditure,  $C_t^w$ , the latter not appearing as an argument on either consumers' utility or the aggregate production function. The ratio of wasteful

public expenditures to output,  $C_t^w/Y_t$ , is exogenously given. Public capital accumulates according to,

$$K_{t+1}^g = I_t^g + (1 - \delta^g)K_t^g, \quad (6)$$

with  $\delta^g \in (0, 1)$  being the depreciation rate. We assume that public capital is bounded above by  $\eta^t K_0$ , with  $\eta \geq 0$ , for all  $t$ .

All revenues are assumed to be raised through taxes. The government can collect resources either by taxing proportionally total factors' rents at a rate  $\tau_t$ , or through lump-sum transfers from the private to the public sector,  $X_t$ . For analytical convenience, we will focus on  $v_t$ , the transfers to output ratio  $X_t = v_t Y_t$ . Debt issuing is not allowed in the economy, balances the government's budget every period,

$$\begin{aligned} G_t &= I_t^g + C_t^g + C_t^w = \tau_t Y_t + v_t Y_t, & (7) \\ \varkappa_t &= G_t/Y_t = \tau_t + v_t, & (8) \end{aligned}$$

with  $\varkappa_t \in [0, 1]$  being the total size of the public sector at period  $t$ . We denote by  $\varkappa_{i,t}$ ,  $\varkappa_{c,t}$  and  $\varkappa_{w,t}$  the ratio of  $I_t^g$ ,  $C_t^g$  and  $C_t^w$  to output, respectively, with  $\varkappa_{j,t} \geq 0$ ,  $j = i, c, w$ , and  $\sum_j \varkappa_{j,t} = \varkappa_t$ .

The government decides on: *i*) investment on public capital,  $I_t^g$ , *ii*) public consumption,  $C_t^g$ , and *iii*) the way to finance expenditures, i.e.,  $\tau_t$  and  $v_t$ . We assume these decisions are taken simultaneously. One of  $\tau_t$  and  $v_t$  could be negative, but they must be positive on the aggregate, since  $\varkappa_t \geq 0$ . When  $\tau_t < 0$ , the government is said to be subsidizing factors income while, if  $v_t < 0$ , the government is transferring a fraction  $v_t$  of output to the consumers.

A fiscal policy  $\Pi$  is defined as a sequence of  $\{\pi_t\}_{t=0}^\infty$ , with  $\pi_t = \{\varkappa_{i,t}, \varkappa_{g,t}, \varkappa_{w,t}, \tau_t, v_t\}$ . Accordingly to (7) and (8), a *feasible fiscal policy* is a policy  $\Pi \in \Lambda$ , with

$$\Lambda \equiv \left\{ \pi_t = \{\varkappa_{i,t}, \varkappa_{g,t}, \varkappa_{w,t}, \tau_t, v_t\}_{t=0}^\infty : \varkappa_{j,t} \geq 0, j = \{i, c, w\}, \sum_j \varkappa_{j,t} = \tau_t + v_t \in [0, 1] \right\},$$

for all period  $t$ . A *stationary, feasible fiscal policy* is a vector of constants,  $\pi = (\varkappa_i, \varkappa_g, \varkappa_w, v, \tau)$  with  $\varkappa_j \geq 0$ ,  $j = \{i, g, w\}$ , and  $\varkappa_i + \varkappa_g + \varkappa_w = v + \tau \leq 1$ , for all period  $t$ . The set of feasible and stationary policies is denoted by  $\bar{\Lambda}$ .

## 2.3 Households

We assume zero population growth and population size is normalized to one. The representative consumer is the owner of physical capital, and allocates

her resources between consumption,  $C_t$ , and investment in physical capital,  $I_t$ . Private physical capital accumulates over time according to,

$$K_{t+1} = I_t + (1 - \delta)K_t, \quad (9)$$

where  $K_{t+1}$  denotes the stock of physical capital at the end of time  $t$ , with  $K_0 > 0$  and  $\delta$  denotes the depreciation rate for physical capital. Decisions are made every period to maximize the discounted aggregate value of the time separable, logarithmic utility function

$$\text{Max}_{\{C_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [\ln C_t + \varphi \ln C_t^g], \quad (10)$$

subject to the resource constraint,

$$C_t + K_{t+1} - (1 - \delta)K_t + X_t \leq (1 - \tau_t)(w_t L_t + r_t K_t), \quad (11)$$

where  $\beta$  is the discount factor, between zero and one, and  $\varphi \geq 0$  determines the relative appreciation for public and private consumption.

The consumer takes fiscal policy and factor prices as given when deciding how to split her current income between consumption and savings. The optimally condition for the consumer is,

$$\frac{C_{t+1}}{C_t} = \beta [(1 - \delta) + (1 - \tau_{t+1})r_{t+1}], \quad (12)$$

together with the budget constraint (11), the transversality condition

$$\lim_{t \rightarrow \infty} \beta^t K_{t+1} \frac{\partial U}{\partial C_t} \equiv \lim_{t \rightarrow \infty} \beta^t K_{t+1} \frac{1}{C_t} = 0, \quad (13)$$

and  $K_{t+1} \geq 0$ ,  $C_t \geq 0$ , for any period  $t$ .

The optimal time allocation of private consumption does not depend upon public consumption because of the separability of the utility function, make our results somewhat restrictive. In addition to the convenience of starting with a simple model to better grasp the intuitions behind our results, separability is needed for the competitive equilibrium being analytically tractable. Modelling nontrivial interactions between private and public consumption requires numerical solution techniques, and it is left for future research.

### 3 Equilibrium conditions and the balanced growth path

#### 3.1 The competitive equilibrium

For a feasible fiscal policy  $\Pi \in \Lambda$ , a  $\Pi$ -competitive equilibrium ( $\Pi$ -CE) is defined as follows:

**Definition 1** *Given initial conditions  $K_0, K_0^g > 0$ , a  $\Pi$ -CE for the overall economy is a vector of time series  $\{C_t, C_t^g, C_t^w, K_{t+1}, K_{t+1}^g, I_t, I_t^g, L_t, Y_t, r_t, w_t\}_{t=0}^\infty$  and a fiscal policy  $\Pi \in \Lambda$ , such that, given  $\{r_t, w_t\}_{t=0}^\infty$ : (i)  $\{L_t, K_{t+1}\}_{t=0}^\infty$  solve the profit maximizing problem of firms [i.e., (3)-(4) hold], (ii)  $\{C_t, K_{t+1}\}_{t=0}^\infty$  maximize the utility of households [i.e., (13),  $C_t, K_{t+1} \geq 0$  and (12) and (11)] hold, (iii) the technology constraints (2), (9), (6) hold and (iv) markets clear every period:*

$$L_t = 1, \quad (14)$$

$$Y_t = C_t + C_t^g + C_t^w + I_t + I_t^g. \quad (15)$$

In fact, marginal utility at the origin equal to infinity guarantees that strict inequalities will hold for  $K_{t+1} > 0, C_t > 0$  at all time periods, which we use in what follows.

