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**TWO PROBLEMS OF THE TAYLOR RULE AND
A PROPOSAL: THE TRACKING RULE**

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Abstract:

This paper deals with some problems related to the application of monetary policy following the Taylor Rule in the theoretical context of a “3-equation model”. The first problem arises if the real interest rate does not affect the equilibrium income level itself –as in the IS curve- but its rate of growth –as in the dynamic IS that we propose. Secondly, the Taylor Rule is incapable of reaching the inflation target when the central bank does not correctly estimate its parameters (the neutral interest rate and potential income) or these parameters vary. Our objective is to propose an alternative to the Taylor Rule which overcomes both problems. This alternative has been called the Tracking Rule, because instead of trying to estimate the neutral interest rate or the potential output, the central bank “tracks” these values based on the economy’s evolution, particularly on variations in the inflation and unemployment rates. After justifying the dynamic IS and explaining the logic of this rule in detail, the paper compares the Tracking Rule with the Taylor Rule, simulating both of them in the context of different types of shock in the modified three equation model. The results, measured by a loss function, show that the Tracking Rule is superior in every single case. It is particularly interesting to evaluate central bank reactions derived from the two rules when the economy suffers a large contractive shock such as the current crisis. The results show that, with the same shock, the economy is more likely to fall into the liquidity trap when the Taylor Rule is applied.

Key words: Monetary Policy; Taylor Rule; Liquidity Trap; Simulations.

JEL codes: E52, E58

1. Introduction:

In the last few years, macroeconomic policies have primarily been analysed by a “three equation model” comprising an IS equation, a Phillips curve with expectations and a monetary policy rule, usually the Taylor Rule¹.

In this theoretical context, this paper’s rationale is two-fold. Firstly, the usual IS curve is not a satisfactory way of representing how an economy’s real sector operates in the appropriate

¹ A classical presentation of this kind of models is Clarida, Galí and Gertler (1999). See also Carlin and Soskice (2006) and Galí (2008). On the other hand, Asso, Kahn and Lesson (2007) show the theoretical and applied impact of the Taylor Rule.

term for this macroeconomic policy analysis. As real economies grow at a certain rate with a constant interest rate, it is evident that the real interest rate does not affect the equilibrium income level itself, but its rate of growth. We have called this relationship between the economy's growth rate and the real interest rate the dynamic IS. Secondly, it is well known that the Taylor Rule is incapable of reaching the inflation target when the central bank does not correctly estimate its parameters (the neutral interest rate and potential income) or these parameters vary².

Considering these two problems, the Taylor Rule may well be an inappropriate guide for central banks, so we have tried to look for and study an alternative rule. The result is the proposed Tracking Rule, so called because instead of maintaining the estimations of the neutral interest rate and the NAIRU constant, they are modified according to the variations registered in the inflation rate and the level of employment.

After justifying the dynamic IS and explaining the logic of this rule in detail, the paper compares the Tracking Rule with the Taylor Rule, simulating both of them in the context of different types of shock in the modified three equation model. The results, measured by a loss function, show that the Tracking Rule is superior in every single case.

It is particularly interesting to evaluate central bank reactions derived from the two rules when the economy suffers a large contractive shock such as the current crisis. The results show that, with the same shock, the economy is more likely to fall into the liquidity trap when the Taylor Rule is applied. Other examples of use of the Taylor Rule for analysing both the origins of the crisis and the appropriate monetary policy at this time, can be found in Taylor (2008, 2009). In a different theoretical context, Benhabib, Schmitt-Grohé and Uribe (2001) already analysed the risks of falling into a liquidity trap associated to the Taylor Rule.

The paper is organised as follows. Section 2 briefly describes the model used and section 3 justifies dynamic IS in more detail. Section 4 explains the problems associated to the Taylor Rule and presents the Tracking Rule proposed as an alternative. The paper then presents the simulations used to compare the two rules: section 5 simulates shocks in which the economy does not face the possibility of falling into a liquidity trap, and section 6 simulates a contractive demand shock which could lead to such a situation, evaluating its likelihood with each rule. The paper ends with our main conclusions.

2. Description of the model:

2.1. Equations:

In order to analyse how the two alternative monetary policy rules work, we use a simple economic model which is similar to the "three equation model", but adapted to a dynamic context in which productivity, the population and the income are growing in the long term. The model comprises the following four equations: dynamic IS, Phillips curve, the relation between

² Taylor (1998, page 52), Woodford (2003, pages 286-288), Reis (2003).

the level of employment and the economic rate of growth, and monetary policy rule. There is also a social welfare function to evaluate monetary policy.

The first equation, called dynamic IS, is an expression in which the income growth rate, not its level, depends on the real interest rate, as follows:

$$g_t = D - br_{t-1} + \varepsilon_t^D \quad (1)$$

Where g is the GDP growth rate, D and b are two positive constants, r is the real interest rate and ε_t^D is an exogenous demand shock.

The second equation is the usual expression of the Phillips curve in which \dot{P} is the inflation rate, n the percentage of employment relative to the active population and \bar{n} that percentage when the economy is at the NAIRU. Finally, a is a positive parameter and ε_t^S represents possible exogenous supply shocks:

$$\dot{P}_t = \dot{P}_{t-1} + a \left(\frac{n_t - \bar{n}}{\bar{n}} \right) + \varepsilon_t^S \quad (2)$$

The third equation relates the level of employment to the income growth rate. This equation is necessary because the interest rate affects the growth rate, but inflation depends on the level of employment. Denoting the income level as Y , labour productivity as A and the active population as L , we have:

$$n_t = \frac{Y_t}{A_t L_t}$$

If we assume that productivity and the active population grow, respectively, at rates γ and l , we obtain the following equation in differences:

$$n_t = n_{t-1} [1 + g_t - (\gamma + l)] \quad (3)$$

The fourth equation is one of the following two monetary policy rules:

- The first is the Taylor Rule which, in terms of the real interest rate, would be³:

³ The original Taylor Rule is expressed in terms of the output gap, but it is equivalent to the employment gap.

$$r_t = \bar{r} + 0.5(\dot{P}_t - \dot{P}^T) + 0.5\left(\frac{n_t - \bar{n}}{\bar{n}}\right) \quad (4a)$$

where \bar{r} is the reference interest rate used by the central bank and \dot{P}^T is the inflation target.

- The second is the Tracking Rule, and its primary characteristic is that the central bank modifies the interest rate set in the previous period according to the changes occurring in the level of employment, the inflation rate, and the difference between the latter and the target rate:

$$r_t = r_{t-1} + \alpha_1 \frac{n_t - n_{t-1}}{n_{t-1}} + \alpha_2 (\dot{P}_t - \dot{P}^T) + \alpha_3 (\dot{P}_t - \dot{P}_{t-1}) \quad (4b)$$

α_1 , α_2 and α_3 are positive parameters.

