It's a Small Small Welfare Cost of Fluctuations

Franck Portier and Luis A. Puch*†

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Abstract

Lucas [1987] has shown that in a representative agent framework, the potential welfare gain from stabilizing consumption around its mean is small. We provide an example and some insight for why Lucas' measure is an upper bound of the welfare cost of fluctuations in walrasian economies.

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1 Introduction

In a seminal contribution, Lucas [1987] has shown that in a representative agent framework, the potential welfare gain from stabilizing consumption around its mean is small. Let us recall briefly Lucas's argument. If aggregate consumption follows a log linear process around a deterministic trend, $c_t = (1+\mu)^t e^{-\frac{1}{2}\sigma_z^2} z_t$, where $\{z_t\}$ is a stationary stochastic process with a stationary distribution given by $\ln z_t \rightsquigarrow N(0, \sigma_z^2)$, then the cost of instability can be computed as the percentage increase in consumption, uniform across all dates and values of the shocks, required to leave the consumer indifferent between consumption instability and a perfectly smooth consumption path. With a CRRA utility function with risk aversion

*Respectively Université de Toulouse for Portier and FEDEA, Universidad Complutense and ICAE for Puch.

†Corresponding author: Franck Portier, Batiment F, Manufacture des Tabacs, Université des Sciences Sociales, 21 Allée de Brienne, F-31000 Toulouse, France. Tel: +33 (0)5 61 12 88 40, Fax: +33 (0)5 61 12 86 37, e-mail: fportier@cict.fr. Franck Portier is affiliated to LEERNA, GREMAQ, IDEI, Institut Universitaire de France and CEPR.

coefficient ν, this cost is given by $\frac{1}{2}\nu\sigma_z^2$. With $\sigma_z = 0.013$ (Lucas's estimate), and $\nu = 5$, the welfare cost of fluctuations is only 0.042% of average consumption.

The Lucas' way of computing the welfare cost of fluctuations is perfectly valid when the observed process of consumption is the only information available to the economist. However, it should be properly implemented when a general equilibrium model has been specified. Firstly, if the mean of the ergodic distribution of consumption in the stochastic economy is equal to the deterministic steady state level of consumption, then this measure corresponds to a steady-state welfare comparison. A steady-state welfare comparison is not economically meaningful in a dynamic model with state variables, as the transition from one steady state to the other can reverse the welfare comparison results. Secondly, the mean of the ergodic distribution of consumption is in general different from the deterministic steady state level of consumption, because of the model non linearity, so that the level effect of fluctuations will not be accounted for if one proceeds a-theoretically in a structural model.

As well understood in the dynamic macroeconomic literature (see for instance Cooley and Hansen [1992] and Krussell and Smith [1999]), non-linearities and transition matters for welfare evaluations. In this note, using a simple general equilibrium model, we propose to evaluate the welfare cost of fluctuations in a comprehensive way, meaning that we will take into account non-linearities and transition. We will prove that in the specific walrasian model that we use, the welfare cost of fluctuations is zero. We will also give numerical results showing that, with a more realistic calibration, this cost is smaller than $\frac{1}{2}\nu\sigma_z^2$, which is what one would obtain disregarding non-linearities and transitions.

2 The Model

We use the simplest analytical version of the RBC model, that is the model with full depreciation, log utility, Cobb-Douglas production function and log-normal shocks to technology. The economy is competitive, populated by one firm that hires labor and capital services to produce a unique good, and by one household that supplies labor and capital services, accumulates capital and consumes. Output is produced with capital K and labor h according to:

$$Y_t = e^{\varepsilon_t} K_t^{\alpha} h_t^{1-\alpha} \tag{1}$$

ε_t is normally distributed, *i.i.d.*, with zero mean and standard deviation σ_ϵ. Preferences are given by the following intertemporal utility function U:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t + \gamma \log(1 - h_t)\right)$$

Capital fully depreciates every period, so that the aggregate resource constraint of the economy is

$$Y_t = C_t + K_{t+1}$$

In such an economy, one can compute analytically the competitive allocations (cf. McCallum [1989]). In particular, equilibrium capital is given by $K_{t+1} = \alpha\beta Y_t$ and employment is $h_t = \widetilde{h}$ constant. Using the notation $x = \log X$, equilibrium capital and consumption can be rewritten:

$$\begin{aligned} c_t &= \theta_c + \varepsilon_t + \alpha k_t & (2) \\ k_t &= \theta_k + \varepsilon_{t-1} + \alpha k_{t-1} & (3) \end{aligned}$$

with $\theta_c = \log\left((1-\alpha\beta)\widetilde{h}^{1-\alpha}\right)$ and $\theta_k = \log\left(\alpha\beta\widetilde{h}^{1-\alpha}\right)$. $k_{ss} = \frac{\theta_k}{1-\alpha}$ and $c_{ss} = \theta_c + \frac{\alpha\theta_k}{1-\alpha}$ are the non stochastic steady state values of logged capital and logged consumption.