#### 3.2 Balanced growth path

A balanced growth path, *bgp*, is a competitive equilibrium trajectory along which aggregate variables grow at a zero or positive constant rate. Under  $\alpha + \theta + \phi = 1$  and given a stationary and feasible fiscal policy  $\Pi \in \bar{\Lambda}$ , it is easy to show from equilibrium conditions that  $Y_t, C_t, K_t, K_t^g, C_t^g, C_t^w$  and  $X_t$  all grow at the same constant rate, denoted  $\gamma$  hereinafter, while bounded variables, such as  $\tau_t, r_t$  and  $v_t$ , remain constant. As a consequence, the ratios  $c_t = C_t/K_t, k_t^g = K_t^g/K_t, y_t = Y_t/K_t, c_t^g = C_t^g/K_t, c_t^w = C_t^w/K_t$  and  $x_t = X_t/K_t$  also remain constant. This condition is similar to those in Barro (1990), Rebelo (1991) and Jones and Manuelli (1997), among others, who show that cumulative inputs must present constant returns to scale in the private production process (i.e.,  $\alpha + \theta + \phi = 1$ ) and  $r_t$  must be constant and high enough for level variables to display a positive and steady growth rate in equilibrium. We assume in what follows that  $\alpha + \theta + \phi = 1$ .

Conditions characterizing a  $\Pi$ - $CE$  can be particularized for a  $bgp$ -equilibrium to,<sup>5</sup>

$$\gamma + \delta = (1 - \varkappa_i - \varkappa_g - \varkappa_w)y - c, \quad (16)$$

$$1 + \gamma = \beta[(1 - \delta) + (1 - \tau)r], \quad (17)$$

$$\gamma + \delta^g = \varkappa_i y (k^g)^{-1}, \quad (18)$$

$$r = \alpha y, \quad (19)$$

$$y = A(k^g)^\theta, \quad (20)$$

$$\varkappa_w = c^w / y, \quad (21)$$

$$\tau = \varkappa_i + \varkappa_g + \varkappa_w - v. \quad (22)$$

a system of equations in  $\gamma, c, k^g, y, r, c^g, c^w$ . Condition (16) comes from the global constraint of resources, (17) is the condition on the intertemporal substitution of consumption, (18) is the public investment rule, (19) is the gross return on capital accumulation, (20) is the production function, (21) is the rule for wasteful expenditures, and (22) is the government budget constraint. The solution to this system must be obtained numerically.

In parallel to Definition 1, a  $\Pi$ - $bgp$  is defined:

**Definition 2** *Given a stationary fiscal policy  $\Pi \in \bar{\Lambda}$ , a  $\Pi$ - $bgp$  is a vector  $(\gamma, c, k^g, y, r, c^g, c^w)$  satisfying (16)-(22) with  $c > 0$  and  $k^g > 0$ .*

The common growth rate property allows us to write the transversality condition (13) along a  $bgp$  as  $\lim_{t \rightarrow \infty} \beta^t (1 + \gamma) \frac{1}{c} = 0$ , which will clearly be satisfied by any  $\Pi$ - $bgp$ .

### 3.3 The full depreciation equilibrium

We assume in what follows that both types of capital fully depreciate every period. This assumption enables us to obtain an analytical characterization of  $\Pi$ - $CE$  and  $\Pi$ - $bgp$  allocations, which is one of the purposes of this paper.

#### 3.3.1 Characterizing a $\Pi$ - $CE$

The simplicity of the model allows for the  $\Pi$ - $CE$  to be analytically characterized. Regarding the decision rules for  $C_t$  and  $K_{t+1}$ , we make a linear guess

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<sup>5</sup>We rid off the subindex  $t$  when considering values along the  $bgp$ .

for their dependence on output:  $C_t = aY_t$  and  $K_{t+1} = bY_t$ . Taking these linear rules to: (12), (11), (8), (3), (4), (2) together with  $L_t = 1$ , we get,

$$C_t = [(1 - \varkappa_i - \varkappa_g - \varkappa_w) - \beta\alpha(1 - \tau)]A(K_t^g)^\theta (K_t)^{1-\theta}, \quad (23)$$

$$K_{t+1} = \alpha\beta(1 - \tau)A(K_t^g)^\theta (K_t)^{1-\theta}, \quad (24)$$

and combining (6), (2),  $L_t = 1$  and the fact that  $I_t^g = \varkappa_i Y_t$ , we get

$$K_{t+1}^g = \varkappa_i A(K_t^g)^\theta K_t^{1-\theta}, \quad (25)$$

a set of three equations characterizing the propagation mechanism for  $C_t$ ,  $K_{t+1}$  and  $K_{t+1}^g$  along a  $\pi$ - $CE$ .

Since  $C_t$ ,  $K_{t+1}$  and  $K_{t+1}^g$  need to be strictly positive in a  $\Pi$ - $CE$ , the following conditions must hold: A1) from (23),  $\varkappa_i + \varkappa_g + \varkappa_w + \beta\alpha(1 - \tau) < 1$ , A2) from (24),  $\tau < 1$ ; A3) from (25),  $\varkappa_i > 0$ . A1)-A3) are called *feasibility conditions*. A1) means that there must be some resources left to consume, after using available resources in the form of public expenditures and to provide private capital for next period (net of subsidies); condition A2) shows that households need positive rents, net of taxes, to consume and invest; condition A3) means that public capital is essential in the private production function.