The primary objective of this paper is to analyse the economy's dynamic performance under different shocks with each of these two rules.

2.2. Welfare function and shocks:

The welfare function is a quadratic loss function which depends on inflation deviations from target and the output gap, where δ is the discount factor and λ_p and λ_{OG} represent the weightings of the inflation and income targets:

$$PS = \sum_{t=0}^T \delta^t \left[\lambda_p (\dot{P}_t - \dot{P}_t^T)^2 + \lambda_{OG} OG_t^2 \right] \quad (5)$$

Possible economic shocks are divided according to two main criteria. Taking into account the shock's origin, we distinguish between shocks affecting the growth rate of aggregate demand, inflationist shocks, a structural change affecting the NAIRU value and a change in the productivity or active population growth rates.

Depending on the shock's duration, a first group would comprise transitory shocks, represented through an autoregressive process. For example, for a demand shock it would be: $\varepsilon_t^D = \rho * \varepsilon_{t-1}^D$, where $0 < \rho < 1$. A second group would include permanent shocks, in which the changes in one of the model's exogenous variables are so long-lasting that $\rho = 1$.

2.3. Equilibrium position:

For the model to be at equilibrium, the four endogenous variables (the growth rate, the inflation rate, the employment level and the interest rate) must remain stable. This only occurs in the following conditions:

- When the employment level is as corresponds to the NAIRU; otherwise, the inflation rate will vary.
- The growth rate must be equal to the sum of the productivity and active population growth rates; otherwise, the level of employment will not remain constant. This sum is called “potential growth rate” (\bar{g}):

$$\bar{g} = \gamma + l \quad (6)$$

- For the above condition to be met, the interest rate must be such that, according to the IS, the growth rate is at its potential value. This equilibrium interest rate is called “neutral interest rate” (\bar{r}):

$$\bar{r} = \frac{D - \bar{g}}{b} \quad (7)$$

If these three conditions are met, the inflation will remain stable, albeit at any value. The economy, therefore, can reach equilibrium even though inflation is not at the rate targetted by the central bank.

3. Justification of dynamic IS:

The intuition underlying the idea of a dynamic IS is that the natural status of an economy in which part of the income is saved is to growth at a certain rate, and that changes in the interest rate affect that growth rate and not the equilibrium income level. We will try to express this idea and to compare it with an alternative formulation.

1. The most simple way of writing the IS equation is:

$$Y_t = a - br_t \quad (8)$$

Where Y is the equilibrium income value at the end of the multiplier process. We call this equation the “static IS”.

For this formulation to show the growth undergone by real economies, there would have to be continuous drops in the real interest rate. Then, in order to adapt this formulation to a growing economy, we can suppose that potential GDP (Y_t^{POT}) grows at an exogenous trend (θ) and that the interest rate affects –with a lag- the output gap. We could write this “IS with trend” as:

$$OG_t = \frac{Y_t - Y_t^{POT}}{Y_t^{POT}} = a - br_{t-1} \quad (9)$$

Once we consider the trend, we obtain the potential income at period t, as follow:

$$Y_t^{POT} = Y_0^{POT} (1 + \theta)^t \quad (10)$$

And the final expression of the IS would be:

$$Y_t = Y_t^{POT} (1 + a - br_{t-1}) \quad (11)$$

Or:

$$Y_t = Y_0^{POT} (1 + \theta)^t (1 + a - br_{t-1}) \quad (12)$$

This formulation, however, does not well resolve the problem of going from a static idea of the economy to another with continuous growth. According to expression (12), if the interest rate rises during a few periods, the GDP would be beneath its potential level by a constant percentage. Moreover, the interest rate would only have to return to that value for the output gap to be zero. The period of contraction would have made no mark on the economy. In a dynamic economy, however, this process would probably be more complex: for the output gap to close, the economy would have to grow at more than its potential rate for a few periods (the trend, in this formulation), which in turn would require a lower than neutral interest rate.

2. In the dynamic IS, all expenditure items must depend on income. More specifically, for each interest rate, consumption and investment demand values are proportional to the period's income. In other words, in an economy in which income has doubled, the number of cost-effective investment projects for a given interest rate will also have doubled, and the same will have occurred to consumption demand. It has to be assumed, therefore, that economic growth does not alter the consumption/saving preference ratio or the viability of investment projects (in proportion to income) for each interest rate.

For the formulation to be dynamic – in Hicks' terminology, for the period's explanation not to be self-contained – there must be a lag in the mutual dependence of income and expenditure. There are two ways in which this can be formalised. Firstly, this lag could be because the investment and consumption decisions in a period are made by individuals according to the previous period's income, known as the Robertson lag. Denoting total aggregate demand with DA , a schematic formulation would be:

$$Y_t = DA_t \quad (13a)$$

$$DA_t = C_t + I_t \quad (14a)$$

$$\frac{C_t}{Y_{t-1}} = \bar{C} - cr_t \quad (15a)$$

$$\frac{I_t}{Y_{t-1}} = d - er_t \quad (16a)$$

Alternatively, however, it could be assumed that the production of period t is generated with a delay relative to the demand decided in $t-1$, which will depend on the income obtained in $t-1$ – Lundberg lag-. This gives us:

$$Y_t = DA_{t-1} \quad (13b)$$

$$DA_{t-1} = C_{t-1} + I_{t-1} \quad (14b)$$

$$\frac{C_{t-1}}{Y_{t-1}} = \bar{C} - cr_{t-1} \quad (15b)$$

$$\frac{I_{t-1}}{Y_{t-1}} = d - er_{t-1} \quad (16b)$$

We prefer the second possibility, because it considers the delay between the real interest rate and the economic growth rate which we find in real economies⁴. Indeed, from equations (13b) to (16b), we obtain:

$$\frac{Y_t}{Y_{t-1}} = \bar{C} + d - (c + e)r_{t-1} \quad (17)$$

With $D = \bar{C} + d - 1$ and $b = (c + e)$, we obtain the expression of dynamic IS:

⁴ Ball (1997) uses a similar equation, also called dynamic IS, and he also mentions this lag.

$$g_t = D - br_{t-1} \quad (1)$$

3. This expression is significantly different from the IS in (12). In this expression, we can see that direct determination of the output gap according to the period's interest rate is "superimposed" on the long-term exogenous trend. However, in a dynamic IS, the interest rate determines the rate of growth, and the output gap evolves according to the difference between the real and potential growth rates. We can see this in the following expression of a period's income from (1):

$$Y_t = Y_{t-1}(1 + D - br_{t-1}) \quad (18)$$

If we assume that income is at its potential level in the initial period, from when there is a constant interest rate, we have the following expression of income at period t:

$$Y_t = Y_0^{POT} (1 + D - br)^t \quad (19)$$

If the interest rate is neutral in the dynamic IS, the economy grows at its potential rate and, with a zero output gap, income will also be at its potential value in all periods. If the interest rate is higher than the neutral rate, however, the economy will grow at less than its potential rate and there will be a growing negative output gap. The difference lies in the fact that when the interest rate moves away from the "neutral" rate in (12)⁵, there is also an output gap, but it remains constant over time. Furthermore, in order to eliminate the output gap with the dynamic IS, the interest rate would have to be below the neutral level for a time, while it would only have to return to its neutral level with expression (12).