3 The Measure of the Welfare Cost of Fluctuations

Assume that there exist a way of shutting down the sources of fluctuations, by driving the variance of ε down to zero.[1] The measure we propose for the welfare cost of fluctuations can be understood as the outcome of the following thought experiment of structural change: let

[1] As it is standard in this context we are restricting ourselves to the question about the welfare gains of eliminating business cycles which is truly a hypothetical one. The limitation of doing so is that the exercise is silent about the design of policy that would stabilize the economy.

us assume that the economy has been one with shocks from $-\infty$ to -1, and that in period $t = -1$, all future shocks (from 0 to eternity) are eliminated by setting $\varepsilon_t = 0 \;\; \forall\, t \geq 0$. One can evaluate the welfare gain of this structural change by comparing the expected discounted lifetime utility of the representative agent in two economies: an economy A that starts with initial condition k_0 and in which shocks are not shut down; an economy B that starts with initial condition k_0 and without shocks. Those welfare measures are denoted $W^A(k_0)$ and $W^B(k_0)$. The conditional welfare cost of fluctuations $\mathcal{C}(k_0)$ is then defined as the percentage increase in consumption, uniform across all dates and values of the shocks, required to leave the consumer indifferent between consumption path A and B. By repeating this experiment for many different starting points k_0, drawn in the adequate distribution (the ergodic one of the economy with shocks), one will get an unconditional measure of the welfare cost of fluctuations $\mathcal{C} = E\left[\mathcal{C}(k_0)\right]$.

4 A Small Small Cost

In our economy, the unconditional variance of consumption is $\frac{1}{1-\alpha^2}\sigma_\varepsilon^2$ and the utility is logarithmic, so that the welfare cost can be measured as $\mathcal{C}_{\ell SS} = \frac{1}{2}\frac{1}{1-\alpha^2}\sigma_\varepsilon^2$, if one adapts the Lucas' approach to this setting. Here $\mathcal{C}_{\ell SS}$ is a measure that disregards non-linearities and compares steady-states.

Let us now implement our comprehensive measure of the welfare cost of fluctuations in the same model. Let us first compute W^A, the expected intertemporal utility in the world with shocks. Iterating equation (3) backward, we have

$$k_t = \frac{1-\alpha^t}{1-\alpha}\theta_k + \alpha^t k_0 + \sum_{i=0}^{t} \alpha^i \varepsilon_{t-i} \tag{4}$$

Note that this equation implies that $E_{k_0}[k_t] = \frac{1-\alpha^t}{1-\alpha}\theta_k + \alpha^t k_0$, where E_{k_0} is the mathematical expectation conditional on the information relevant in period $t = -1$ (which is simply k_0). Given (2), we also have $E_{k_0}[c_t] = \theta_c + \frac{\alpha(1-\alpha^t)}{1-\alpha}\theta_k + \alpha^{t+1}k_0$. We can therefore compute $W^A(k_0)$

as follows. Using (2) and (3), we have

$$\begin{aligned} W^A(k_0) &= E_{k_0}\left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t + \gamma\log(1-h_t)\right)\right] \\ &= \sum_{t=0}^{\infty}\beta^t E_{k_0}\left[c_t\right] + \frac{\gamma}{1-\beta}\log\widetilde{h} \\ &= \frac{1}{1-\beta}\left[\theta_c + \frac{\alpha\beta\theta_k}{1-\alpha\beta}\right] + \frac{\alpha}{1-\alpha\beta}\,k_0 \end{aligned}$$

$W^B(k_0)$ is now computed, as the intertemporal utility associated to a deterministic transition path that starts from k_0. Along this transition path, one has $k_t = \theta_k + \alpha k_{t-1}$ and $c_t = \theta_c + \alpha k_t$. Straightforward algebra shows that

$$\begin{aligned} W^B(k_0) &= \left[\sum_{t=0}^{\infty}\beta^t\left(\log C_t + \gamma\log(1-h_t)\right)\right] \\ &= \frac{1}{1-\beta}\left[\theta_c + \frac{\alpha\beta\theta_k}{1-\alpha\beta}\right] + \frac{\alpha}{1-\alpha\beta}\,k_0 \end{aligned}$$

so that $W^B(k_0) = W^A(k_0)$ for all k_0. Therefore, $\mathcal{C}(k_0) = 0$ for all k_0 and

$$\mathcal{C} = 0$$

In this economy with risk averse agents, the welfare cost of fluctuations is not only small, but null.

5 Discussion and Extension to Non-Analytical Models

In this economy, the welfare cost of fluctuations is null even though agents are risk averse. Technically, as the economy is log-linear, the mean of consumption is by Jensen inequality larger than its deterministic counterpart, because the average level of TFP is larger in the stochastic economy that it is in the deterministic one. In this analytical model, the mean effect on consumption exactly offsets the cost of the variability of consumption. The increase in the mean of capital and consumption is a result that is also found in models with more general preferences and without full depreciation, as shown in Den Haan and Marcet [1990].