Given  $K_0$ ,  $K_0^g > 0$ , a fiscal policy  $\Pi$  and feasible conditions A1)-A3), there exists a unique  $\Pi$ - $CE$  allocation, the sequence  $\{C_t, K_{t+1}\}_{t=0}^\infty$  obtained from the linear system (23)-(24) belonging to it.<sup>6</sup> The characteristics of the dynamics of  $C_t$ ,  $K_{t+1}$  and  $K_{t+1}^g$  are given by the eigenvalue structure of the coefficient matrix of the state-space representation of the above system, in logged variables [ $\hat{c} = \ln(C)$ ,  $\hat{k} = \ln(K)$ ,  $\hat{k}^g = \ln(K^g)$ ],

$$\begin{aligned} \hat{c}_t &= d_{1,t} + \theta \hat{k}_t^g + (1 - \theta) \hat{k}_t, \\ \begin{pmatrix} \hat{k}_{t+1}^g \\ \hat{k}_{t+1} \end{pmatrix} &= \begin{pmatrix} d_{2,t} \\ d_{3,t} \end{pmatrix} + \begin{pmatrix} \theta & 1 - \theta \\ \theta & 1 - \theta \end{pmatrix} \begin{pmatrix} \hat{k}_t^g \\ \hat{k}_t \end{pmatrix}, \end{aligned} \quad (26)$$

where  $d_{1,t}$ ,  $d_{2,t}$  and  $d_{3,t}$  do not depend on state variables. That matrix has a zero eigenvalue and a second eigenvalue equal to one. The zero eigenvalue reflects the absence of transitional dynamics, while the unit eigenvalue is inherent to endogenous growth models,<sup>7</sup> implying that the ratios  $K_t^g/K_t$ ,

<sup>6</sup>See Marrero and Novales (2003) and Glomm and Ravikumar (1994,1999).

<sup>7</sup>See King and Rebelo (1988) and Caballé and Santos (1993).

$C_t/K_t$  stay forever at constant levels,  $k^g$  and  $c$ , respectively. Consequently, given  $K_0, K_0^g > 0$ , the previous system provides us with the values of  $C_0, K_1$  and  $K_1^g$  under either tax policy, the three variables growing from that time on at the common rate  $\gamma$  given by (29).

### 3.3.2 Characterizing the $\Pi$ -*bgp*

The full depreciation  $\Pi$ -*bgp* equilibrium is obtained by solving (16)-(22) for  $\{\gamma, c, k^g, y, r, c^g, c^w\}$  under  $\delta = \delta^g = 1$ :

$$k^g = \frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)}, \quad (27)$$

$$r = \alpha A \left[ \frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)} \right]^\theta, \quad (28)$$

$$\gamma = A\varkappa_i^\theta [\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)]^{1-\theta} - 1, \quad (29)$$

$$y = A \left[ \frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)} \right]^\theta, \quad (30)$$

$$c = \frac{A\varkappa_i [(1 - \varkappa_i - \varkappa_g - \varkappa_w)(1 - \alpha\beta) - \alpha\beta v]}{[\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)]^\theta}, \quad (31)$$

$$c^g = \varkappa_g y = \varkappa_g A \left[ \frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)} \right]^\theta, \quad (32)$$

$$c^w = \varkappa_w y = \varkappa_w A \left[ \frac{\varkappa_i}{\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)} \right]^\theta \quad (33)$$

where  $c, k, k^g > 0$  so long as A1)-A3) hold.

## 4 Taxing or subsidizing factors' rents?

A benevolent planner maximizes households welfare under competitive equilibrium conditions and the need to finance a fixed ratio of wasteful public expenditures,  $\varkappa_w \geq 0$ , each period. She decides on *i*) the public investment/output ratio, *ii*) the public consumption/output ratio and *iii*) how to split the financing of required public resources between lump-sum and income taxes.

Hence, public expenditures are chosen as a constant fraction of output each period, as it is natural in a growth model. It is also a rule commonly used in actual budgetary policy. There are four externalities in our model economy, each linked to a specific parameter: *i*) public consumption in the utility function,  $\varphi$ , *ii*) public capital in the production process,  $\theta$ , *iii*) the spillover effect of private capital,  $\phi$ , *iv*) the constraint to finance each period a fixed fraction of wasteful public expenditures to output,  $\varkappa_w$ .<sup>8</sup>

Condition (8) allows us to focus on the choice of  $\varkappa_i$ ,  $\varkappa_g$  and  $v$ . On the other hand, from (8) and A1)-A3), the feasible set for the planner's problem is, given  $\varkappa_w \in [0, 1)$ ,

$$\Gamma \equiv \{\zeta = (\varkappa_i, \varkappa_g, v, \tau) \in \Lambda : \varkappa_i + \varkappa_g + \varkappa_w + \beta\alpha(1 - \tau) < 1, \tau < 1, \varkappa_i > 0\}. \quad (34)$$

We next find that subsidizing factors' rents is by no means a definite result. On the contrary, raising taxes on factors' income as part of an optimal fiscal policy is a more pervasive result than it seems. We will show that a positive level of wasteful expenditures affects the optimal composition of public expenditures, but it also influences the optimal composition of taxes. High enough levels of  $\varkappa_w$  could make positive income taxes enhance welfare. Moreover, it could even be the case that the fraction of resources collected from a distorting source be higher than that coming through lump-sum taxation.

## 4.1 Maximizing steady-state growth

In an economy without transitional dynamics, the consumption path is determined by applying to initial consumption the growth-rate along the *bgp*, so that the influence of steady-state growth on welfare is obvious. We first characterize the fiscal policy leading to a higher long-run growth rate, leaving the discussion on the implied welfare levels for the next section. As it is usually the case under endogenous growth, growth maximizing policies are rather unrealistic, since they imply a huge sacrifice in consumption to produce fast capital accumulation and growth. This section is hence ancillary, being useful just to better interpret the results regarding welfare maximization.<sup>9</sup>

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<sup>8</sup>Since  $\varkappa_i$  and  $\varkappa_g$  are chosen optimally, the relevant externality comes through  $\varkappa_w$ , which is taken as given by the government.

<sup>9</sup>Along the paper, an asterisk denotes a value obtained under a growth-maximizing strategy.

**Proposition 3** *Optimal choices to maximize the steady-state growth rate are  $x_g^* = 0$ ,  $\varkappa_i^* = \theta(1 - \varkappa_w)$ ,  $v^* = (1 - \theta)(1 - \varkappa_w) \frac{1 - \alpha\beta}{\alpha\beta} > 0$ ,  $\tau^* = \varkappa_w + (1 - \varkappa_w) \left(1 - \frac{1 - \theta}{\alpha\beta}\right)$ .*

**Proof.** See Appendix (part 1) ■

The level of  $\varkappa_i^*$  is the same as in Barro (1990). Since  $v^*$  is always positive, the ratio  $s^*$  of distortionary to lump-sum tax rates,

$$s^* = \frac{\tau^*}{v^*} = \frac{\varkappa_w(1 - \theta) - (1 - \alpha\beta - \theta)}{(1 - \theta)(1 - \varkappa_w)(1 - \alpha\beta)} = s^*(\alpha, \beta, \theta, \varkappa_w) \quad (35)$$

shares the same sign as  $\tau^*$ .