Another difference is found when comparing expressions (18) and (11). In the proposed dynamic IS, the interest rate has an impact on the relationship between a period's income and the previous period's effective income – which has an impact on consumer and investment decisions (aggregate demand). In the usual IS, however, Say's Law appears to be applicable, as the reference is always to potential income rather than the economy's actual income.

In our opinion, this more complex description of the economy's evolution when the interest rate varies is also more realistic. Therefore, the use of dynamic IS is particularly important because it enables us to explore the difficulties involved in regulating an economy in

⁵ The interest rate with which the output gap is zero. In this case: $\bar{r} = \frac{a}{b}$.

which the interest rate affects the growth rate but inflation rate variations depend on the difference between effective and potential income levels.

4. The problems of the Taylor Rule and a proposed solution:

4.1. Theoretical problems of the Taylor Rule:

The logic of the Taylor Rule is that the central bank uses a reference interest rate which is established providing that the inflation rate is on target and the economy registers its potential income level. It would be reasonable to believe, therefore, that this reference interest rate will be such that this situation is maintained over time (neutral interest rate). When this condition is not met, the authorities change the nominal interest rate in an attempt to obtain a real rate which is higher or lower than the neutral rate, depending on the problem to be solved.

Is such a rule an appropriate guide for monetary policy? In our opinion, it is not, because of two problems associated to the Taylor Rule.

The first problem arises if the model used by the authorities does not include a dynamic IS, because the neutral interest rate they will attempt to estimate to be used as a reference for monetary policy will not be appropriate. Indeed, the central bank will estimate an interest rate such that the economy will register potential income in the medium term⁶, when it should – with a dynamic IS – estimate the rate that will maintain the economy at potential growth.

Furthermore, the terms of the rule only include the income level, but not the growth rate. This means that monetary policy will react the same to a given output gap, irrespective of the economic growth rate. Assuming a negative output gap, the central bank should, however, apply a more expansionary monetary policy if the economy is registering a low growth rate than if the economy is growing at a high rate. In the first case, the economy's cyclical status would be worsening, whereas in the second it would be undergoing a correction process.

The second problem facing the authorities when attempting to apply a Taylor-like rule is that they cannot observe the two reference values for the application of monetary policy – the neutral interest rate and the equilibrium employment level – so they have to estimate their most likely values. This estimation, however, is subject to some uncertainty⁷, so errors can lead to unsatisfactory results.

Orphanides y Williams (2002) believe that the uncertainty regarding the estimation of these two variables could be due to not knowing which is the true theoretical model, to the level of information available in real-time (when interest rate decisions are made), and to the

⁶ This definition is the same as used by Greenspan (1993), Blinder (1998, page 32) and Woodford (2003, page 248).

⁷ The difficulty involved in estimating these two variables is empirically documented. For the monetary union, for instance, Crespo Cuaresma, Gnan and Ritzberger-Grünwald (2005) present a broad summary of the estimates of the neutral interest rate obtained by different procedures and the large differences found in the results obtained. Benati and Vitale (2007) estimate both the neutral interest rate and the NAIRU, showing that they are both subject to a high degree of variability over time and that their estimated value is affected by significant uncertainty. Similar conclusions are reached for the United States by Wu (2005).

existence of different estimation methods. The problem is actually greater: even if the central bank has been able to correctly estimate these two variables in a given period, any economic shock would alter at least one of the two values. As a result, the central bank would be attempting to compensate for the effects of a shock with a rule based on a reference which the same shock has changed.

Assume, for instance, a negative demand shock which moves the IS downwards. If the central bank wishes to compensate for this drop in the aggregate demand growth rate, so that the economy can continue to grow at its potential rate, it will have to establish a lower interest rate. This is the only way to stimulate the growth rate enough while the unemployment rate remains constant. The neutral interest rate will therefore have changed.

If this shock is transitory, for instance lasting for only one period, the neutral interest rate will rapidly return to its initial value and the error associated to the Taylor Rule will not be very significant, especially if we consider the delay with which monetary policy is effective. These one-period shocks, however, are not the most important from a monetary policy perspective, and changes in aggregate demand are more likely to disappear gradually, if they do so at all. The variation in the neutral interest rate will last longer, then, and the costs derived from incorrectly estimating the equilibrium interest rate will increase as shocks persist.

Likewise, any shock on the supply side will affect the equilibrium values that the central bank is using as a reference for application of its monetary policy. Consider an inflationist shock ($\varepsilon_t^S > 0$). To maintain inflation constant will require a higher unemployment rate; the NAIRU will therefore change and this is not considered by the Taylor Rule. As before, if the shock lasts for a single period, the cost of this will be insignificant, but it will tend to grow as the supply shock persists. Another example could be a structural change in how the labour or goods markets operate. In this case, the NAIRU would be permanently changed.

The following section provides a detailed analysis of the problems derived from these estimation errors.

4.2. Consequences of an error in the estimation of the neutral interest rate or the NAIRU:

An error when estimating these reference values creates two types of problem: the inflation rate that characterises the economy's equilibrium could be other than the target rate, and the adjustment process towards this equilibrium could be too slow and costly or give rise to an unstable dynamic process.

This can be seen by assuming first that there is a permanent shock and analysing the economy's equilibrium (if reached). We then consider the consequences of this error for the dynamic adjustment process and the case of a transitory shock.