We have also explored such models, and have computed our comprehensive measure of the welfare cost of fluctuations in an economy similar to the one we have presented above, but that does not have an analytical solution. More specifically, we assume that $\beta = .99$, $\alpha = .33$, that preferences are given by $u(c) = (1-\nu)c^{1-\nu}$ with $\nu = 3$, that capital depreciates at rate $\delta = .025$ and that TFP follows an $AR(1)$ process with autoregressive parameter .95 and standard-deviation of the innovation σ. We consider three level of variability of the shock: $\sigma \in \{0.01, 0.02, 0.03\}$. Using a Parameterized Expectations Approach to solve these models, we find (see Table 1) that our comprehensive measure is three to four times smaller than the $\mathcal{C}_{\ell SS}$ measure. It is most likely that this ranking is robust to alternative calibration, showing that in walrasian economies the welfare cost of fluctuations is even smaller than Lucas' evaluation.

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Table 1: Alternative Measures of the Welfare Cost of Fluctuations

σ	s.d. of $\log(C/C_{SS})$	$\mathcal{C}_{\ell SS}$	$\mathcal{C}$
0.01	0.0316	0.15%	0.042%
0.02	0.0634	0.60%	0.19%
0.03	0.0937	1.31%	0.44%

Appendix

A Measuring Costs

To obtain a structural (model driven) measure of the welfare cost of fluctuations, we compare the economies with and without fluctuations starting from the same set of initial conditions $S = (K, \theta)$. The measure we compute can be understood as the outcome of the following thought experiment of structural change: let us assume that we have been in an economy with shocks from $-\infty$ to $T-1$, and that from T to ∞, fluctuations are eliminated by setting $\varepsilon_t = 1 \;\; \forall\; t \geq T$. We evaluate the welfare gain of this structural change by comparing the expected intertemporal utility of the representative agent in two economies: an economy A that starts with initial condition S_{T-1} and in which shocks are not shut down; an economy B that starts with initial condition S_{T-1} and without shocks. The conditional (on S_{T-1}) welfare cost of fluctuations $\mathcal{C}(S_{T-1})$ is then defined as the percentage increase in consumption, uniform across all dates and values of the shocks, required to leave the consumer indifferent between consumption path A and B. By repeating this experiment for many different starting points, drawn in the ergodic distribution of the economy with shocks, one will get an unconditional measure of the welfare cost of fluctuations $\mathcal{C} = E\left[\mathcal{C}(S_{T-1})\right]$. More formally, the measure we propose is given by

$$\int_{S_{T-1}} \int_{\mathcal{E}} \sum_{j=0}^{\infty} \beta^j \left[\log\left(C^A_{T+j}\left(1 + \mathcal{C} \times C_{SS}\right)\right) + \gamma \log(1 - h^A_{T+j})\right] \; \mathrm{d}g(\mathbf{e}) \; \mathrm{d}f(S)$$
$$= \int_{S_{T-1}} \sum_{j=0}^{\infty} \beta^j \left[\log C^B_{T+j} + \gamma \log(1 - h^B_{T+j})\right] \; \mathrm{d}f(S). \tag{5}$$

where $\mathbf{e}$ is an infinite sequence of ε and f the ergodic joint density of (K, θ) in the economy with shocks. Note that $\mathcal{C}$ is expressed in percentage points of the non stochastic steady-state level of consumption C_{SS}. Next we explain the way in which we implement the unconditional measure of the welfare cost of fluctuations that we refer to as a comprehensive one.

B Computation of the Welfare Cost of Fluctuations

The first step consists of computing the solution of the model. We use a Parameterized Expectations Approach (see Marcet and Lorenzoni [1998] for a presentation) to obtain approximated decision rules. We then simulate the solution of the model over 45.000 periods and build upon an empirical estimate of the invariant distribution $f(K, \theta)$ of capital stock and productivity to obtain an evenly spaced grid of 50×50 points in the $K \times \theta$ space. Then we draw initial conditions (K_{-1}, θ_{-1}) in that probability distribution of the economy with shocks to compute a 1500 periods deterministic transition to the non stochastic steady state.[2] These paths are denoted $\left\{C^B_{K_{-1},\theta_{-1}}(t), h^B_{K_{-1},\theta_{-1}}(t)\right\}_{t=0}^{1499}$.

We proceed in a similar way for the economy with shocks. Specifically, this amounts to simulating 1000 stochastic paths starting from the same initial conditions from which no-uncertainty transition paths have been computed. These paths are denoted correspondingly $\left\{C^A_{K_{-1},\theta_{-1}}(t), h^A_{K_{-1},\theta_{-1}}(t)\right\}_{t=0}^{1499}$.

We then evaluate $\mathcal{C}$, our comprehensive measure, using equation (5). We also compute the measure $\mathcal{C}_{\ell SS} = \frac{1}{2}\sigma_z^2$ that corresponds to a non structural evaluation, with $\sigma_z^2 = E\left[\left(C^A - E(C^A)\right)^2\right]$.

[2] We extensively compute transition paths for all the cells of the 50 by 50 (K, θ) matrix, and then weight the utility of each of these paths with the density of its initial conditions.