**Proposition 4** *There exists a threshold  $\varkappa_w^{1*}$  for  $\varkappa_w$ ,*

$$\varkappa_w^{1*} = \frac{1 - \alpha\beta - \theta}{1 - \theta} > 0, \quad (36)$$

*such that  $s^*, \tau^* > 0$  whenever  $\varkappa_w > \varkappa_w^{1*}$ .*

**Proof.** It comes directly from (35) ■

Taxing, rather than subsidizing, production factors is desirable in order to maximize long-run growth whenever wasteful public expenditures are large enough. The value of the  $\varkappa_w^{1*}$ -threshold is inversely related to  $\alpha$ ,  $\theta$  and  $\beta$ . Hence, the higher the values of  $\varkappa_w$ ,  $\alpha$ ,  $\beta$  and  $\theta$ , the more likely it is that public expenditures will be partly financed through distortionary taxation when attempting to maximize steady-state growth.

The fact that private consumption cannot possibly become negative places an upper bound on the possibility of raising lump-sum taxes, one of the reasons explaining why taxing factors' income may become growth-maximizing. This result shows the trivial nature of the growth-maximization problem, since it leads to exhaust all private resources to finance capital accumulation, leaving no resources left for consumption. In this situation, it might even be the case that the share of total resources collected through distortionary taxation could be higher than that coming from lump-sum taxation. This is precisely the case when  $\varkappa_w > \varkappa_w^{2*}$ , with  $\varkappa_w^{2*} = 1 - \frac{\alpha\beta}{(1 - \theta)(2 - \alpha\beta)} > \varkappa_w^{1*}$ , then  $s^* > 1$ .

## 4.2 Maximizing welfare

Given  $\varkappa_w \geq 0$  and  $K_0, K_0^g > 0$ , the absence of transition under full depreciation of both types of capital reduces the problem of choosing  $\varkappa_i, \varkappa_g, v$  and  $\tau$  to maximize welfare to,

$$\max_{\zeta \in \Gamma} V(C_0, \gamma) = \left[ \frac{1}{1-\beta} (\ln C_0 + \varphi \ln C_0^g) + \frac{\beta(1+\varphi)}{(1-\beta)^2} \ln(1+\gamma) \right], \quad (37)$$

subject to (29). Plugging the expressions of  $C_0, C_0^g$  and  $\gamma$  into (37) we get,

$$V(\zeta) = \frac{1}{1-\beta} \left[ \begin{aligned} & \ln [(1 - \varkappa_i - \varkappa_g - \varkappa_w) - \beta\alpha(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)] + \varphi \ln \varkappa_g + \\ & (1 + \varphi) \ln Y_0 + \\ & + \frac{\beta(1+\varphi)}{(1-\beta)} \ln \left\{ A \varkappa_i^\theta [\alpha\beta(1 - \varkappa_i - \varkappa_g - \varkappa_w + v)]^{1-\theta} \right\} \end{aligned} \right]. \quad (38)$$

**Proposition 5** *Optimal choices for expenditure ratios and tax rates, in order to maximize welfare are  $\varkappa_g^+ = \frac{\varphi}{1+\varphi} (1-\beta) (1-\varkappa_w)$ ,  $\varkappa_i^+ = \beta\theta (1-\varkappa_w)$ ,  $\varkappa^+ = 1 - (1-A)(1-\varkappa_w)$ , where  $A = \beta\theta + \frac{\varphi}{1+\varphi} (1-\beta)$ ,  $v^+ = \frac{(1-\theta-\alpha)+\beta\alpha\theta+\varphi(1-\theta)(1-\alpha\beta)}{\alpha(1+\varphi)} (1-\varkappa_w)$  and  $\tau^+ = 1 - (1-\varkappa_w) \frac{1-\theta}{\alpha}$ .*

**Proof.** See Appendix (part 2) ■

Since  $v^+$  is always positive, the ratio  $s^+$  of distortionary to lump-sum tax rates,

$$s^+ = \frac{\tau^+}{v^+} = \frac{[\alpha - (1 - \varkappa_w)(1 - \theta)] (1 + \varphi)}{(1 - \varkappa_w) \{(1 - \theta) [1 + \varphi(1 - \beta\alpha)] - \alpha(1 - \beta\theta)\}}, \quad (39)$$

shares the same sign as  $\tau^+$ .

**Proposition 6** *There is a threshold level  $\varkappa_w^{1+}$  for  $\varkappa_w$ ,*

$$\varkappa_w^{1+} = 1 - \frac{\alpha}{1-\theta}, \quad (40)$$

above which  $\tau^+ > 0$ .

**Proof.** It comes directly from (39) ■

This proposition shows that for a large enough level of wasteful government expenditures,  $\varkappa_w > 1 - \frac{\alpha}{1-\theta}$ , taxing factors' income, rather than

subsidizing them, is optimal. This is an interesting result because, first, if the government is efficient, in the sense that wasteful public expenditures are relatively low, subsidizing factors' income is optimal; second, the level of wasteful expenditures defining an efficient government is just a function of technological parameters. Moreover, taxing factors' income might be optimum even for small levels of  $\varkappa_w$ . For instance, in the absence of the spillover externality from private capital,  $\alpha = 1 - \theta$ , taxing factors income becomes optimal for any positive level of wasteful public expenditures. The level of the  $\varkappa_w^{1+}$ -threshold just depends on technological externalities: it is decreasing in  $\alpha$  and  $\theta$ , and, fixing any of them, is increasing in  $\phi$ . So, the higher  $\alpha$  or  $\theta$ , the smaller the spillover externality, and more likely that taxing factors' income becomes optimal.

It might even be optimal to raise through factor taxes a higher revenue than through lump-sum transfers. That will be the case whenever  $\varkappa_w > \varkappa_w^{2+} = 1 - \frac{\alpha}{[2(1-\theta) + \alpha\beta\theta] - \alpha \frac{1+\beta\varphi}{1+\varphi}}$ , in which case  $s^+ > 1$ . The  $\varkappa_w^{2+}$ -threshold now depends on all structural parameters: it is decreasing in  $\alpha$  and  $\theta$ , increasing with  $\varphi$  and increasing with  $\beta$  if  $\theta > \frac{\varphi}{1+\varphi}$ .

Finally, the next corollary shows that it is more likely that taxing factors' income can be optimal when maximizing welfare than when attempting to maximize long-run growth.

**Corollary 7** *We have, for any economy, that  $\varkappa_w^{1+} < \varkappa_w^{1*}$ .*

**Proof.** It is straightforward from the expressions of  $\varkappa_w^{1+}$  in (40) and  $\varkappa_w^{1*}$  in (36) ■

### 4.3 Special cases

Having shown that distortionary taxation can be part of an optimal fiscal policy, the relevant question relates to which specific aspects of our model drive the optimality of factor income taxes. For a full interpretation of our results, we consider in this section extreme cases of our model economy.