Using \bar{r}^{BC} to denote the "neutral" interest rate estimated by the central bank and \bar{n}^{BC} to denote the employment level that the central bank believes is compatible with stable inflation, the Taylor Rule would be as follows:

$$r_t = \bar{r}^{BC} + 0.5(\dot{P}_t - \dot{P}^T) + 0.5\left(\frac{n_t - \bar{n}^{BC}}{\bar{n}^{BC}}\right) \quad (20)$$

Earlier, we saw that equilibrium would be characterised by a constant interest rate and a constant employment level, that is $r = \bar{r}$ and $n = \bar{n}$; if these two conditions are substituted in the monetary policy rule:

$$\bar{r} = \bar{r}^{BC} + 0.5(\dot{P} - \dot{P}^T) + 0.5\left(\frac{\bar{n} - \bar{n}^{BC}}{\bar{n}^{BC}}\right) \quad (21)$$

This expression shows that, whenever there is an error in the estimation of the interest rate or the employment level associated to the NAIRU by a central bank which applies its monetary policy according to the Taylor Rule, the economy will stabilise at an inflation rate $(\bar{\dot{P}})$ which differs from the target rate. This “inflationist bias” will be equal to:

$$\bar{\dot{P}} - \dot{P}^T = \frac{(\bar{r} - \bar{r}^{BC}) - 0.5\left(\frac{\bar{n} - \bar{n}^{BC}}{\bar{n}^{BC}}\right)}{0.5} \quad (22)$$

So inflation will be higher than the target rate whenever the central bank estimates a too low neutral interest rate or NAIRU.

The problem could be even greater, however, if the error is in an upwards direction and the central bank has also established a low inflation target; in this case, the equilibrium inflation rate could be negative or could even require a negative nominal interest rate to reach the real neutral interest rate. The economy would then face the problem of the liquidity trap. Specifically, the nominal interest rate that the central bank should establish at equilibrium (\bar{i}) would be:

$$\bar{i} = \bar{r} + \bar{\dot{P}} = \bar{r} + \dot{P}^T + \frac{(\bar{r} - \bar{r}^{BC}) - 0.5\left(\frac{\bar{n} - \bar{n}^{BC}}{\bar{n}^{BC}}\right)}{0.5} \quad (23)$$

Is there a high risk of this rate being negative? It can certainly not be ruled out. If the estimated real neutral interest rate is 2% and the inflation target is also 2%, the real neutral interest rate would only have to fall by 1.35 percentage points for the equilibrium nominal interest rate to be negative, or it would only have to fall by one percentage point if the central bank was overestimating the NAIRU by a bit more than one point.

We will be returning later to this possible economic instability problem. For the time being, assume that the economy returns to a new equilibrium after the shock. It is important to

note that, besides this equilibrium being characterised by an inflation rate different than target, the adjustment process will also be a slow one, as a result of using references which are no longer valid.

This monetary policy maladjustment would certainly have a cost in terms of greater inflation or employment rate deviations relative to the central bank's targets, as shown by our simulations.

This also shows that, although the difference between the actual and target inflation rates would not be maintained at long-term equilibrium if the shock is not permanent (or if the central bank corrects its reference values after a few periods), the problem could extend to any shock which persists some time. They all affect the interest rate or equilibrium employment (or both) for several periods, in which the Taylor Rule would be setting an interest rate that is not the adequate for the economy's circumstances.

4.3. An alternative monetary policy rule:

What type of rule could prevent these two problems? The alternative proposed here is a modification of the Taylor Rule characterised by two principal ideas.

- First, the concept of neutral interest rate that the central bank uses as a reference for its monetary policy is derived from a dynamic IS.
- Secondly, the central bank does not estimate this neutral interest rate or the NAIRU. Instead of this, the rule "tracks" these values based on the economy's evolution, particularly on variations in the inflation and unemployment rates.

This Tracking Rule is formally derived from the original Taylor Rule:

$$r_t = \bar{r} + 0.5(\dot{P}_t - \dot{P}^T) + 0.5\left(\frac{n_t - \bar{n}}{\bar{n}}\right) \quad (4a)$$

The two terms which are unknown to the central bank are the neutral interest rate and the difference between the effective employment rate and that associated to the NAIRU. How can they be identified with the information available?

If the interest rate applied in the previous period was the neutral rate, the growth rate will be at its potential value and the employment rate will remain the same. On the other hand, a discrepancy between the actual interest rate and the neutral rate will lead to a change in employment, so this variation can be used to know the difference between the current and the neutral interest rate.

Specifically, according to the IS, the period's growth rate depends on the previous period's real interest rate:

$$g_t = D - br_{t-1}$$

Replacing the interest rate with the neutral rate, we obtain the potential growth rate:

$$\bar{g} = D - b\bar{r}$$

But if these two expressions are subtracted:

$$\bar{r} = r_{t-1} + \frac{g_t - \bar{g}}{b} \quad (24)$$

Finally, according to (3), the difference between the actual and the potential growth rate is equal to the percentage variation in the employment level. Then:

$$\bar{r} = r_{t-1} + \frac{1}{b} \frac{n_t - n_{t-1}}{n_{t-1}} \quad (24b)$$

A similar reasoning applies to the NAIRU. If the current employment rate is indeed the economy's NAIRU, inflation would remain stable. However, any difference between the current unemployment rate and the NAIRU would be reflected in a change in the inflation rate. Based on the Phillips curve:

$$\left(\frac{n_t - \bar{n}}{\bar{n}} \right) = \frac{1}{a} (\dot{P}_t - \dot{P}_{t-1}) \quad (25)$$

Replacing (24b) and (25) in the Taylor Rule, we obtain the Tracking Rule:

$$r_t = r_{t-1} + \alpha_1 \frac{n_t - n_{t-1}}{n_{t-1}} + \alpha_2 (\dot{P}_t - \dot{P}^T) + \alpha_3 (\dot{P}_t - \dot{P}_{t-1}) \quad (4b)$$

Where $\alpha_1 = \frac{1}{b}$, $\alpha_2 = 0.5$ and $\alpha_3 = \frac{0.5}{a}$. Giving parameter b a value of 0.8 and a value of 0.4 to parameter a ⁸:

$$r_t = r_{t-1} + 1.25 \frac{n_t - n_{t-1}}{n_{t-1}} + 0.5 (\dot{P}_t - \dot{P}^T) + 1.25 (\dot{P}_t - \dot{P}_{t-1}) \quad (4c)$$

⁸ These values are taken from the calibration of a three-equation model by Aarle, Garretsen and Huart (2004, table 1, page 418) for the monetary union.

As we can see, the logic of the rule would be to start with the previous period's interest rate and change it whenever there are signs that the economy is not at the desired equilibrium. Specifically, the central bank should alter the previous period's interest rate :

1. When employment is varying, as this shows that the growth rate is not at its potential value, so the current interest rate is not the neutral rate. For example, if the unemployment rate is rising, this means that the interest rate is higher than the neutral rate, and it should be reduced by the central bank. With this term, it "tracks" the neutral interest rate.
2. When the rate of inflation is varying, as this shows that the unemployment rate, however constant, is not the equilibrium rate. For example, the central bank should increase the interest rate whenever inflation accelerates, as this shows that the unemployment rate is lower than the NAIRU, which the central bank is "tracking" with this term of the rule.
3. When inflation is at other than target level, to prevent the economy from stabilising with a constant, but undesired, employment and inflation rate.

As this rule does not include an estimate of the neutral interest rate or NAIRU, it is not affected by the problem of the long-term equilibrium of the economy being characterised by other than target inflation. Indeed, we saw earlier that, at equilibrium, $r_t = r_{t-1} = \bar{r}$, $n_t = n_{t-1} = \bar{n}$ and $\dot{P}_t = \dot{P}_{t-1}$, and substituting in (4b), we have that:

$$r_t = r_{t-1} + 1.25 \frac{n_t - n_{t-1}}{n_{t-1}} + 1.5 (\dot{P}_t - \dot{P}^T) + 1.25 (\dot{P}_t - \dot{P}_{t-1})$$

$$\bar{\dot{P}} - \dot{P}^T = 0 \tag{26}$$

For the Tracking Rule to be defined as better than the Taylor Rule, we also have to compare its ability to take the economy to equilibrium without major fluctuations. The following sections therefore simulate different shocks, comparing the economy's trajectory when the authorities apply the Taylor Rule or the Tracking Rule. We can anticipate that one important conclusion is that this rule mitigates the excessively cautious monetary policy derived from the Taylor Rule.

It is also important to note some similarities between this rule and the generalization of the Taylor Rule proposed by Orphanides (2007) to tackle the problem of uncertainty concerning the real values of neutral interest rate and potential income. This rule is as follows:

$$i_t = (1 - \phi_i)(\bar{r} + \dot{P}^T) + \phi_i i_{t-1} + \phi_p (\dot{P}_t - \dot{P}^T) + \phi_Y (Y_t - \bar{Y}) + \phi_{\Delta Y} (\Delta Y_t - \Delta \bar{Y}_t)$$

Where ϕ_i , ϕ_p , ϕ_Y and $\phi_{\Delta Y}$ are positive parameters, Y is the income and \bar{Y} is the potential income. Taylor Rule would be a particular case of this general rule, where $\phi_i = \phi_{\Delta Y} = 0$. According to Orphanides, however, when the uncertainty about estimations of neutral interest rate and potential income is high, we should use the values $\phi_i = 1$ and $\phi_Y = 0$, so

$$i_t = i_{t-1} + \phi_p(\dot{P}_t - \dot{P}^T) + \phi_{\Delta Y}(\Delta Y_t - \Delta \bar{Y}_t)$$

This rule is similar to the Tracking Rule as it uses the interest rate of the previous period and not the estimation of the “neutral” interest rate. Nevertheless some important differences can be appreciated:

- The potential growth rate still appears in this rule, although it cannot be observed. It is for that reason that, in our rule, the difference between real growth and potential growth is substituted by the variation in the employment level, which, in fact, can be observed.
- Orphanides introduces this term as an approximation for the differences between effective income and potential income, which is more difficult to calculate. However, this term is included in the Tracking Rule to “track” the neutral interest rate based on the dynamic IS.
- While Orphanides removes the term of the output gap, we “track” this gap by including the changes in inflation. Our simulations show that the stabilizing properties of the Rule are enhanced through the inclusion of the changes in inflation above mentioned.

5. Comparison of the two rules when there is no liquidity trap risk:

This and the following section present the results of simulations made to show the dynamic adjustment and equilibrium reached by the economy after different shocks. It is initially considered that the shock is permanent, but we afterwards continue by considering that the shock follows an auto-regressive process.

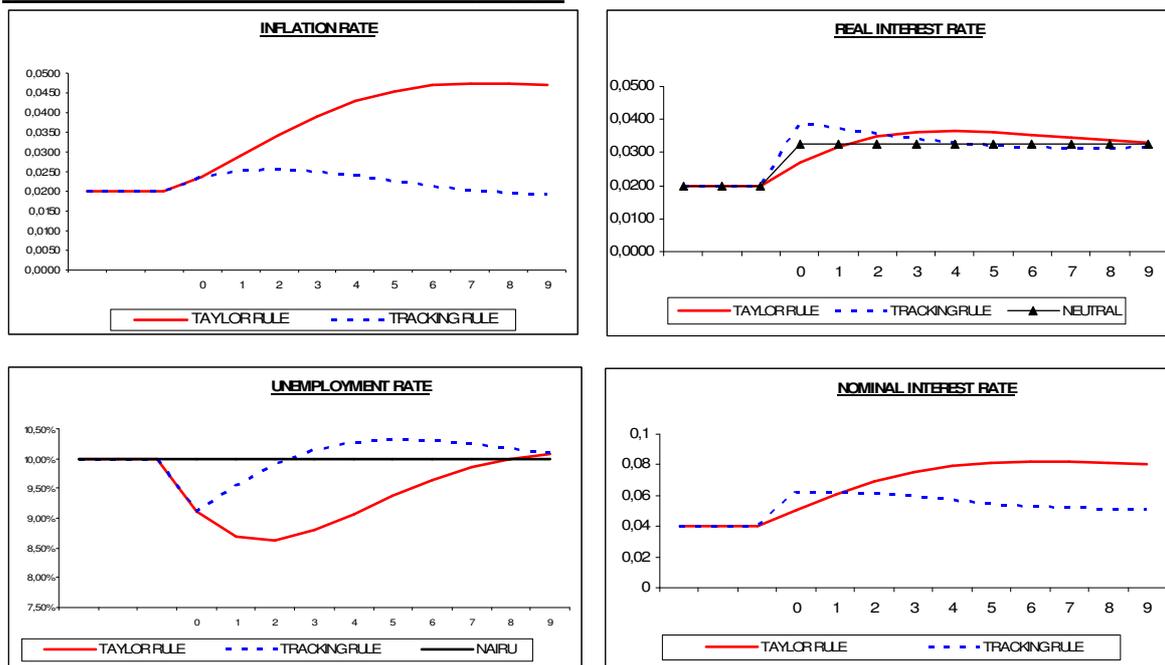
We study two main cases. The first, which is approached in this section, is one in which the economy is stable with both rules, and we will want to know the inflation deviations from target and income deviations from its potential, both at final equilibrium and during the adjustment process. The second case, which is found in the following section, analyses the risk of the economy finally falling into a liquidity trap with each of the rules, after a significant drop in demand.

The first shock simulated is a one percentage point increase in demand growth. If the original situation was equilibrium, the central bank should rise the interest rate so that the economy returns to its potential growth rate.