#### 4.3.1 The spillover externality

If we assume  $\phi = 0$ , then  $\alpha = 1 - \theta$ , and optimal expenditure ratios and tax rates would be:  $\varkappa_g^+ = \frac{\varphi}{1+\varphi} (1 - \beta) (1 - \varkappa_w)$ ,  $\varkappa_i^+ = \beta\theta (1 - \varkappa_w)$ ,  $v^+ = \frac{\beta\theta(1-\theta) + [\varphi - \theta - \beta(1-\theta)]}{(1-\theta)(1+\varphi)} (1 - \varkappa_w)$  and  $\tau^+ = \varkappa_w$ . The important result in this

case is that taxing factors income is always optimal, with independence of the output elasticities of public and private capital. The absence of the spillover externality eliminates the main reason to subsidize private investment. Subsidies are then never optimal, no matter how important the externality of public capital may be. This result shows, first, that it is not necessary for optimality of factor income taxes that the government be constrained by a high level of wasteful public expenditures. Secondly, that the presence of a public capital externality is not, by itself, enough to imply the optimality of income taxes.

### 4.3.2 The public capital externality

Under  $\theta = 0$ , optimal expenditure ratios and tax rates would be:  $\varkappa_g^+ = \frac{\varphi}{1+\varphi} (1 - \beta) (1 - \varkappa_w)$ ,  $\varkappa_i^+ = 0$ ,  $v^+ = \frac{(1-\alpha)+\varphi(1-\alpha\beta)}{\alpha(1+\varphi)} (1 - \varkappa_w)$  and  $\tau^+ = 1 - \frac{1-\varkappa_w}{\alpha}$ , and taxing factor income will be optimal for any wasteful expenditure ratio above  $\varkappa_w^0 = 1 - \alpha$ . The government does not invest in public capital, but it still purchases some consumption commodity and pays for wasteful expenditures. Taxes are optimal in this setup just for a large enough ratio of wasteful expenditures. When neither type of aggregate capital appears as a production input, we collapse to the *AK* model, since  $\theta = \phi = 0$  and  $\alpha = 1$ , so that  $y_t = Ak_t$ . Optimal expenditure ratios and tax rates then are,  $\varkappa_g^+ = \frac{\varphi}{1+\varphi} (1 - \beta) (1 - \varkappa_w)$ ,  $\varkappa_i^+ = 0$ ,  $v^+ = \frac{\varphi(1-\beta)}{1+\varphi} (1 - \varkappa_w)$  and  $\tau^+ = \varkappa_w$  and income taxes become again optimal for  $\varkappa_w$  above 0.

### 4.3.3 The consumption externality

If the consumption externality was not present,  $\varphi = 0$ , optimal expenditure ratios and tax rates would be:  $\varkappa_g^+ = 0$ ,  $\varkappa_i^+ = \beta\theta (1 - \varkappa_w)$ ,  $v^+ = \frac{(1-\theta-\alpha)+\beta\alpha\theta}{\alpha} (1 - \varkappa_w)$  and  $\tau^+ = 1 - (1 - \varkappa_w) \frac{1-\theta}{\alpha}$ , so that optimal public consumption would then be zero.

In economies with a spillover of private capital,  $\alpha < 1 - \theta$ , it will be optimal to tax factors' income if the ratio of wasteful expenditures is above the general threshold in proposition 5,  $\varkappa_w^{1+} = 1 - \frac{\alpha}{1-\theta}$ , the consumption externality not being necessary for distortionary taxation to be optimal. In our model economy, this result has an alternative lecture: performing in an optimal way, any increase in  $\varkappa_g$  will be financed with lump-sum taxes.

#### 4.3.4 Expenditures as a proportion of output

Let us now suppose that wasteful public expenditures are constant over time, as opposed to all other aggregate variables in the economy, which grow at a constant rate  $\gamma$ . If wasteful expenditures are initially very large, lump-sum financing might bring consumption to a very low level, preventing the possibility of subsidizing private production inputs. Income taxes would then be needed, showing that having a public expenditures target in the form of a given proportion of output is not a necessary condition for factor taxes to be part of an optimal fiscal policy. However, in a growth economy it makes sense that wasteful public expenditures also grow at a positive rate.

### 5 Interpretation of results

In our model economy, the four possible uses of public funds are: i) public investment, ii) public services, iii) wasteful expenditures and iv) subsidies to factors' rents. On the revenue side, the government decides on which percentage of resources to raise through distortionary and non-distortionary taxation, so as to maximize consumers' welfare.<sup>10</sup> The interaction between technological and fiscal externalities is central to understanding why the government may end up taxing factor rents.

Externalities emerging from the presence of aggregate private capital and the stock of public capital in the production function, both tend to produce underinvestment in the competitive equilibrium solution, relative to the social optimum. Not perceiving the spillover externality of aggregate private capital, the consumer takes into account a private marginal productivity of capital below the social product of capital, leading to underinvestment. In the competitive solution the consumer also ignores that the stock of public capital next period, another input in the production function, is a given proportion of current output and hence, a function of current investment. This is a second factor that produces a gap between private and social marginal products of physical capital, leading again to underinvestment. Because of these externalities, a government will generally be interested in subsidizing private investment and, whenever possible, the general tendency will be to finance these subsidies through non distortionary taxation.

Deciding on the suitable policy, the purpose of the government is precisely

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<sup>10</sup> Although decisions on revenue raising and spending are taken simultaneously.

to reduce the effects of these externalities. Whenever the public investment to output ratio is chosen optimally, the government is able to mitigate the under investment driven by the public capital externality. Thus, the public capital externality is relevant to decide upon the optimal public expenditure policy and the total amount of resources to be raised, but not to determine whether subsidizing productive factors is optimal or not.

At this respect, the spillover externality plays a more relevant role, being necessary for subsidies on production factors to be optimal. In general, raising public expenditure crowds out private consumption. Moreover, for initial periods, this crowding out is larger under lump-sum than under income taxes, their difference turning more important the larger is the percentage of wasteful public expenditure to output to be financed each period (the fiscal externality).<sup>11</sup>

Thus, when both, the fiscal and the spillover externality, are positive in the model two factors are moving in opposite directions. For any spillover degree, there exists a wasteful public investment output ratio above which taxing factor rents is always optimal, thus the government reduces the excess of crowding-out of private consumption relative to private investment. In fact, the absence of the spillover externality guarantees that taxing factor rents is optimal for any positive wasteful public expenditure ratio.

## 5.1 A numerical example

We keep  $\varphi = .40$  and  $\beta = .90$  constant in Table 1, and change all other parameters, to find stationary equilibrium values and characterize optimal fiscal policy. All variables are shown as a percentage of total output. The first four columns show the parameterization used, including the ratio of wasteful expenditures to output in the first column. Columns five and six show private consumption and investment decisions, followed by optimal public decisions on consumption and investment, all measured as a proportion of total output. Together with wasteful expenditures, these four decisions exhaust total output. Next columns show lump-sum taxes and distortionary taxes, both as a proportion of total output, followed by their net value.

Total investment amounts to  $I + I^g = \beta(1 - \varkappa_w)Y$ , being therefore constant for constant wasteful expenditure ratios. Public and private invest-

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<sup>11</sup>This claim is easy to show, when, under the optimal public expenditure policy, we compare  $C_0$  in (23) under alternative extreme tax scenarios (lump-sum taxes and factors' income taxes). See Marrero and Novales (2003) for more details on this point.

ment is high in our economy because of the full depreciation assumption on both types of capital. This is the same reason why consumption levels are relatively small.<sup>12</sup> Public investment is increasing in  $\phi$ , as it should be expected, while private investment is decreasing in  $\alpha$  and  $\phi$  through their sum,  $I = \beta (1 - \varkappa_w) [1 - (\alpha + \phi)] Y$ .

[INSERT TABLE 1 ABOUT HERE]

Since the government chooses the optimal tax structure, a relevant question is the relationship between different types of financing and expenditures. In particular, since the government faces the constraint to pay for wasteful expenditures, a significant issue is the possible correspondence between resources raised through lump-sum taxes and the wasteful component of public expenditures. A possible view would think on financing wasteful expenditures through lump-sum taxes. If these are not too large, the government might still be able to raise additional resources to finance public investment, as well as to subsidize private investment. If, on the other hand, wasteful expenditures exhaust a good percentage of output, the government might face the need to raise taxes on factors income, rather than subsidizing them, in order to finance public investment.

Our results show this intuition to be partially wrong: Figure 1 and 2 shows that, as a function of  $\varkappa_w$ , optimal lump-sum taxes,  $v^+/\varkappa_w$ , fall very quickly as a proportion of wasteful expenditures. For reasonable values of wasteful expenditures, optimal lump-sum taxes are well above the level needed to pay for them as well as for public consumption. This is even more so in economies with a small spillover effect from aggregate private capital. Both effects combine to make governments be more sensible towards providing public capital, which suggest raising additional revenues. The fact that lump-sum taxes are so large, relative to this expenditure component, shows that the government is additionally interested in financing subsidies on private capital accumulation through lump-sum taxes. In this situation, the positive distortion is financed through a non-distorting mechanism. Additionally, taxing factor incomes may turn out to be optimal for almost any level of wasteful expenditures.

[INSERT FIGURE 1 ABOUT HERE]

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<sup>12</sup>With  $\beta$  smaller than 0.3, total consumption and investment would be, respectively, around a 75% and 25% of total output, which are quiet similar to that observed for main Occidental Economies.

[INSERT FIGURE 2 ABOUT HERE]

Optimally of taxes also arises for very different values of  $\alpha$ , the output elasticity of private investment. The more relevant factor determining optimality of income taxes seems to be that the spillover of aggregate private capital not be too important. However, as the last row shows, even with a large spillover externality, taxing factor income will be optimal under important wasteful expenditures. In fact, taxes are a significant proportion of output in this case.

## 6 Final Remarks

In an economy with a spillover effect from aggregate private capital, we have characterized the optimal fiscal policy from the point of view of a benevolent planner, assuming that it is constrained to pay for wasteful public expenditures. These are given as a fixed proportion of output. We assume the government cannot issue any debt, so it has three decisions to make: *i*) investment on public capital, *ii*) public consumption and *iii*) the combination of lump-sum and income taxes to finance public expenditures. Public capital enters as an input in the aggregate technology, while public consumption affects consumers' preferences. Subsidies might be interesting because of the presence of the two technological externalities: the spillover from private capital, and the presence of public capital as a production input.

We have shown that optimally of factor income taxes may be a more pervasive result than usually thought. It is a somewhat natural result in the absence of the private capital spillover, but it may also arise when that spillover externality is significant. The constraint to pay for a relatively high level of wasteful public expenditures can be enough to impose taxes, rather than subsidies, on private production factors as part of an optimal fiscal policy. Taxing income may be optimal for reasonable parameterization, with the public capital externality and the spillover effect from aggregate private capital, in spite of the fact that both externalities tend to favor subsidizing production factors.

Taxing factor incomes can turn out to be optimal for any level of wasteful government expenditures, as shown, in a limit example, by the no spillover case. However, without wasteful expenditures, subsidizing, rather than taxing factor income would be optimal under private capital spillover in the production function. Whenever that externality is present, a given level of

wasteful expenditures is needed for distortionary taxation to be optimal. Public capital as a production input reinforces the role of the private capital spillover to make subsidies on factor incomes to be optimal. That raises the level of wasteful public expenditures needed for factor income taxes to be optimal.

The presence of public consumption as an argument in the utility function is an element in favor of raising public resources via taxes. Although the first choice would be to raise lump-sum taxes, that might leave too few resources for consumption, and imposing income taxes might be better. However, we have shown this externality in preferences not to be necessary for the optimality of income taxes.

There is a strong dichotomy in the economy we have considered: public consumption depends only on parameters in preferences, while public investment depends mostly on parameters in the production technology. From the point of view of government revenues, we get that, as a percentage of output, distortionary taxes depend just on technology parameters, while lump-sum taxes also depend on parameters in preferences. That determines that an increase in public consumption will be financed through lump-sum taxes. One extension to this research should go in the direction of considering a non-separable utility function in private and public consumption, to avoid this dichotomy.

On the other hand, the payment of interests due to the outstanding debt is a way of wasteful public expenditure. Thus, the lack of discipline today in the current public budget will make raise wasteful public expenditures in the future and thus taxing rather than subsidizing factor rents turning optimal. As a second extension to this research, issuing debt must be allowed in order to analyze the relationship between public deficit and optimal taxation.

We have shown that it is more likely that taxes be part of an optimal fiscal policy if the government cares about maximizing welfare than about maximizing growth. This suggests that long-run effects tend to be favorable to subsidizing production factors, while short-run considerations tend to push for taxing factor incomes. Hence, a second extension should look at an economy with non-trivial transition. A simple way of doing that would consist on explicitly including leisure in the utility function. An alternative approach would consider less than full depreciation. A continuity argument suggests that our results will be robust to high depreciation rates. On the other hand, the lower the depreciation rates of private and public capital, the slower the transition would be, placing a heavier weight on short-run effects

and thus favoring taxes on factor incomes.