Assume that the central bank uses the Taylor Rule and maintains the neutral interest rate constant at the value estimated before the shock (2%). The central bank increases the nominal interest rate, but not as much as it would have if it had perceived the change in the neutral real interest rate, which has risen to 3.25%. As a result, it does not prevent the unemployment rate from continuing to fall and the inflation rate from continuing to rise (Graph 1). The increase in employment and greater rise in inflation will lead to correcting monetary policy to make it more restrictive, as required by the economy, and this will eventually lead to a return to the equilibrium employment level and growth rate. However, not only will this be a slow process, because the error in estimating the neutral interest rate is maintained, but, as shown by expression (22), the inflation rate will stabilise at higher than the central bank's target: in this case, 4.5% versus the initial 2%.

In other words, the application of the Taylor Rule to solve a situation of disequilibrium (too high growth and a rising output gap) has generated another disequilibrium (a too high inflation rate).

GRAPH 1: A PERMANENT DEMAND SHOCK



SOURCE: Model's simulations.

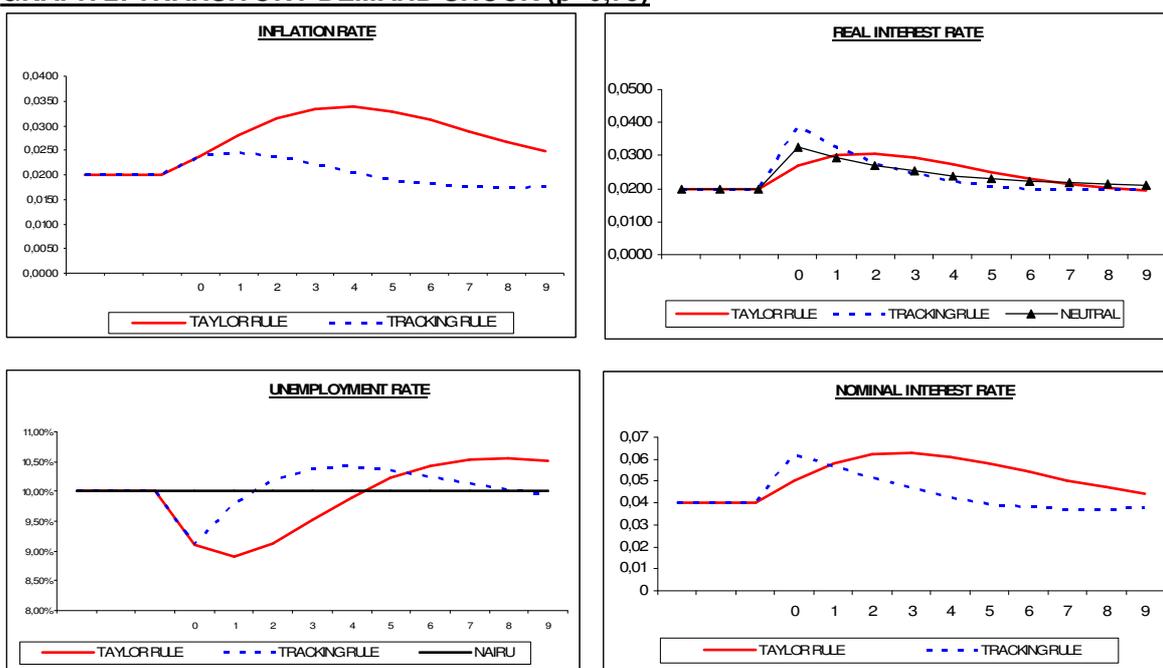
What happens with a tracking rule? In the first period, the central bank increases the interest rate more than with the Taylor Rule, because the central bank's reaction to higher than target inflation and to unemployment falling below the Nairu, as represented in the Taylor Rule, is also considered in the Tracking Rule in which the variation in employment also causes

the interest rate to rise further, so the central bank is anticipating the increase in the neutral interest rate.

This reduction is sufficient for the growth rate to fall beneath its potential value, so the unemployment rate starts to rise. Monetary policy also adapts faster to the restrictive tone required in the following periods, because its reference is not a fixed interest rate, but the previous period's interest rate, which is rising. As we see in Graph 1, this does not only prevent inflation from rising earlier, but also ensures that the economy returns to its original equilibrium.

Finally, the question is how these two monetary policy rules compare if the shock is not permanent. As is to be expected, the cost of using the Taylor Rule is smaller when the persistence of shocks is smaller, for two reasons. First, because the inflationist bias created at equilibrium (expression (22)) disappears. Secondly, because the shorter the shock, the shorter is the duration of the central bank's error in the references used for its monetary policy. However, as shown on Graph 2 and Table 1, this cost does not vanish completely, as the rise in the growth rate (above the potential level) and inflation (above target) and the reduction in the unemployment rate (beneath equilibrium) is greater than with the Tracking Rule.

GRAPH 2: TRANSITORY DEMAND SHOCK ($\rho=0,75$)



SOURCE: Model's simulations.

TABLE 1: SOCIAL WELFARE LOSS AFTER A DEMAND SHOCK (+1p.p.)*

Persistence (ρ)	Taylor Rule	Tracking Rule
1	0.502**	0.014
0.75	0.07	0.012
0.5	0.0329	0.01
0	0.132	0.009

* Value of the social loss function (5) assuming that $\lambda_p = \lambda_{OG} = 0.5$.

** When the shock is permanent ($\rho=1$) the social loss is always rising with the Taylor Rule, because the equilibrium inflation rate is not the central bank's target. The figure in the table is for the period in which the equilibrium is reached.

SOURCE: Model's simulations.