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## 7 Appendix

### 7.1 Part 1: proof of Proposition 3

**Proof.** From (29), it is easy to see that, *i*) for any given values of  $x_i$ ,  $x_g$  and  $x_w$ ,  $\gamma$  is strictly increasing in  $v$ , *ii*) for any given  $x_i$ ,  $x_w$  and  $v$ ,  $\bar{\gamma}$  is inversely related to  $x_g$ . Hence, to maximize steady-state growth, we set  $x_g^* = 0$  and choose  $v$  as large as possible, to approach  $v^* = \frac{(1-\varkappa_w-\varkappa_i)(1-\alpha\beta)}{\alpha\beta} > 0$  defined by (34). By continuity, we set  $v = v^*$  into (29) and the problem reduces to choosing  $\varkappa_i$  so as to maximize,  $\bar{\gamma}'$ ,

$$\bar{\gamma}' = A\varkappa_i^\theta \left[ \alpha\beta \left( 1 - \varkappa_i - \varkappa_w + \frac{(1 - \varkappa_w - \varkappa_i)(1 - \alpha\beta)}{\alpha\beta} \right) \right]^{1-\theta} - 1.$$

Since  $\bar{\gamma}'$  is strictly concave in  $\varkappa_i$ , condition  $\partial\bar{\gamma}'/\partial\varkappa_i = 0$  gives us the value of  $\varkappa_i$  maximizing steady-state growth,

$$\varkappa_i^* = \theta(1 - \varkappa_w),$$

and substituting into (8), we get the growth-maximizing levels of income and lump-sum tax rates ■

### 7.2 Part 2: proof of Proposition 5

**Proof.** It is easy to show that  $V$  is not monotonic in either  $\varkappa_i$ ,  $\varkappa_c$  and  $v$ . Moreover, for any given  $v$ , we have  $\lim_{\varkappa_i, \varkappa_c \rightarrow 0^+} V = \lim_{\varkappa_i, \varkappa_c \rightarrow 1^-} V = -\infty$ , so the welfare-maximizing levels of  $\varkappa_i$ ,  $\varkappa_c$  fall in the interior of  $[0, 1]^2$ . This is not the case for  $v$ , whose optimal value may fall above 1 or below 0.  $V(\zeta)$  defined from (38) is strictly concave in  $\zeta$ , so condition  $\nabla V(\zeta) = 0$  is necessary and sufficient for a global maximum,

$$\frac{\partial V}{\partial \varkappa_i} = -\frac{(1 - \beta\alpha)}{1 - \varkappa - \beta\alpha(1 - \varkappa + v)} + \frac{\beta(1 + \varphi)\theta}{(1 - \beta)\varkappa_i} - \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)(1 - \varkappa + v)} \quad (40)$$

$$\frac{\partial V}{\partial v} = -\frac{\beta\alpha}{1 - \varkappa - \beta\alpha(1 - \varkappa + v)} + \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)(1 - \varkappa + v)} = 0, \quad (42)$$

$$\frac{\partial V}{\partial \varkappa_g} = -\frac{(1 - \beta\alpha)}{1 - \varkappa - \beta\alpha(1 - \varkappa + v)} + \frac{\varphi}{\varkappa_g} - \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta)(1 - \varkappa + v)} = 0, \quad (43)$$

where  $\varkappa = \varkappa_i + \varkappa_g + \varkappa_w$ . Adding up (42) and (43), we get,

$$v = (1 - \varkappa) \left( \frac{1}{\alpha\beta} - 1 \right) - \frac{\varkappa_g}{\varphi} \frac{1}{\alpha\beta}, \quad (44)$$

$$1 - \varkappa + v = 1 - \tau = \left( 1 - \varkappa - \frac{\varkappa_g}{\varphi} \right) \frac{1}{\alpha\beta}, \quad (45)$$

while combining (41) and (43), we get the relationship between the welfare maximizing levels of  $\varkappa_i$  and  $\varkappa_g$ ,

$$\varkappa_i^+ = \beta \frac{1 + \varphi}{1 - \beta} \frac{\theta}{\varphi} \varkappa_g^+. \quad (46)$$

Using (45) in (42) we get,

$$-\frac{\beta\alpha}{1 - \varkappa - \left( 1 - \varkappa - \frac{\varkappa_g}{\varphi} \right)} + \frac{\beta(1 + \varphi)(1 - \theta)}{(1 - \beta) \left( 1 - \varkappa - \frac{\varkappa_g}{\varphi} \right) \frac{1}{\beta\alpha}},$$

which leads to,

$$\begin{aligned} & \left( 1 - \varkappa_i - \varkappa_g - \varkappa_w - \frac{\varkappa_g}{\varphi} \right) [(1 - \beta) + \beta(1 + \varphi)(1 - \theta)] \\ &= \beta(1 + \varphi)(1 - \theta)(1 - \varkappa_i - \varkappa_g - \varkappa_w), \end{aligned}$$

from where we get the relationship,

$$\varkappa_g^+ = \frac{\varphi}{1 + \varphi} \frac{1 - \beta}{(1 - \beta) + \beta(1 - \theta)} (1 - \varkappa_i^+ - \varkappa_w).$$

Combining this expression with (46), we get,

$$\varkappa_g^+ = \frac{\varphi}{1 + \varphi} (1 - \beta) (1 - \varkappa_w),$$

and,

$$\varkappa_i^+ = \beta\theta(1 - \varkappa_w).$$

Then,

$$\begin{aligned} \varkappa^+ &= \varkappa_i^+ + \varkappa_g^+ + \varkappa_w = 1 - \left[ 1 - \beta\theta - \frac{\varphi}{1 + \varphi} (1 - \beta) \right] (1 - \varkappa_w) = \\ &= 1 - (1 - A)(1 - \varkappa_w); \quad A = \beta\theta + \frac{\varphi}{1 + \varphi} (1 - \beta), \end{aligned}$$

and

$$\begin{aligned}
v^+ &= (1 - \varkappa^+) \left( \frac{1}{\alpha\beta} - 1 \right) - \frac{\varkappa_g^+}{\varphi} \frac{1}{\alpha\beta} = \\
&= (1 - \varkappa_w) \left[ \left( 1 - \beta\theta - \frac{\varphi}{1 + \varphi} (1 - \beta) \right) \left( \frac{1}{\alpha\beta} - 1 \right) - \frac{1}{\alpha\beta} \frac{1 - \beta}{1 + \varphi} \right] = \\
&= \frac{(1 - \theta - \alpha) + \beta\alpha\theta + \varphi(1 - \theta)(1 - \alpha\beta)}{\alpha(1 + \varphi)} (1 - \varkappa_w).
\end{aligned}$$