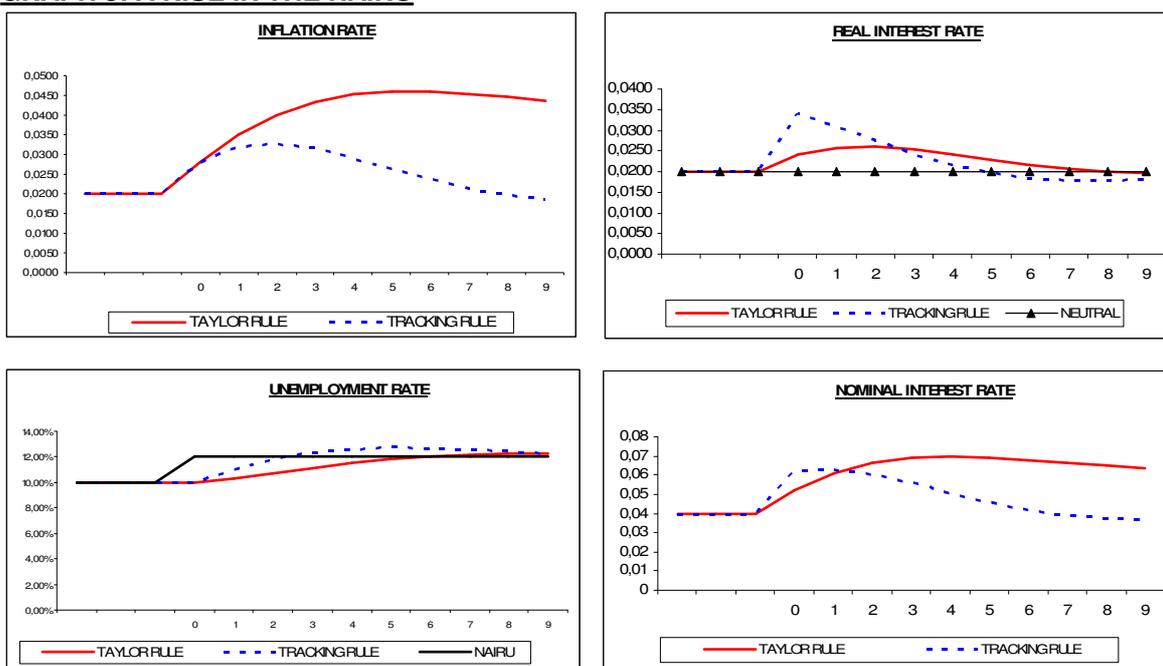
We have also studied what would happen if shocks originate on the supply side. We specifically simulated a 2-percentage point increase in the NAIRU which was not anticipated by the central bank. As a result of the shock, the initial effect will be an increase in inflation to 2.8%, and if the central bank applies the Taylor Rule, both the nominal and real interest rates will increase. Specifically, the nominal interest rate would go from 4% to 5.2% and the real interest rate would be 2.4%.

This central bank reaction is a move in the right direction and will cause an increase in the employment rate, which is required in order to stabilise inflation. Inflation, however, will continue to grow, because this new unemployment rate is still below the economy's new NAIRU, although it is above the NAIRU estimated by the central bank. This is very important, because even though monetary policy should continue to be restrictive, by calculating the Taylor Rule with the wrong NAIRU (too low), the nominal interest rate will rise less than it should. Inflation will continue to accelerate and, when the economy returns to the neutral interest rate, it will stabilise at higher than the initial rate of inflation (4.3%), as shown on Graph 3, because the unemployment rate is lower than the NAIRU during nearly the entire adjustment process.

If the Tracking Rule is applied, the same differences occur as in the case of demand shocks: the nominal interest rate will grow faster, as the central bank will immediately react to the fall in the NAIRU through accelerating inflation. Thanks to this more active monetary policy reaction, inflation will start to fall earlier, so both the nominal and real interest rates would increase more rapidly. Eventually, the economy would stabilise again at the target inflation rate.

Table 2 shows the differences in the value of the Welfare Function derived from application of the Taylor Rule or the Tracking Rule when the supply shock has different degrees of persistence.

GRAPH 3: A RISE IN THE NAIRU



SOURCE: Model's simulations.

TABLE 2: SOCIAL WELFARE LOSS AFTER A RISE IN THE NAIRU (+2p.p.)*

Persistence (ρ)	Taylor Rule	Tracking Rule
1	0.477**	0.074
0.75	0.0935	0.063
0.5	0.0572	0.052
0	0.0376	0.044

* Value of the social loss function (5) assuming that $\lambda_p = \lambda_{OG} = 0.5$.

** When the shock is permanent ($\rho=1$) the social loss is always rising with the Taylor Rule, because the equilibrium inflation rate is not the central bank's target. The figure in the table is for the period in which the equilibrium is reached.

SOURCE: Model's simulations.

6. Monetary policy and the present crisis. The liquidity trap risk with both rules:

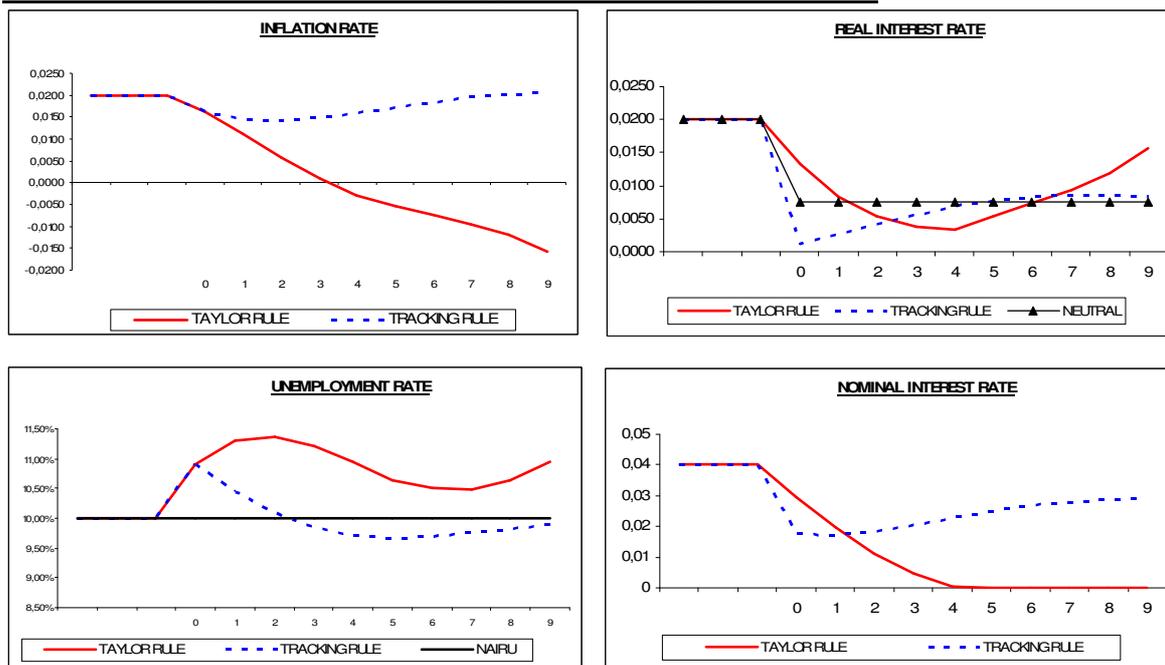
One of the characteristics of the present crisis is that economies are suffering a large and persistent drop in aggregate demand. In the model, this could be seen as a permanent contractive demand shock. We would like to see the results of the application of each of the two rules in these circumstances and, in particular, the likelihood in each case of the economy falling into a liquidity trap.

This analysis is performed in two complementary manners. We first simulate a fall in the demand growth rate, to analyse the differences occurring in the economy's dynamics depending on the rule that is applied. This will show us why the liquidity trap risk is greater with the Taylor Rule. We then attempt to evaluate the minimum size of demand shock required for the economy to fall into a liquidity trap with each rule, finding that, with the Tracking Rule, this situation occurs with shocks twice the size as with the Taylor Rule.

We therefore start by assuming that the economy is at equilibrium and that the aggregate demand growth rate falls by one percentage point, also reducing the neutral interest rate to 0.75%.

How would the central bank reaction in one case or the other? The nominal interest rate will fall with both rules, but we have seen here that this reaction will be more cautious with the Taylor Rule in the first few periods. As a result, the unemployment rate will continue to grow and inflation will continue to fall, although it would already be recovering with the Tracking Rule.

GRAPH 4: A NEGATIVE DEMAND SHOCK AND THE LIQUIDITY TRAP



SOURCE: Model's simulations.

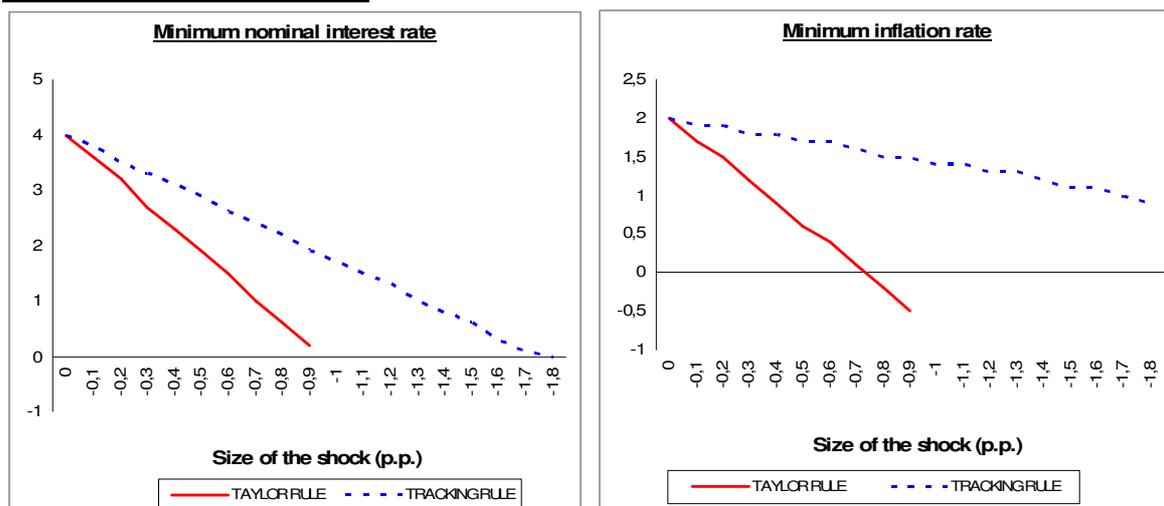
The problem with this is not only that the economy takes longer to return to equilibrium, and not even that this equilibrium would involve a lower than target inflation rate. The problem is that, as inflation falls more with the Taylor Rule, the cut required in the nominal interest rate for the real rate to be sufficiently beneath the neutral rate is also greater. However, as nominal interest rates have a lower limit for monetary policy – they cannot be negative –, the delay with which the central banks has acted has increased the risk of the economy falling into a liquidity trap with nominal interest rate equal to zero, deflation, rising real interest rates and rising unemployment rates, in which case the economy would not return to equilibrium (Graph 4).

The Tracking Rule does not prevent the economy from falling into a liquidity trap on every occasion, however, because if the shock is very large, the real interest rate reduction required could be too much to be attained with a positive nominal rate. However, as the central bank acts faster with this rule and inflation therefore is less reduced, the economy is less likely to fall into a liquidity trap with a given shock, as a smaller reduction is required in the nominal interest rate. In other words, the size of the negative shock required for the economy to fall into a liquidity trap is smaller with the Taylor Rule than with the Tracking Rule.

The second part of our analysis consists precisely of evaluating the size a demand shock needs to be in order to fall into a liquidity trap with the Taylor Rule and the Tracking Rule, and it is shown on Graph 5. With both rules, we have simulated different cuts in the demand growth rate, increasing the size of the shock by 0.1 percentage point at a time (abscissa). In each case, we show both the minimum nominal interest rate and the minimum inflation rate registered during the adjustment process after the shock (ordinate). The conclusions derived from this graph are clear:

- For any shock size, both inflation and the nominal rate fall more in any period with the Taylor Rule than with the Tracking Rule.
- A 1-point shock would be sufficient for the economy to fall into a liquidity trap with the Taylor Rule. Indeed, inflation would be negative in some periods with a drop of less than one percentage point.
- The threshold for the economy to fall into a liquidity trap with the Tracking Rule is significantly higher, up to 1.9 percentage points.

GRAPH 5: MINIMUM NOMINAL INTEREST RATE AND INFLATION RATE AFTER A NEGATIVE DEMAND SHOCK



SOURCE: Model's simulations.

7. Conclusions:

The objective of this paper was to propose an alternative to the Taylor Rule which overcomes the monetary policy application problems derived from the fact that the NAIRU and neutral interest rate values used in the rule are not the real figures, and from the use of a neutral interest rate concept derived from static rather than dynamic IS. This alternative has been called the Tracking Rule.

Our analysis of how the model works with the two rules and the results of the simulations shows that the use of the Tracking Rule has clear advantages over the Taylor Rule:

1. When shocks are permanent, the Taylor Rule stabilises the economy with an other than target inflation rate. This difference is proportional to the error when estimating the neutral interest rate and the NAIRU. As they are not estimated with the Tracking Rule, this error does not occur and the inflation rate at which the economy stabilises is always the target rate.
2. As a result of using fixed references, the interest rate variations required after a shock occur more slowly with the Taylor Rule than with the Tracking Rule. This means that the economy takes longer to return to equilibrium, with greater deviations from target inflation and the potential income level. Social losses in the form of the welfare function,

therefore, are greater with the Taylor Rule than with the Tracking Rule, with permanent or transitory shocks.

3. If this slower monetary policy reaction takes place during a permanent and persistent drop in the demand growth rate, the likelihood of a negative inflation rate is greater with the Taylor Rule.
4. The size of the negative shock that would cause the economy to fall into a liquidity trap is significantly smaller with the Taylor Rule than with the Tracking Rule.

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