Then, the optimal tax rate on factors income,

$$\begin{aligned}
\tau^+ &= \varkappa^+ - v^+ = \\
&= 1 - (1 - \varkappa_w) \left[ 1 - \beta\theta + \frac{(1 - \theta - \alpha) - \alpha\varphi(1 - \beta) + \beta\alpha\theta + \varphi(1 - \theta)(1 - \alpha\beta)}{\alpha(1 + \varphi)} \right] = \\
&= 1 - (1 - \varkappa_w) \frac{1 - \theta}{\alpha}.
\end{aligned}$$

■

## 8 Tables and Graphics

Figure 1: Optimal  $v/\varkappa_w$  vs.  $\varkappa_w$  under alternative parameterization (1)

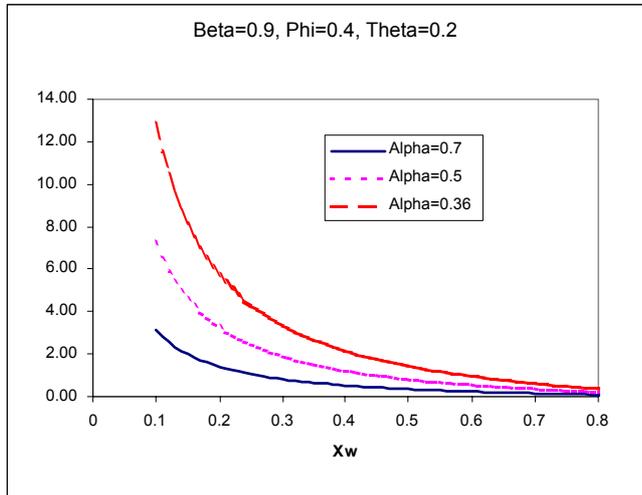


Figure 2: Optimal  $v/\varkappa_w$  vs.  $\varkappa_w$  under alternative parameterization (2)

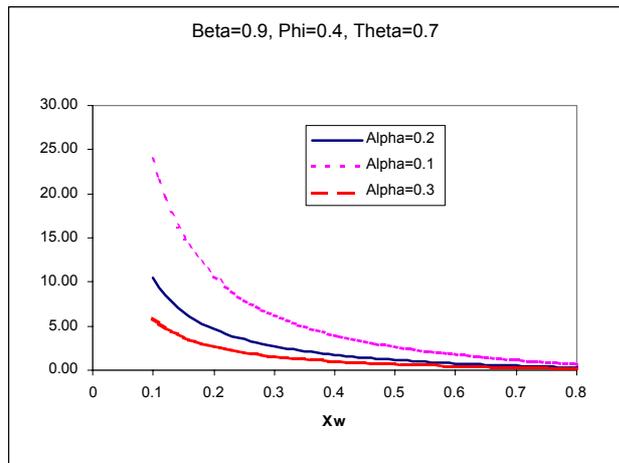


Table 1: Optimal fiscal policy and stationary equilibrium values

Xw	$\alpha$	$\phi$	$\theta$	% of total output					% of wasteful expenditures				
				C	I	Ig (*)	Cg (*)	Cw	Lump-Sum Taxes (*)	Distortional Taxes (*)	TOTAL TAXES	Lump-Sum Taxes	Distortional Taxes
0.15	0.75	0.00	0.25	6.1%	57.4%	19.1%	2.4%	15.0%	21.6%	15.0%	36.6%	1.4	1.0
0.15	0.75	0.10	0.15	6.1%	65.0%	11.5%	2.4%	15.0%	25.2%	3.7%	28.9%	1.7	0.2
0.15	0.75	0.25	0.00	6.1%	76.5%	0.0%	2.4%	15.0%	30.8%	-13.3%	17.4%	2.1	-0.9
0.15	0.40	0.00	0.60	6.1%	30.6%	45.9%	2.4%	15.0%	48.3%	15.0%	63.3%	3.2	1.0
0.15	0.40	0.35	0.25	6.1%	57.4%	19.1%	2.4%	15.0%	95.9%	-59.4%	36.6%	6.4	-4.0
0.15	1.00	0.00	0.00	6.1%	76.5%	0.0%	2.4%	15.0%	2.4%	15.0%	17.4%	0.2	1.0
0.30	0.75	0.00	0.25	5.0%	47.3%	15.8%	2.0%	30.0%	17.8%	30.0%	47.8%	0.6	1.0
0.30	0.75	0.10	0.15	5.0%	53.6%	9.5%	2.0%	30.0%	20.8%	20.7%	41.5%	0.7	0.7
0.30	0.75	0.25	0.00	5.0%	63.0%	0.0%	2.0%	30.0%	25.3%	6.7%	32.0%	0.8	0.2
0.30	0.40	0.00	0.60	5.0%	25.2%	37.8%	2.0%	30.0%	39.8%	30.0%	69.8%	1.3	1.0
0.30	0.40	0.35	0.25	5.0%	47.3%	15.8%	2.0%	30.0%	79.0%	-31.3%	47.8%	2.6	-1.0
0.30	1.00	0.00	0.00	5.0%	63.0%	0.0%	2.0%	30.0%	2.0%	30.0%	32.0%	0.1	1.0
0.15	0.50	0.20	0.30	6.1%	53.6%	23.0%	2.4%	15.0%	59.4%	-19.0%	40.4%	4.0	-1.3
0.15	0.30	0.20	0.50	6.1%	38.3%	38.3%	2.4%	15.0%	97.3%	-41.7%	55.7%	6.5	-2.8
0.15	0.40	0.20	0.40	6.1%	45.9%	30.6%	2.4%	15.0%	75.5%	-27.5%	48.0%	5.0	-1.8
0.3	0.50	0.20	0.30	5.0%	44.1%	18.9%	2.0%	30.0%	48.9%	2.0%	50.9%	1.6	0.1
0.3	0.30	0.20	0.50	5.0%	31.5%	31.5%	2.0%	30.0%	80.2%	-16.7%	63.5%	2.7	-0.6
0.3	0.40	0.20	0.40	5.0%	37.8%	25.2%	2.0%	30.0%	62.2%	-5.0%	57.2%	2.1	-0.2

Note: bold letters shows those cases where taxing factor rents is optimal

(\*) Optimal policy